ONION-SHELL MODEL FOR COSMIC RAY ELECTRONS AND RADIO SYNCHROTRON EMISSION IN SUPERNOVA REMNANTS

by

R. Beck, L. O'C. Drury, H. Völk, Max-Planck-Institut für Kernphysik, Postfach 10 39 80, 6900 Heidelberg, W. Germany,

T.J. Bogdan, National Center for Atmospheric Research*, High Altitude Observatory, P.O. Box 3000, Boulder, CO. 80307, USA.

ABSTRACT

The spectrum of cosmic ray electrons, accelerated in the shock front of a supernova remnant (SNR), is calculated in the test-particle approximation using an "onion-shell" model. Particle diffusion within the evolving remnant is explicitly taken into account. The particle spectrum becomes steeper with increasing radius as well as SNR age. Simple models of the magnetic field distribution allow a prediction of the intensity and spectrum of radio synchrotron emission and their radial variation. The agreement with existing observations is satisfactory in several SNR's but fails in other cases. Radiative cooling may be an important effect, especially in SNR's exploding in a dense interstellar medium.

1. Introduction. Diffusive acceleration of relativistic particles in the shock fronts of supernova remnants (SNR's) is thought to be an important mechanism to replenish cosmic rays in the interstellar medium. In case of strong, steady, plane shocks, the particle energy spectrum $N(E) \propto E^{-q}$ becomes a power law with a slope of $q = 2$. In reality, intensity and slope of the spectrum are changed by various effects:
(i) The acceleration time is limited by the SNR age, leading to a high-energy cutoff. (ii) The SNR Mach number decreases with time. (iii) The particle energy decreases due to (adiabatic) expansion. (iv) Radiative cooling sets in below $\sim 10^6$ K temperature. (v) The back-reaction of the accelerated particles modifies strength and structure of the shock. (vi) The accelerated particles excite magnetohydrodynamic waves which produce a shock precursor. (vii) The accelerated particles (particularly the electrons) are subject to energy loss processes due to collisions and radiation.

The full particle transport equation can be solved only for rather special cases (1,2,3), which we cannot use here. Numerical studies of the hydrodynamical version of the time-dependent problem have been started only recently (4), but do not provide information about the particle spectrum. A test-particle model for the cosmic ray production by recurrent passages of SNR shock fronts has been discussed (5). Models for cosmic ray acceleration in a single SNR have been calculated (6,7), in which effects (i), (ii), (iii) and - schematically - (vi) are taken into account. The lifetime of the SNR is

---

* NCAR is sponsored by the NSF

(technically) split into equally spaced intervals corresponding to a series of shells, characterized by the Mach number of the bounding shock. Suprathermal particles with 1 keV energy are continuously injected into the shock front, accelerated up to the cutoff, according to the instantaneous shock Mach number in a shell, and adiabatically expanded with the gas until the gas pressure reaches the external pressure. Particles remain confined within their shell until being released at the end of the SNR evolution. The sum of the particle spectra from all these "onion shells" yields a power-law with q = 2.1-2.3 between 10^6 and several 10^{13} eV, only slightly varying with different choices of the injection energy and the other input parameters. Allowing for energy-dependent subsequent escape from the galaxy, the spectral index of the galactic radio emission is consistent with this model.

2. The Model. We have extended the "onion-shell" model to study the particle spectrum as a function of time and position. Instead of confining the particles to a shell we take account of both the intrinsic width of the acceleration region and of the subsequent diffusion by introducing a smearing length scale λ given by:

$$\lambda^2(t,E) = \left(\frac{\kappa(E)}{\dot{R}(t)}\right)^2 + \kappa(E)\ (t-t_o)$$

Here $\dot{R}(t)$ is the expansion velocity of the shock front; $(t-t_o)$ is the time since the formation of the shell at the shock. κ is the energy-dependent diffusion coefficient; it roughly increases proportional to particle energy. Due to wave generation by accelerated nucleons κ is taken to be given by the gyroradius limit; for 1 MeV electrons and 3 μG magnetic field strength κ becomes $\sim 10^{19}$ cm^2s^{-1}. At low energies (< 1 GeV) the electrons still remain concentrated in their original shell while at the highest energies (> 1000 GeV) electrons essentially diffuse across the whole SNR.

The SNR is assumed to expand freely until the ejecta have swept up an equal mass of interstellar material. This initial phase is followed by an adiabatic Sedov phase, where $R \propto t^{2/5}$. After passage of the shock each shell expands with the downstream flow (8) until the time of observation t_F. Any point in the spectrum follows from the integral over the contributions from all shells at a fixed observation radius.

Hence it is possible to approximately compute the electron spectrum at any given SNR age and at any given radius.

The accelerated electrons emit synchrotron radio waves in the magnetic field of the SNR. Two extreme field models have been investigated, which bracket the actual situation: (I) constant field strength, (II) field compression $B \propto \rho$. The distribution of relativistic electrons and of the magnetic field allows a prediction of the radio synchrotron emissivity. For comparison with radio observations, the emission from all shells along the line of sight has to be summed.

3. Results. Models have been calculated for different SNR ages (represented by the Mach number M_F at the time of observation) and for different external gas densities. The explosion was assumed to eject 1 $M_\odot$ with 10 000 km/s velocity. A field strength of 3 μG and a pressure of 1.5 10^{-12} dyne/cm^2 in the interstellar medium has been adopted.

Particles with low energies (< 1 GeV) are supplied by all shells created at different SNR ages. High-energy particles, however, are produced only during the early stages of the SNR evolution when the Mach number is high. This result still holds after extensive adiabatic expansion, i.e. for old adiabatic SNR's. Therefore, a steepening of the particle spectrum with increasing distance from the center is expected, except for the highest energies where diffusion dominates.

The spectral index for the electron energy spectrum between 10^6 and 10^{12} eV varies varies with radius. In young SNR's (final Mach number $M_F \geq 3$) almost no variation occurs. A spectral steepening occurs in the outer region of old adiabatic SNR's ($M_F \lesssim 2$). The corresponding radio spectral index between 100 MHz and 10 GHz frequency for $B \propto \varrho$ (Fig. 1) refers to the emission along the line of slight radius R so that variations with R are somewhat smoothed out. The assumption B = const does not change the curves of Fig. 1 significantly. The spectral index of the integral radio emission varies with SNR age, from $\alpha \cong 0.60$ in young SNR ($M_F > 5$) to $\alpha \cong 0.65$ in old SNR ($M_F < 1.5$), which is consistent with the result given (6).

These general conclusions hold if the SNR remains adiabatic during its evolution. In fact, for an external hot interstellar medium gas density of $3 \times 10^{-3} cm^{-3}$ radiative cooling of the outer shell becomes important only in the latest phases ($M_F \lesssim 1.1$) when the downstream

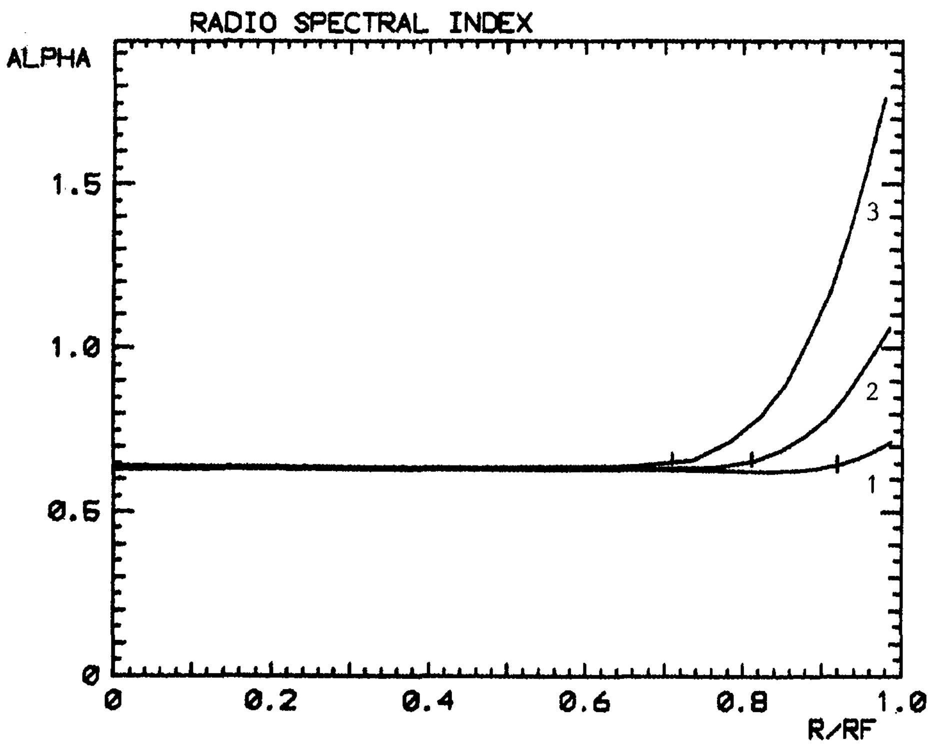

Fig. 1: The variation of radio synchrotron spectral index α with radius relative to the final radius R_F of the shock front for three models:
(1) Final Mach number $M_F = 3$ ($R_F = 90$ pc), (2) $M_F = 2$ ($R_F = 117$ pc),
(3) $M_F = 1.5$ ($R_F = 142$ pc). The maximum radio intensity is marked. At fixed external pressure the curves are independent of the external density.

temperature drops to 10^6 K. For larger densities, say $n \sim 3 \times 10^{-2} cm^{-3}$, shell cooling sets in earlier ($M_F \lesssim 3$). Then the outer shock tends to become isothermal. The increased compression ratio then yields a harder spectrum, at least for high energies, leading to a radial decrease of the spectral index. Although this effect is not explicitly treated here, it is an obvious diagnostic possibility regarding the evolutionary state of a given SNR, with radial increase (decrease) indicating an adiabatic (radiative) phase of the compressed outer shell.

4. Comparison with observations. The mean spectral index of SNR's, including also flat-spectrum (Crab-type) SNRs, is $\alpha = 0.45 \pm 0.15$ (9) which is not inconsistent with the prediction of the "onion-shell" model. The same data do not indicate a correlation between SNR spectral index and diameter(10), while in a more recent catalogue (11) a statistically weak flattening with increasing diameter is visible. Since the diameter depends mainly on the initial velocity and the external gas density and is far from being a unique measure of SNR age, these observations do not conflict with our model.

The determination of radio spectral index within a SNR requires an accurate subtraction of the background emission which is difficult especially for large SNRs in the galactic plane. The remnant of Tycho's supernova is a favourite object because it is small and radio-bright. The spectral index of the integrated emission is $\alpha = 0.61 \pm 0.03$ without a significant variation across the remnant (12).This agrees with the prediction of our model for young SNR's. The large, probably old, SNR G65.2+5.7 in Cygnus reveals a spectral steepening from $\alpha \cong 0.4$ in the inner part to $\alpha \cong 0.6$ in the main shell of emission, followed by an increase to $\alpha \cong 1.0$ beyond, (13), as expected in view of our model.

A larger sample of SNRs with radial spectra has now become available (Fürst, private communication). In most cases, no spectral index variation across the SNR is detected. In some sources, however, the spectrum seems to flatten with radius, indicating the limits of the model presented here.

References

1. Drury, L. O'C., Axford, W.I., Summers, D.: 1982, MNRAS 198, 833.
2. Bogdan, T.J., Lerche, I: 1985, MNRAS 212, 413.
3. Webb, G.M., Bogdan, T.J., Lee, M.A., Lerche, I.: 1985, MNRAS (submitted).
4. Dorfi, E.: 1984, Adv. Space Res. 4, No. 2-3, 205.
5. Blandford, R.D., Ostriker, J.P.: 1980, Astrophys.J. 237, 793.
6. Bogdan, T.J., Völk, H.J.: 1983, Astron.Astrophys. 122, 129.
7. Moraal, H., Axford, W.I.: 1983, Astron.Astrophys. 125, 204.
8. Kahn, F.D.: 1975, Proc. 14th Int. Cosmic Ray Conf., München, Vol. 11, 3566.
9. Clark, D.H., Caswell, J.L.: 1976, MNRAS 174, 267.
10. Lerche, I.: 1980, Astron.Astrophys. 85, 141.
11. Green, D.A.: 1984, MNRAS 209, 449.
12. Klein, U., Emerson, D.T., Haslam, C.G.T., Salter, C.J.: 1979, Astron.Astrophys. 76, 120.
13. Reich, W., Berkhuijsen, E.M., Sofue, Y.: 1979, Astron.Astrophys. 72, 270.