INTRODUCTION: Supernova explosions are the most violent and energetic events in the galaxy and have long been considered probable sources of Cosmic Rays (1). Recent shock acceleration models (2,3), treating the Cosmic Rays (CR's) as test particles in a prescribed Supernova Remnant (SNR) evolution, indeed indicate an approximate power law momentum distribution \( f_{\text{source}}(p) \propto p^{-\alpha} \) for the particles ultimately injected into the Interstellar Medium (ISM). This spectrum extends almost to the momentum \( p = 10^6 \text{ GeV/c} \), where the break in the observed spectrum occurs. The calculated power law index \( \alpha \approx 4.2 \) agrees with that inferred for the galactic CR sources. The absolute CR intensity can however not be well determined in such a test particle approximation.

It is important to know the intensity which results if the CR's are selfconsistently included in the dynamics of the SNR, and whether the predicted spectral characteristics are approximately conserved in such a nonlinear treatment. In the average ISM the CR energy density is comparable to the thermal and magnetic energy densities. Thus, in their sources the CR's must also be a significant dynamical element that manifests itself through its pressure as well as its high energy current (4,5). In fact the usual estimates for the observationally required CR production efficiency of SNR sources are too low by about a factor 4 which raises the overall production efficiency from the traditional 2 to 4 percent of the average SNR energy input rate to about 8 to 16 percent. The reason is simply that not the observed intensity

\[ f_{\text{obs}}(p) \propto p^{-4.75} \]

but the source intensity \( f_{\text{source}} \), obtained from \( f_{\text{obs}} \) by correction for rigidity dependent escape \( \tau_e(p) \propto p^{-\xi} \) must be replenished after a CR lifetime \( \tau_e(p \approx 1 \text{ GeV/c}) \) of about \( 2.10^7 \text{ yrs.} \). Taking \( \xi = 0.55 \) (6,7,8) and a cutoff momentum of \( 10^6 \text{ GeV/c} \), the additional input rate corresponding to the hatched region in Fig. 1 is about a factor of 2.75 larger than the one corresponding to \( f_{\text{obs}} \), leading to a factor

\[ (4.75-4)/(4.75-4-\xi) = 3.75 \]

increase in the overall efficiency requirement. According to the test particle models, most of the acceleration must take place prior to the phase in which the SNR radius is about a factor 4 below...
its radius at particle release. Therefore the instantaneous early conversion efficiency must be about 4 times higher than the required average acceleration efficiency and correspond to 32 to 64 percent of the total energy input rate per supernova. Thus one must expect an extensive CR reaction on SNR dynamics. This is not only of interest for CR origin but also for the overall structure of the ISM in galaxies.

This paper discusses simplified models for SNR evolution with CR's. Although ultimately this must be calculated numerically, such models are needed to clarify the numerical results. They are important for parameter studies and provide a first iteration and rough guidance for the interpretation of observations. Since the present theory only calculates macroscopic CR quantities we do not get spectral information about the particle distribution, but information on the most important unknown, the overall efficiency of the acceleration process.

2. Phases of SNR evolution: In the initial phase of a SNR the cooling ejecta sweep up external interstellar material until a reverse shock runs into the interior and heats it to high temperatures. Even though the reverse shock will certainly accelerate particles from the ejecta and may in fact push the maximum particle energy from a disk SNR easily to $10^6$ GeV with interesting consequences for the chemical and isotopic composition in this energy range, it is not included in the model presented here. We consider the subsequent phases where the outer shock is driven by a uniform hot interior generating a uniform dense shell with a thickness $\Delta R$ that is small compared to the remnant radius $R(t)$ (Fig. 2). The local physical variables in the shell are assumed to have their postshock values. Such an approximation is by no means new. For the adiabatic phase it has been used e.g. by Chernyi (9) and it is standard for the so-called pressure modified snowplow phase after the onset of shell cooling. We apply it here for particle acceleration and backreaction on the gas dynamics in a hydrodynamic approximation for the CR, or High Energy Component as the energy containing part of the CR spectrum below $10^6$ GeV/c might be more appropriately called (10).

![Fig. 2: Density field in a 2-shell model for a SNR, where upstream ISM, shell, and interior parameters are denoted by the suffices 1, 2, and 3 respectively. All mass is concentrated in the shell, all internal energy in the interior.](image)

Mass balance is then given by

$$\frac{d}{dt} (M_s) = \frac{d}{dt} (M_3) + 4\pi R^2 \rho I \frac{dR}{dt}$$  \hspace{1cm} (1)$$

where the total mass $M_s$ is assumed to be concentrated in the shell and $dM_s/dt \sim T_{3}^{5/2}$ denotes the mass evaporation rate from cold clouds in the hot interior with temperature $T_3$. Momentum balance is that of the shell
\[
\frac{d}{dt} \left( M_s \cdot v_2 \right) = 4 \pi R^2 \cdot \left( p_3 - p_1 \right)
\]

where \( v_2 \) is the postshock mass velocity, and the total pressure \( p = p_g + p_c \) is the sum of gas pressure \( p_g \) and CR pressure \( p_c \). In the adiabatic phase where gas cooling is unimportant overall energy conservation - neglecting any other CR energy losses than adiabatic ones - reads as

\[
\frac{d}{dt} \left\{ \frac{1}{2} M_s v_2^2 + \frac{4 \pi R^3}{3} \cdot \frac{p_g}{(\gamma - 1)} + \frac{4 \pi R^3}{3} \cdot \frac{p_c}{(\gamma_c - 1)} \right\} = 4 \pi R^2 \cdot \frac{dR}{dt} \cdot \left[ \frac{p_{g1}}{(\gamma - 1)} + \frac{p_{c1}}{(\gamma_c - 1)} \right]
\]

Here \( \gamma = \frac{5}{3} \), and \( \frac{4}{3} < \gamma_c < 5/3 \) are the effective adiabatic indices of thermal gas, and CR's, respectively. CR energy balance can be written as

\[
\frac{d}{dt} \left\{ \frac{4 \pi R^3}{3} \cdot \frac{p_c}{(\gamma_c - 1)} \right\} = 4 \pi R^2 \cdot v_2 \cdot \left( p_{c2} - p_{c3} \right) - 4 \pi R^2 \cdot \left\{ F_{c2} - \frac{dR}{dt} \cdot \frac{p_{c2}}{(\gamma_c - 1)} \right\}
\]

where \( F_c = v \cdot p_c \cdot \gamma_c / (\gamma_c - 1) - (\partial p_c / \partial r) \cdot R / (\gamma_c - 1) \) is the local CR energy flux density, \( \bar{R} \) the mean CR diffusion coefficient (10), and we have assumed \( v(r,t) = v_2(t) \cdot r / R(t) \) in rough correspondence to the selfsimilar Sedov solution.

For the particle acceleration at the shock (Fig. 2) we assume a quasi-stationary state, i.e. steady fluxes of mass, momentum and total energy in the shock frame plus local Energetic Particle Hydrodynamics (11). Due to the finite acceleration time this is a far reaching assumption indeed. However, adiabatic cooling in the expanding SNR introduces a finite postshock gradient in CR pressure which couples the acceleration at the shock to the interior dynamics. Among other effects this is equivalent to an instantaneous cutoff of the accelerated spectrum at that particle momentum where the acceleration time becomes equal to the SNR lifetime.

Radiative cooling of the shock heated gas sets in for \( T \ll 10^6 \text{K} \). Unless cloud evaporation were so efficient that cooling set in first in the interior (12), cooling leads to a dense shell and an adiabatic interior: \( p_g \sim R^{-5} \) and \( v_2 \sim \Delta R / \Delta t \). Of course the CR's do not cool radiatively. Due to the now lower shock speed some of the highest energy particles escape, but the rest still continues to be accelerated, driving the remnant. The higher energy particles should then be accelerated across the dense shell where they are scatterfree due to ion-neutral damping of hydromagnetic waves (13). Neglecting \( p_{g2} - p_{g3} \) across the thin shell, continuity of mass and momentum flux allows one to simply determine the pressure \( p_{c2} \) of accelerated particles

\[
p_{c2} = \rho_1 \cdot (dR/dt)^2 + p_{c1} + (p_{g1} - p_{g3})
\]

in terms of the shock velocity and overall gas pressure contrast.
3. **Preliminary conclusions:** (i) the CR's take away internal energy from the gas with earlier onset of cooling, while on the other hand (ii) the CR's cool much less due to their softer equation of state so that they can push the shell from the interior longer than the gas (iii) coupling with the interior introduces an effective time dependence of acceleration through a cutoff in the spectrum (iv) disregard of the sweep-up phase makes the model somewhat inconclusive regarding the relative contribution of gas and CR pressure during the earliest phases (v) After cooling the effective shock compression ratio increases, leading possibly to harder momentum spectra for older shocks with intriguing consequences for the radial (synchrotron) spectral index variation in SNR's.

**References**