

THE USE OF THE McILWAIN L-PARAMETER TO ESTIMATE COSMIC RAY VERTICAL
CUTOFF RIGIDITIES FOR DIFFERENT EPOCHS OF THE GEOMAGNETIC FIELD

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ABSTRACT

A useful relationship employing the McIlwain L-parameter to estimate vertical cutoff rigidities has been derived for the twenty-five year period 1955-1980.

1. Introduction. It is intuitively pleasing to utilize the dipolar geometry inherent in the McIlwain L-parameter to order cosmic ray cutoff rigidities. However, in some areas of the world, secular changes in the geomagnetic field between 1955 and 1980 have been large enough to produce significant differences in both the vertical cutoff rigidities and in the L-value for a specified position. In this paper we show that these changes are complimentary, and it is possible to derive a relationship between the L-value and vertical cutoff rigidity that can be used for the twenty-five year period, 1955-1980.

2. Background. The trajectory-tracing process is generally recognized as the most accurate method for calculating cosmic ray cutoff rigidities. Since cutoff rigidities are a function of latitude, longitude, altitude, zenith angle, azimuthal angle, and field model, using the trajectory-tracing method for a large number of positions and directions is impractical. For this reason, cosmic radiation data from many experiments are often ordered by the cutoff rigidity values in the vertical direction.

One method of estimating vertical cutoff rigidities was suggested by Smart and Shea (1967) who derived three equations for the relationship between the McIlwain L-parameter (McIlwain, 1961) and (1) the upper calculated cutoff, (2) the lower calculated cutoff and (3) the effective cutoff rigidities.* These three equations were derived using cutoff rigidities calculated for the Finch and Leaton (1957) field for Epoch 1955.0 and the Jensen and Cain (1962) field for Epoch 1960. The equations thus derived for the upper calculated cutoff, the lower calculated cutoff and the effective cutoff rigidities were essentially the same for both field models.

At the time of this original work, cosmic ray physicists did not recognize that the secular changes in the geomagnetic field were sufficiently large over a relatively small time period (on the order of 25 years) to significantly affect the detection, at the surface of the earth, of galactic cosmic radiation above 1 GV. It was not until Shea (1971) suggested

* In the paper of Smart and Shea (1967), these rigidity values were called main cone cutoff, Stormer cone cutoff and effective cutoff, respectively. Since new terminology for cosmic ray cutoffs has been agreed upon by scientists working in this area (Cooke et al., 1985) we will use these newer terms throughout this paper.

that the decrease in vertical cutoff rigidity at Huancayo over a 20-year period might possibly be observed as an increase in the background radiation measured by a stable neutron monitor at this location (since verified by Cooper and Simpson, 1979), that it became apparent that the secular changes in the geomagnetic field might be sufficiently large in some areas of the world that changes in cutoff rigidities, and consequently measured changes in cosmic radiation, would occur.

In view of the changes in the main geomagnetic field and the related changes in the calculated cutoff rigidities, we feel it is necessary to re-examine the use of the McIlwain L-parameter to estimate cosmic ray vertical cutoff rigidities for the 25-year period 1955-1980.

3. Method. The world grid of vertical cutoff rigidities calculated each 5° in latitude and 15° in longitude for Epochs 1955, 1965 and 1980 (Shea et al., 1968; Shea and Smart, 1975, 1983) and the McIlwain L-values (McIlwain, 1961) calculated for these same locations comprised the basic data sets used for this analysis. All calculations were made for an altitude of 20 km above the surface of the earth as defined by the international reference ellipsoid. Both the vertical cutoff rigidities and the McIlwain L-values were calculated using the geomagnetic field coefficients for the appropriate Epoch (i.e., 1955, 1965 and 1980).

Expressing the cosmic ray cutoffs by an equation of the form $R=KL^\gamma$ where R is the cutoff rigidity, L is the McIlwain L-value, K is a constant and γ is an exponent, K and γ were evaluated by a least-squares fit of the (1) upper calculated cutoff rigidity, (2) effective cutoff rigidity and (3) lower calculated cutoff rigidity. Each of the equations derived for each Epoch, together with the RMS error for each set of calculations, is given in Table 1. It is important to note that vertical cutoff rigidities < 0.20 GV were omitted from these calculations. Since the cosmic ray equator and the equator defined by the minimum L-value do not coincide, all grid points within a band ± 5 degrees of either equator (or between the two equators) were also omitted. Figure 1 illustrates the locations of each of these equators for Epoch 1980.

Table 1. Equations to estimate cutoff rigidities for various Epochs

Epoch	$R_U =$	$R_C =$	$R_L =$
1955	$16.727 L^{-2.0054}$ RMS = 6.28 %	$16.192 L^{-2.0177}$ RMS = 5.48 %	$14.992 L^{-1.9986}$ RMS = 6.61 %
1965	$16.722 L^{-2.0212}$ RMS = 6.70 %	$16.222 L^{-2.0418}$ RMS = 5.52 %	$14.942 L^{-2.0296}$ RMS = 6.98 %
1980	$16.717 L^{-2.0206}$ RMS = 7.06 %	$16.222 L^{-2.0441}$ RMS = 5.74 %	$14.823 L^{-2.0311}$ RMS = 7.27 %
Composite 1955-1980	$16.762 L^{-2.0174}$ RMS = 6.67 %	$16.237 L^{-2.0353}$ RMS = 5.64 %	$14.912 L^{-2.0185}$ RMS = 7.19 %

4. Discussion. From an inspection of the equations in Table 1 it is evident that the constants K and γ for each of the three vertical cut-off rigidities are essentially independent of Epoch. The root mean square values are also similar, with the slightly larger RMS values for 1980 attributed to the evolution of the magnetic field and an increasing divergence between the cosmic ray equator and the minimum L equator. The area between these two equators has increased approximately 10 percent between 1955 and 1980.

Since these equations were almost identical we combined the data for all three Epochs in an effort to determine a suitable equation for the upper calculated cutoff, the effective cutoff and the lower calculated cutoff for the entire 25-year period from 1955 to 1980. Again all locations with cutoffs less than 0.20 GV were omitted from the analysis. Since the location of the cosmic ray equator changed between 1955 and 1980 (Shea et al., 1983), and different equatorial grid locations had been removed for each Epoch, we removed all locations within $\pm 5^\circ$ of any of the three equators (i.e., if a particular location had been removed for the analysis for one Epoch, it was removed from all three data sets for the composite analysis). Again the constants K and γ were determined by the method of least squares. The results for this composite set of over 1875 data

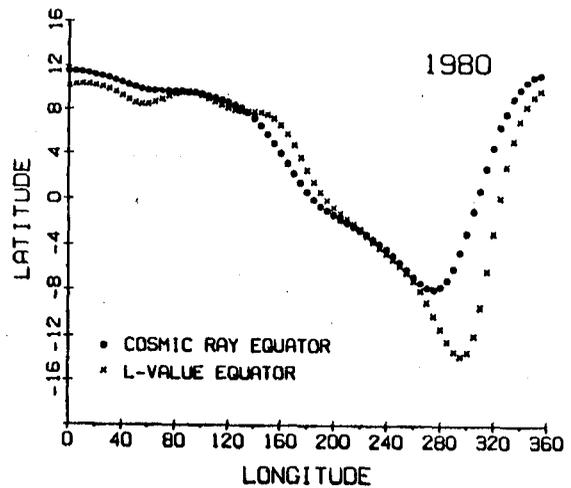


Figure 1. Geographic location of the cosmic ray equator and the minimum L equator for 1980.

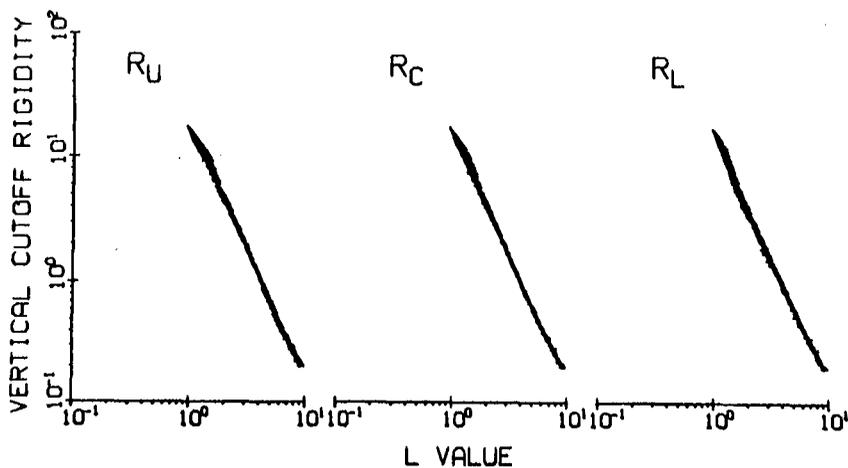


Figure 2. The upper calculated vertical cutoff rigidity (R_U), effective vertical cutoff rigidity (R_C), and lower calculated vertical cutoff rigidity (R_L), plotted as a function of the McIlwain L-value. The data set is a composite of the world grid locations for 1955, 1965 and 1980.

points are graphically illustrated in Figure 2; the equations are given in the bottom line of Table 1. From these results we feel that it is possible to use these three equations to estimate the upper, effective and lower cutoff rigidities for the entire period 1955 to 1980 provided the L-values are calculated using the field model for the same time period that the cutoffs are needed.

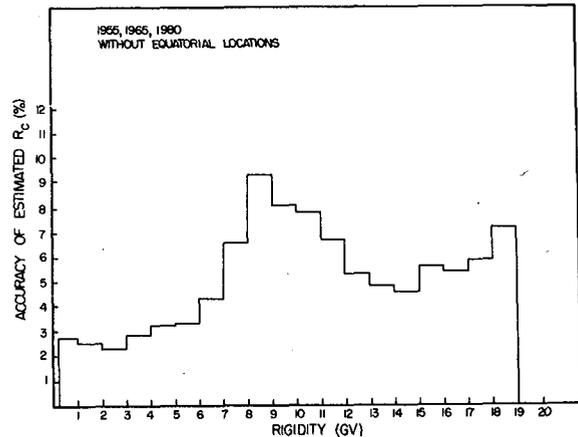


Figure 3: Accuracy of estimated vertical cutoff rigidity.

Figure 3 illustrates the accuracy that can be expected for an estimate of the effective vertical cutoff rigidity as a function of rigidity. These data were obtained by calculating for each location the percentage difference (in rigidity) between the cutoff rigidity value determined by the trajectory-tracing procedure and the value estimated by using the L-value approximation equation. These individual percentages were then averaged as a function of intervals of estimated cutoff rigidity.

5. Conclusion. From this analysis we conclude that the McIlwain L-parameter can be used to estimate upper calculated, effective, and lower calculated vertical cutoff rigidities for the period 1955 to 1980.

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