THE EFFECT OF POWER LAW BODY FORCES ON A THERMALLY-DRIVEN FLOW BETWEEN CONCENTRIC ROTATING SPHERES

MICHELE G. MACARAEG

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ABSTRACT

A numerical study is conducted to determine the effect of power-law body forces on a thermally-driven axisymmetric flow field confined between concentric co-rotating spheres. This study is motivated by Spacelab geophysical fluid-flow experiments, which use an electrostatic force on a dielectric fluid to simulate gravity; this force exhibits a $(1/r)^5$ distribution. Meridional velocity is found to increase when the electrostatic body force is imposed, relative to when the body force is uniform. Correlation among flow fields with uniform, inverse-square, and inverse-quintic force fields is obtained using a modified Grashof number.
I. Background

Geophysical fluid flow experiments (Hart, 1976; Fowlis and Fichtl, 1977) designed to run aboard future shuttle flights will rely on a radial body force created by imposing an electric field across a dielectric fluid to simulate gravity. The fluid will be contained between concentric, rotating spheres with latitudinal "side" walls to confine experimental hardware (see figure 1). The dielectric force is proportional to \((1/r)^5\), (Hart, 1976). The effect of this power law on the flow field relative to a uniform radial body force or one following an inverse square law is investigated here. [Effects of varying rotation rates and boundary temperatures for a uniform gravitational field can be found in the literature (Douglass, 1975; Miller and Gall, 1983; Macaraeg, 1984).] This study utilizes a numerical model based on the incompressible Navier-Stokes equations in a vorticity/stream function formulation, discretized by a pseudospectral representation in the latitudinal direction and a finite difference representation of radial dependencies. For details of the numerical model see Macaraeg (1985).

II. Results of Numerical Study

The gravitational distribution is varied by changing the exponent for \(1/r\) (where \(r\) is the radial coordinate) in the buoyancy term of the vorticity equation (Macaraeg, 1985), as given below:

\[
\sin \theta \left( \frac{1}{r} \right)^n \text{Gr} E^2 \frac{\partial T}{\partial \theta},
\]

\(1\)
where $Gr$ is the Grashof number, $E$ is the Ekman number, $T$ is temperature, and $\theta$ is the latitudinal coordinate. Values of 0, 2, and 5 are given to $n$ to simulate a uniform gravitational field, and fields proportional to $(1/r)^2$ and $(1/r)^5$, respectively. $Gr$ is a function of a reference gravitational acceleration ($g_{\text{ref}}$):

$$Gr = \frac{g_{\text{ref}} \alpha \Delta T r_0^3}{\nu^2},$$

where $\alpha$ and $\nu$ are the coefficients of thermal expansion and kinematic viscosity, respectively, and $\Delta T$ is the maximum temperature difference on the boundaries. $g_{\text{ref}}$ is calculated so that a mean value of gravity is maintained across the gap:

$$g_{\text{ref}} = g_{\text{mean}} (r_o - r_i) \left[ \int_{r_i}^{r_o} \left( \frac{r_o}{r} \right)^n \frac{dr}{r} \right]^{-1}$$

where $r_o$ and $r_i$ are the outer and inner sphere radius.

In figure 2 a plot of maximum values of stream function versus rotation rate is given. Three families of curves are shown corresponding to maximum temperature differences on the solid boundaries of 1, 2, and 3°C. Within each family $\psi_{\text{max}}$ increases with $n$. For the parameter range studied the effects of the gravitational distribution appear secondary relative to temperature and rotation.

However, figure 3 exhibits some characteristics of the flow which are influenced by the gravitational distribution. Figures 3a and 3b are stream function contours for a uniform field ($n=0$) and a field produced by the dielectric ($n=5$), respectively. Although overall features of the flow fields appear to be identical, the dielectric force produces a shift in the internal
meridional recirculation relative to a uniform gravitational field. Note that
the inner cell is stronger in the case of the dielectric than the inner
recirculating cell of the uniform field. Conversely, the outer recirculating
cell is stronger for \( n = 0 \). Since the inverse quintic power law of the dielectric
produces stronger forces towards the inner sphere which drop off quickly towards
the outer sphere, the above trend is expected.

To correlate the data, the Grashof number, (Eq. 2), which represents the
ratio of buoyancy to viscous forces was plotted against \( \psi_{\text{max}} \) for the same
cases given in figure 2. The data is badly scattered. This Grashof number is
the parameter obtained from nondimensionalizing the equations and imposes the
proper condition in the buoyancy term, i.e. that the mean value of gravity
across the gap be constant for varying \( n \). However, it is clearly not the proper
similarity parameter for this flow.

A modified Grashof number (\( \text{Gr}' \)) is defined which takes into account the
gravitational distribution as given in equation (4),

\[
\text{Gr}' = \alpha \frac{\Delta T}{r_0 r_1^2} g_{\text{mean}} \int_{r_1}^{r_0} \left( \frac{r_0}{r} \right)^n dr .
\]  

The scatter reduces considerably. Figure 5 is a plot of \( \psi_{\text{max}} \) versus \( \text{Gr}' \). The
data collapses into three sets corresponding to a rotation rate of 1, 2, and 3
radians per second, respectively. For a given rotation rate the data
distributes itself so that the highest temperature differences on the boundary
correspond to the highest values of \( \psi_{\text{max}} \). For a given \( \Delta T \) the data shows that
as \( n \) increases from 0 to 5, \( \psi_{\text{max}} \) also increases.
References


Figure 1. Numerical Model Schematic
Figure 2. $\psi_{\text{max}}$ vs. $\Omega$
Figure 3. Stream Function Contours. (contour increment = $3 \times 10^{-3}$)
MAX STREAM FNC vs. GRASHOF NO.

Figure 4. $\psi_{\text{max}}$ vs. Gr.
\[ \psi_{\text{max}} \]

**MAX STREAM FNC vs. MODIFIED GRASHOF NO.**

Figure 5. \( \psi_{\text{max}} \) vs. \( \text{Gr}' \).
A numerical study is conducted to determine the effect of power-law body forces on a thermally-driven axisymmetric flow field confined between concentric co-rotating spheres. This study is motivated by Spacelab geophysical fluid-flow experiments, which use an electrostatic force on a dielectric fluid to simulate gravity; this force exhibits a \((1/r)^p\) distribution. Meridional velocity is found to increase when the electrostatic body force is imposed, relative to when the body force is uniform. Correlation among flow fields with uniform, inverse-square, and inverse-quintic force fields is obtained using a modified Grashof number.