SINGLE PARTICLE MOMENTUM AND ANGULAR DISTRIBUTIONS
IN HADRON-HADRON COLLISIONS AT ULTRAHIGH ENERGIES

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1. Introduction. The forward-backward charged multiplicity distribution \( P(n_F, n_B) \) of events in the 540 GeV \( \bar{p}p \) collider has been extensively studied by the UA5 Collaboration. It was pointed out that the distribution with respect to \( n = n_F + n_B \) satisfies approximate KNO scaling and that with respect to \( Z = n_F - n_B \) is binomial [1]. The geometrical model of hadron-hadron collision interprets [2] the large multiplicity fluctuation as due to the widely different nature of collisions at different impact parameters \( b \). For a single impact parameter \( b \), the collision in the geometrical model should exhibit stochastic behavior. This separation of the stochastic and non-stochastic (KNO) aspects of multiparticle production processes gives conceptually a lucid and attractive picture of such collisions [3], leading to the concept of partition temperature \( T_P \) and the single particle momentum spectrum to be discussed in detail below.

2. Description of Model. Assuming the separation of stochastic from non-stochastic aspects of collision to remain valid as \( n \rightarrow \infty \), we expect [1] that the distribution in the two-dimensional \( (n_F/n) - (n_B/n) \) plane would become more and more concentrated in a narrow region. For 540 GeV \( \bar{p}p \) collisions this region is in the form of an ellipse as shown in Fig. 1(a). When \( \bar{n} \) becomes large, it becomes thinner and eventually collapses into a line segment (Fig. 1(b)). This line segment is a collection of points, at each of which \( n_F = n_B \), and both \( n_F \) and \( n_B \) fluctuate only to the extent of \( \sqrt{n_F} \) (i.e., like a stochastic distribution). For example, if \( n = 2 \times 10^6 \), then \( n_F \) could easily be as small as \( 0.5 \times 10^6 \) or as large as \( 2 \times 10^6 \). But in either case, one can predict that \( n_B = n_F \) with fractional
errors of the order \((\bar{n})^{-\frac{1}{2}} \sim 10^{-3}\).

Accepting this picture for very high energies, we see that for fixed \(n_F\), the distribution of \(n_B\) is stochastic. How then is the energy partitioned in the backward hemisphere? We shall assume that the energy partition for each hemisphere for a fixed \(z = (n_F + n_B)/\bar{n}\) is also stochastic but subject to a number of conditions: (a) that the total energy of all outgoing particles on each side is \(E_0h\), (b) Bloch-Nordsieck factor \(d^3p/E\) for each particle, and (c) transverse momentum (\(p_T\)) cutoff factor \(g(p_T)\). In other words, the probability distribution for central particles on each side will be taken as

\[
\delta(\sum E_i - E_0h) \prod (d^3p_i/E_i)g(p_{Ti})
\]

where \(E_0 = \sqrt{s}/2\), \(E_0(1-h) = \) total energy of all leading particles, and \(i = 1,2,...\) ranges over all the particles (positive, negative, and neutral) on one side minus the leading particles. \(h\) is a parameter that describes the fraction of \(E_0\) that fragments into all particles in the central region.

Now the mathematical problem (1) is well-known in statistical mechanics as describing a microcanonical ensemble. By the well-known Darwin-Fowler method the single particle distribution of such an ensemble is given by the canonical ensemble:

\[
dn = K (d^3p/E) g(p_T) \exp(-E/T_p)
\]

where \(T_p\) will be called the partition temperature and \(K\) is a normalization constant. Notice that all particles, positive, negative, and neutral, kaons, nucleons as well as pions, share the same \(T_p\).

3. Comparison with Experimental Angular Distribution at 540 GeV. As Fig. 1(a) shows, at the 540 \(\bar{p}p\) collider, the distribution is still an ellipse. We shall nevertheless test the validity of (2) at 540 GeV by evaluating the single particle angular distribution from it. We write

\[
dn/d\eta = 2\pi K \sin^2 \theta \int_0^{E_0h} p^2(dp/E)g(\sin \theta)\exp(-E/T_p)
\]

where \(\eta = \) pseudo-rapidity, \(\cosh \eta = \csc \theta\), and

\[
g(\sin \theta) = \exp(-\alpha \sin \theta).
\]
Fig. 1 Schematic diagram for forward-backward multiplicity distribution at very high energies. (a) The contour lines represent $P(n_F, n_B)$ at constant fractions of its maximum value at $\sqrt{s} = 540$ GeV where $\bar{n} \approx 29$. (b) The same contour lines degenerate to straight lines for extremely large $\bar{n}$.

Fig. 2 Calculated and experimental $dn/d\eta$ vs. $\eta$ at $\sqrt{s} = 540$ GeV.

Fig. 3 Calculated $dn/d\eta$ versus $\eta$ at $\sqrt{s} = 2$ TeV and 40 TeV.
We take $\alpha$ to be equal to $5.25(\text{GeV/c})^{-1}$. Only pions are included in this calculation. The angular distribution is evaluated from (3) and compared with the results [4] of UA5 in Fig. 2. It is found that the UA5 curve for each multiplicity $n$ is well fitted by (3) for one value of $T_p$. We emphasize that there are no adjustable parameters in this computation, the cutoff $\alpha$ having been taken from experiments [5] concerning $p_T$ distributions. The parameter $h$ and normalization constant $K$ are both determined from the curves themselves.

We conclude that the angular distribution (3) that results from (2) is in excellent agreement with experiment. Furthermore, we believe (2) would give a complete description of the single particle momentum distribution for central particles.

4. **Extrapolation to Higher Energies.** We made extrapolations of the angular distribution to $\sqrt{s} = 2 \text{ TeV}$ (Tevatron) and 40 TeV (SSC). The assumptions made in these computations are as follows: (i) The values of $\alpha$ for these energies are taken to be 5.0 and 4.4 $(\text{GeV/c})^{-1}$, respectively [5]. (ii) The parameter $h$ is taken to be a function of the KNO variable $z = n/\bar{n}$ only. (iii) The values of $\bar{n}_{ch}$ for these energies are taken to be 41 and 78, respectively, by extrapolation. The results are presented in Fig. 3.

5. **Acknowledgments.** This work is supported in part by the U.S. DOE Grant No. DE-FG09-84ER40160, and by the U.S. NSF Grant No. PHY 81-09110 A01.

**References**