GENERAL CONSEQUENCES OF THE VIOLATED FEYNMAN SCALING

G. Kamberov
Department of Mathematics and Mechanics, Sofia State University, Bld. A. Ivanov 5, Sofia, Bulgaria

E. Popova
Institute of Nuclear Research and Nuclear Energy, Bld. Lenin 72, Sofia, Bulgaria

1. Introduction. The problem of scaling of the hadronic production cross sections represents an outstanding question in high energy physics especially for interpretation of cosmic ray data. A comprehensive analysis of the accelerator data\(^1\) leads to the conclusion for existence of broken Feynman scaling. It was proposed that the Lorentz invariant inclusive cross sections for secondaries of a given type approach constant in respect to a broken scaling variable \(x_{\alpha}\). Thus, the differential cross sections measured at accelerator energy \(\sqrt{s_{\alpha}}\) can be extrapolated to higher cosmic ray energies

\[
\frac{d^3\sigma}{d\eta d\phi dE} = g(x_{\alpha}, p_T) = f(x(s/s_0)^{\alpha}/2, p_T) (s/s_0)^{\alpha/2},
\]

where \(x = x(s/s_0)^{\alpha}\), \(\alpha = 0.26\).

This assumption leads to some important consequences. In this paper we'll discuss the distribution of secondary multiplicity that follows from the violated Feynman scaling, using a similar method of Koba et al\(^2\).

2. Derivation of multiplicity distribution in the case of broken Feynman scaling. The distribution of secondary particles in high energy hadron interactions is an object from the first to the recent observations with accelerator facilities. On the other hand, assuming Feynman scaling it was theoretically derived by Koba et al\(^2\) that asymptotically \(n\), \(n(a)\) is only a function of \(n/\bar{n}\)

\[
\left(\frac{n}{\bar{n}}\right) = \frac{\sigma_{\text{tot}}}{\sigma_{\text{tot}}(a)} = \frac{1}{z^n} \Psi(n/\bar{n}),
\]

where \(\Psi(a)\) is the cross section for multiplicity being \(a\) at CMS energy \(\sqrt{s}\), \(\bar{n}\) is the average multiplicity and \(\Psi(z)\) is independent function of \(z\) except through the variable \(z=n/\bar{n}\). The shape of multiplicity distribution has been obtained in a variety of models with rather different theoretical inputs (uncorrelated cluster model\(^3\), geometrical models\(^4\), quark parton model\(^5\), dual parton models\(^6,7\)). The theoretical predictions for multiplicity distribution have been found to be approximately true from \(\sqrt{s} = 1.5\) GeV up to ISR energy \(\sqrt{s} = 63\) GeV\(^8\) where violation of Feynman scaling was observed. When studying the multiplicity distribution at the collider region at \(\sqrt{s} = 540\) GeV the KNO scaling does not necessarily hold for part of the phase space corresponding to higher multiplicities\(^9\). There is a clear indication of an increa-
sing high multiplicity tail. It causes that many of the original models have been amended to accommodate the observed scaling violation by assuming that between the collider and ISR energies some new physical mechanism (rescattering, three gluon coupling) characterized by higher multiplicity started becoming important. We will examine what follows from breaking of Feynman scaling. Let us assume scaling on \( x \) for the distribution functions integrated over the transverse momentum

\[
\gamma(q)(x_{s1}, \ldots, x_{sq}) = \int g(q)(x_{s1}, \ldots, x_{sq}, p_{tq}) \, dp_{t1} \cdots dp_{tq} = \int \left[ g(q) \left( \frac{x_{s1}}{s}, \ldots, \frac{x_{sq}}{s} \right) \right] \, \frac{dx_{s1}}{x_{s1}^{1-\alpha}} \cdots \frac{dx_{sq}}{x_{sq}^{1-\alpha}} \left( \frac{p_{t1}^2 + m^2}{x_{s1}^2 (s/4)^{1-\alpha}} \right) \left( \frac{p_{tq}^2 + m^2}{x_{sq}^2 (s/4)^{1-\alpha}} \right).
\]

There is needed only that the transverse momentum is limited as \( \sqrt{s} \) goes to infinity. In eq. 3 are used functions which incorporate a particular semi-inclusive cross sections. We can derive the moments of multiplicity distribution in an analogical way to that of Koba et al. Thus, for secondaries with rest mass \( m \) we set

\[
\langle n(n-1) \ldots (n-q+1) \rangle = \sum_n P_n(s) \, n(n-1) \ldots (n-q+1) =
\]

\[
= \int g(q)(x_{s1}, \ldots, x_{sq}, p_{tq}) \left\{ \frac{dx_{s1}}{x_{s1}^{1-\alpha}} \left( \frac{p_{t1}^2 + m^2}{x_{s1}^2 (s/4)^{1-\alpha}} \right) \left( \frac{p_{tq}^2 + m^2}{x_{sq}^2 (s/4)^{1-\alpha}} \right) \right\} \times \left[ \ln(x_{s1} + \frac{p_{t1}^2 + m^2}{x_{s1}^2 (s/4)^{1-\alpha}}) \right] \frac{dx_{s1}}{x_{s1}} \cdots \frac{dx_{sq}}{x_{sq}}
\]

\[
= \ln \left( \frac{p_{t1}^2 + m^2}{x_{s1}^2 (s/4)^{1-\alpha}} \right) \left\{ g(q)(0, p_{t1}, s_{2s}^2, p_{t2}, \ldots, s_{sq}^2, p_{tq}) + \frac{w}{\ln \left( \frac{p_{t1}^2 + m^2}{x_{s1}^2 (s/4)^{1-\alpha}} \right)} \frac{d^2 p_{t1}}{x_{s1}^2 + \frac{p_{t1}^2 + m^2}{(s/4)^{1-\alpha}}} \cdots \frac{d^2 p_{tq}}{x_{sq}^2 + \frac{p_{tq}^2 + m^2}{(s/4)^{1-\alpha}}} \right\}
\]

(4)

where the integral

\[
W = \int \frac{dx_{s1}}{x_{s1}} \frac{d^2 p_{t1}}{x_{s1}^2 + \frac{p_{t1}^2 + m^2}{(s/4)^{1-\alpha}}} \cdots \frac{d^2 p_{tq}}{x_{sq}^2 + \frac{p_{tq}^2 + m^2}{(s/4)^{1-\alpha}}} \]

converges. After integration of eq. 4 we obtain
\[ <n(n-1)\ldots(n-q+1)> = \int \frac{\ln((s/4)^{1-\alpha})}{p_{t_1}^2 + m^2} \ldots \frac{\ln((s/4)^{1-\alpha})}{p_{t_q}^2 + m^2} \left[ g(q)(0, p_{t_1}, \ldots, 0, p_{t_q}) \right] dq_0^{1-\alpha} \]

\[ + \frac{1}{\ln 1-\alpha} \int dp_{t_1}^2 \ldots dp_{t_q}^2 \frac{v(q)}{q} \frac{1}{(\ln 1-\alpha)q} + \frac{1}{\ln 1-\alpha} \]

where \( (\ln 1-\alpha)q \) means terms that at most go like \( (\ln 1-\alpha)q \).

Consequently the same asymptotic behaviour has not only the mean value of any \( q \)-order polynomial of \( n \) but the mean value of \( n \) as well. Taking into account eq. 3 we can set

\[ \int n^q p_{n}(s) \frac{d n}{d s} = \frac{\sum q^q(n)(0, \ldots, 0)}{\sum q^q(0)} (\ln 1-\alpha)q + (\ln 1-\alpha)q-1 \times \]

Dividing eq. 5 by \( \frac{\sum q^q(0)}{\sum q^q(0)} [\hat{f}(1)(0)] q \) we obtain

\[ \int z^q z(n)(s) \frac{\hat{f}(1)(0)}{\sum q^q(0)} \ln 1-\alpha \frac{d z}{d s} = \frac{\hat{f}(1)(0)}{\sum q^q(0)} + \frac{1}{\ln 1-\alpha} \]

where

\[ z = \frac{\hat{f}(1)(0)}{\sum q^q(0)} \frac{1}{\sum q^q(0)} \ln 1-\alpha \]

We assume that the function

\[ \frac{\hat{f}(1)(0)}{\sum q^q(0)} \frac{1}{\sum q^q(0)} \ln 1-\alpha \]

is determined uniquely by the moments (5). Thus, to the highest order in \( \sum q^q(0) \ln 1-\alpha \) we have the following broken scaling result

\[ \frac{\hat{f}(1)(0)}{\sum q^q(0)} \frac{1}{\sum q^q(0)} \ln 1-\alpha = \frac{1}{n} \ln 1-\alpha \]

where the mean multiplicity as function of \( \sqrt{s} \) is

\[ <n> = \frac{\hat{f}(1)(0)}{\sum q^q(0)} \frac{1}{\sum q^q(0)} \ln 1-\alpha \]

3. Comparison of the broken Feynman scaling results for the multiplicity distribution with experimental data in accelerator energy range. In figure 1 we have compared the multiplicity distribution function specified by the semi-inclusive cross sections from FNAL which are published in the paper of Kafka et al. /14/ for multipartical production up to above two times larger than the mean value of multiplicity. It is seen that the distribution of relative multiplicity scales at least for the range of not very large multiplicities which are responsible for the typical events in cosmic ray experiments.

As far as the average multiplicity is concerned the assumption for breaking of Feynman scaling gives a good agreement with the acceleration observations in wide energy range up to 10^5 GeV. The energy dependence of mean multiplicity according eq. 9 is compared in figure 2 with accelerator data taken from Carlson /15/.
Fig. 1 Comparison of our parametrisation of the multiplicity distribution (made on the basis of FNAL data) with SPS data for $\eta<3.5$. The latter are representative of the shape of the full distribution.

We can conclude that for the purposes of cosmic ray investigation scaling of multiplicity distribution derived from broken Feynman scaling can be assumed in order to prescribe the semi-inclusive cross sections at very high energies.

References
5. Takagi F., Zeit. fur Phys. 113, 301, 1982
8. Thome W. et al., Nucl. Phys. 8129, 365, 1977
15. Per Carlson XI Intern. Winter Meet. on Fundamental Phys., Toledo, 1983