A NEW METHOD OF DIFFERENTIAL STRUCTURAL ANALYSIS OF $\chi$-FAMILY BASIC PARAMETERS

L.G. MELKUMIAN, S.V. TER-ANTONIAN
Yerevan Physics Institute, Markarian st. 2
375036, Yerevan, Armenia, USSR

YU.A. SMORODIN
Lebedev Physics Institute, Leninsky Prospekt 53
117312 Moscow, USSR

ABSTRACT

The maximum likelihood method is used for the first time to restore parameters of electron-photon cascades registered on X-ray films. The method allows to carry out a structural analysis of the $\chi$-quanta family darkening spots independent of the $\chi$-quanta overlapping degree and to obtain maximum admissible accuracies in estimating the energies of the $\chi$-quanta composing a family. The parameter estimation accuracy weakly depends on the value of the parameters themselves and exceeds by an order the values obtained by integral methods.

In our previous papers /1,2/ we have proposed for the first time to use the maximum likelihood method for the solution of the traditional inverse problem of X-ray emulsion experiments. The essence of the method consisted in the reconstruction of source parameters (the energy $E$, zenith and azimuth angles $\Theta$ and $\Psi$, the spot center coordinates $X$, $Y$) of the darkening spot on the X-ray emulsion film according to the matrix of darkening densities $\{p_{ij}\}$ measured by a densitometer in the cells $(i, j)$ $i = 1, \ldots, N_i$, $j = 1, \ldots, N_j$. The scanning cell dimensions $\Delta X$, $\Delta Y$ and the number of cells $n_i \cdot n_j = N$ are fixed. It has been shown in /1,2/ that the maximum likelihood method gives consistent, efficient and nonshifted estimates for the parameters $E, \Theta, \Psi, X, Y$ in the wide range of scanning cell dimensions $\Delta X = \Delta Y = 10, 20, \ldots 50, 75 \mu$km. The accuracy of parameter estimation slightly depends on the value of the parameters themselves and exceeds by an order the values obtained by the momenta method and by integral methods.

The results of /1,2/ are obtained for the darkening spots caused by a single $\chi$-quantum (electron) with the energy $E$ in the range $2 \times 10^0$ TeV and zenith angle in the interval $1 \times 0.5$.

In /3/ the results of solution of the inverse problem for the general case - the number of the darkening spot...
sources (γ-quanta) is greater than 1 with the strongly marked structure of the spot overlapping, are given.

Remaining within the framework of notations accepted in /1, 2/, one may present the likelihood function for the realized density matrix \{p_{ij}\} at the unknown but fixed vector of parameters \( \vec{\eta} = \eta(\xi_1, ..., \xi_n, x_1, ..., x_n, y_1, ..., y_n, \Theta, \Psi) \) in the form

\[
\phi(\{p_{ij}\} | \vec{\eta}) = \prod_{i=1}^{N_i} \prod_{j=1}^{N_j} f_e(n_{ij} / \vec{\eta}),
\]

where \( n_{ij} = p_{ij} \cdot S \) is the number of electrons in the cell \((i, j)\); \( S = \Delta x \cdot \Delta y \) is the cell area; \( N_i, N_j \) is the corresponding number of scanning cells along the X and Y axes; \( f_e(n_{ij} / \vec{\eta}) \) is the probability of \( n_{ij} \) electrons having been registered in the cell \((i, j)\) at the given \( \vec{\eta} \) vector of parameters; \( \ell = 1 \), if \( n_{ij} < m_{\text{min}} \); \( \ell = 0 \), if \( m_{\text{min}} \leq n_{ij} \leq m_{\text{max}} \); \( \ell = 1 \), if \( n_{ij} > m_{\text{max}} \). The boundary values of the electron number are defined by the operation properties of the film and are chosen by us as in /1-3/ \( m_{\text{min}} = m_{\text{max}} / S = 0.5 \mu \text{km}^2 \), \( m_{\text{min}} = m_{\text{min}} / S = 0.002 \mu \text{km}^2 \). The distribution function \( f_e(n_{ij} / \vec{\eta}) \) is assumed to be Poisson with the mean value \( \vec{n} = e_{\text{cm}} \cdot \vec{\eta} + \vec{n}_\phi \), where \( \vec{n} \) is the number of γ-quanta (electrons), \( \vec{n}_m(\xi_m, x_m, y_m, \Theta, \Psi) \) is the average number of electrons in the cell \((i, j)\) generated by the source with the energy \( E_m \), whose trajectory crosses the film at the point \( x_m, y_m \) at the zenith and azimuth angles \( \Theta \) and \( \Psi \) respectively; \( \vec{n}_\phi \) is the average number of "background" electrons (\( \rho_\phi = \vec{n}_\phi / S = 0.04 \mu \text{km} \)). The determination of the \( \vec{n} \) vector components is performed numerically by minimizing the likelihood function negative logarithm:

\[
\mathcal{L}_o \equiv \min \Delta(\vec{n})_{\gamma} \equiv \min \left\{ -\ln \phi(\{p_{ij}\} / \vec{n}_\gamma) \right\}
\]

The inverse problem may also be solved in the case when the number of darkening spot sources \( m_{\gamma} \) is unknown. In this case one should minimize the functional

\[
\mathcal{L}_1 \equiv \min \left\{ \min \Delta(\vec{n}) \right\}_{m_{\gamma}}, \quad m_{\gamma} = n_o, n_o + \Delta n, ..., n',
\]

where \( n_o \) and \( n' \) determine the boundaries of search for the darkening source number.

The inverse problem was solved by the maximum likelihood method basing on the density matrix \{\( p_{ij} \) \} generated by the Monte-Carlo method at the given vector of parameters \{\( p_{ij} \) \}. The average number of equivalent electrons in the cell \((i, j)\) was calculated according to the cascade theory axial approximation. Fluctuations were assumed to be Poisson-distributed. The BESM-6 computer main memory allowed one to analyze the rasters with the cell number \( N_i \cdot N_j \leq 7000 \) and the summary number of unknown parameters \( 3(n_\gamma + 1) \leq 99 \).

Consider the possibilities of the elaborated method at
1) \( n_\chi = 1 \), 2) \( n_\chi = 2 \), that is of interest for calibrating the emulsion technique on \( \Lambda^- \) decays; 3) \( n_\chi > 2 \) for processing \( \chi^- \)-families; 4) \( n_\chi = ? \), i.e. when the number of the darkening spot sources is the component of parameter vector and is unknown.

1. \( n_\chi = 1 \)

The simulated rasters parameter reconstruction was carried out for discrete series of \( \chi^- \)-quanta energy \((E_\chi = 2, 5, 10, 20, 50, 100 \, \text{TeV})\) \( \Delta x = \Delta y = 10, 20 \, \mu \text{km} \) cell dimensions and various thicknesses of absorbers \((d)\).

The relative accuracy of estimates of \( E, \Theta, \Psi \) made a few per cent. The accuracy of the definition of the spot center coordinates is \( 2-5 \, \mu \text{km} \).

2. \( n_\chi = 2 \)

For the darkening spot pair the calculations were performed at various energies \( E_1 = 2 \cdot E_2 (2, 4, 5, 10, 20 \, \text{TeV}) \) and distances between the spot centers \( \delta = (0.1 \div 100 \, \text{km}) \) at the scanning step of \( 10, 20, 50 \, \mu \text{km} \).

The results may be formulated as follows:

a) the accuracy of determining the zenith \( \Theta \) and azimuth \( \Psi \) angles differs insignificantly from the single spot results and makes \( \xi_{\cos \Theta} < 5 \div 7 \% \);

b) the accuracy of determining the spot center coordinates is \( \xi_x \approx \xi_y \approx \Delta x/2 \), the distances between centers are \( \xi_{\Theta} \approx \Delta x \);

c) the accuracy of determining the energy for the overlapping spots is not worse than 10% at the summary energy reconstruction accuracy of 5-7%.

3. \( n_\chi > 2 \)

For the overlapping families of \( \chi^- \)-quanta the inverse problem was solved for the energies distributed uniformly in the 2-20 TeV interval. The number of \( \chi^- \)-quanta was varied in the region 2-20 over the area 0.25 mm.

The accuracies of determining the parameter vector components at \( 3 \leq n_\chi \leq 10 \) were independent of the \( n_\chi \) number and on the average equalled the corresponding values at \( n_\chi = 2 \).

The accuracy of determining the zenith angle cosine made 4 \div 5 \% and also was independent of \( n_\chi \).

4. \( n_\chi = ? \)

The \( n_\chi \) value varied in the interval 1 \div 5 due to the complexity of solving the problem at the unknown number of the darkening spot sources.

No noticeable variation of the estimation accuracy was observed. Note also that at the known values of \( n_\chi \leq 10 \) the efficiency of solving the inverse problem equalled 90 \div 100 \%. At the unknown \( n_\chi \) the efficiency decreased to 80\%, and the computer counting time grew 5 \div 7 times.
REFERENCES