1. Introduction
The distorsion of the electron cascade development under LPM effects is now currently admitted (1, 2, 3); it consists in an increase of depths of showers origin, of shower maximum $T_{\text{max}}$, a decrease of the number of particles at maximum $N_{\text{max}}$ and results in a flattening and a widening of the cascade transition curve. Connected with the influence of multiple Coulomb scattering on basic electromagnetic processes (bremsstrahlung, pair production), this effects appears at high energy with a threshold dependent on the density of the medium (more than 10 TeV for lead, more than $10^6$ TeV in air).

We examine here, consequently, the electromagnetic components of hadron induced showers in lead and EAS in air, calculated for the same hadronic cascades in the different alternative, including or not LPM effect.

2. Analytical representation of cascade curve
We have used in lead our Monte-Carlo data (1) to estimate from numerical values of $\gamma$-induced showers the different moments at fixed primary photon energy $E_0$

$$p_n(E_o) = \int_0^\infty t^n N_e(E_o,t) \, dt$$

The longitudinal spread $\tau$ and the integral track length $S_o$ have therefore been obtained from the relations between appropriate moments

$$\left[\tau(E_o)\right]^2 = \frac{p_2(E_o) - [p_1(E_o)]^2}{p_o(E_o)}$$

and

$$S_o(E_o) = \sqrt{2\pi \tau(E_o) N_{\text{max}}(E_o)} = \int_0^\infty p(E_o,t) \, dt$$

We found convenient to describe the data including LPM effect by the following formula (replacing Greisen's formula when $E_o > 1$ TeV in lead):

$$N_e(E_o,t) = 15.3 [1 + \alpha_2 B_o^2]^{-1} E_o^{0.9773} \exp \left[-(t - T_{\text{LPM}})^2/2\tau^2\right]$$

with $T_{\text{max}} = T_{\text{LPM}}$, $\tau = \tau_{\text{LPM}}$ and $B = \ln (E_o/10^3)$, where

$$T_{\text{LPM}} = 4.98 + \ln E_o + \alpha_1 [\ln E_o]^{\beta_1}$$

with $\alpha_1 = 0.0001477$, $\beta_1 = 4.64$

and

$$\tau_{\text{LPM}} = 3.78 (1 + \alpha_2 B_o^2) E_o^{\delta}$$

with $\alpha_2 = 0.008446$ and $\delta = 2.7936$

This formula is inserted in our Monte-Carlo programm of hadronic
cascade for all $\gamma$ rays of different energy emitted at different depth.

Similar procedure has been adopted for air, from Monte-Carlo data (2) leading to the formula for $E_o \geq 10^5$ TeV

$$N_e(E_o, t) = 0.00825 E_o (100-0.88 B_o -1.62 B^2_o) \exp \left[-(t-T_o)^2/2\tau^2\right]$$

where $B_o = \ln (E_o/10^8)$, $T_o = 1.363B_o + 21.09$

$$\tau = 5.2 \text{ if } E_o \leq 10^{10} \text{ GeV}$$

$$\tau = 5.2 - 0.426 \ln (E_o/10^{10}) \text{ if } E_o \geq 10^{10} \text{ GeV}$$

($E$ in GeV, $t$ in c.u.). This formula is also inserted in our hybrid Monte-Carlo-analytic simulation in air in place of Greisen's formula.

3. Simulation in lead calorimeters

The model used for production of secondary hadrons is SBM extended in lead following HE 4.1-9 with $\langle N \rangle = 3.2$. The energy lost in disintegration of the struck nucleus is $E_D (\text{MeV}) = 124N + 30$, the number of tracks $N$ being obtained from $N = 3.46E^{0.33}A^{0.19}$ (4). The Monte Carlo procedure gives the quantities $E_{\text{ion}}, E_{\text{out}}$ at different depth of a calorimeter 1000 g/cm$^{-2}$ deep built with lead plates of 50 g/cm$^{-2}$.

$$E_{\text{ion}} = (N_1 + N_2 + 0.75 N_3) 32 \times 7.4 \text{ (MeV)}$$

$(N_1 = \frac{1}{2}(N(100+N(200))), N_2 = \frac{1}{2}(N(300)+N(400))/2)$...

is the energy lost by secondary particles and $\gamma$ initiated cascades, $E_D$ is the total energy spent by disintegration, $E_{\text{out}}$ is the energy leaking out the considered slide (or the bottom of the calorimeter) estimated as the sum of the individual energies of outgoing hadrons. The behaviour of those quantities with depth are given in fig. 1 for incoming protons of $10^5$ GeV. $E_{\text{ion}}$ is given with and without LPM. It can be ascertained that for short calorimeters ($\sim 3\lambda$) $E_{\text{ion}}$ is 1.6 times lower at $10^6$ GeV when LPM is taken into account and a systematic underestimation, rising with energy, occurs when the primary $E_0$ is estimated from $E_x$ without consideration of LPM. A first approach of the amended Grigorov spectrum is shown in fig. 2. Similar consequences will also be detailed for emulsion chamber data.

4. EAS with LPM

EAS induced by proton have been simulated between $10^9-10^{11}$ GeV, for scaling model (5) and CKP model. For 1st model according to the small number of more energetic secondaries secondaries an important distorsion occurs in cascade curve (fig. 3 - 4) at $10^{10}$ and $10^{11}$ GeV. The discrepancy is not visible at $10^9$ GeV. For CKP model, LPM can be neglected even at $10^{10}$ GeV. If we postulate, following (6) the validity of scaling at such energies, the primary energy near $10^{11}$ GeV estimated from the Fly's eye could be underestimated by 30% without LPM correction.

5. Conclusion

LPM effect implies higher intensities near $10^6$ GeV, estimated from direct measurements. The tenancy of fig. 2 where Grigorov
Fig. 1

Fig. 2
amended spectrum is nearer of EAS data (7) could be stronger, if we consider in nuclear model the decrease of inelasticity with primary energy (HE 4,1-9,10). In atmosphere, it's difficult to know at present if scaling model is valid at so high energy, but we have considered here at least, the extremal distorsion due to LPM.

References
7. Wdowczyk J., 1984, 9th ECRS, Kosice

![Fig. 3](image1.png)

![Fig. 4](image2.png)