1. Introduction

Nucleon decay detectors at large depths offers now a total area larger than 1000 m$^2$ to registerate muons of energy exceeding 1 TeV. Near complete high energy muon families are detected in those arrays and we have started an extensive 3D Monte-Carlo simulation in view to understand the spatial distribution of those events and the possible link with elementary act or primary composition.

As pion or kaon parents have a very small decay probability at so high energy, multimuon phenomena occurs at high altitude where the atmospheric density is small after the most energetic collisions.

2. Monte-Carlo modelling and target diagrams

For a first attempt, we described the multiple production of secondary hadrons by the scale breaking model (1). The energy of secondaries are generated from the SBM fit of the invariant cross section

$$E \frac{d^3 \sigma}{dp^3} = f \left(x \left( \frac{s}{s_0} \right), \frac{p_t}{s_0} \right)^{\alpha/2}$$

The average multiplicity obtained is comparable to the fit of Thome, slightly more important than in usual scaling model. At energy exceeding the collider possibility, i.e., more than 155 TeV, we assumed

$$n_s \sim \left( \frac{s}{s_0} \right)^{\alpha/2}$$

with $\alpha/2 = 0.13$, taking into account different reviews on $p$-$p$ data (2)(3).

An important violation (especially for multimuon production) of KNO scaling has been incorporated in the calculation, as shown in fig. 1, where the fluctuations of multiplicity $\Psi(z)$ are quite larger than usually assumed in Slattery's distribution.

Transverse momenta are generated following

$$\frac{dN}{dp_t} = (p_t/p_o)^{1.5} \exp (-p_t/p_o)$$

where $<p_t> = 2.5 p_o$ and $<p_t> = 0.0151 \ln E + 0.24$.

15% of the pions are treated like kaons and charged pions are
submitted to probability charge exchange with probability 1/3. The zenith angle of emission is calculated for each secondary following its longitudinal and transverse momenta. A rotation matrix provides the director cosines of outgoing particles versus those of the incoming hadron.

More recently, we have described the multiple production by a multicluster independent model, in better agreement with short range order correlations, especially adapted to nucleon-air collision, presented in the papers 4.1-9, 10. For nucleus primaries, we have used superposition models, or, as in p-air nucleus collisions, assumed a rise of central rapidity density correlated with the number of "participating nucleons" following the rules edicted by Faessler (4).

Pions and kaons decays, with the subsequent deviations are also treated by the Monte-Carlo method like 2-body decays, with respect to individual lifetime and masses. Earth magnetic field is also taken into account with some approximations for near vertical muons (5). A 3D Monte-Carlo method incorporating muon bremsstrahlung, pair production, nuclear interaction, ionisation and multiple Coulomb scattering (6) is used finally to propagate the muons at large depths in the rock.

For each muon family, we have obtained the coordinates, the director cosines and the energy of the muons recorded at depths 1800, 4200, 5000 mwe corresponding approximately to the levels of Soudan, Momestake or Frejus, and Mont-Blanc installations, respectively.

3. Topological aspects of multimuons

The average number of muons $\bar{n}_\mu$ versus primary energy are plotted on fig. 2. At very high energy, heavy nuclei would become more efficient to produce multimuons however. This must be weighted according to their abundance.

It is shown on fig. 3, how events with large separation $r_{ij}$ and large $n_\mu$ would be more likely generated by heavy nuclei. (The plot contains 50 events generated by iron of $5 \times 10^6$ GeV and nucleons of $10^7$ GeV, assumed to have the same frequency). The separation $r_{ij}$ of a multimuon is taken as

$$ r_{ij} = \frac{1}{n_\mu} \sum ((x_i-x_j)^2 + (y_i-y_j)^2)^{1/2} $$

The aspect of individual target diagramms (fig. 4) suggests reduction of the average radius, as well as separation when primary energy is rising (respectively 1.8 and 3.5 m for vertical protons of $10^7$ GeV).

4. Discussion

Pseudo-random target diagramms suggest that multimuons initiated by nucleons would have stronger concentration in the delimited area. A criterium to recognize nucleon primary could be proposed such as $n_i/2$ contained in a circle of radius $r_i/3$. On the opposite events with large $r_{ij}$ and large $n_\mu$ would have to be associated to heavies. A more accurate procedure will be deduced from a larger statistics of simulation.
Fig. 1: Fluctuations assumed for multiplicity.

Fig. 3: $\bar{n}_i$ versus $n_\mu$ for protons and nuclei.
- proton primaries $10^7$ GeV
- iron primaries $5 \times 10^6$ GeV

Fig. 2: $\bar{n}_\mu = f(E)$

Fig. 4: Individual target diagrams with $\bar{n}_i = 8$.
a. vertical proton $10^7$ GeV
b. vertical iron $5 \times 10^6$ GeV.
Comparison of the intensity obtained assuming the JACEE spectrum is in tolerable agreement up to 5 muons, with Soudan, Homestake and Nusex results (7); more interesting will be the comparison for $n > 5$ which will give information in the decade $10^6$-$10^7$ GeV, depending together on primary spectrum intensity behaviour and composition. Energy calibration by an EAS surface array could be promising, for instance, a multimuon event would be associated to a surface muon size ($> 1$ GeV) of $6.10^4$ muons instead of $2.10^4$ according to a primary iron at nucleon of $5.10^6$ GeV.

The introduction of the multicluster independent model doesn't modify very much the profile of multimuons events, when compared to SBM. According to the high energy threshold, the modification of the plateau of rapidity in central region for nucleon-nucleus and nucleus-nucleus collision have no appreciable effects, those muons being produced by secondaries of high rapidity.

The multimuon radius is enlarged with zenith angle, but always reduced with primary energy, becoming lower than 2 meters for an incoming proton of $10^7$ GeV ($\theta = 22^\circ.5$) at 4200 mwe (the threshold corresponding to 3 TeV).

References