ON MUON ENERGY SPECTRUM IN MUON GROUPS UNDERGROUND

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ABSTRACT

A method is described which was used to measure muon energy spectrum characteristics in muon groups underground using \( \mu \)-e decays recording. The Baksan Telescope's experimental data on \( \mu \)-e decays intensity in muon groups of various multiplicities are analysed. The experimental data indicating very flat spectrum does not however represent the total spectrum in muon groups. Obviously the muon energy spectrum depends strongly on a distance from the group axis. The "core attraction" effect makes a significant distortion, making the spectrum flatter. After taking this into account and making corrections for this effect the integral total spectrum index in groups has a very small dependence on muon multiplicity and agrees well with expected one: \( \beta = \beta_{\text{expected}} = 1.75 \).

1. Introduction. It has been shown (1) that \( \mu \)-e decays method can be used to measure an effective exponent of muon energy spectrum in muon groups underground, which is expected (2) to be much flatter than the total muon flux spectrum*). Unfortunately, such measurements are affected by so called "core attraction" effect, which preferably selects the central part of the muon group making the spectrum of recorded muons flatter, as compared with the spectrum in the group as a whole. This effect depends on the ratio of detecting area size to the muon group dimension, on the form of lateral distribution function and on muon multiplicity spectrum. To make the corrections for "core attraction" effect we have to use the above mentioned parameters which we take from (1) and (7). Because the exponent of multiplicity spectrum is not a constant at small multiplicities, we have to do the analysis for different multiplicities separately.

For the total muon flux the relations between experimentally observed quantities \( R_{at} \) and \( R_{loc} \) and muon energy spectrum underground are well established and understood (3, 4, 5). By definition, \( R_{at} \) is the ratio of stopping to throughgoing muon fluxes normalized to 100 g/sm² target. \( R_{loc} \) is defined in similar way, but for cases when muon from atmosphere does

*) First experimental evidence of this has been presented in (5).
not stop and \( \mu - e \) decays come from pions originated in muon induced hadronic cascade.

\[
\begin{align*}
R_{at} &= 100 a \cdot \frac{dJ(0)/dE_\mu}{J(>0)} \quad 11/1; \quad R_{at} = 100 a \cdot \frac{J}{E_0} \quad 11/1' \\
R_{loc} &= C \cdot \left( \frac{E_\mu}{E_0} \right)^{0.75} \quad 12/1; \quad R_{loc} = C \cdot E_\mu^{0.75} (\delta - 0.7)^{-0.8} \quad 12/1'
\end{align*}
\]

\[
dJ(E_\mu)/dE_\mu \approx (E_\mu + E_0)^{-\delta} \quad 13/1
\]

The eqs. /1-2/ give these relations in general way. Here, \( E_\mu \) is muon energy underground, \( a = 2.34 \cdot 10^{-3} \text{ GeV} \cdot \text{sm}^2/\text{g} \) is mean ionization energy losses at our depth, \( c = 2.10^{-5} \text{ GeV} \cdot \text{m}^2/\text{g} \) (adjusted by experiment). The eqs. /1'/ and /2'/ are valid if the spectrum underground is expressed by eq. /3/, \( E_0 \) is a specific energy equal to 210 GeV for our depth. Eq. /4/ corresponds to purely power law muon spectrum \( E_\mu^{-\delta} \) on the surface. Nevertheless, if the spectrum is not strictly a power law eqs. /1'/ and /2'/ can be used to determine the effective exponent \( \gamma \) near the depth energy threshold. But, in the case of muon groups such a procedure would be erroneous because of distortion due to "core attraction".

2. Experiment. In this experiment the \( \mu - e \) decays were recorded in the second horizontal 200 m² and 40 tons scintillation layer of Baksan Telescope (4). Other 7 layers were used to determine the number of parallel muon trajectories \( (m) \) and to distinguish between "atmospheric" and "locally produced" events. The latter procedure is made with high confidence (~99%) for low multiplicities, but becomes difficult for \( m > 30 \). The selection of \( m > 1 \) events was made by the off-line computer. All necessary data associated with these events were printed and analysed visually together with 10-beam oscilloscope slides of decay's electrons pulses. The only trigger used in the experiment was a delayed pulse in the mentioned scintillator layer with rate \( \approx 30 \text{ h}^{-1} \). Data accumulated during 200 h of observation were included in the analysis. A flux of muons throughgoing the target layer in groups has been measured in a special short run with the same conditions and data processing but without requirement of \( \mu - e \) decay.

3. Calculation of \( R_{at} \) and \( R_{loc} \) in muon groups. To do this the assumptions as in (1, 2, 7) and a number of consecutive integrations have to be made:

\[
\int(N)dN \quad 14/1; \quad N' = N \cdot \Delta(R) \quad 15/1; \quad \Delta(R) = \int \frac{F(x)}{dS} / \int_0^{\infty} 2\pi z F(z) dz \quad 16/1
\]

\[
F'(E_\mu, z) = E_\mu^{-\delta} \cdot z \cdot \frac{E_\mu}{E_0}^{0.7} \quad 17/1; \quad F(z) = \int_{E_0}^{\infty} F'(E_\mu, z) dE_\mu \quad 18/1
\]

\[
G_m(R) dR = 2\pi R dR \int_0^{\Delta(R)} \int_0^{\Delta(R)} \boldsymbol{\varphi}(N') \cdot z \cdot \frac{N'}{m!} \quad 19/1
\]
Here in /4/: \(N\) is the mean number of muons at a given depth from definite primary, \(\gamma(N)dN\) is a distribution of \(N\) assuming standard energy spectrum and chemical composition of primaries.

In /5/: \(\Delta(R)\) is a fraction of \(N\) which happened to be inside detecting area, \(R\) is a distance between centre of area and muon group core.

In /6/: \(\Delta(R)\) is calculated using lateral distribution function \(F(r)\), where \(r\) is a distance between area element \(dS\) and core position.

Eq. /7/ represents the assumed lateral-energy distribution of muons, \(E_{\mu}\) is muon energy at the surface, \(\beta\) is integral exponent of power law muon spectrum there, \(E_t\) is the minimal energy to penetrate to a given depth. For our depth \(r_0=6.2\) m if \(\beta=1.75\). Eq. /8/ represents muon lateral distribution.

In /9/: \(G_m(R)\) is distribution of impact parameter \(R\) for a given multiplicity \(m\). Poisson fluctuations of \(m\) for a given \(N'\) are assumed.

Fig.1 shows these normalized distributions for several \(m\). Curve \((m\geq 1)\) represents the case without core attraction: \(G_1(R)\sim 2\pi R \Delta(R)\). One can see that for \(m>3\) the distortion of distribution of \(R\) becomes standard, for \(m=1\) the distortion is small, but of the opposite sign.

In /10/: by integrating \(F'(E_{\mu}, r)\) over detecting area \(S\), \(r\) being the distance between \(dS\) and the group core, then integrating over \(R\) we get the muon energy spectrum inside the detecting area for a given recorded multiplicity \(m\).

Eq. /11/ represents the normalized muon energy spectrum at a given \(R\). Eq. /12/ is relation between muon energy underground \((E_{\mu})\) and on the surface \((E_{\mu})\).

Next steps are as follows: using /12/ transform /10/ into spectrum underground; put this spectrum into eqs. /1-2/ and obtain the expected quantities \(R_{at}\) and \(R_{loc}\) for assumed \(\beta\); select \(\beta\) which fits the experimental values of \(R_{at}\) and \(R_{loc}\). So, for each \(m\) we obtain two values of \(\beta\): \(\beta_1\) related to \(R_{at}\), and \(\beta_2\) related to \(R_{loc}\). If all the procedure is reliable and the surface muon spectrum follows power law, then \(\beta_1\) and \(\beta_2\) should coincide.

4. Results. All the data are presented in Table. Denoted as \(\beta^*\) and \(\beta^*\) indices are exponents obtained without taking into account the "core attraction" effect, but simply using eqs. /1*\ and /2*. It is seen, that corrected exponents \(\beta\) are bigger than noncorrected \(\beta^*\). On the other hand, one can see
Fig. 1. Distribution of impact parameter R for various multiplicities.

that $\beta_1$ and $\beta_2$ are closer to each other than $\beta_1^*$ and $\beta_2^*$ which indicates a good fit to the power law spectrum, what can be expected. Deviations from this for $m=11-30$ has probably statistical origin, for $m=2$ it could be expected because of primary nucleon energy for $m=2$ is not big enough(7).

Taking all events with $m\geq 3$ and using both $\beta_1$ and $\beta_2$ we obtain the integral spectral index of muons in groups as

$\beta=1.67\pm 0.11$ for $E_\mu \geq 200$ Gev. This result is in agreement with EAS-data (6).

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References
2. Chudakov A.E. Proc. 16 ICRC, Kyoto, 10, 192, (1979)
7. Budko E.V. et al. This conference, HE 5.1-12