

# BOUNDING SOLUTIONS OF GEOMETRICALLY NONLINEAR VISCOELASTIC PROBLEMS

by

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## Abstract

Integral transform techniques such as the Laplace transform, provide simple and direct methods for solving viscoelastic problems formulated within a context of linear material response and using linear measures for deformation. Application of the transform operator reduces the governing linear integro-differential equations to a set of algebraic relations between the transforms of the unknown functions, the viscoelastic operators, and the initial and boundary conditions. Inversion, either directly or through the use of the appropriate convolution theorem, provides the time domain response once the unknown functions have been expressed in terms of sums, products or ratios of known transforms. When exact inversion is not possible approximate techniques, such as suggested by Schapery<sup>1</sup>, may provide accurate results.

The overall problem becomes substantially more complex when nonlinear effects must be included. We consider here situations where a linear material constitutive law can still be productively employed but where the magnitude of the resulting time dependent deformations warrants the use of a nonlinear kinematic analysis. The governing equations will be nonlinear integro-differential equations for this class of problems. Thus traditional as well as approximate techniques, such as cited above, cannot be employed since the transform of a nonlinear function is not explicitly expressible.

Rogers and Lee<sup>2</sup> considered such a problem in an investigation of the finite deflection of a viscoelastic cantilever beam. Employing an analogy to an associated elastic problem they derived a solution to the viscoelastic problem in a form involving a time convolution of a nonlinear space and time dependent integral function. Numerical evaluation was accomplished using Picard's method of successive substitutions. Newton-Cotes quadratures were employed to approximate the spatially dependent integral relationship; a mean value based finite difference formula was used for the time convolution.

Solution procedures of this type are generally well suited for computer implementation. However they can become computationally inefficient when the response must be determined over an extended time period. Each increment in time requires a reevaluation of the convolution integrals. Thus the entire deformation history must be retained in memory during the calculations. Since each completed set of computations adds another set of results to this history this results in an ever increasing memory requirement. In addition the total number of computations which must be performed during the succeeding iteration is also increased.

In this regard an approximation technique proposed by Schapery<sup>3</sup> can provide an attractive alternative. Commonly referred to as the "quasi-elastic" approximation it has most recently been employed by Vinogradov<sup>4</sup> and Vinogradov and Vijeweera<sup>5</sup> in studies of the behavior of eccentrically loaded viscoelastic cantilever beams.

The method is based on the observation that the solution procedure developed by Rogers and Lee may be interpreted as a sequence of short time interval "quasi-elastic" solutions. This suggests that approximate solutions may be generated by replacing the original viscoelastic problem by an "equivalent" time dependent elastic one. In this replacement problem the elastic properties are equated to the instantaneous values of the relaxation moduli or creep compliances of the viscoelastic material.

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The inherent numerical advantage provided by this technique is that it can eliminate the potentially inefficient convolution integral calculations. Thus the speed and efficiency at which the time dependent response is calculated will become independent of elapsed time. The obvious potential disadvantage of the technique is that since it is an approximation significant differences may exist between the actual response of the viscoelastic body and those predicted quasi-elastically. In addition, the quasi-elastic method does not provide a direct method for assessing whether or not the errors which may be incurred are conservative.

In this paper we demonstrate that the quasi-elastic approximation technique can be modified to provide both upper and lower bound predictions for the class of viscoelastic problems under consideration. To accomplish this the solution of the actual viscoelastic problem is first formulated in terms of a Volterra type integral equation of the second kind. An upper or lower bound solution is then established by replacing the desired unknown function which appears under the integral sign by an appropriate approximating function. The approximating function selected is one which inherently bounds the desired unknown function and is also independent of the integration variable. The kernel of the integral operator can then be integrated formally casting the approximate problem into the typical quasi-elastic format. Provided the kernel of the original integral operator is positive semi-definite this quasi-elastic solution must bound the actual solution in the same manner that the approximating function bounded the desired unknown function.

Illustrated on the following page are comparisons of results obtained using the bounding technique to the exact solution for the transversely loaded viscoelastic cantilever beam problem considered by Rogers and Lee. Additional examples of the application of the technique, including procedures to be used when nonlinear boundary terms appear in the original integral equation, are provided in the paper.

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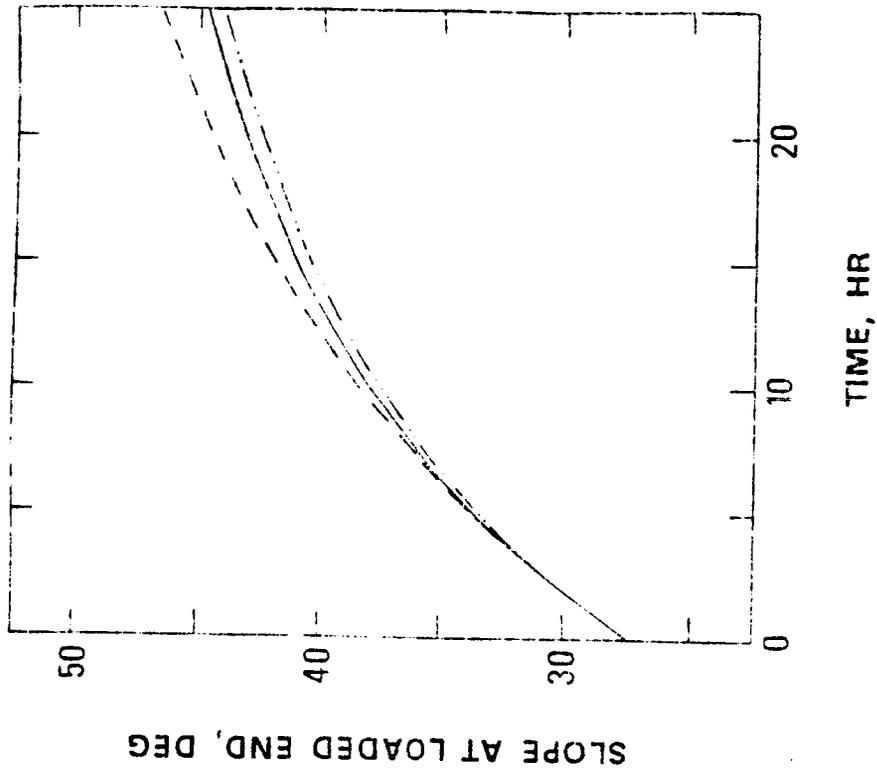


Fig. 2 Viscoelastic response at end point of transversely loaded cantilever

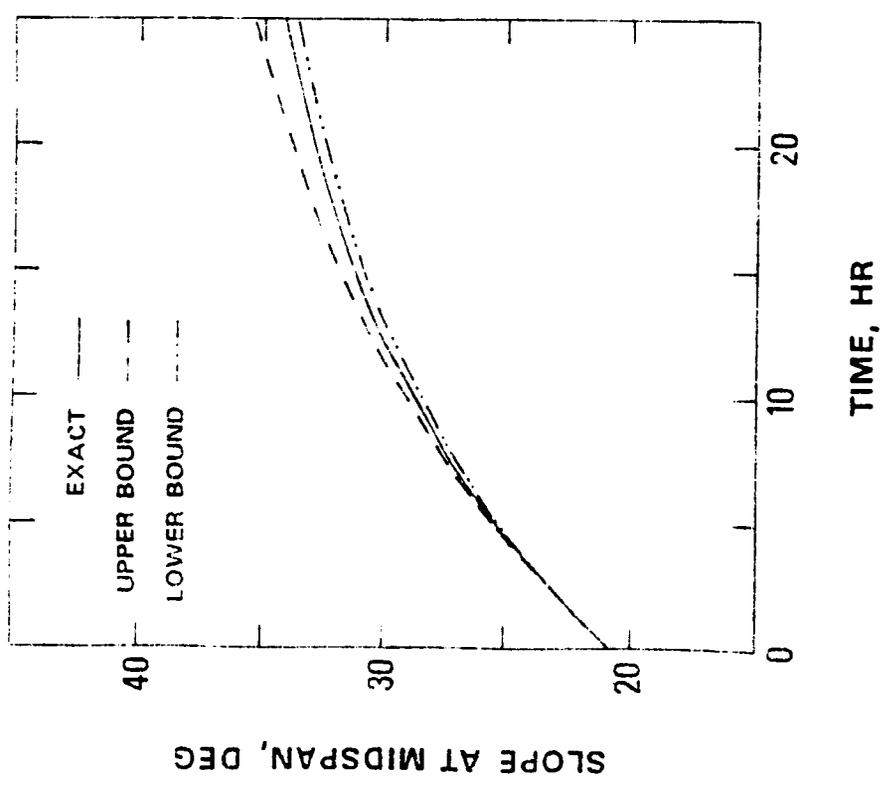


Fig. 1 Viscoelastic response at midspan of transversely loaded cantilever.