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# CRITERIA FOR SCALING HEAT EXCHANGERS TO MINIATURE SIZE

Philipp B. Rudolf von Rohr and Joseph L. Smith, Jr.
Massachusetts Institute of Technology
Cryogenic Engineering Laboratory
Cambridge, MA 02139

ABSTRACT

The purpose of this work is to highlight the particular aspects of miniature heat exchangers performance and to determine an appropriate design approach. A thermodynamic analysis is performed to express the generated entropy as a function of material and geometric characteristics of the heat exchangers. This expression is then used to size miniature heat exchangers.

Key words: Miniature heat exchanger design; laminar flow heat exchanger optimization; second law analysis.

## 1. Introduction

In recent years a large number of applications have been developed for small superconducting devices being cooled around 4.2 K . Their viability will depend on the availability of closed cycle, efficient refrigerators. An extended survey [1] has shown that small scale refrigerators, for loads around 1 watt, are not commercially available.

Different approaches have been undertaken to build reliable and efficient closed-cycle refrigerators. In particular, Vuillemier, Gifford-McMahon and Stirling cycles were investigated. These cycles are efficient at temperatures greater than 10 K. The main problems in these pressure-cycling refrigerators for liquid helium temperatures are rooted in the poor heat capacity of regenerators materials at low temperatures. Preliminary studies [2] show that for small loads Collins cycle refrigerators are advantageous.

As part of a program to develop an efficient, closed cycle Collins type refrigerator for 1 watt at 4.2 K, we investigate the performance of miniature helium plate heat exchangers (Fig. 1). Due to the mass flows involved, these heat exchangers operate in laminar flow conditions.

We will show that the axial conduction in the solid wall has a considerable impact on the performance of low temperature miniature heat exchangers.

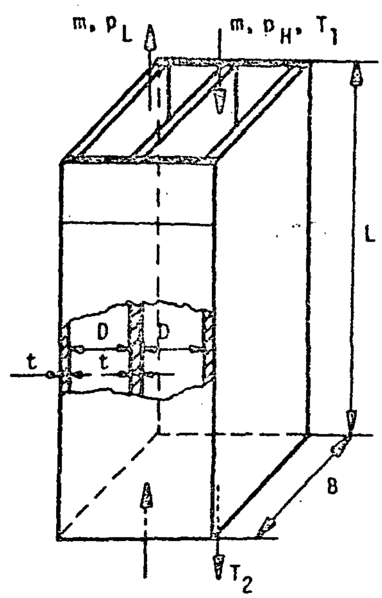


Figure 1: Plate heat exchanger

## 2. Thermodynamic analysis

The three major contributions to irreversibilities in heat exchangers are:

2.1. Entropy generation due to friction, $S_p$
2.2. Entropy generation due to temperature difference between the heat exchanger streams, $S_T$
2.3. Entropy generation due to heat conduction in the solid wall, $S_c$

Their sum will furnish the generated entropy for the isolated heat exchanger:

$$S = S_p + S_T + S_c \cdot \qquad (1)$$

### 2.1. Entropy generation due to friction

Assuming that helium behaves as a perfect gas, we find that the entropy generation due to flow friction is given by:

$$S_p = mR[(\Delta p_L/p_L) + (\Delta p_H/p_H)] \, , \qquad (2)$$

where: m = mass flow
R = gas constant
$p_L$ = low pressure
$p_H$ = high pressure.

Since the high pressure stream friction loss is negligible, $S_p$ is given by:

$$S_p = mR(\Delta p_L/p_L) \, , \qquad (3)$$

where:

$\Delta p_L = [m^2/(\rho B^2 D^2)](f/2Re)(L/D)$

$f/4Re$ = friction factor [3]

$Re$ = Reynolds number

$\rho$ = density

$B, D, L$ = geometric parameters (Fig. 1).

Substituting $\Delta p_L$ into eq. (3), we find:

$$S_p = (m^2 R/\rho_L)(f/2Re)[L/(B^2 D^2)] . \qquad (4)$$

It may be immediately seen from eq. (4) that entropy generation by friction will increase with increasing heat exchanger length L and decreasing breadth B and wall-to-wall distance D.

### 2.2. Entropy generation due to finite temperature difference

The generated entropy due to finite temperature difference is:

$$S_T = m^2 c_p{}^2 [(2D/(k_f Nu) + t/k_w)[(T_1 - T_2)/(T_1 T_2)](1/(BL)) , \qquad (5)$$

where: $c_p$ = specific heat at constant pressure of the fluid

$k_f$ = fluid thermal conductivity

$k_w$ = wall thermal conductivity

$Nu$ = Nusselt number.

The derivation of eq. (5) is summarized in the Appendix.

It may be seen from eq. (5) that the entropy generation will decrease with decreasing wall-to-wall distance D and wall thickness t and increasing plate breadth B and heat exchanger length L.

### 2.3. Entropy generation due to heat conduction in the solid wall

The temperature gradient in the heat exchanger walls is linear and the heat conductivity $k_w$ and the crossectional area $A_s$ are constant; then the entropy generation $S_c$ associated with the heat transfer by conduction, $Q_c$ is:

$$S_c = Q_c(T_1 - T_2)/(T_1 T_2) , \qquad (6)$$

where:

$$Q_c = (T_1 - T_2) k_w A_s / L . \qquad (7)$$

It may be seen from eqs. (6) and (7) that the entropy generation by axial wall conduction will increase with increasing crossectional area $A_s = 3Bt$ and decreasing heat exchanger length L.

### 2.4. Overall entropy generation

The generated entropy in a laminar counterflow heat exchanger, assuming helium behaves as perfect gas and with constant material properties, is derived from eqs. (1),(3),(4),(5) and (6):

$$S = m^3R[f/(2\rho_L Re p_L)][L/(B^2D^3)] + m^2{c_p}^2[(2D/k_fNu) + t/k_w)][(T_1-T_2)/(T_1T_2)][1/(BL)] + k_w[(T_1-T_2)^2/(T_1T_2)](Bt/L) . \qquad (8)$$

It will be useful, for the purposes of simplifying the expression of S and to recognize familiar dimensionless parameters, to obtain a dimensionless form for eq. (8). This is achieved by dividing eq. (8) by $[(k_f/\upsilon m]$, that is constant at pressures below 20 atm [4]. Equation (8) then becomes:

$$S/[(k_f/\upsilon)m] = [(k_W/k_f)(t/L)(1.5/Re)(T_1-T_2)^2/(T_1T_2))] + [((T_1-T_2)^2/(T_1T_2))(Pr^2Re/Nu)(D/L)] + [(k_f/k_W)(t/L)(RePr^2/2)((T_1-T_2)^2/(T_1T_2))] + [(R/c_p)(PrRe)(f\,\upsilon^2/(8\rho_L\rho_L))(L/D^3)] . \qquad (9)$$

This is the entropy S generated in a counterflow laminar heat exchanger, valid for ideal gases in the laminar flow range with Re larger than 100 [3].

## 3. Heat exchanger sizing

The geometrical parameters chosen are the wall thickness t, the breadth B, the wall-to-wall distance D and the heat exchanger length L. Material properties are defined by $k_W$.

Results will show that the entropy generation decreases with decreasing t and increasing L and B, with $k_W = k_W(B)$ and $D = D(L)$.

### 3.1 General methodology for miniature heat exchanger sizing

Equation (9) shows that the wall thickness t needs to be small for minimum entropy generation. Consequently its value will be determined by structural requirements.

The first derivative of S with respect to $k_W$ is set to zero:

$$k_W = 0.82RePrk_f . \qquad (10)$$

Equation (10) has a form that is similar to the optimized wall thermal conductivity for a concentric tube heat exchanger [5].
For B considerably larger than D, the Reynolds number may be expressed as:

$$Re = 2m/(B\upsilon) . \qquad (11)$$

Substituting eq. (10) into eq. (9), we find:

$$S/[(k_f/\upsilon)m] = 1.22Pr((T_1-T_2)^2/(T_1T_2))(t/L) + (Pr^2Re/Nu)(T_1-T_2)^2/(T_1T_2)(D/L) + (PrRe)(R/c_p)(f\upsilon^2/(8\rho_L\rho_L))(L/D^3) . \qquad (12)$$

The minimum of the entropy generation with respect to the wall-to-wall distance D is then determined similarly:

$$D = [((R/c_p)(f\nu^2/(2p_L\rho_L))(Nu/Pr))/((T_1-T_2)^2/(T_1T_2))]^{.25}[L]^{.5}. \quad (13)$$

Equation (13) is substituted into eq. (12); S is then expressed as a function of L and Re, where Re is defined by eq.(11):

$$S/[(k_f/\nu)m] = c_1(t/L) + c_2Re(L)^{-.5}, \quad (14)$$

where:

$$c_1 = 1.22Pr((T_1-T_2)^2/(T_1T_2))$$
$$c_2 = 1.05(Pr^{1.75}/Nu^{.75})((T_1-T_2)^2/(T_1T_2))^{.75}(R/c_p)^{.25}(f\nu^2/(p_L\rho_L))^{.25}.$$

The function S is monotonic decreasing with increasing L and decreasing Re. The remaining parameters B and L may be evaluated by considering space limitation, heat leak to the surrounding and manufacturing costs.

### 3.2. Applications for helium heat exchangers

We take into consideration a laminar counterflow plate heat exchanger for a two-expander-saturated-vapor-pressure cycle refrigerator [6] with a cooling load of 1 watt at 4.2 K. With a reasonable expander efficiency, the mass flow in the balanced heat exchanger between 300 K and 60 K is 0.1 g/s, while the mass flow in the heat exchanger between 30 K and 14 K is 0.085 g/s.

The generated entropy as a function of the length L, using the Reynolds number as a parameter, is shown in Figures 2 and 3 for the two heat exchangers.

The heat exchangers breadth B is computed from eq. (10) for two Reynolds numbers :
Figure 2: for Re = 100, B = 0.171 m and for Re = 300, B = 0.057 m
Figure 3: for Re = 150, B = 0.294 m and for Re = 300, B = 0.147 m.

The optimized wall-to-wall distance D is computed from eq. (13):
Figure 2: for 0.1m< L < 1.0m    0.26mm < D < 0.82mm
Figure 3: for 0.1m< L < 1.0m    0.06mm < D < 0.19mm .

Figures 2 and 3 also show the entropy generation due to axial wall conduction. Its contribution to the total entropy generation is particularly significant for the cold miniature heat exchanger and, in general, for low Reynolds numbers.

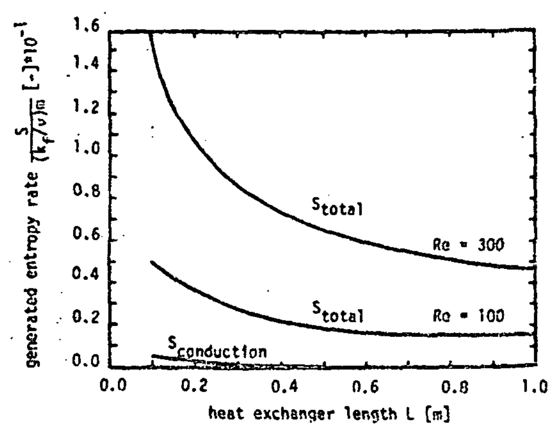


Figure 2: Entropy generation rate vs. length of heat exchanger
for two Reynolds numbers
($T_1$ = 300 K, $T_2$ = 60 K, $p_L$ = 2 bar, $p_H$ = 20 bar)
($\dot{m}$ = 0.1 g/s, t = 0.2 mm)

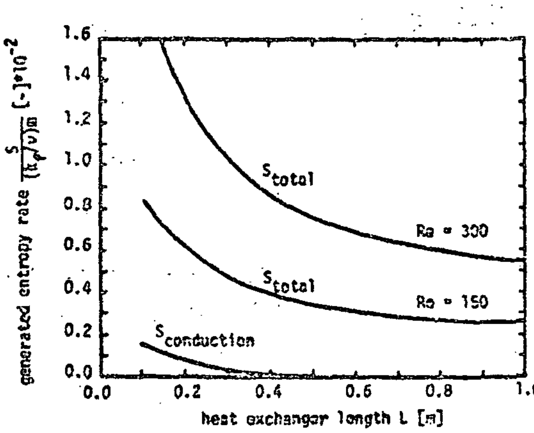


Figure 3: Entropy generation rate vs. length of heat exchanger
parameter: Reynolds number Re
($T_1$ = 30 K, $T_2$ = 14 K, $p_L$ = 2 bar, $p_H$ = 20 bar)
($\dot{m}$ = 0.085 g/s, t = 0.2 mm)

## 4. Conclusions

The thermodynamic analysis performed in this work consists of:
- expressing the entropy generation contributions due to the axial wall conduction, due to the pressure loss and due to the finite temperature difference as a function of four geometrical parameters and one material characteristic.
- minimizing the generated entropy and thereby reducing the set of independent parameters.

In miniature heat exchangers, axial conduction in the wall may become an important phenomenon.

It is shown that the miniature heat exchanger design may not be performed by simply scaling down standard heat exchangers because of the short heat exchanger length and the laminar flow.

Acknowledgments

Ph. B. Rudolf von Rohr was supported by the "Schweizerischer Nationalfonds".

## Appendix

The temperature difference $\Delta T$ between the streams compared to the axial temperature difference $(T_1-T_2)$ is assumed to be small, then the heat transfer rate through the heat exchanger surface A is:

$$Q = mc_p(T_1-T_2) = h\Delta TA \ . \qquad (A1)$$

Assuming the same convective heat transfer coefficients $h_{co}$ for the high and low pressure streams, we find:

$$h_{co} = (Nuk_f)/D. \qquad (A2)$$

Thus the overall heat transfer coefficient h is:

$$1/h = 2/h_{co} + t/k_w. \qquad (A3)$$

Considering the fluid heat conductivity $k_f$ independent of the pressure, eq. (A2) shows that the wall-to-wall distance D is equal for both channels.

The finite temperature difference is derived from eqs. (A2),(A3) and (A1):

$$\Delta T = (mc_p/A)(T_1-T_2)[(2D/(k_fNu)) + (t/k_w)]. \qquad (A4)$$

The entropy generation due to the finite temperature difference $\Delta T$ is:

$$S_T = mc_p\Delta T(T_1-T_2)/(T_1T_2). \qquad (A5)$$

By substituting eq. (A4) into eq. (A5), the entropy generation $S_T$ becomes:

$$S_T = m^2c_p^2[(2D/(k_fNu)) + (t/k_w)][(T_1-T_2)^2/(T_1T_2)][1/(BL)]. \qquad (A6)$$

## Nomenclature

| | | | |
|---|---|---|---|
| A | area | Re | Reynolds number |
| B | plate width | S | entropy |
| $c_1$ | constant in eq. (14) | t | plate thickness |
| $c_2$ | constant in eq. (14) | T | temperature |
| $c_p$ | specific heat of gas | $\Delta$ | difference |
| D | plate spacing | $\nu$ | viscosity |
| f | friction factor | $\rho$ | density |
| h | heat transfer coefficient | | |
| $h_{co}$ | convective heat transfer coefficient | | |
| k | thermal conductivity | | |
| L | heat exchanger length | | |
| m | mass | | |
| Nu | Nusselt number | | |
| p | pressure | | |
| Pr | Prandtl number | | |
| Q | heat transfer rate | | |
| R | gas constant | | |

Subscripts

| | |
|---|---|
| 1 | warm end of exchanger |
| 2 | cold end of exchanger |
| c | wall conduction |
| f | fluid |
| H | high pressure passage |
| L | low pressure passage |
| P | frictional pressure drop |
| s | cross section |
| T | gas heat transfer |
| w | wall |

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