A TECHNIQUE TO MEASURE THE SIZE OF PARTICLES IN LASER DOPPLER VELOCIMETRY APPLICATIONS*

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*Material presented by Dr. J. D. Insolinger at the workshop was extracted from this paper, which has been written for the ASME 1985 winter meeting.
A method to measure the size of particles in LDV applications is discussed. Since in LDV the velocity of the flow is associated with the velocity of particles to establish how well they follow the flow, in the present method the interferometric probe volume is surrounded by a larger beam of different polarization or wavelength. The particle size is then measured from the absolute intensity scattered from the large beam by particles crossing the fringes. Experiments using polystyrene particles between 1.1 and 3.3 μm and larger glass beads are reported. It is shown that the method has an excellent size resolution and its accuracy is better than 10% for the particle size studied.

NOMENCLATURE

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<th>Symbol</th>
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<tr>
<td>b₀</td>
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<tr>
<td>d</td>
<td>particle diameter</td>
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<td>I</td>
<td>intensity</td>
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<td>scattering cross-section</td>
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<td>parallel polarization</td>
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<td>V</td>
<td>particle visibility</td>
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Greek Symbols

<table>
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<tr>
<td>α</td>
<td>size parameter</td>
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<tr>
<td>γ</td>
<td>intersection angle of small laser beams</td>
</tr>
<tr>
<td>θ</td>
<td>angle between laser beam and axis of collecting lens</td>
</tr>
<tr>
<td>μ</td>
<td>laser wavelength</td>
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<tr>
<td>Ω</td>
<td>solid angle of collection</td>
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246
INTRODUCTION

A technique to measure the size of particles in laser Doppler velocimetry (LDV) systems is described here. In typical LDV applications the velocity of the flow is inferred from the measurement of the velocity of very small particles seeded in the fluid. These particles are chosen to be small (0.5 μm to 1 μm) and when possible neutrally buoyant to insure that their velocity is the same as the velocity of the flow. There are, however, many situations of interest where large particles might be generated, and since they don't necessarily follow the flow they will introduce an error in the velocity measurement. Examples of these situations are flows with very large acceleration, sooting environments, wind tunnels with large amounts of oil droplets, and in general, two phase flows where it is of interest to distinguish the velocity of the two phases. Many LDV electronic processors have built-in saturation circuits to reject signals with excessive amplitudes. This, however, will not prevent large particles from being measured. Since the intensity profile of the laser beams is typically Gaussian, large particles moving through the edge of the beams will scatter light with an acceptable signal level. Therefore, the saturation circuits mentioned above will only accept large particles when they cross through the edge of the Gaussian laser beams. This will not eliminate velocity bias but it might introduce additional errors since the large particles are detected over a different volume which could have a different velocity. The method discussed here eliminates the above limitation and provides for a mechanism to obtain the size distribution of the measured particles. In its simplest form this method can be used to produce a velocity histogram of small particles and one of large particles where the separation can be arbitrarily selected.

In this method two small laser beams of a given wavelength and polarization are crossed in the middle of a large laser beam of the same wavelength but normal polarization. The two small beams interfere where they cross, and this interference pattern identifies the center of the large beam. This defines a region of almost uniform intensity within the large beam where the light scattered by the individual particles can be related to their size. Only particles crossing through the interference pattern are measured and their velocity is obtained from the Doppler signal. Their size is obtained from the absolute scattered light since the incident intensity is known.

The method which is referred to as Polarization Intensity Maximum (PIMAX) was described in Reference (1) where it was used to measure the size and velocity of droplets in a spray. For LDV applications it is necessary to show that the PIMAX technique can also discriminate the size of particles down to a
fraction of a micron. Numerical and experimental studies were conducted to 
demonstrate such ability and they are reported here.

The method could also be implemented in two-color LDV systems. Here two 
small beams of one color cross in the middle of two crossing large beams of 
another color forming two fringe patterns perpendicular to each other. As 
before, the small beams identify the center of the large beams from which the 
size information is obtained.

The model presented here is for spherical particles with a diameter of 
the same order as the wavelength of the incident beams. Therefore, the full 
Mie solution (2) is necessary to analyze the scattering patterns. Results are 
presented for flows seeded with known size particles.

DESCRIPTION OF OPTICAL TECHNIQUE

The method discussed here bases the size measurement on the absolute 
intensity scattered by the particle crossing the probe volume, and the 
velocity measurement on the classical Doppler signal. A similar method could 
be described for a single beam velocimeter but the approach used here is for 
LDV. In situ single particle counters are limited because of the nonuniform 
profile (typically Gaussian) of laser beams. Under this condition a particle 
crossing the middle of the beam will scatter more light than a similar 
particle crossing through the edge. Therefore, the relationship between size 
and scattered light is not unique.

To circumvent this problem, two small beams of a given wavelength or 
polarization are crossed in the middle of a larger beam of different wave-
length or polarization identifying a region of almost uniform intensity and, 
therefore, removing the Gaussian ambiguity. The crossing beams will interfere 
and a fringe pattern will be formed in the middle of the large beam 
(Figure 1). Signals exhibiting a sinusoidal modulation correspond to parti-
cles that cross the fringe pattern and therefore the middle of the large 
beam. Both size and velocity of individual particles can be extracted from 
this signal. The method and results reported here correspond to the two-
polarization (PIMAX) system.

If we refer to the small beam as 1 and the large beam as 2, the intensity 
profiles in the probe volume can be separated by their polariza-
ation and given by:

\[
I_1 = I_{o1} \exp \left( \frac{-2}{b_{o1}^2} \right) \frac{2}{x^2 + y^2 + z^2/4} \cdot \left[ \cosh \left( \frac{2xyz}{b_{o1}^2} \right) + \cos \left( \frac{4\pi x \sin(y)}{z} \right) \right]
\]

and

\[
I_2 = I_{o2} \exp \left( \frac{-2}{b_{o2}^2} \right) (x^2 + y^2)
\]

(1)
where $I_0$ is the center intensity, $\gamma$ is the intersection angle, $b_0$ the waist radius, $\lambda$ the laser wavelength, and $x, y, z$ the coordinates. The $z$ dependence of the large beam is negligible. If we also assume that $2\pi \cos \gamma = 0$ (which is an excellent assumption since a pinhole in the receiver will limit the value of $z$), the intensity scattered by a spherical particle is given by:

$$I_{s1} = 2I_0 K_1(d, n, \theta, \Omega, \lambda, P)G_1 \exp\left[-\frac{2}{b_0^2} (x^2 + y^2) \right] \left[ 1 + \cos 2 \frac{\pi x \lambda}{X} \right]$$

(3)

and

$$I_{s2} = I_0 K_2(d, n, \theta, \Omega, \lambda, S)G_2 \exp\left[-\frac{2}{b_0^2} (x^2 + y^2) \right]$$

(4)

where $K$ is the scattering cross-section as obtained by the Lorentz-Mie theory (3). It is a function of a complex size function $d$, the index of refraction $n$, the collection angle $\theta$, the solid angle of collection $\Omega$, the wavelength $\lambda$ and the polarization. Here it is assumed that the small beams have a polarization parallel to the scattering plane while the polarization of the big beam is perpendicular. $G$ is the gain function of the instrument, and $V$ is the visibility of the measured particle.

It should be pointed out that the visibility of the particle is not an adequate parameter to obtain the size, given that many of the particles of interest are below 3 $\mu$m, and in general, many of the particles will be sub-micron.

Equation (3) gives the signal response of the laser Doppler velocimeter which will establish the detectability of the signal. The processing logic will be the following: signals exhibiting sinusoidal modulation will have crossed the fringe pattern which is located in the middle of the large beam. Therefore, signals validated by the laser velocimeter correspond to particles crossing the middle of the large beam and their scattered light can be inverted to size since the Gaussian ambiguity is removed. In Equation (4) the exponent can therefore be approximated as unity and we get

$$I_{s2} = I_0 K_2 G_2$$

(5)

The $K_2$ coefficients will be obtained from Lorentz-Mie scattering using a modified version of the numerical program developed by Dave (4). Equation (5) can then be solved for the particle size which is contained in $K_2(d, n, \theta, \Omega, \lambda, S)$. 

249
NUMERICAL COMPUTATIONS OF THE SCATTERING FUNCTIONS

To obtain the particle diameter, d, it is necessary to know the functional relationship of \( K_2(\mathbf{d}, n, \theta, \Omega, \lambda, S) \) as given by Equation (4). This function can be quite complex and it is necessary to find the conditions under which the ambiguities, if any, are within tolerable error margins. The computations were made in a PDP11 computer. Parametric studies were conducted to establish optimum experimental conditions. These parameters include the angle of collection (\( \theta \)), the solid angle of collection (\( \Omega \)), and index of refraction \( (n_1 - \text{Im}(n_2)) \). The results show that the best optical characteristics are near forward scattering angles of collection (\( \theta < 5^\circ \)).

Since the index of refraction of the particles present in LDV applications may be quite different, it is important to establish conditions which are less sensitive to these variations. Both real \( (n_1) \) and imaginary \( (n_2) \) parts of the refractive indices were varied to check the sensitivity of these parameters. Figures 2 and 3 show the scattered intensity as a function of the particle size parameter \( a \) for different values of \( n_1 \) and \( n_2 \) at different scattered angles.

It can be concluded from these calculations that shallow angles of collection (\( < 5^\circ \)) offer the most favorable conditions.

The calibration of the system was conducted with known size polystyrene particles, therefore the scattering function corresponding to these particles was also evaluated. The results are shown on Figure 4 and, as before, the shallow angles offer the best conditions.

APPARATUS AND EXPERIMENTAL FACILITY

Figure 5 shows a schematic of the optical system used in the experiments. The system consists of a transmitter and a receiver positioned 5° off axis. The transmitter uses a 7 mW He-Ne laser. The laser beam is focused on a diffraction grating (DG) by lens \( L_1 \), and the three major orders are collimated by lens \( L_2 \). The zero order beam goes through a beam compressor formed by \( L_4 \) and \( L_5 \) and its polarization is rotated by PR. Lens \( L_3 \) focuses and crosses all three beams to form a probe volume like the one shown in Figure 1. The light scattered by particles crossing the probe volume is collected by \( L_6 \) and focused onto the photomultipliers by \( L_7 \). A cube polarizer (CP) efficiently divides the scattered light into its two polarization components. In addition, a polarization filter was placed in front of the PMT which looks at the big beam (BPMT) to reduce the crosstalk between the two polarizations.

The size of the probe volume was typically 600 \( \mu \text{m} \) for the large beam and 86 \( \mu \text{m} \) for the small ones. The fringe spacing was 10.7 \( \mu \text{m} \). To reduce the size of the probe volume we used a slit in front of the BPMT. The width of this slit was typically equal to the diameter of the small probe volume (100 \( \mu \text{m} \)) and its length was about four times longer to avoid signal masking. Thus, the probe volume was about 10\(^{-5} \text{ cm}^3\) which in general is quite adequate for LDV applications.
The outputs of the photomultipliers are input to an electronic breadboard which measures the Doppler period and the peak intensity of each scattering center. A description of the electronics can be found in Reference (5).

THE PARTICLE GENERATOR

The particle size of interest is that corresponding to typical LDV applications. We used polystyrene particles of 1.1 μm, 1.7 μm, 2.7 μm and 3.3 μm in diameter. These are latex particles made by Dow Chemicals of good size uniformity and of spherical shape. The particles come suspended in water with a concentration of 10% by weight. A few droplets of the particle suspension were introduced and diluted in a nebulizer. An air compressor provided the air flow to produce a mist carrying the polystyrene particles out of the nebulizer and into a heated chamber. There the water was evaporated and the particles were sprayed over the probe volume.

Different size particles could be introduced into the nebulizer thus producing monodisperse, bimodal, trimodal and quadrumodal distributions. The compressor was also used to trap the particles after they passed through the probe volume to avoid contaminating the surrounding environment. Figure 6 shows a photograph of the optics and the particle generator.

An alternative apparatus was also constructed to introduce glass beads into the probe volume. The glass beads come in dry containers and their size distribution was reasonably broad. A test tube with a stopper was used to contain the glass beads. Two glass tubes protruded through the stopper and flex tubing was connected at the end of each glass tube. One of the flex tubes was connected to a nitrogen tank via a pressure regulator and a flow meter. The other flex tube discharged the glass beads into the probe volume. The flow rate of nitrogen could be regulated very carefully and thus the flow velocity at the exit of the flexible tube could be reproduced.

EXPERIMENTAL RESULTS

The system was calibrated and tested with polystyrene latex particles of uniform and known size distributions. The size and uniformity of the particles were checked with a microscope and agreed with the manufacturer's specifications.

Size and velocity distributions of the polystyrene particles flowing out of the heating chamber were obtained with the optical system described above. Figure 7a shows the distributions corresponding to 1.74 μm and 3.3 μm. The calibration high voltage of 500V was established based on the 1.74 μm. The arrow with the 3.3 μm mark indicates the value predicted by the Mie calculations. In Figure 7b an intermediate size of 2.7 μm was added and the calibration high voltage was changed to 550V to decrease the size range. As before the calibration was based on the 1.74 μm, and the arrows point at the numerically predicted values. The 3.3 μm was measured very accurately, but the measurement of the 2.7 μm particles was off by two bins. This error was consistent and very repeatable. We attribute it to the oscillations found in the scattering function. Figure 7c shows the distributions corresponding to four different latex particles. As before the calibration was obtained with
the 1.74 \mu m particles. The calibration high voltage is 550V. It must be pointed out that the size histograms are divided into 53 equal size bins. Therefore, distributions on the large diameter end will appear broader than those in the small diameter end.

These results indicate that the technique and instrument can accurately discriminate the particles by their size in LDV applications. Unfortunately, since all the particles are rather small and the flow velocity is also small and constant, all the particles moved at about the same velocity. Therefore, no velocity/size discrimination could be established. To illustrate this last point we used very large glass beads which would not follow the flow. Two different size classes were simultaneously introduced into the test tube and entrained into a constant nitrogen flow. These particles were sprayed over the probe volume and their size and velocity histograms are shown on Figure 8. Notice the bimodal size distribution and the corresponding bimodal velocity distribution. Although not obvious from this figure the small particles are moving faster than the big ones. This is better illustrated in Figure 9 which shows a velocity/size distribution of the entrained small and large glass beads. It can be deduced that the small glass beads are moving at about 13 m/s while the large ones are moving at about 8 m/s.

This particle size range is not expected in LDV applications but the experiment illustrates the kind of error that can be incurred in the flow velocity measurement if no consideration is given to the particle size.

An aspect of the PIMAX technique that needs to be considered is the crosstalk between the two polarizations. To establish this crosstalk a series of tests were conducted. First, the polarization ratio of the transmitted laser beams was measured with and without particle extinction. To produce a substantial particle interference a fuel spray was used to attenuate the laser beam by 10%. Very little change in the polarization ratio was measured which is of no surprise since the light scattered in the forward direction carries the same polarization as the incident radiation. A second set of tests was also conducted. Monodisperse droplets produced by a Berglund-Liu generator scattered light which could be analyzed by the receiving optics. One PMT measured the S-polarized light while the other measured the P-polarized light. We blocked the large beam which is S-polarized and illuminated the particles with the small P-polarized beams. The scattered light was measured in the PMT intended for the S-polarized light. The crosstalk was about 4% at large (20°) angles and smaller at shallow angles. This crosstalk could be virtually eliminated by placing a polarizing filter in front of the PMT. It should be noted that the intensity into the small and large beams may not necessarily be the same and, therefore, the crosstalk will be proportional to the intensity ratio. For the small particles of interest in LDV and at shallow angles of collection this crosstalk is negligible.

Obviously the two-color system (IMAX) is not subject to this crosstalk. In general, it is easier to separate the different colors of the scattered light than it is to separate the different polarizations. Yet budget constraints may impose the use of single color lasers in which case PIMAX is a viable technique.
CONCLUSIONS

A method has been presented and results discussed to measure the size of particles in LDV applications. The accuracy and resolution of the method were illustrated by measuring polystyrene particles between 1.1 μm and 3.3 μm in diameter. The effect of crosstalk between the two polarizations was measured and established to be very small.

It was shown, using large glass beads, that different size particles move at different velocities and, therefore, the velocity of the flow cannot be established from the particle velocity unless consideration to the particle size is given.

ACKNOWLEDGEMENTS

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REFERENCES


Figure 1. Probe volume of two polarization PIMAX technique.
Figure 2. Perpendicularly polarized scattered light intensity as a function of the size parameter $\alpha$

$\theta = 0^\circ$: (x-x-x ($\bar{n} = 1.56 - 12.9 \times 10^{-7}$), ---
($\bar{n} = 1.66 - 12.9 \times 10^{-7}$), 0 - 0 - 0
($\bar{n} = 1.96 - 12.9 \times 10^{-7}$) $\theta = 5^\circ$: (----- $\bar{n} = 1.66$),
$\theta = 10^\circ$: (Δ - Δ - Δ $\bar{n} = 1.66$)
Figure 3. Perpendicularly polarized scattered light intensity as a function of the size parameter $\alpha$ for different refractive indices ($\Re = 1.75$, $\Re = 1.75 - 10.29$) at angles $0^\circ$, $5^\circ$ and $10^\circ$. 
Figure 4. Integrated intensity function for $\theta = 5^\circ$ and F/5 lens for polystyrene microspheres ($n = 1.59$) as a function of diameter.
**Figure 5. Schematic of PIMAX system**

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<td>$L_7$</td>
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CP : Cube Polarizer
DG : Diffraction Grating
100 lines/mm
PR : Polarization Rotator
Figure 6. Photograph of PIMAX transmitter and particle generator.
Figure 7a. Size and velocity histogram of 1.74 μm and 3.3 μm polystyrene spheres in air. Photomultiplier tube high voltage = 500 V.
Figure 7b. Size and velocity histogram of 1.74 μm, 2.7 μm, and 3.3 μm polystyrene spheres in air. Photomultiplier high voltage = 550 V.
Figure 7c. Size and velocity histogram of 1.1 μm, 1.74 μm, 2.7 μm, and 3.3 μm polystyrene spheres in air. Photomultiplier high voltage = 550 V.
Figure 8. Size and velocity histogram of small and large glass beads entrained in an air jet.
Figure 9. Velocity distribution of small and large glass beads entrained in an air jet.