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Linearity Optimization in a Class of Analog Phase Modulators

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SYMBOLS

A	amplitude constant
A_o	phasor amplitude constant
AM	amplitude modulation
$A(\Psi)$	amplitude function of approximated phase-modulated signal
C_o	gain constant
C_1	fundamental-frequency Fourier coefficient
D	total harmonic distortion voltage ratio
$e(t)$	ideal phase-modulated signal
j	imaginary
K_i	weighting coefficients in power series
MMSE	minimum mean square error
MTHD	minimum total harmonic distortion
n	circuit order
PM	phase modulation
Re[ ]	real part of [ ]
t	time
THD	total harmonic distortion
$v(t)$	approximation to $e(t)$
α	radian frequency of modulating signal
$\overline{\varepsilon^2}$	mean-square-error function
ϕ_o	phase angle $v(t)$
$\phi_o(t)$	phase angle of $v(t)$ with Ψ time-varying
Ψ	modulating variable
Ψ_D	design-maximum phase deviation
$\Delta\Psi$	peak sinusoidal phase deviation
$\Delta\Psi_D$	design-maximum peak sinusoidal phase deviation
ω	radian frequency of carrier signal

ACKNOWLEDGMENT

The author wishes to acknowledge with deep appreciation the contributions of the late Lewis R. Wilson to the work reported here. Mr. Wilson handled all aspects of the computer programming involved in optimum coefficient generation and also the total harmonic distortion comparisons.

SUMMARY

This paper examines the ultimate modulating linearity attainable with a phase modulation technique based on the linear addition of quadrature phase carrier signals which have been multiplied by precisely defined nonlinear transformations of the modulating signal. This technique, sometimes described as complex phasor phase modulation, is a logical extension of the Armstrong method and can provide exceptionally good linearity for phase deviations as large as 5 radians. For example, it is shown that a total harmonic distortion ratio of -55 dB at a peak sinusoidal phase deviation of 5 radians is theoretically possible by using a relatively simple fifth-order nonlinear processor.

INTRODUCTION

In the early 1970's, a landing-guidance system under development required a phase modulator having good modulating linearity for phase deviations exceeding π radians without resorting to frequency multiplication. Several methods for accomplishing this objective were examined, and the one ultimately chosen is the subject of this paper.

The complex phasor phase modulation technique is an extension of the Armstrong method (ref. 1, pp. 603 to 605) and follows logically from an expansion of the ideal phase modulation waveform given by equation (1)

$$e(t) = \mathrm{Re}\left[A_o\, e^{j(\omega t + \Psi)}\right] \tag{1}$$

into quadrature phase components. Equation (1) can be rewritten as either

$$e(t) = A_o \cos \omega t \cos \Psi - A_o \sin \omega t \sin \Psi \tag{2}$$

or

$$e(t) = A_o\left(1 - \frac{\Psi^2}{2!} + \frac{\Psi^4}{4!} - \ldots\right)\cos \omega t - A_o\left(\Psi - \frac{\Psi^3}{3!} + \frac{\Psi^5}{5!} - \ldots\right)\sin \omega t \tag{3}$$

The functional configuration shown in figure 1 can be designed to perform the operations indicated by equations (2) and (3). Implementation of equation (2) requires nonlinear processors to transform Ψ to $\sin \Psi$ and $\cos \Psi$. Although sine and cosine nonlinear function modules are available, they are two-quadrant devices, and their applicability is therefore limited to phase deviations not exceeding $\pi/2$ radians.

In lieu of direct sine/cosine transformation, the synthesis of sine/cosine functions from infinite power series of Ψ is theoretically possible. This alternative is not appealing; however, it does suggest a third possibility: the approximation of the infinite series with a practically finite number of nonlinear terms. If the nonlinear processors in figure 1 can generate powers of Ψ through the fifth, the first-zone output signal (around ω) can be expressed as

$$v(t) = A\left(K_0 - K_2\Psi^2 + K_4\Psi^4\right)\cos\omega t - A\left(K_1\Psi - K_3\Psi^3 + K_5\Psi^5\right)\sin\omega t \tag{4}$$

which may be written in the form $A(\Psi)\cos\left[\omega t + \phi_o(\Psi)\right]$ with

$$A(\Psi) = A\left[\left(K_0 - K_2\Psi^2 + K_4\Psi^4\right)^2 + \left(K_1\Psi - K_3\Psi^3 + K_5\Psi^5\right)^2\right]^{1/2} \tag{5}$$

and

$$\phi_o(\Psi) = \tan^{-1}\left(\frac{K_1\Psi - K_3\Psi^3 + K_5\Psi^5}{K_0 - K_2\Psi^2 + K_4\Psi^4}\right) \tag{6}$$

This paper shows that remarkedly good linearity between ϕ_o and Ψ in equation (6) can be obtained if the coefficients are optimized. These "optimum" coefficients differ significantly from those in equation (3). This paper addresses the following questions:

(1) What optimization criteria should be used?

(2) What are the "optimum" coefficients in equation (6), based on the criteria established in (1) above?

(3) What total harmonic distortion performance can be expected when the optimum coefficients are used?

Since the initial work on this project was done over a decade ago (ref. 2), significant improvements in the components needed to implement the radio-frequency and nonlinear processors shown in figure 1 have occurred (ref. 3 and ref. 4, p. 13), and further improvements are likely. Therefore, this study is limited to defining the ultimate performance capabilities of the technique without regard to hardware imperfections. Circuit orders of 2 through 5 will be considered, where order denotes the highest power of Ψ having a nonzero coefficient in equation (6).

The fact that the envelope of the synthesized approximation to $e(t)$ is not constant but is a function of Ψ as given by equation (5) is largely ignored, and it is assumed throughout most of the paper that hard limiting can satisfactorily remove all amplitude fluctuations. A final section describes the incidental envelope variations that inevitably accompany this phase modulation technique for the purpose of facilitating the design of a suitable limiter.

ANALYSIS

Phase Function Optimization

The approach employed here was to postulate reasonable and practically applicable optimization criteria, determine the optimum K_i set based on each of these criteria, and determine the linearity between ϕ_o and Ψ for each K_i set on a common basis. The optimization criteria utilized here fall into the general categories of minimum mean square error (MMSE) and minimum total harmonic distortion (MTHD) for Ψ, a sinusoidal signal. The final choice between the various approximations was based on total harmonic distortion (THD) performance. That is, for $\Psi(t) = \Delta\Psi \sin \alpha t$, the Fourier coefficients of $\phi_o(t)$ were computed as a function of $\Delta\Psi$ for each K_i set. It should be emphasized that THD minimization to determine the optimum K_i set and the THD comparison between the various approximations (K_i sets) are quite different operations. In the first case, $\Delta\Psi$ is fixed at $\Delta\Psi_D$ and the THD is minimized with respect to the K_i values, whereas in the second case, the K_i values are fixed and $\Delta\Psi$ is the independent variable.

The mean-square-error (MSE) function is defined as

$$\varepsilon^2 = \frac{1}{2\Psi_D} \int_{-\Psi_D}^{\Psi_D} \left[\phi_o(\Psi) - C_o\Psi\right]^2 d\Psi \tag{7}$$

where $C_o\Psi$ is the desired linear relationship between ϕ_o and Ψ. In general, the mean-square-error function is minimized with respect to the K_i values; however, other constraints or conditions may be imposed. An initial optimization approach, designated $(\mathrm{MMSE})_1$, set K_0 and K_1 in equation (6) equal to unity, and the coefficient C_o in equation (7) was made equal to the first derivative of $\phi_o(\Psi)$ in equation (6) evaluated at 0, or unity. Thus, for an nth-order circuit, the procedure determines the $n - 2$ remaining coefficients. Computer programs were developed which minimized the integral expression in equation (7) by a numerical procedure related to the Newton-Raphson method (ref. 5, p. 221) generalized to an arbitrary number of independent variables. The integrals were evaluated numerically by means of Simpson's rule. In figures 2 through 5 the optimum $(\mathrm{MMSE})_1$ coefficients for circuit orders 2 through 5 are plotted as a function of Ψ_D, the design-maximum phase deviation.

The normalized root mean square error, or RMSE, provides a physically meaningful measure of the deviation of $\phi_o(\Psi)$ from the desired linear relationship $C_o\Psi$. Normalized RMSE is defined as

$$\text{Normalized RMSE} \triangleq \frac{\sqrt{\overline{\varepsilon^2}}}{2\Psi_D} \tag{8}$$

and is plotted versus Ψ_D in figure 6 for the purpose of indicating the circuit order required to achieve a specified value of normalized RMSE for values of Ψ_D up to 5 radians.

Total harmonic distortion ratio, a widely accepted measure of system nonlinearity, is defined as

$$D \triangleq \sqrt{\frac{\text{Total harmonic distortion "power"}}{\text{Fundamental "power"}}} \tag{9}$$

when the excitation is sinusoidal, that is, when $\Psi(t) = \Delta\Psi \sin \alpha t$. The THD ratio may be expressed in terms of the mean square value of ϕ_o and C_1, the fundamental-frequency Fourier coefficient, as

$$D^2 = \frac{\overline{\phi_o^2} - C_1^2}{C_1^2} \tag{10}$$

If $\Psi(t)$ is sinusoidal with period $2\pi/\alpha$, $\phi_o(t)$ also has a period of $2\pi/\alpha$ and possesses both odd and half-wave symmetry. The total "power" in $\phi_o(t)$ is

$$\overline{\phi_o^2} = \frac{\alpha}{2\pi} \int_{-\pi/\alpha}^{\pi/\alpha} \phi_o^2(t)\, dt \tag{11}$$

and the fundamental-frequency "power" is

$$C_1^2 = \left[\overline{\frac{\alpha}{4\pi}} \int_{-\pi/\alpha}^{\pi/\alpha} \phi_o(t) \sin \alpha t\, dt \right]^2 \tag{12}$$

An integral expression for $D^2(\Delta\Psi)$ is obtained upon combining equations (6), (10), (11), and (12), and letting $\Psi(t) = \Delta\Psi \sin \alpha t$. This expression was evaluated numerically by using the $(MMSE)_1$ optimum K_i sets corresponding to selected values of Ψ_D. The results are plotted in figures 7 through 10 for circuit orders of 2 through 5. The THD performance of the Armstrong (linear) approach is shown as a dashed line for comparison purposes. The THD was plotted up to values of $\Delta\Psi$ numerically equal to the Ψ_D values producing approximately 1 percent normalized MSE in figure 6. The following observations can be made about these curves:

(1) The THD performance is much improved relative to the Armstrong method.

(2) In all cases, THD increases very rapidly when $\Delta\Psi$ exceeds Ψ_D.

(3) As $\Delta\Psi$ becomes less than Ψ_D, THD fluctuates generally downward; however, THD is always less than the value corresponding to $\Delta\Psi = \Psi_D$.

(4) When $\Delta\Psi \ll \Psi_D$, THD decreases monotonically with $\Delta\Psi$.

(5) Values of THD as low as -55 dB can be obtained with a fifth-order circuit for a peak sinusoidal phase deviation of 5 radians, as shown in figure 10.

In addition to $(MMSE)_1$ minimization, several other optimization criteria were postulated and evaluated but were rejected in favor of $(MMSE)_1$. Additional MMSE constraints were imposed in an attempt to further reduce the normalized MSE. In $(MMSE)_2$ minimization, K_1 in equation (6) was made an additional independent variable and equation (7) was minimized, with $C_o = 1$. For $(MMSE)_3$ minimization, C_o in equation (7) was made the additional independent variable and equation (7) was minimized, with $K_1 = 1$. In both cases, the reduction in normalized RMSE was trivial; however, the THD behavior was considerably different from that of $(MMSE)_1$, as discussed shortly. In THD minimization, the integral expression for $D^2(\Delta\Psi_D, K_i)$, obtained by combining equations (10), (11), and (12), was minimized with respect to the K_i values, with $\Delta\Psi_D$ being a parameter. Thus, D^2 was minimized for $\Psi(t) = \Delta\Psi_D \sin \alpha t$. Upon obtaining the optimum K_i set for each value of $\Delta\Psi_D$, $D^2(\Delta\Psi)$ was computed as in the $(MMSE)_1$ case.

The salient results of comparing $D^2(\Delta\Psi)$ for all four optimization criteria can be summarized as follows (refer to fig. 11):

(1) For $\Delta\Psi << \Delta\Psi_D$ (or Ψ_D), $(MMSE)_1$ is slightly superior to the other three cases.

(2) For intermediate values of $\Delta\Psi$, say $\Delta\Psi = 1/2\Delta\Psi_D$, $(MMSE)_2$, $(MMSE)_3$, and MTHD all exhibit an undesirable characteristic - THD increases considerably relative to the THD value corresponding to $\Delta\Psi = \Psi_D$ (or $\Delta\Psi_D$).

(3) For values of $\Delta\Psi$ approaching $\Delta\Psi_D$, MTHD always gives the lowest THD, as it should; however, the improvement is small - typically less than 5 dB.

Figure 11 illustrates these effects and is representative of the behavior observed with other circuit orders and values of $\Delta\Psi$ and Ψ_D. Thus, the small reduction in THD at values of $\Delta\Psi$ approaching Ψ_D ($(MMSE)_2$ and $(MMSE)_3$) and $\Delta\Psi_D$ (MTHD) was more than offset by the undesirable behavior at intermediate values of $\Delta\Psi$ and the larger THD at small values of $\Delta\Psi$. Therefore, $(MMSE)_1$, the simplest both conceptually and computationally, was judged to be the best choice for calculating the weighting coefficients in equation (4).

Concomitant Envelope Variation

The envelope function of equation (5) was totally disregarded in linearizing the phase function of equation (6). Thus, it should not be surprising to find that appreciable envelope variation accompanies this phase modulation method. Well-designed hard limiting, as indicated in figure 1, can greatly reduce residual envelope variations; however, AM to PM conversion in the limiter (ref. 6, p. 51) should be strictly avoided.

The degree of envelope variation to be encountered with this PM method is therefore of interest. This information is contained in figure 12, which shows $A(\Psi)/A(0)$ (see eq. (5)) as a function of the independent variable Ψ for various circuit orders and values of Ψ_D. It is seen that $A(\Psi)$ is a monotonically increasing function of Ψ over the range of variables considered, and that higher circuit orders always result in less envelope variation.

EXPERIMENTAL RESULTS

As noted in the Introduction, this phase modulation technique was applied to a practical situation. A fifth-order circuit for $\Psi_D = 5$ radians was built, evaluated, and successfully used in the intended application. Unfortunately, the apparatus available for evaluating the modulating linearity of the experimental phase modulator did not possess adequate linearity to make a definitive measurement. The theory predicted that the experimental model should exhibit THD not exceeding -55 dB (see fig. 10); however, measurements made with the test apparatus available resulted in

THD values of -40 dB. This discrepancy is at least partially, and perhaps largely, the result of distortion generated within the phase demodulator used to evaluate the experimental phase modulator. Thus, the extent to which a practical circuit matches the ideal remains undefined and may well be better than the measured value of 1 percent, or -40 dB. Even if the -40-dB value is correct, it is the opinion of the author that the hardware improvements mentioned in the Introduction would result in better THD performance.

CONCLUDING REMARKS

This paper has shown that the complex phasor phase modulation technique is capable of exceptionally good modulation linearity for phase deviations as large as 5 radians. A simple procedure was developed to determine the optimum weighting or gain coefficients in the nonlinear processors required to implement the technique. These optimum coefficients are shown graphically for circuit orders up to 5 and phase deviations up to 5 radians. The total harmonic distortion performance for a complex phasor modulator using the optimum coefficients is defined. Finally, the unavoidable amplitude modulation accompanying the technique is quantified.

Although this paper is basically theoretical in nature, the ideas contained herein were applied, and practical circuits were constructed. For those familiar with analog and radio-frequency signal-processing components, it will be apparent that the critical items in this concept are (1) the analog multipliers needed to implement the nonlinear processors and (2) the balanced modulators (also analog multipliers) used to perform the carrier modulation operations. Since the other hardware items needed to implement the concept can be considered to be essentially perfect for this application, the ultimate performance will be dependent on the analog modulators.

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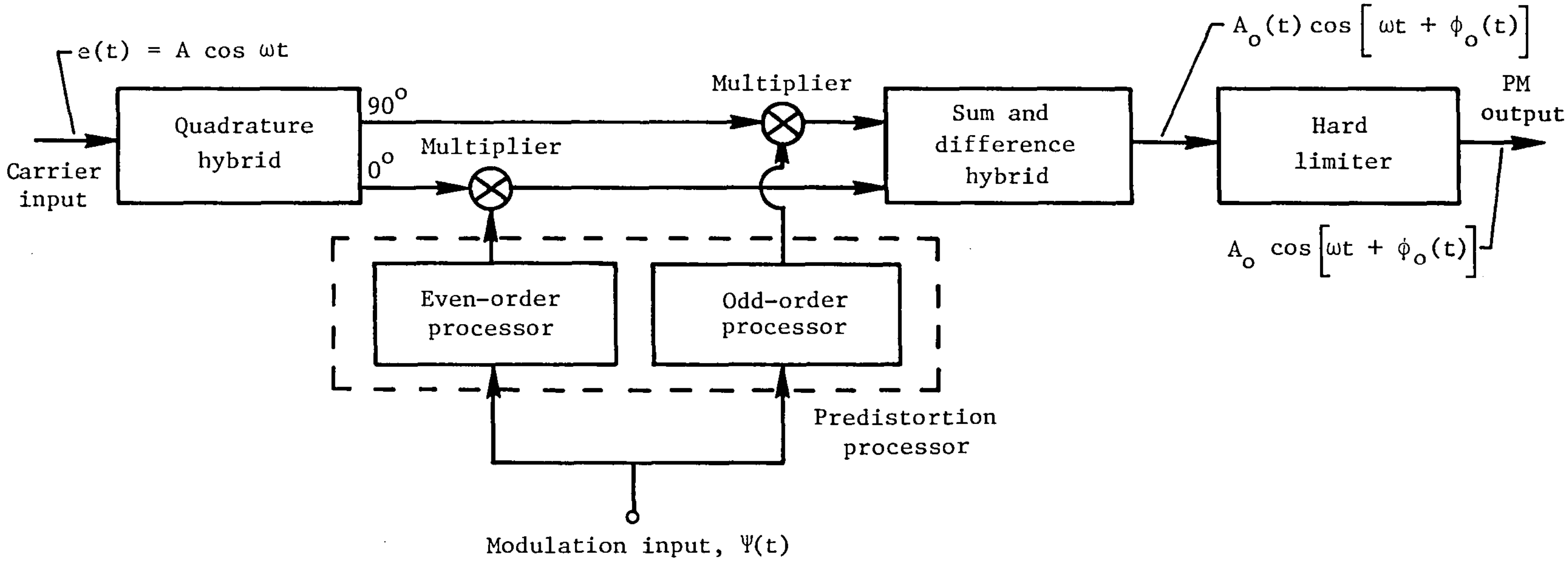


Figure 1.- Functional diagram of complex phasor phase modulator.

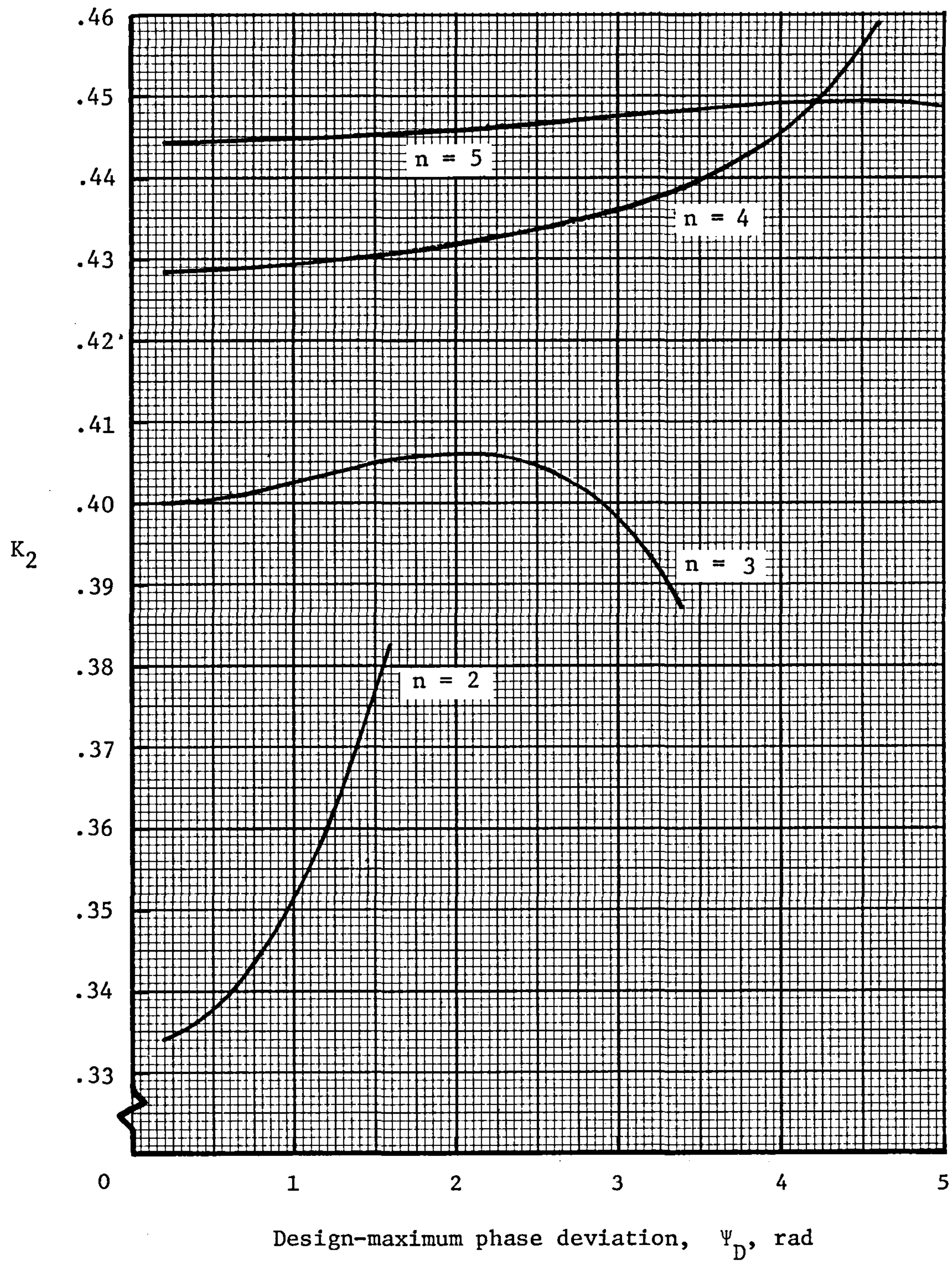


Figure 2.- $(MMSE)_1$ K_2 values. Circuit orders 2 through 5.

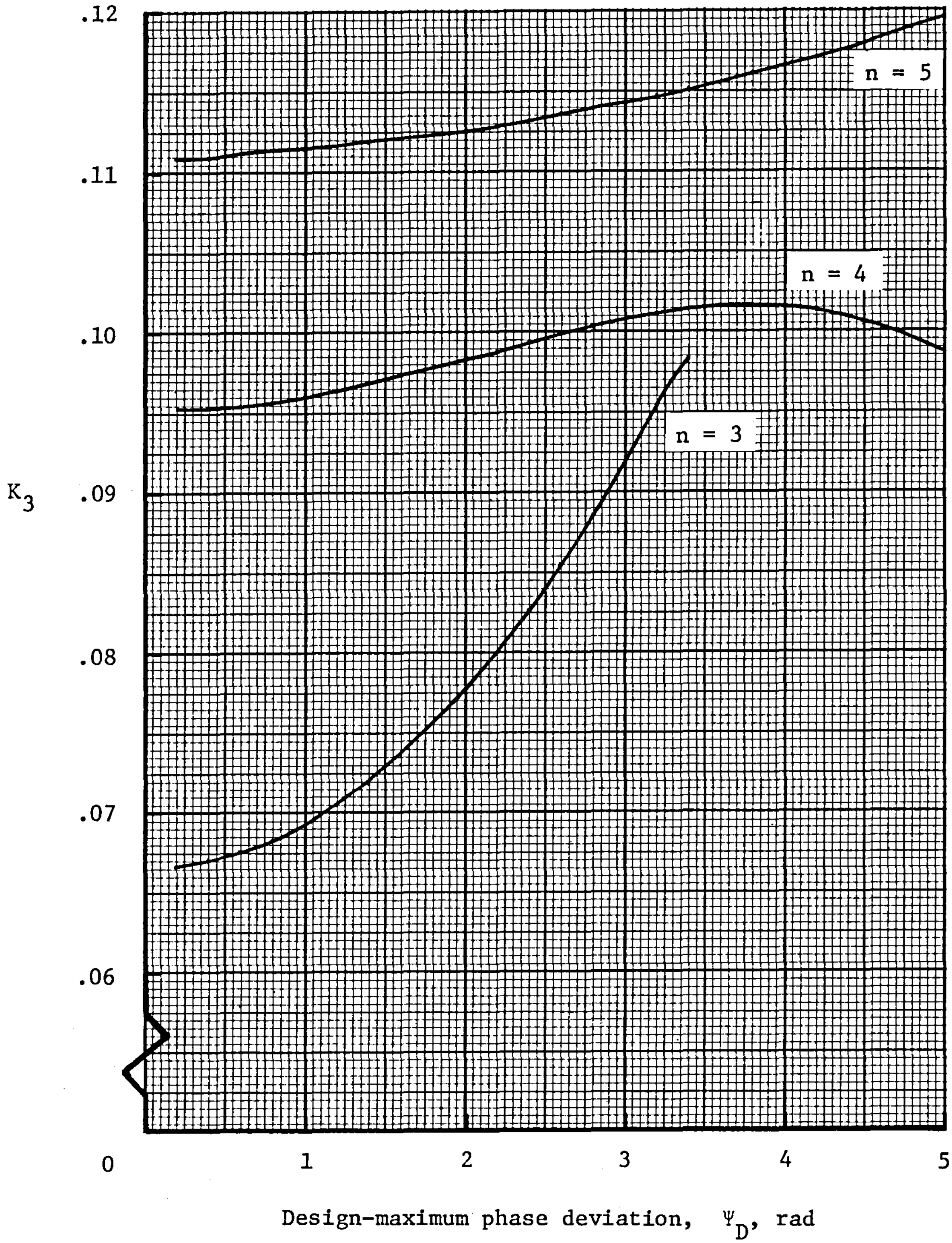


Figure 3.- $(MMSE)_1$ K_3 values. Circuit orders 3 through 5.

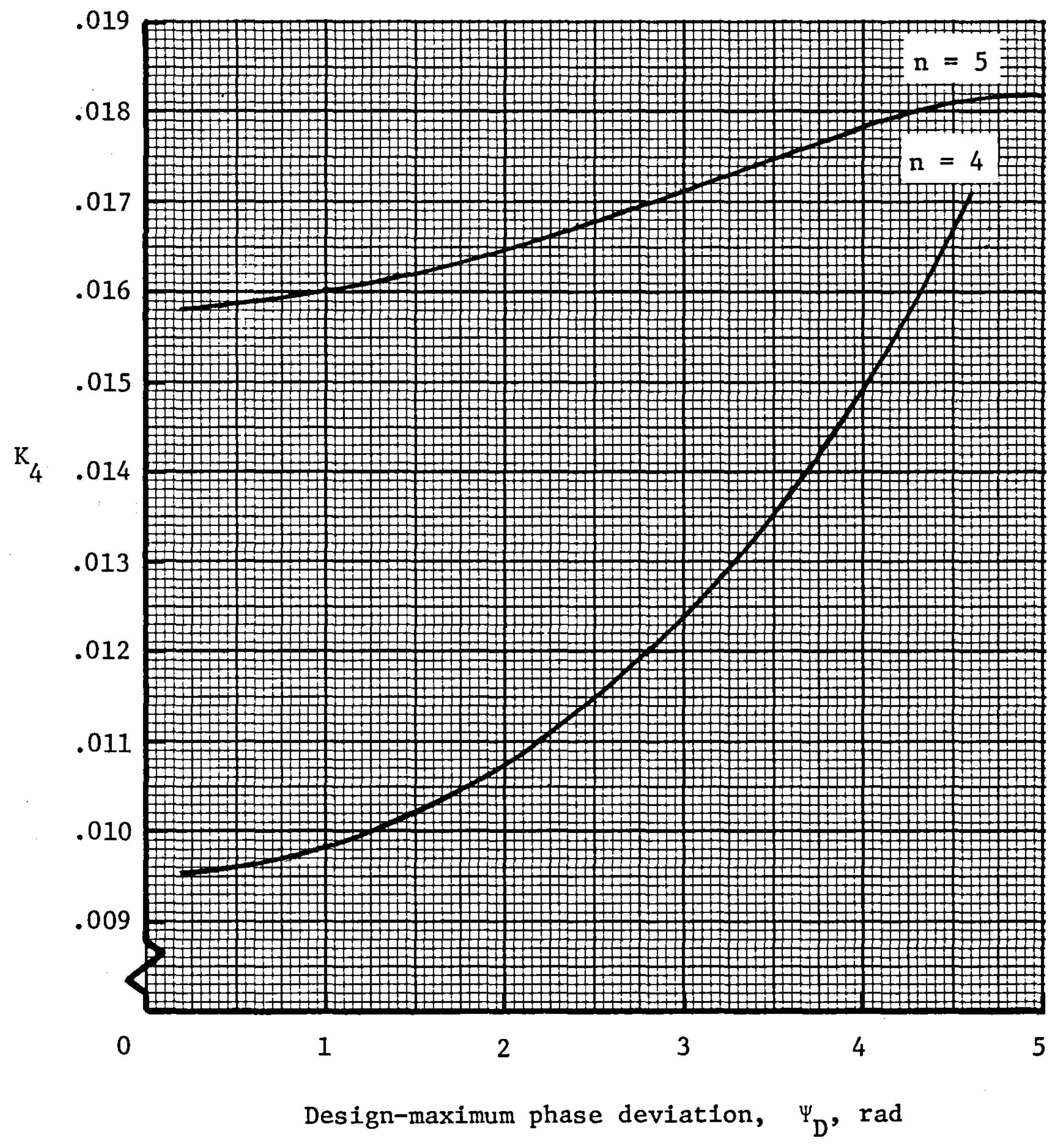


Figure 4.- $(MMSE)_1$ K_4 values. Circuit orders 4 through 5.

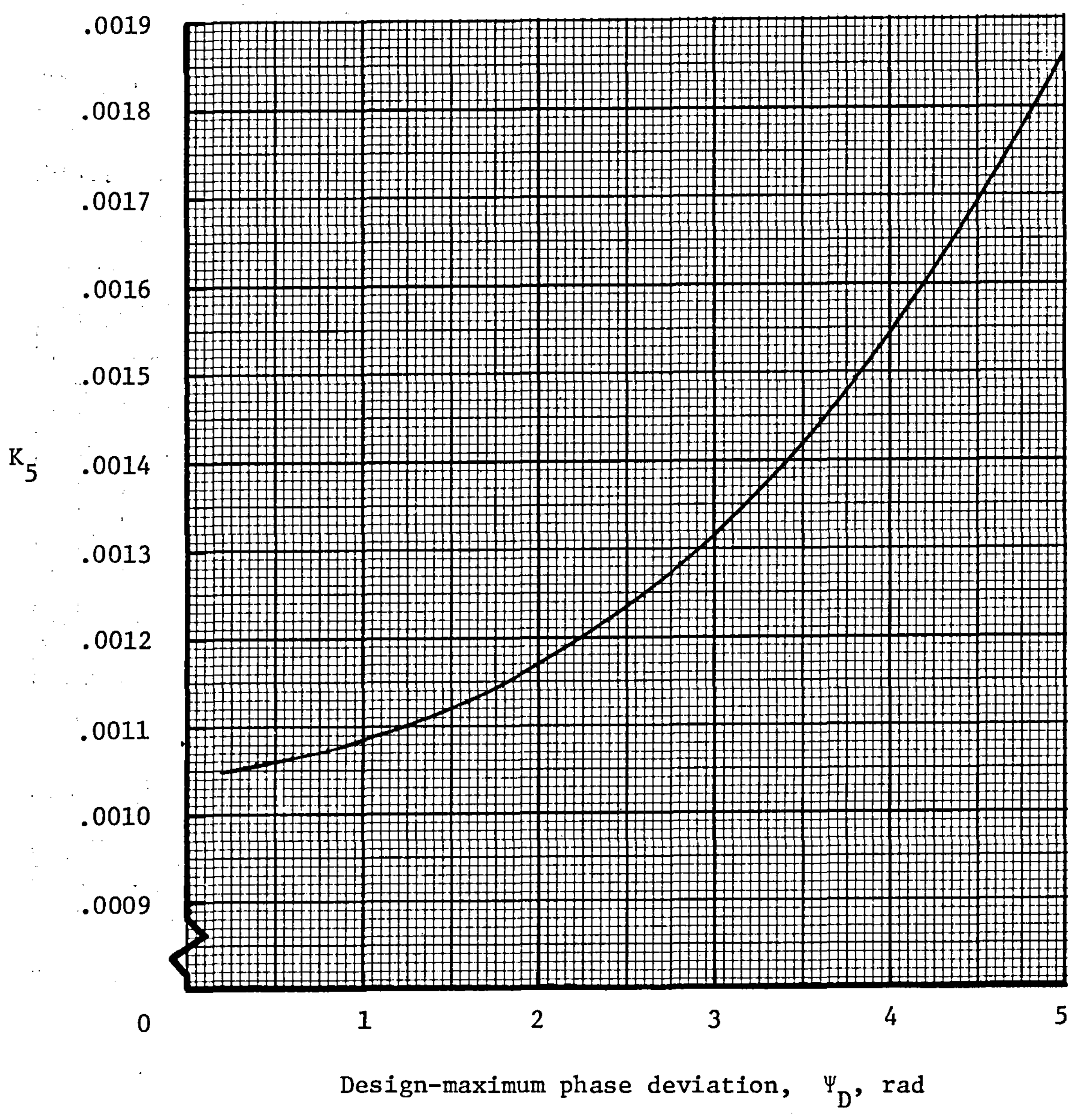


Figure 5.- $(MMSE)_1$ K_5 values. Circuit order 5.

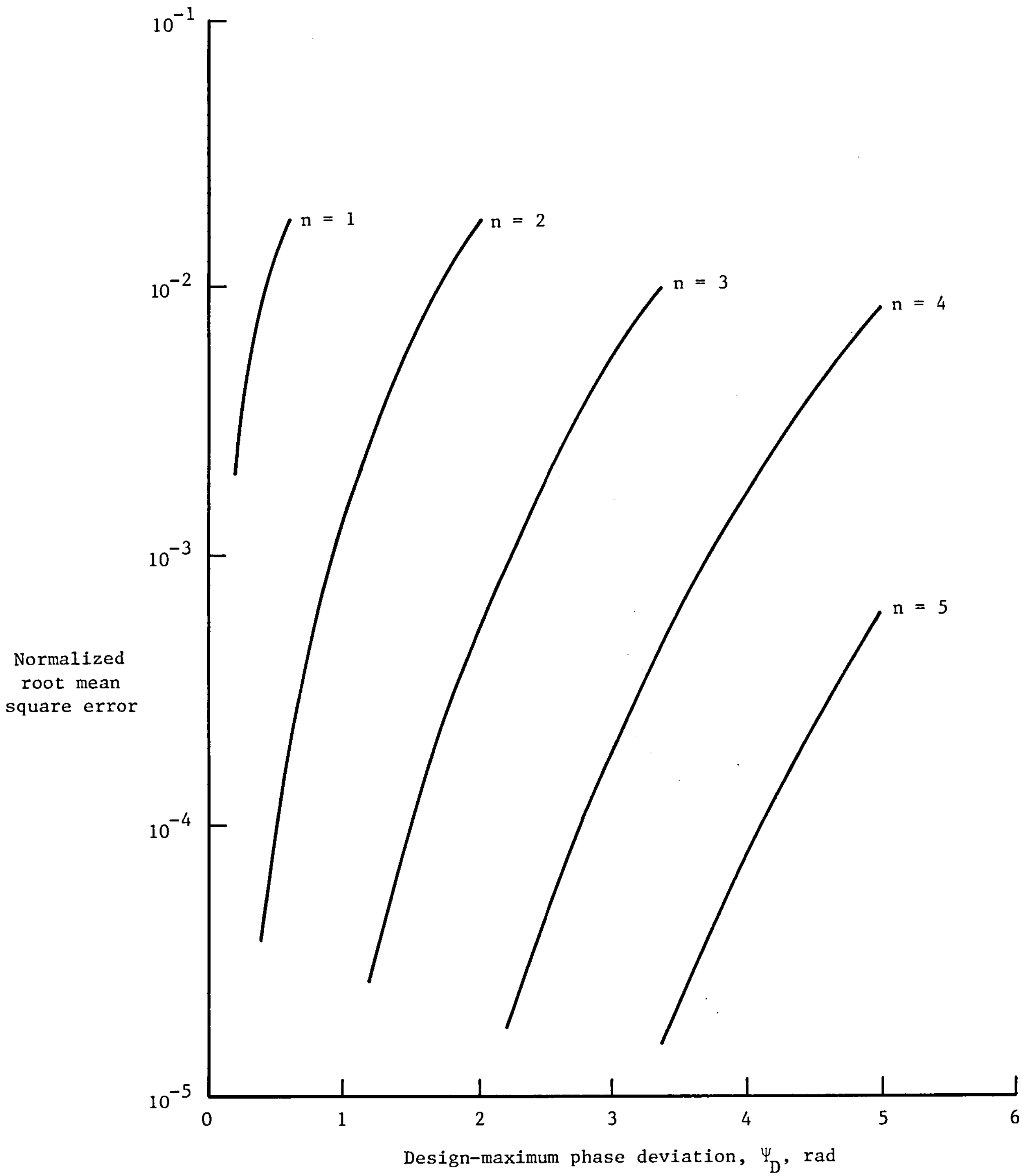


Figure 6.- Normalized root mean square error versus design-maximum phase deviation.

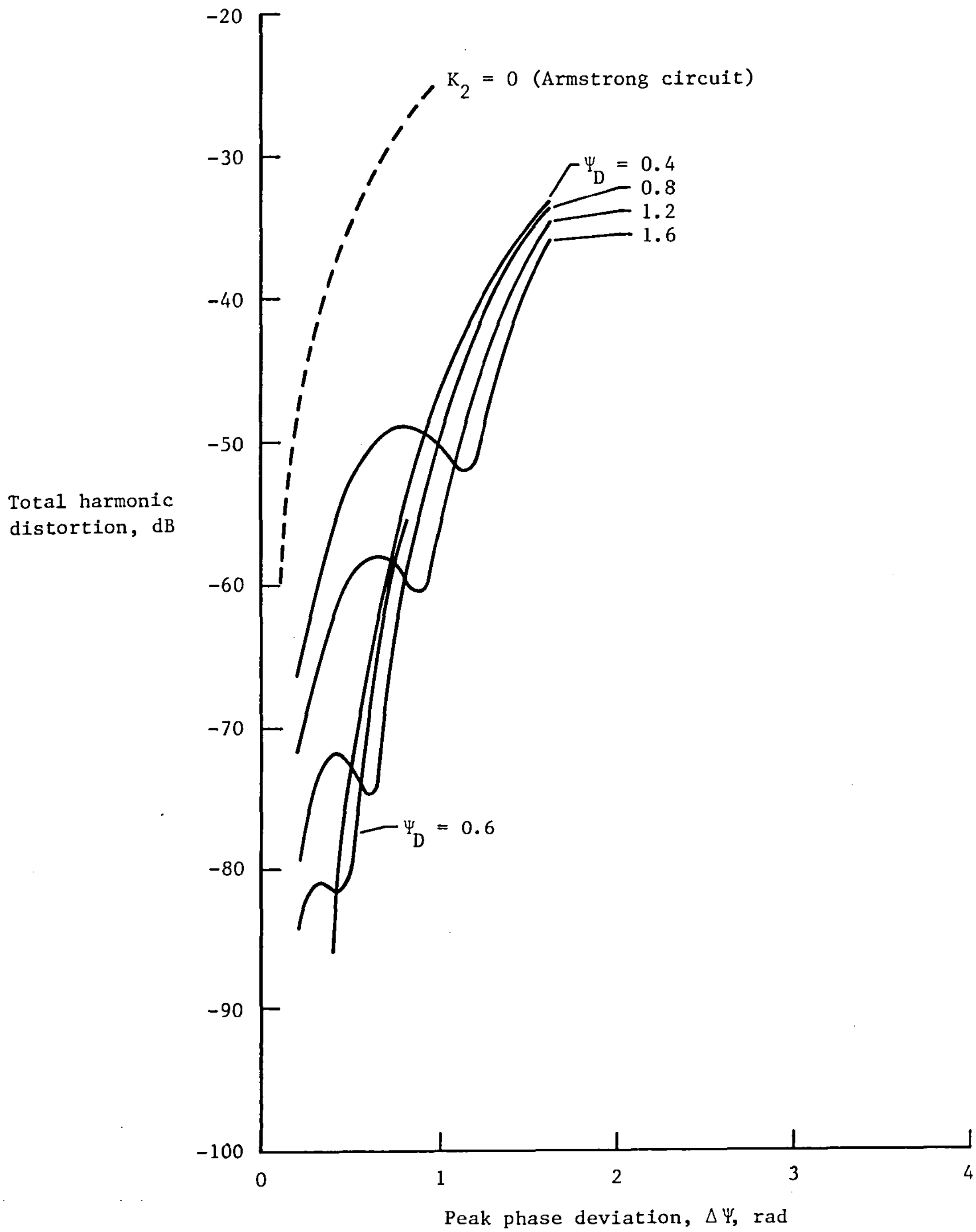


Figure 7.- Total harmonic distortion versus peak phase deviation. Second-order circuit; $(MMSE)_1$ coefficients.

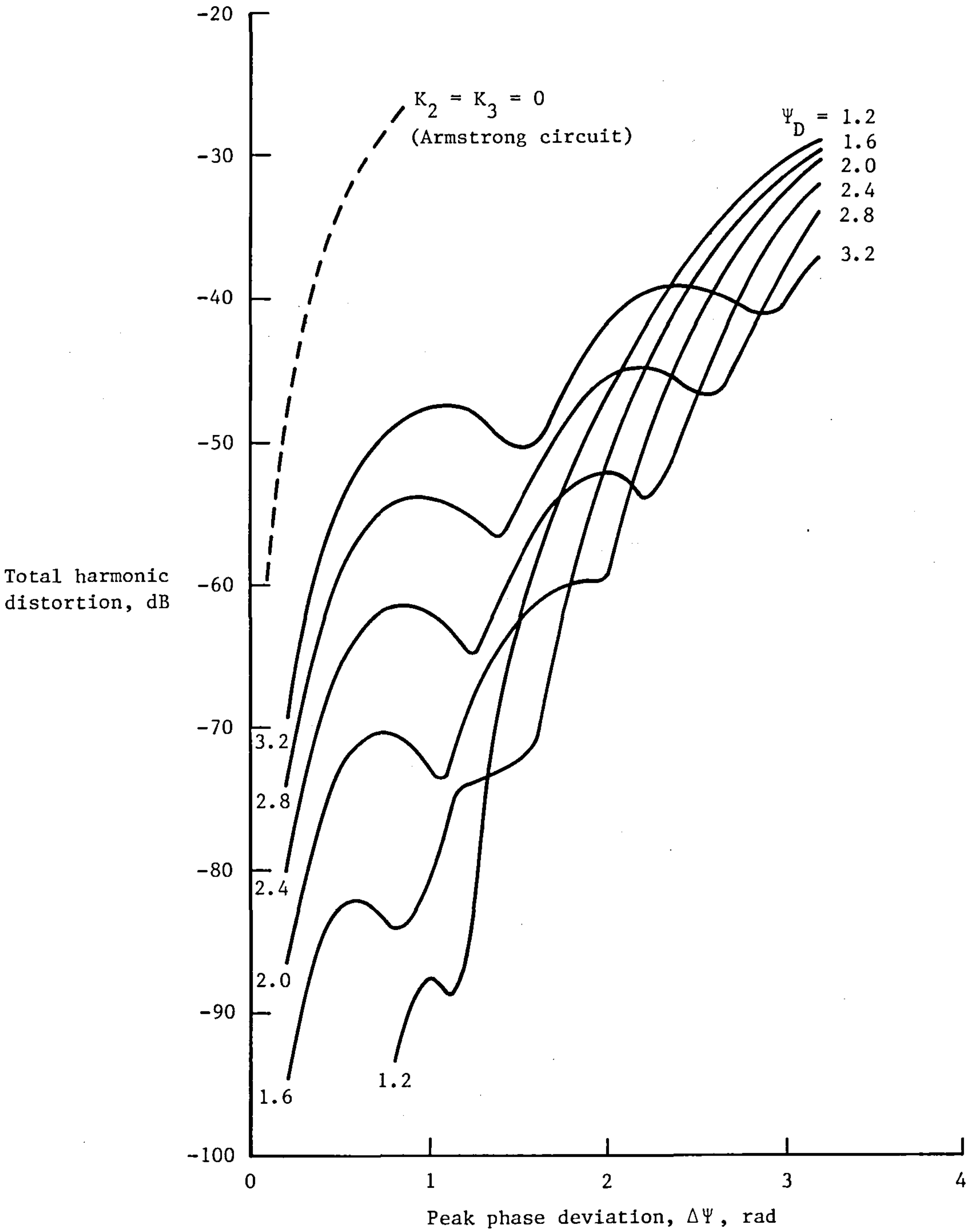


Figure 8.- Total harmonic distortion versus peak phase deviation. Third-order circuit; $(MMSE)_1$ coefficients.

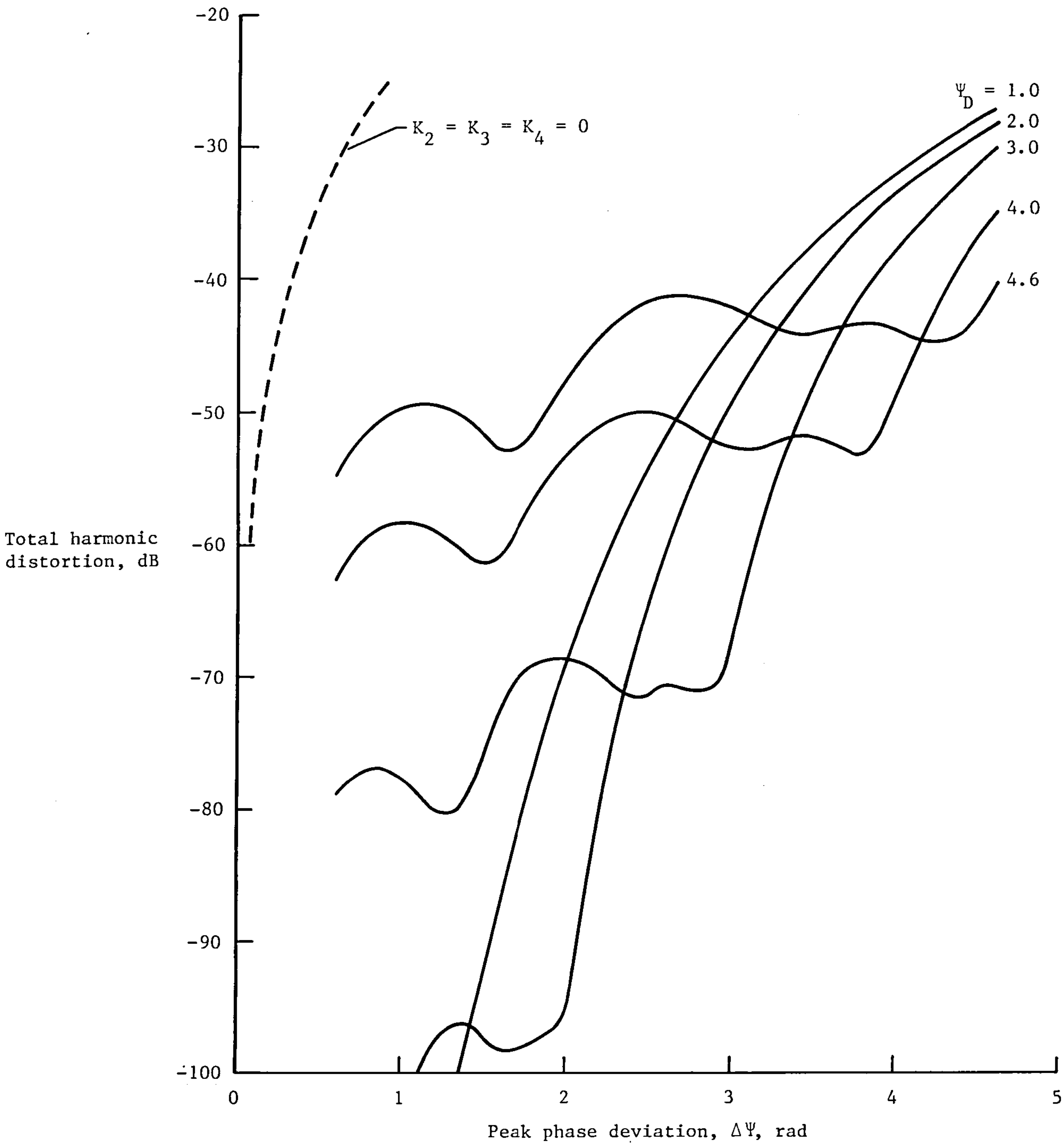


Figure 9.- Total harmonic distortion versus peak phase deviation. Fourth-order circuit; $(MMSE)_1$ coefficients.

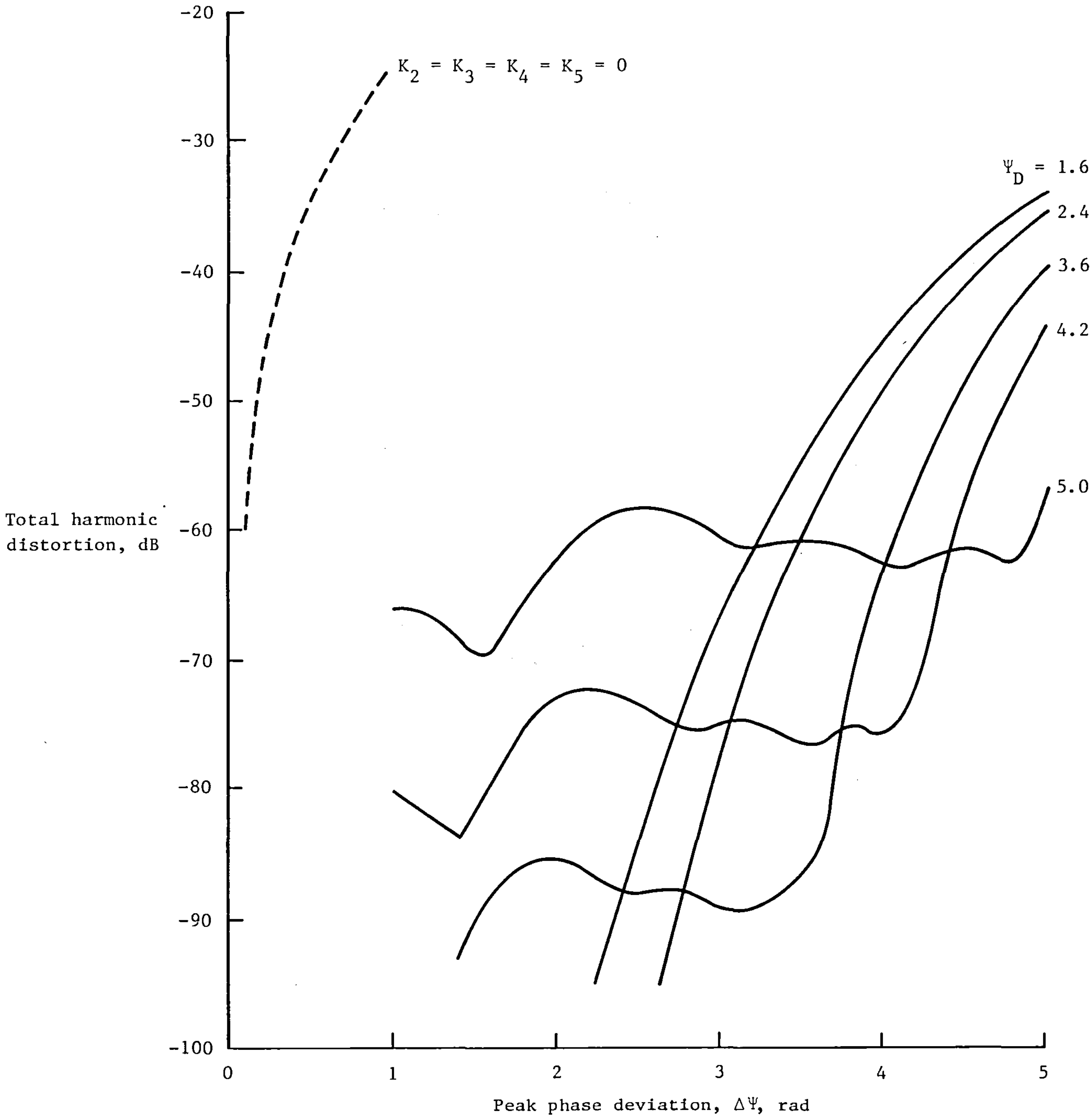


Figure 10.- Total harmonic distortion versus peak phase deviation. Fifth-order circuit; $(MMSE)_1$ coefficients.

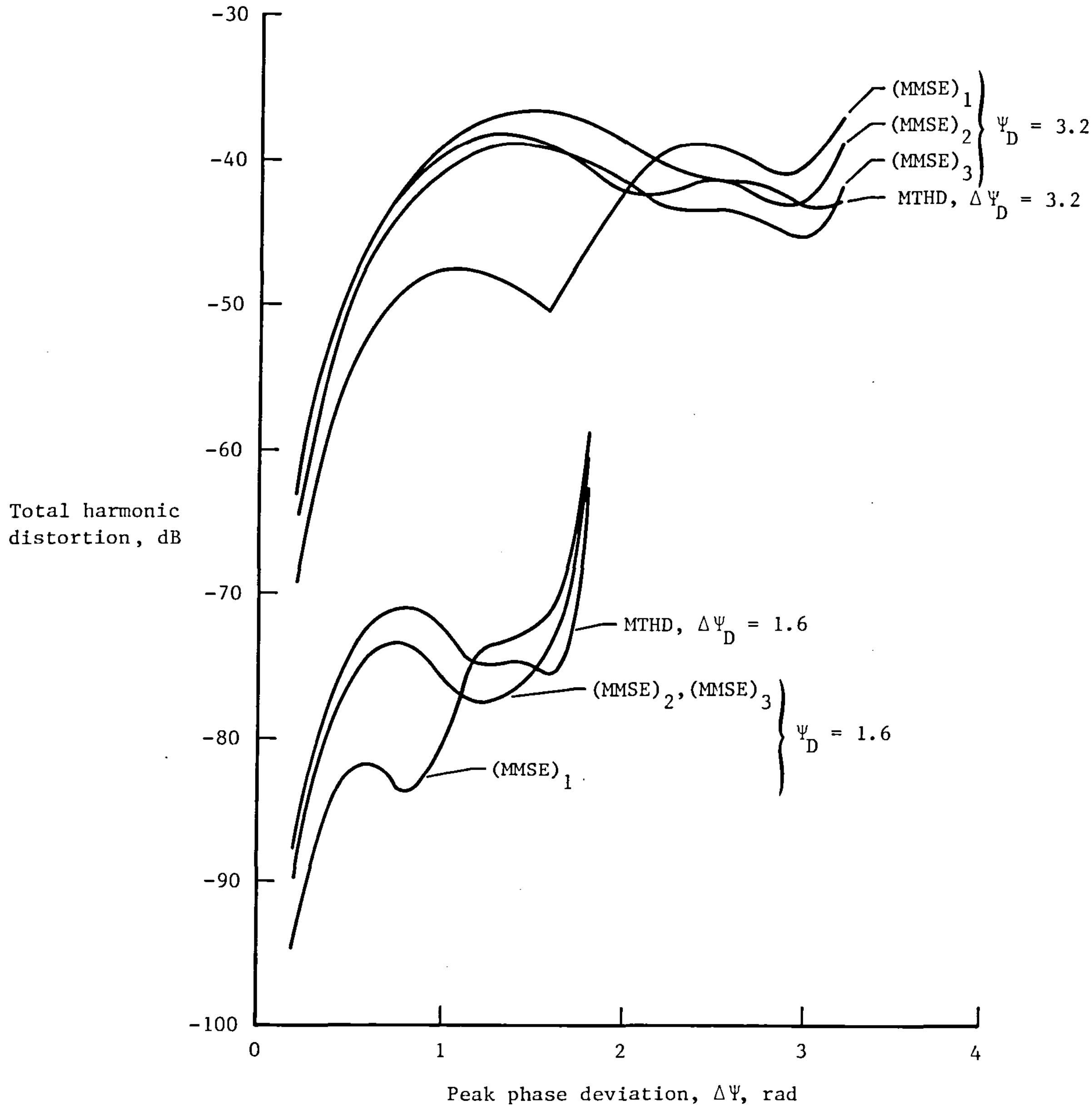
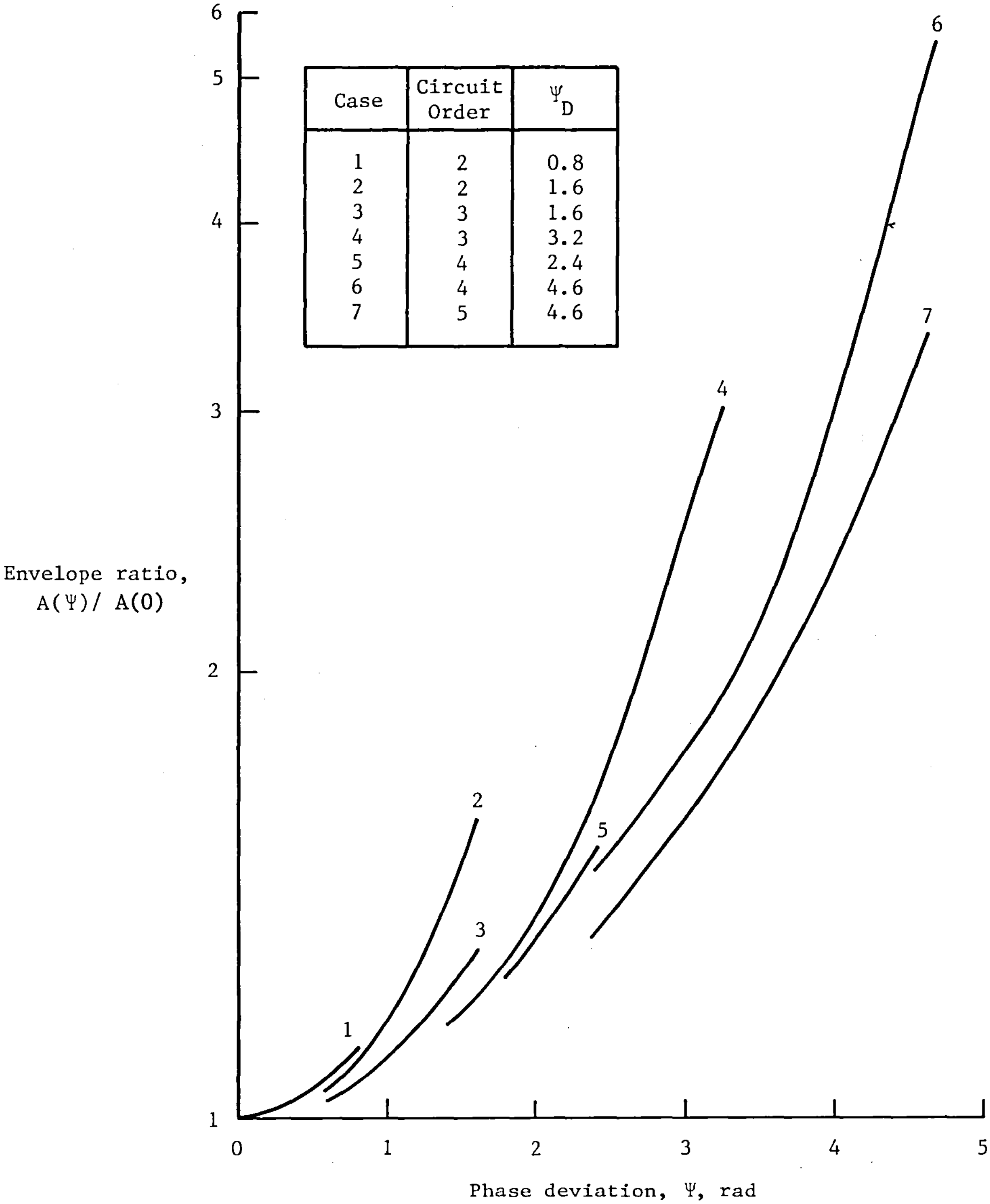


Figure 11.- Total harmonic distortion for various optimization criteria versus peak phase deviation. Third-order circuit.



Case	Circuit Order	Ψ_D
1	2	0.8
2	2	1.6
3	3	1.6
4	3	3.2
5	4	2.4
6	4	4.6
7	5	4.6

Figure 12.- Envelope variation with phase deviation.
Numbers next to curves denote cases.

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16. Abstract

This paper examines the ultimate modulating linearity attainable with a phase modulation technique based on the linear addition of quadrature phase carrier signals which have been multiplied by precisely defined nonlinear transformations of the modulating signal. Optimum gain coefficients are derived and plotted to permit implementation of analog phase modulators capable of exceptionally good linearity for phase deviations as large as 5 radians.

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