COUPLED ROTOR-BODY VIBRATIONS WITH INPLANE DEGREES OF FREEDOM

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Abstract

In an effort to understand the vibration mechanisms of helicopters, the following basic studies are considered. A coupled rotor-fuselage vibration analysis including inplane degrees of freedom of both rotor and airframe is performed by matching of rotor and fuselage impedances at the hub. A rigid blade model including hub motion is used to set up the rotor flaplag equations. For the airframe, 9 degrees of freedom and hub offsets are used. The equations are solved by harmonic balance. For a 4-bladed rotor, the coupled responses and hub loads are calculated for various parameters in forward flight. The results show that the addition of inplane degrees of freedom does not significantly affect the vertical vibrations for the cases considered, and that inplane vibrations have similar resonance trends as do flapping vibrations.

Notation

a = slope of lift curve, rad^{-1}

A = ratio of rotor mass to moment of inertia, $M \bar{x} R/I_y$

$\begin{Bmatrix} a \\ b \end{Bmatrix}_F$ = cosine and sine harmonics of F

b = number of blades

$\hat{C}_z$ = conventional thrust coefficient, thrust/$\rho\pi\Omega^2R^4$

C_x, C_y, C_z, C_M, C_L = vibratory portion of nondimensional longitudinal force, lateral force, thrust, pitch and roll moment over σa

$\bar{C}_x, \bar{C}_y, \bar{C}_z, \bar{C}_M, \bar{C}_L$ = steady portion

$\bar{d}_F$ = offset between focus and center of fuselage, divided by R

$\bar{d}_P$ = offset between focus and center of pylon, divided by R

$\bar{d}_r$ = offset between hub and center of pylon, divided by R

EI = beam cross-section bending stiffness

$\{F\}$ = vector of harmonics of C_M, C_L, C_x, C_y, C_z

$\bar{g}$ = nondimensional acceleration of gravity, g/Ω^2R

g_z, g_y, g_m, g_L = plunge, lateral, pitch and roll structural damping, $= 2\zeta$

$\bar{h}$ = offset between hub and focus, divided by R

$[H]$ = fuselage receptance

$[I]$ = identity matrix

$\bar{I}_{y1}$ = pitch inertia moment of pylon, divided by M_pR^2

I_{x1} = roll inertia moment of pylon, divided by M_pR^2

$\bar{I}_{y2}$ = pitch inertia moment of fuselage, divided by M_FR^2

$\bar{I}_{x2}$ = roll inertia moment of fuselage, divided by M_FR^2

$K_{\alpha c}$ = fuselage pitch-spring-restraint stiffness, N-m/rad

$K_{\alpha s}$ = fuselage roll-spring-restraint stiffness, N-m/rad

ℓ = length of the beam, m

m = mass per unit beam length, kg/m

M_C = lumped mass on the center of the fuselage, kg

M_P = mass of pylon, kg

M_F = mass of fuselage, $M_C + m\ell$, kg

M_f = mass of whole fuselage, $M_C + m\ell + M_P$, kg

p = first flap frequency divided by R

P = p^2

$\bar{r}_{pm}$, $\bar{r}_{pL}$ = radius of gyration of pylon in pitch, roll, divided by R

$\bar{r}_{Fm}$, $\bar{r}_{FL}$ = radius of gyration of fuselage in pitch, roll, divided by R

R = rotor radius, m

R_V = beam mass divided by whole airframe mass, $m\ell/(m\ell + M_C + M_P)$

$[S_i]$ = general matrices

[T] = transformation matrix

W = rotor stiffness parameter[11]

$\bar{x}$ = distance along fuselage, tail to nose, or distance along radius of rotor, root to tip, divided by R

x,y,z = rotating coordinates fixed on the blade

x',y',z' = rotating coordinates if flapping and lead-lag are zero

X,Y,Z = fixed fuselage coordinates

$\bar{X},\bar{Y},\bar{Z}$ = dimensionless displacements, X/R, Y/R, Z/R

$\bar{Y}_F$, $\bar{Z}_F$ = dimensionless fuselage elastic degree of freedom in vertical and lateral directions

Z = rotor stiffness parameter[11]

[Z] = rotor impedance

α_C, α_{CF} = pitch angle of hub, fuselage, positive nose up, rad

$\bar{\alpha}_C$ = steady hub pitch angle, rad

α_S, α_{SF} = roll angle of hub, fuselage, positive advancing side down, rad

$\bar{\beta}$ = equilibrium flapping angle, $\beta_O + \beta_S \sin\psi + \beta_C \cos\psi$, rad

β_O = coning angle, rad

β_S = lateral cyclic flap angle, rad

β_C = longitudinal cyclic flap angle, rad

β_{pc} = pre-cone angle

γ = Lock number

λ = inflow ratio

μ = advance ratio

μ_P = ratio of mass of pylon to mass of fuselage, $M_P/(M\ell + M_C)$

μ_{mc} = ratio of lumped mass to the uniformly distributed mass, $M_C/m\ell$

μ_{mp} = ratio of mass of pylon to the uniformly distributed mass, M_p/m_ℓ

σ = rotor solidity

$\{\vartheta\}$ = vector of control variables

$\bar{\theta}$ = equilibrium pitch angle, $\theta_O + \theta_S \sin\psi + \theta_C \cos\psi + \theta_\beta(\bar{\beta}-\beta_{pc}) + \theta_\zeta \bar{\zeta}$

θ_O, θ_S, θ_C = collective and cyclic pitch, rad

θ_β, θ_ζ = pitch-flap and pitch-lag coupling ratios

ψ = azimuth angle, nondimensional time,

$\bar{\omega}$ = natural frequency of fuselage, divided by Ω

$\bar{\omega}_{xy}$ = frequency of "y" motion with "x" boundary condition, divided by Ω; y = z,y,m,L plunge, lateral, pitch, roll, x = c,f cantilevered, free

Ω = rotor speed, rad/sec

$(\overset{*}{\ })$ = d( )/dψ

$(\cdot)$ = d( )/dt

C_{do} = blade profile drag coefficient

ζ = lag angle, positive forward, rad

$\bar{\zeta}$ = equilibrium lag angle, rad

$\tilde{\zeta}$ = small perturbation of lag angle

$\tilde{\beta}$ = small perturbation of flapping angle

Introduction

Helicopter vibration reduction has become more and more important in recent years because of human factors and expanded operational capabilities. Unlike the conventional fixed-wing aircraft, the helicopter suffers an intrinsic, severe vibration source - the main rotor. The main rotor is connected flexibly to the fuselage by a hub-pylon system which makes the problem sophisticated. The fuselage motions due to rotor vibrations can cause the hub to move in all degrees of freedom which, in turn, can alter the hub loads obtained for a fixed-hub condition. This alteration can often be an order-of-magnitude change. Therefore, what we are studying is a feedback or coupled system.

The concept of performing a coupled rotor/airframe vibration analysis by impedance matching goes back about 20 years, Reference 1. That reference points out two important facts. First, a coupled rotor/airframe analysis can be performed in a rigorous manner by separate calculation of rotor and fuselage impedances followed by a matching of forces and displacements at the hub. Second, the rotor impedance need only be calculated for a single blade and then appropriately transformed to apply to any number of blades. In 1974, Staley and Sciarra treated the vertical vibrations of a coupled rotor and fuselage, including the effect of vertical hub motions.[2] They used a rigid-body mass as a model for rotor impedance and showed that hub motions could create order-of-magnitude changes in hub loads. In Reference 3, Hohenemser and Yin further investigate the effects of rotor-body coupling. Their model for rotor impedance is based on a rotor representation that includes two masses (each equal to one-half of the total rotor mass) connected by a spring to represent the first flapping frequency. Thus, Reference 3 contains a more sophisticated rotor impedance than does Reference 2. Reference 3 presents some very interesting conclusions that pertain to fuselage design. Particularly, it notes that under certain conditions it may be desirable to tune a fuselage frequency to the blade passage frequency in order to eliminate hub loads. Also, it outlines a method of computing the complete rotor impedance by finite elements and transfer matrices. Other work on the importance of hub impedance may be found in References 4-6.

When one considers the rather crude models that have been used for hub impedance (rigid mass, no aerodynamics, etc.) one might wonder why more sophisticated models were not used. The answer is straightforward. These were only the initial investigations into this effect. Furthermore, although most analysts realized the importance of detailed blade modeling (blade modes, unsteady aerodynamics, periodic coefficients, etc.) for fixed hub loads, it was not clear in the beginning which of these effects would be important for finding the role of hub motion on loads. Because of the high frequencies involved (4/rev, 8/rev), many felt that inertial terms would dominate.

Reference 7 offers a sophisticated (but linear) rotor flapping model that allows for a detailed investigation of both rotor loads and impedance (even in the presence of periodic coefficients). The method, generalized harmonic balance, involves a computer-based manipulation of equations that allows many degrees of freedom, many modes, and many harmonics. In Reference 8, Hsu and Peters apply this method to a flexible rotor and then use impedance matching to include plunge, pitch, and roll of the hub. This combined solution technique proves to be very efficient on two counts. First, the calculation for only one blade can be used for n-blades (as in Reference 1). Second, wholesale changes in fuselage properties can be made without a requirement to re-calculate rotor properties. It is interesting that other investigators who began with a full-blown, coupled analyses later changed to the impedance matching technique, References 9-10.

The next step, outlined in this paper, is to add inplane loads and inplane motions to the work of Reference 8. To do this, we need to consider a model for the inplane blade dynamics. Our plan is to begin with a rigid-blade rotor analysis, as outlined in Reference 11, and then to add hub motions to it. Later, we plan to do the same for the elastic flap-lag model of Reference 12. The work reported here is the former of these and is based on a Master of Science Thesis by the first author, Reference 13.

Rotor Model

The rotor model used here is that of Reference 9 but with the addition of hub motions. Fig. 1 shows the rotor model used in this paper.

The equations of motion of this system can be obtained from LaGrange's method with appropriate linearization about an equilibrium condition, $\bar{\beta}$. The aerodynamic terms are obtained from inviscid, linear, quasi-steady strip theory with the small-angle assumptions. Details of the derivation are given in Reference 11, upon which this paper is based. They can be expressed in matrix form as follows.

$$\left\{\begin{matrix}\ddot{\beta}\\ \ddot{\zeta}\end{matrix}\right\} + \left[C(\Psi)\right]\left\{\begin{matrix}\dot{\beta}\\ \dot{\zeta}\end{matrix}\right\} + \left[K(\Psi)\right]\left\{\begin{matrix}\beta\\ \zeta\end{matrix}\right\}$$

$$= \left[Q(\Psi)\right]\left\{\begin{matrix}\theta_0\\ \theta_s\\ \theta_c\\ \lambda\\ \bar{\beta}\\ \beta_{pc}\\ \bar{\zeta}\\ \bar{\alpha}_c\end{matrix}\right\} + \left[\bar{A}(\Psi)\right]\left\{\begin{matrix}\alpha_c\\ \alpha_s\\ \bar{X}\\ \bar{Y}\\ \bar{Z}\end{matrix}\right\} + \left[\bar{B}(\Psi)\right]\left\{\begin{matrix}\dot{\alpha}_c\\ \dot{\alpha}_s\\ \dot{\bar{X}}\\ \dot{\bar{Y}}\\ \dot{\bar{Z}}\end{matrix}\right\} + \left[\bar{D}(\Psi)\right]\left\{\begin{matrix}\ddot{\alpha}_c\\ \ddot{\alpha}_s\\ \ddot{\bar{X}}\\ \ddot{\bar{Y}}\\ \ddot{\bar{Z}}\end{matrix}\right\} \quad (1a)$$

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Where,

$$[C(\Psi)] = \begin{bmatrix} \begin{array}{l} \gamma/8 \\ +\gamma/6\,\mu S\Psi \end{array} & \begin{array}{l} \frac{\gamma}{6}\lambda + \frac{\gamma}{6}\bar{\beta}\mu C\Psi \\ -\frac{\gamma}{4}\bar{\theta} - \frac{\gamma}{8}\bar{\theta}\mu S\Psi \\ +2\bar{\beta} \end{array} \\ \hline \begin{array}{l} \frac{\gamma}{8}\bar{\theta} + \frac{\gamma}{6}\mu\bar{\theta}S\Psi \\ -\frac{\gamma}{3}\lambda - \frac{\gamma}{3}\mu\bar{\beta}C\Psi \\ -2\bar{\beta} \end{array} & \begin{array}{l} \frac{\gamma}{6}\lambda\bar{\theta} + \frac{\gamma}{6}\mu\bar{\theta}\bar{\beta}C\Psi \\ +\frac{\gamma}{3}\frac{C_{d_0}}{a}\mu S\Psi + \frac{\gamma}{4}\frac{C_{d_0}}{a} \end{array} \end{bmatrix}$$

$$[\bar{A}(\Psi)] = \begin{bmatrix} \begin{array}{l} \frac{\gamma}{6}\mu \\ +\frac{\gamma}{4}\mu^2 S\Psi \end{array} & 0 & 0 & 0 & 0 \\ \hline \begin{array}{l} -\frac{\gamma}{2}\mu^2\bar{\beta}C\Psi \\ -\frac{\gamma}{2}\mu\lambda \\ +\frac{\gamma}{6}\mu\theta_0 \\ +\frac{\gamma}{6}\mu\theta_s S\Psi \\ +\frac{\gamma}{6}\mu\theta_c C\Psi \\ +\frac{\gamma}{4}\mu^2\theta_0 S\Psi \\ +\frac{\gamma}{4}\mu^2\theta_s S^2\Psi \\ +\frac{\gamma}{4}\mu^2\theta_c S\Psi C\Psi \end{array} & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$[\bar{D}(\Psi)] \begin{bmatrix} C\Psi & S\Psi & -A\bar{\beta}C\Psi & A\bar{\beta}S\Psi & A \\ \hline \bar{\beta}S\Psi & -\bar{\beta}C\Psi & -AS\Psi & -AC\Psi & 0 \end{bmatrix}$$

$$[\bar{B}(\Psi)] = \begin{bmatrix} \begin{array}{l} \frac{\gamma}{8}C\Psi \\ +\frac{\gamma}{6}\mu S\Psi C\Psi \\ -2S\Psi \end{array} & \begin{array}{l} \frac{\gamma}{8}S\Psi \\ +\frac{\gamma}{6}\mu S^2\Psi \\ +2C\Psi \end{array} & \begin{array}{l} \frac{\gamma}{3}\theta_0 S\Psi \\ +\frac{\gamma}{3}\theta_s S^2\Psi \\ +\frac{\gamma}{3}\theta_c S\Psi C\Psi \\ +\frac{\gamma}{2}\mu\theta_0 S^2\Psi \\ +\frac{\gamma}{2}\mu\theta_s S^3\Psi \\ +\frac{\gamma}{2}\mu\theta_c S^2\Psi C\Psi \\ -\frac{\gamma}{4}\lambda S\Psi \\ -\frac{\gamma}{2}\mu\bar{\beta}S\Psi C\Psi \\ -\frac{\gamma}{6}\bar{\beta}C\Psi \end{array} & \begin{array}{l} \frac{\gamma}{3}\theta_0 C\Psi \\ +\frac{\gamma}{3}\theta_s C\Psi S\Psi \\ +\frac{\gamma}{3}\theta_c C^2\Psi \\ +\frac{\gamma}{2}\mu\theta_0 C\Psi S\Psi \\ +\frac{\gamma}{2}\mu\theta_s C\Psi S^2\Psi \\ +\frac{\gamma}{2}\mu\theta_c C^2\Psi S\Psi \\ -\frac{\gamma}{4}\lambda C\Psi \\ -\frac{\gamma}{4}\mu\bar{\beta}C^2\Psi \\ +\frac{\gamma}{6}\bar{\beta}S\Psi \\ +\frac{\gamma}{4}\mu\bar{\beta}S^2\Psi \end{array} & \begin{array}{l} \frac{\gamma}{6} \\ +\frac{\gamma}{4}\mu S\Psi \end{array} \\ \hline \begin{array}{l} -\frac{\gamma}{3}\lambda C\Psi \\ -\frac{\gamma}{3}\mu\bar{\beta}C^2\Psi \\ +\frac{\gamma}{8}\theta_0 C\Psi \\ +\frac{\gamma}{8}\theta_s S\Psi C\Psi \\ +\frac{\gamma}{8}\theta_c C^2\Psi \\ +\frac{\gamma}{6}\mu\theta_0 S\Psi C\Psi \\ +\frac{\gamma}{6}\mu\theta_s S^2\Psi C\Psi \\ +\frac{\gamma}{6}\mu\theta_c S\Psi C^2\Psi \end{array} & \begin{array}{l} -\frac{\gamma}{3}\lambda S\Psi \\ -\frac{\gamma}{3}\bar{\beta}\mu S\Psi C\Psi \\ +\frac{\gamma}{8}\theta_0 S\Psi \\ +\frac{\gamma}{8}\theta_s S^2\Psi \\ +\frac{\gamma}{8}\theta_c S\Psi C\Psi \\ +\frac{\gamma}{6}\mu\theta_0 S^2\Psi \\ +\frac{\gamma}{6}\mu\theta_s S^3\Psi \\ +\frac{\gamma}{6}\mu\theta_c S^2\Psi C\Psi \end{array} & \begin{array}{l} -\frac{\gamma}{3}\frac{C_{d_0}}{a}S\Psi \\ -\frac{\gamma}{2}\frac{C_{d_0}}{a}\mu S^2\Psi \end{array} & \begin{array}{l} -\frac{\gamma}{3}\frac{C_{d_0}}{a}C\Psi \\ -\frac{\gamma}{2}\frac{C_{d_0}}{a}\mu S\Psi C\Psi \end{array} & \begin{array}{l} -\frac{\gamma}{2}\lambda \\ +\frac{\gamma}{6}\theta_0 \\ +\frac{\gamma}{6}\theta_s S\Psi \\ +\frac{\gamma}{6}\theta_c C\Psi \\ +\frac{\gamma}{4}\mu\theta_0 S\Psi \\ +\frac{\gamma}{4}\mu\theta_s S^2\Psi \\ +\frac{\gamma}{4}\mu\theta_c S\Psi C\Psi \\ -\frac{\gamma}{2}\mu\bar{\beta}C\Psi \end{array} \end{bmatrix}$$

$$[K(\Psi)] = \begin{bmatrix} \begin{array}{l} P \\ +\frac{\gamma}{6}\mu C\Psi \\ +\frac{\gamma}{4}\mu^2 S\Psi C\Psi \\ -\frac{\gamma}{6}\theta_\beta \\ -\frac{\gamma}{4}\mu^2\theta_\beta S^2\Psi \\ -\frac{\gamma}{3}\mu\theta_\beta S\Psi \end{array} & \begin{array}{l} Z \\ +\frac{\gamma}{4}\mu\lambda C\Psi \\ +\frac{\gamma}{4}\mu^2\bar{\beta}(C^2\Psi - S^2\Psi) \\ -\frac{\gamma}{4}\mu^2\theta_\zeta S^2\Psi \\ -\frac{\gamma}{2}\mu^2\bar{\theta}S\Psi C\Psi \\ -\frac{\gamma}{3}\bar{\theta}\mu C\Psi \\ -\frac{\gamma}{3}\mu\theta_s S\Psi \\ -\frac{\gamma}{6}\mu\bar{\beta}S\Psi \\ -\frac{\gamma}{8}\theta_\zeta \end{array} \\ \hline \begin{array}{l} Z \\ +\frac{\gamma}{6}\lambda\theta_\beta \\ +\frac{\gamma}{8}\mu\bar{\theta}C\Psi \\ +\frac{\gamma}{6}\mu\bar{\beta}\theta_\beta C\Psi \\ +\frac{\gamma}{4}\mu\lambda\theta_\beta S\Psi \\ +\frac{\gamma}{4}\mu^2\bar{\beta}S\Psi C\Psi \\ +\frac{\gamma}{4}\mu^2\theta_\beta\bar{\beta}S\Psi C\Psi \\ -\frac{\gamma}{2}\bar{\beta}\mu^2 C^2\Psi \\ -\frac{\gamma}{2}\lambda\mu C\Psi \end{array} & \begin{array}{l} W \\ +\frac{\gamma}{2}\mu^2\bar{\beta}^2 C\Psi S\Psi \\ +\frac{\gamma}{2}\lambda\mu\bar{\beta}S\Psi \\ +\frac{\gamma}{6}\lambda\theta_\zeta \\ +\frac{\gamma}{6}\mu\bar{\beta}\theta_\zeta C\Psi \\ +\frac{\gamma}{4}\mu\lambda\theta_\zeta S\Psi \\ +\frac{\gamma}{4}\mu\lambda\bar{\theta}C\Psi \\ +\frac{\gamma}{4}\mu^2\bar{\theta}\bar{\beta}(C^2\Psi - S^2\Psi) \\ +\frac{\gamma}{2}\frac{C_{d_0}}{a}\mu^2 S\Psi C\Psi \\ -\frac{\gamma}{6}\mu\bar{\theta}\bar{\beta}S\Psi \\ +\frac{\gamma}{4}\mu^2\theta_\zeta\bar{\beta}S\Psi C\Psi \\ +\frac{\gamma}{3}\frac{C_{d_0}}{a}\mu C\Psi \end{array} \end{bmatrix}$$

and $S\Psi = \sin\Psi$ (1b-g)

$C\Psi = \cos\Psi$

$$[Q(\Psi)] = \begin{bmatrix} \begin{array}{l} \frac{\gamma}{8} \\ +\frac{\gamma}{4}\mu^2 S^2\Psi \\ +\frac{\gamma}{3}\mu S\Psi C\Psi \end{array} & \begin{array}{l} \frac{\gamma}{8}S\Psi \\ +\frac{\gamma}{4}\mu^2 S^3\Psi \\ +\frac{\gamma}{3}\mu S^2\Psi \end{array} & \begin{array}{l} \frac{\gamma}{8}C\Psi \\ +\frac{\gamma}{4}\mu^2 S^2\Psi C\Psi \\ +\frac{\gamma}{3}\mu S\Psi C\Psi \end{array} & \begin{array}{l} -\frac{\gamma}{6} \\ -\frac{\gamma}{4}\mu S\Psi \end{array} & \begin{array}{l} -\frac{\gamma}{6}\mu C\Psi \\ -\frac{\gamma}{4}\mu^2 S\Psi C\Psi \\ -P \end{array} & P-1 & 0 & \begin{array}{l} \frac{\gamma}{6} \\ +\frac{\gamma}{4}\mu^2 S\Psi \end{array} \\ \hline \begin{array}{l} -\frac{\gamma}{6}\lambda \\ -\frac{\gamma}{6}\mu\bar{\beta}C\Psi \\ -\frac{\gamma}{4}\mu\lambda S\Psi \\ -\frac{\gamma}{4}\mu^2\bar{\beta}S\Psi C\Psi \\ +\frac{\gamma}{6}\mu\bar{\alpha}_c \\ +\frac{\gamma}{4}\mu^2\bar{\alpha}_c S\Psi \end{array} & \begin{array}{l} -\frac{\gamma}{6}\lambda S\Psi \\ -\frac{\gamma}{6}\mu\bar{\beta}S\Psi C\Psi \\ -\frac{\gamma}{4}\mu\lambda S^2\Psi \\ -\frac{\gamma}{4}\mu^2\bar{\beta}S^2\Psi C\Psi \\ +\frac{\gamma}{6}\mu\bar{\alpha}_c S\Psi \\ +\frac{\gamma}{4}\mu^2\bar{\alpha}_c S^2\Psi \end{array} & \begin{array}{l} -\frac{\gamma}{6}\lambda C\Psi \\ -\frac{\gamma}{6}\mu\bar{\beta}C^2\Psi \\ -\frac{\gamma}{4}\mu\lambda S\Psi C\Psi \\ -\frac{\gamma}{4}\mu^2\bar{\beta}S\Psi C^2\Psi \\ +\frac{\gamma}{6}\mu\bar{\alpha}_c C\Psi \\ +\frac{\gamma}{4}\mu^2\bar{\alpha}_c S\Psi C\Psi \end{array} & \begin{array}{l} \frac{\gamma}{6}\lambda \\ +\frac{\gamma}{6}\mu\bar{\beta}C\Psi \end{array} & \begin{array}{l} -Z \\ +\frac{\gamma}{4}\mu^2\bar{\beta}C^2\Psi \end{array} & Z & \begin{array}{l} -\frac{\gamma}{4}\mu S\Psi \\ -\frac{\gamma}{6} \\ -\frac{\gamma}{4}\mu^2 S^2\Psi \end{array} & \begin{array}{l} -\frac{\gamma}{4}\mu^2\bar{\beta}C\Psi \\ -\frac{\gamma}{4}\mu\lambda \end{array} \end{bmatrix}$$

One can also derive a detailed set of equations for hub loads (pitch moment, roll moment, propulsive force, side force, thrust) in terms of known parameters, unspecified hub motions ($\alpha_c, \alpha_s, \bar{X}, \bar{Y}, \bar{Z}$), and blade motions ($\beta, \zeta$).

$$\begin{Bmatrix} C_M \\ C_L \\ C_H \\ C_Y \\ C_T \end{Bmatrix} = [O(\Psi)] \begin{Bmatrix} \theta_0 \\ \theta_s \\ \theta_c \\ \lambda \\ \bar{\beta} \\ \beta_{pc} \\ \frac{C_{d_0}}{a} \\ \bar{\alpha}_c \end{Bmatrix} + [A(\Psi)] \begin{Bmatrix} \alpha_c \\ \alpha_s \\ \bar{X} \\ \bar{Y} \\ \bar{Z} \end{Bmatrix} + [B(\Psi)] \begin{Bmatrix} \dot{\alpha}_c \\ \dot{\alpha}_s \\ \dot{\bar{X}} \\ \dot{\bar{Y}} \\ \dot{\bar{Z}} \end{Bmatrix}$$

$$+\left[D(\psi)\right]\begin{Bmatrix}\overset{**}{\alpha}_c\\ \overset{**}{\alpha}_s\\ \overset{**}{\bar{X}}\\ \overset{**}{\bar{Y}}\\ \overset{**}{\bar{Z}}\end{Bmatrix}+\left[E(\psi)\right]\begin{Bmatrix}\tilde{\beta}\\ \tilde{\zeta}\end{Bmatrix}+\left[F(\psi)\right]\begin{Bmatrix}\overset{*}{\tilde{\beta}}\\ \overset{*}{\tilde{\zeta}}\end{Bmatrix}+\left[G(\psi)\right]\begin{Bmatrix}\overset{**}{\tilde{\beta}}\\ \overset{**}{\tilde{\zeta}}\end{Bmatrix}$$

$$\{F\}=\begin{Bmatrix}\begin{Bmatrix}a_n\\ b_n\end{Bmatrix}_{C_M}\\ \begin{Bmatrix}a_n\\ b_n\end{Bmatrix}_{C_L}\\ \begin{Bmatrix}a_n\\ b_n\end{Bmatrix}_{C_X}\\ \begin{Bmatrix}a_n\\ b_n\end{Bmatrix}_{C_Y}\\ \begin{Bmatrix}a_n\\ b_n\end{Bmatrix}_{C_Z}\end{Bmatrix}\qquad \{z\}=\begin{Bmatrix}\begin{Bmatrix}a_n\\ b_n\end{Bmatrix}_{\alpha_c}\\ \begin{Bmatrix}a_n\\ b_n\end{Bmatrix}_{\alpha_s}\\ \begin{Bmatrix}a_n\\ b_n\end{Bmatrix}_{\bar{X}}\\ \begin{Bmatrix}a_n\\ b_n\end{Bmatrix}_{\varphi}\\ \begin{Bmatrix}a_n\\ b_n\end{Bmatrix}_{\bar{Z}}\end{Bmatrix}\tag{9c-d}$$

The equations expressed by Eqs. (1) and (2) are systems of ordinary differential equations with periodic coefficients. These can be solved for the periodic response by the harmonic balance method, Reference 3. This method involves operator matrices $[\pi]$ and $[\square]$ which can be used to transform a system of periodic-coefficient differential equations into a set of linear, algebraic equations. For example, the single equation

$$M(\psi)\ddot{X} + C(\psi)\dot{X} + K(\psi)X = F(\psi) \tag{3}$$

(where M, C, and F are periodic), can be transformed into algebraic equations for the unknown Fourier coefficients of x

$$x = a_o + \sum_{n=1}^{N} a_n\cos(n\psi) + b_n\sin(n\psi) \tag{4}$$

$$[\pi(M)][\square]^2\begin{Bmatrix}a_n\\ b_n\end{Bmatrix}_x + [\pi(C)][\square]\begin{Bmatrix}a_n\\ b_n\end{Bmatrix}_x + [\pi(K)]\begin{Bmatrix}a_n\\ b_n\end{Bmatrix}_x = \begin{Bmatrix}a_n\\ b_n\end{Bmatrix}_F \tag{5}$$

$$\begin{Bmatrix}a_n\\ b_n\end{Bmatrix}_x = [\pi(M)\square^2+\pi(C)\square+\pi(K)]^{-1}\begin{Bmatrix}a_n\\ b_n\end{Bmatrix}_F \tag{6}$$

where $[\pi]$ is a function of the Fourier coefficients of its argument. The same operations can be applied to Eqs. (1) and (2) to give equations for the unknown harmonics of blade motions and loads,

$$\{\delta\} = [S_1]\{\theta\} + [S_2]\{z\} \tag{7}$$

$$\{F\} = [S_3]\{\theta\} + [S_4]\{z\} + [S_5]\{\delta\} \tag{8}$$

where $\{\delta\}$ are the harmonics of $\tilde{\beta}$ and $\tilde{\zeta}$, $\{F\}$ are the harmonics of hub loads, $\{z\}$ are harmonics of hub motions, and $\{\theta\}$ are specified rotor parameters.

$$\{\delta\}=\begin{Bmatrix}\begin{Bmatrix}a_n\\ b_n\end{Bmatrix}_{\bar{\beta}}\\ \begin{Bmatrix}a_n\\ b_n\end{Bmatrix}_{\bar{\zeta}}\end{Bmatrix}\qquad \{\theta\}=\begin{Bmatrix}\theta_0\\ \theta_s\\ \theta_c\\ \lambda\\ \bar{\beta}\\ A_k\\ [illegible]\\ \bar{\alpha}_c\end{Bmatrix}\tag{9a-b}$$

Equation (7) can be substituted into Eq. (8) to remove the blade motions. This gives rotor loads in the form

$$\{F\} = [\Theta]\{\theta\} + [Z]\{z\} \tag{10}$$

where

$$[\Theta] = [S_3] + [S_5][S_1] \tag{11a}$$

$$[Z] = [S_4] + [S_5][S_2] \tag{11b}$$

The matrix $[\Theta]\{\theta\}$ represents the rotor loads with a fixed hub (e.g., without feedback due to hub motion), and the impedance matrix $[Z]$ represents the effect of hub motion on rotor loads. The calculation of $[\Theta]$ and $[Z]$ in Eq. (10) need be performed for a single blade only. Subsequently, the corresponding matrices for a b-bladed rotor can be found by simply eliminating all harmonics that are not integer multiples of b. (Complete details are in Reference 3.)

It should be noted here that the present method of calculation of rotor impedance has experimental verification which can be found in Reference 8.

Fuselage Model

The mathematical description of the flexible fuselage includes 9 degrees of freedom. These are: 1) vertical rigid-body, 2) rigid-body pitch, 3) rigid-body roll, 4) rigid-body lateral, 5) rigid-body longitudinal, 6) elastic vertical, 7) elastic lateral, 8) elastic pylon in pitch, and 9) elastic pylon in roll. The model also includes vertical offsets between the fuselage center of mass, the pylon focus, the pylon center of mass, and the rotor center. Fig. 2 illustrates the vertical, longitudinal, and pitch degrees of freedom. The plunge and lateral model is the same as that of the plunge model in Reference 8, which is a uniform beam with a lumped mass M_C added at the center. The mass and inertial moment of the pylon are separated from the fuselage. The offsets are shown in Fig. 2. One can imagine that the lateral and roll directions have a similar schematic as that in Fig. 2 if X, α_c and α_{CF} are replaced by Y, α_s and α_{SF}.

The fuselage equations of motion are obtained from Lagrange's Method and the Rayleigh-Ritz Method. They are given in nondimensional form below (where blank elements are taken to be zero).

$$
\begin{bmatrix}
R^y_{11} & & R^y_{13} & & & & & R^y_{12} & \\
 & R^x_{11} & & R^x_{13} & & & & & R^x_{12} \\
R^y_{31} & & 1 & & & & & R^y_{32} & \\
 & R^x_{31} & & 1 & & & \frac{R_v}{12} & & R^x_{32} \\
 & & & & 1 & \frac{R_v}{12} & & & \\
 & & & & \frac{20}{3} & 1 & & & \\
 & & & \frac{20}{3} & & & 1 & & \\
R^y_{21} & & R^y_{23} & & & & & R^y_{22} & \\
 & R^x_{21} & & R^x_{23} & & & & & R^x_{22}
\end{bmatrix}
\begin{Bmatrix} \ddot{\alpha}_c \\ \ddot{\alpha}_s \\ \ddot{\bar{X}} \\ \ddot{\bar{Y}} \\ \ddot{\bar{Z}} \\ \ddot{\bar{Z}}_F \\ \ddot{\bar{Y}}_F \\ \ddot{\alpha}_{CF} \\ \ddot{\alpha}_{SF} \end{Bmatrix}
+
$$

$$
\begin{bmatrix}
 & & & & & & & & \\
 & & & & & & & & \\
 & & & & & & & & \\
 & & & & & & & & \\
 & & & & & & & & \\
 & & & & & g_z\bar{\omega}_z^2 & & & \\
 & & & & & & g_y\bar{\omega}_y^2 & & \\
 & & & & & & & g_{\alpha_c}\bar{\omega}_{\alpha_c}^2 & \\
 & & & & & & & & g_{\alpha_s}\bar{\omega}_{\alpha_s}^2
\end{bmatrix}
\begin{Bmatrix} \dot{\alpha}_c \\ \dot{\alpha}_s \\ \dot{\bar{X}} \\ \dot{\bar{Y}} \\ \dot{\bar{Z}} \\ \dot{\bar{Z}}_F \\ \dot{\bar{Y}}_F \\ \dot{\alpha}_{CF} \\ \dot{\alpha}_{SF} \end{Bmatrix}
+
$$

$$
\begin{bmatrix}
 & & & & & & & & \\
 & & & & & & & & \\
 & & & & & & & & \\
 & & & & & & & & \\
 & & & & & & & & \\
 & & & & & \bar{\omega}_z^2 & & & \\
 & & & & & & \bar{\omega}_y^2 & & \\
 & & & & & & & \bar{\omega}_{\alpha_c}^2 & \\
 & & & & & & & & \bar{\omega}_{\alpha_s}^2
\end{bmatrix}
\begin{Bmatrix} \alpha_c \\ \alpha_s \\ \bar{X} \\ \bar{Y} \\ \bar{Z} \\ \bar{Z}_F \\ \bar{Y}_F \\ \alpha_{CF} \\ \alpha_{SF} \end{Bmatrix}
= \frac{\bar{g}}{\bar{C}_z}
\begin{Bmatrix} \frac{C_m}{\bar{r}^2_{gm}} - \frac{C_x\bar{h}}{\bar{r}^2_{gm}} \\ \frac{C_L}{\bar{r}^2_{gL}} + \frac{C_Y\bar{h}}{\bar{r}^2_{gL}} \\ C_x \\ C_Y \\ C_z \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix}
\quad \cdots \quad (12)
$$

Using the harmonic-balance method, Eq. (12) can be easily solved as the form below.

$$
\begin{Bmatrix} \{z\} \\ \{z_F\} \end{Bmatrix} = [H][T][F] \qquad (13a)
$$

Where:

$$
\{z_F\} = \begin{Bmatrix} \begin{Bmatrix} a_n \\ b_n \end{Bmatrix}_{\bar{Z}_F} \\ \begin{Bmatrix} a_n \\ b_n \end{Bmatrix}_{\bar{Y}_F} \\ \begin{Bmatrix} a_n \\ b_n \end{Bmatrix}_{\alpha_{CF}} \\ \begin{Bmatrix} a_n \\ b_n \end{Bmatrix}_{\alpha_{SF}} \end{Bmatrix} \qquad (13b)
$$

$\bar{Z}_F$, $\bar{Y}_F$, α_{CF}, α_{SF} are elastic deflections in plunge, lateral, pitch, roll directions respectively.

[H] is receptance (inverse of impedance) of the fuselage.

[T] is a transformation matrix which is defined as

$$
[T] = \begin{bmatrix}
\frac{1}{\bar{r}^2_{gm}} & & -\frac{\bar{h}}{\bar{r}^2_{gm}} & & \\
 & \frac{1}{\bar{r}^2_{gL}} & & \frac{\bar{h}}{\bar{r}^2_{gL}} & \\
 & & 1 & & \\
 & & & 1 & \\
 & & & & 1 \\
 & & & & \\
 & & & & \\
 & & & & \\
 & & & &
\end{bmatrix} \qquad (14)
$$

As before, only integer-multiple harmonics of the blade number (b,2b,...) are retained. Furthermore, higher harmonics may be truncated as deemed appropriate.

The combined rotor/airframe vibrations may be performed by the matching of the impedances from Eq. (10) with those of Eq. (13). This implies the matching of harmonics of both loads {F} and displacements {z} at the hub. Therefore, we have

$$\{F\} = [\theta]\{\theta\} + [Z|0][H][T]\{F\} \qquad (15)$$

$$\{F\} = [I-[Z|0][H][T]]^{-1}[\theta]\{\theta\} \qquad (16)$$

It is noted that these loads include vertical, inplane, and radial loads.

Coupled Response

We now calculate vibrations. To begin, we look at the coupled rotor-fuselage response of a system with the following baseline parameters.

Rotor: 4 blades, $p = 1.09$, $\omega_\zeta = 0.7$ (soft inplane) and $\omega_\zeta = 1.4$ (stiff inplane)

$\gamma = 6.0$, $\bar{C}_z = 0.0144$,
$\bar{C}_L = \bar{C}_M = \bar{C}_x = \bar{C}_Y = 0$,
$\mu = 0.3$, $\bar{\alpha}_c = 0.7$, $\sigma = 0.07$,
$a = 5.73$

$\lambda = 0.0306$, $\hat{C}_z = 0.0058$

Fuselage: $\bar{\gamma}_{Fm} = .379$, $\bar{\gamma}_{FL} = .143$

$$\bar{\gamma}_{pm} = .171,\ \bar{\gamma}_{pL} = .148$$

$$\bar{\omega}_{fz} = 1.45\ \bar{\omega}_{cz},\ \bar{\omega}_{fL} = 1.18\ \bar{\omega}_{fm}$$

$$\bar{\omega}_{fm} = 10.0\ \bar{\omega}_{cm},\ \bar{\omega}_{fL} = 4.47\ \bar{\omega}_{CL}$$

$$\bar{\omega}_{cz} = 1.06,\ \bar{\omega}_{cm} = 0.26$$

$$g_z = g_y = g_m = g_L = 0.02,\ 0.002$$

Frequencies with subscript "c" denote cantilevered modes in which the hub degree of freedom is constrained but the remainder of the fuselage is free to move elastically. Frequencies with subscript "f" denote free modes for which neither the hub nor the fuselage is fixed. The parameters above are very close to those in Reference 8 (for comparison purposes) except for the parameters of inplane characters and offsets.

Results are presented for $g_y = g_z = 0.02$, 0.002, and $g_m = g_L = 0.02, 0.002$. Also shown are curves labeled "without feedback", which give the fixed-hub loads. As mentioned in Reference 8, for the coupled response, the natural frequency with the rotor is different from the frequency without the rotor.

The C_z curve ($g_z = 0.02$) in Fig. 3 is nearly identical to the corresponding curve in Reference 8. Therefore, the rigid, inplane degree of freedom does not affect vertical vibrations very much in the case considered. Figs. 4 and 5 show the lateral and longitudinal forces versus the fuselage bending frequency, which is assumed to be equal for vertical and lateral modes, $\bar{\omega}_{cz} = \bar{\omega}_{cy}$. It is seen that the lateral response is significant. The lateral response, therefore, can be an important consideration in helicopter dynamic design. Figs. 6 and 7 show that pitch and roll loads are not affected by the vertical vibration. Figs. 8-12 show the hub loads as a function of fuselage vertical frequency with a stiff inplane rotor and without offsets. The response is a little bit larger than that of soft inplane mentioned above, but the same conclusions hold.

Figs. 13-17 and Figs. 18-22 present the hub loads versus $\bar{\omega}_{cm} = \bar{\omega}_{cL}/1.18$, the pylon pitch and roll frequencies. Both the soft inplane and stiff inplane cases are shown. Because of aerodynamic coupling, all loads are affected by $\bar{\omega}_{cm}$ and $\bar{\omega}_{cL}$. For the smaller damping, $g_m = g_L = 0.002$, most of couplings are apparent (two resonant peaks), while at large damping they are less noticeable (one resonant peak).

Fig. 23 shows the effect of hub offsets on the vertical vibration. Comparison with Fig. 3 shows that there is little effect of hub offsets for plunge. For the pitch and roll modes, however, the effect of offsets is very significant, as shown in Figs. 24-28. (Compare with Figs. 18-22). In addition to the large change in magnitude due to the offsets, one notices that the resonance point is moved to approximately $\bar{\omega}_{cm} = 0.95$. The reason for this is that the rotor-fuselage coupling due to offsets ($\bar{h}$, $\bar{d}F$) shifts the fuselage natural frequency, so that the resonance with 4/rev is moved.

This phenomenon is illustrated in Fig. 29, which presents the fuselage natural frequency (without the rotor) vs. offsets $\bar{h}$ and $\bar{d}F$. Similarly, Figs. 30-31 show fuselage natural frequencies without the rotor vs. fuselage constrained vertical and pitch frequencies, respectively.

One can further appreciate that the rotor itself has an effect on the system frequencies, therefore, the 4/rev resonances in Figs. 29-31 do not exactly match the 4/rev resonances of the coupled rotor/body system. (See Reference 13 for details.) More calculations have been made, and one can find more figures in Reference 13. A few of the more interesting curves have been presented here.

Conclusions

The conclusions based on the assumptions and results of this study are:

1) Helicopter coupled rotor/fuselage vibrations with inplane degrees of freedom of both rotor and fuselage can be easily solved by harmonic balance and impedance matching and a single-blade analysis.

2) The addition of inplane degrees of freedom does not significantly affect the plunge vibrations for the cases considered, and these cases are for reasonable configurations.

3) The lateral response is significant, it should not be neglected in helicopter vibration analysis.

4) The hub offsets will significantly affect the coupled response.

Acknowledgment

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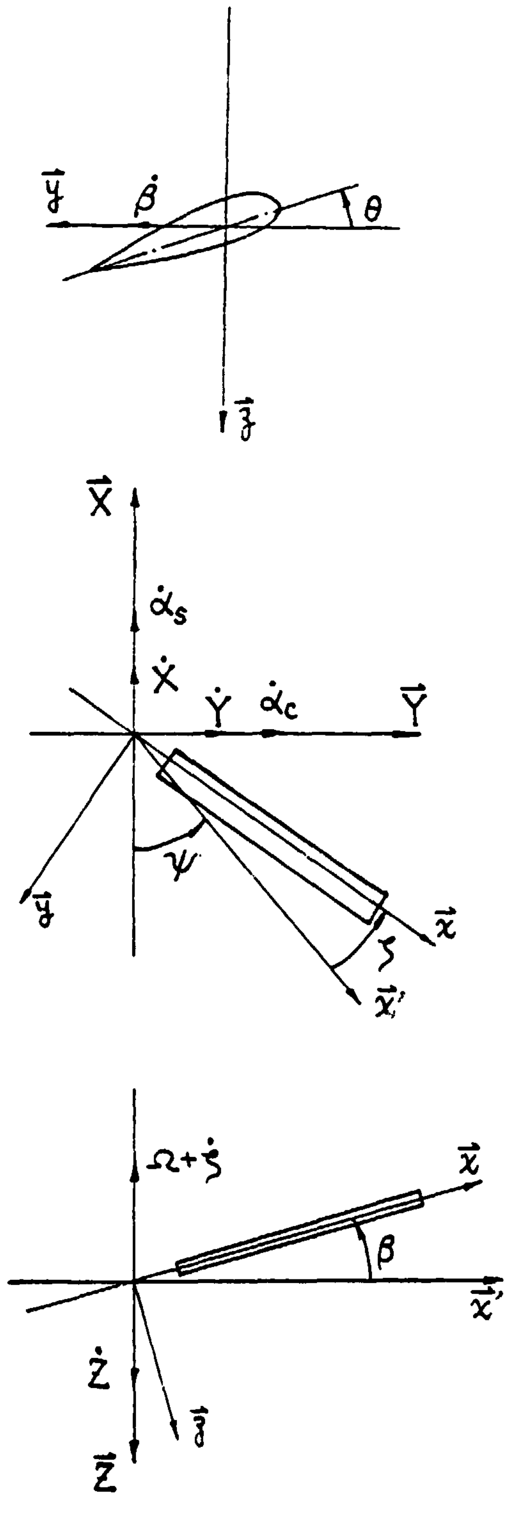

Figure 1. Rotor Model

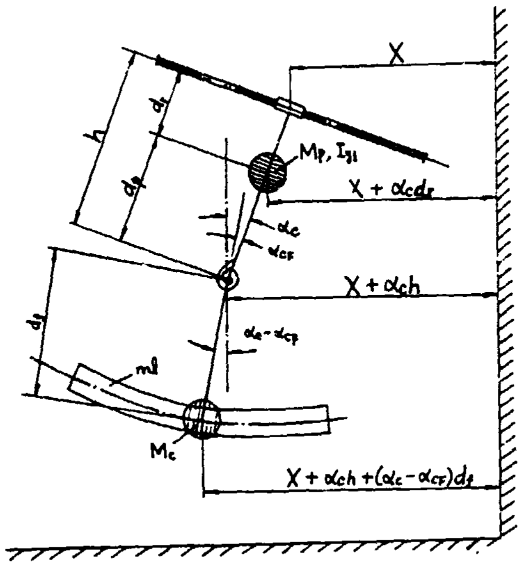

Figure 2. Fuselage Model in Longitudinal and Pitch Direction.

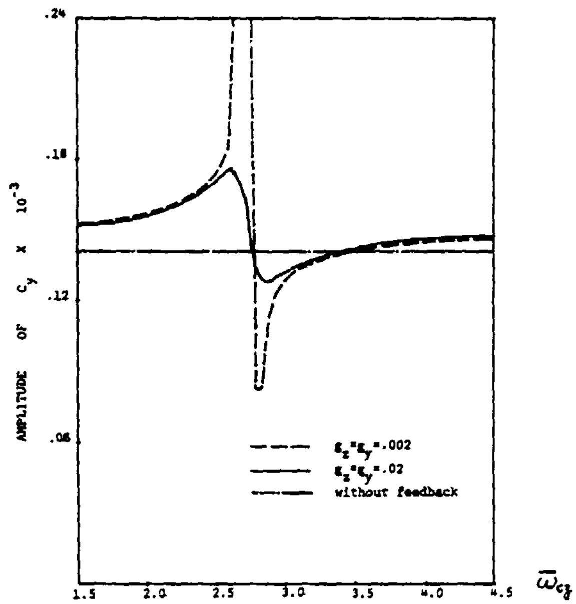

Fig. 4 4/rev lateral loads as a function of fuselage vertical constrained frequency $\omega_\zeta=.7$, $\bar{h}=\bar{d}_p=\bar{d}_f=0$

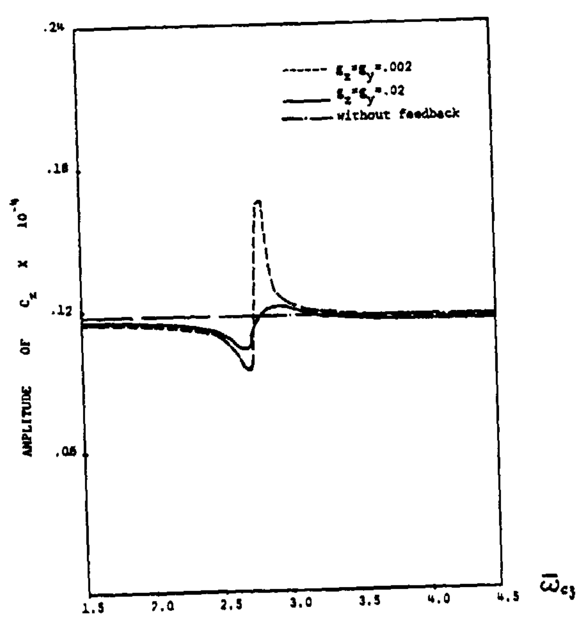

Fig. 3 4/rev vertical loads as a function of fuselage vertical constrained frequency $\omega_\zeta=.7$, $\bar{h}=\bar{d}_p=\bar{d}_f=0$

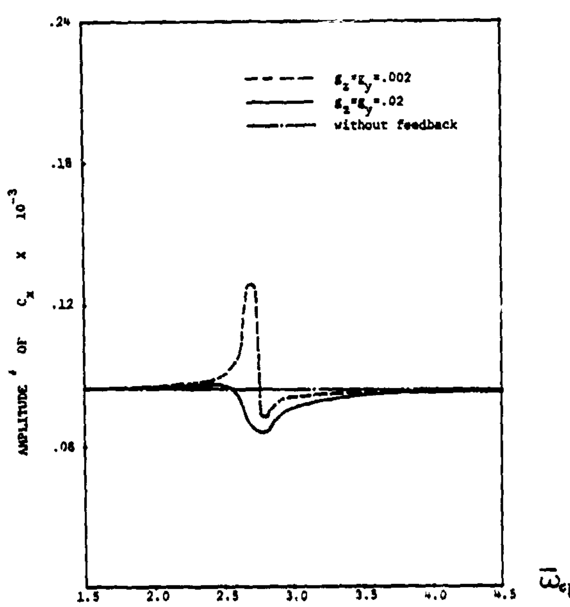

Fig. 5 4/rev longitudinal loads as a function of fuselage vertical constrained frequency $\omega_\zeta=.7$, $\bar{h}=\bar{d}_p=\bar{d}_f=0$

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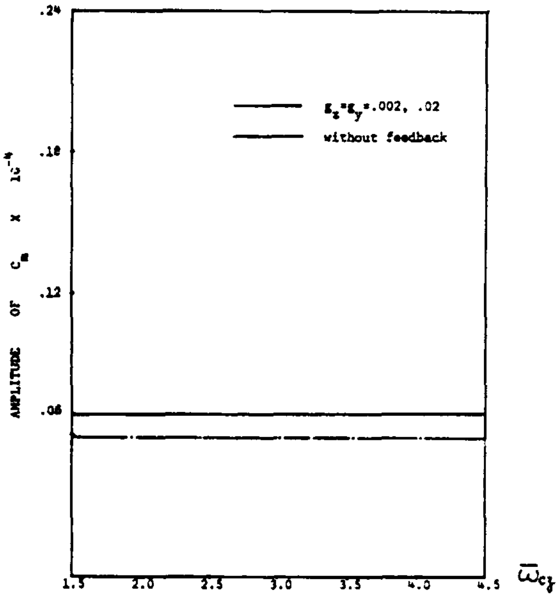

Fig. 6 4/rev pitch loads as a function of fuselage vertical constrained frequency $\omega_\zeta = .7$, $\bar{d}_p = \bar{d}_f = \bar{h} = 0$

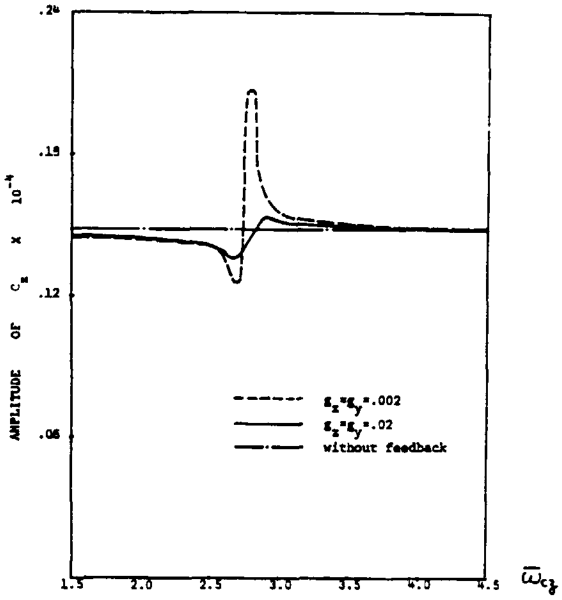

Fig. 8 4/rev vertical loads as a function of fuselage vertical constrained frequency $\omega_\zeta = 1.4$, $\bar{h} = \bar{d}_p = \bar{d}_f = 0$

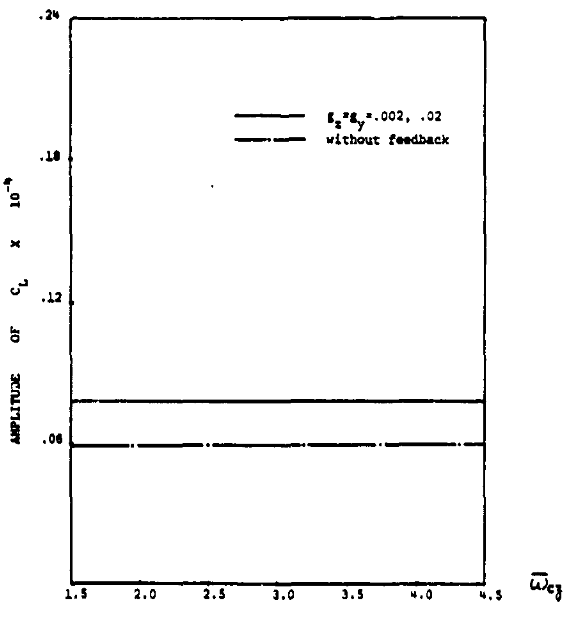

Fig. 7 4/rev roll loads as a function of fuselage vertical constrained frequency $\omega_\zeta = .7$, $\bar{h} = \bar{d}_p = \bar{d}_f = 0$

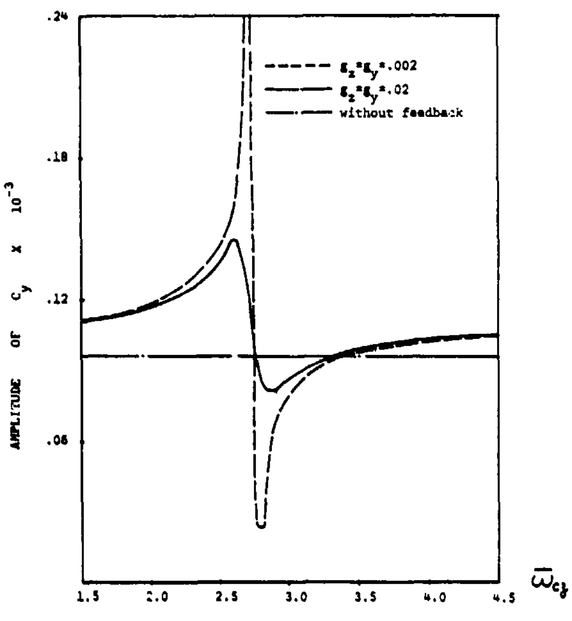

Fig. 9 4/rev lateral loads as a function of fuselage vertical constrained frequency $\omega_\zeta = 1.4$, $\bar{h} = \bar{d}_p = \bar{d}_f = 0$

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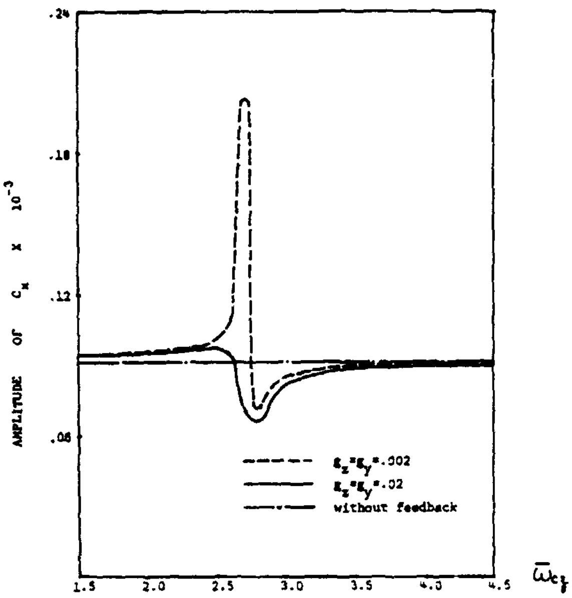

Fig. 10 4/rev longitudinal loads as a function of fuselage vertical constrained frequency $\omega_\zeta=1.4, \bar{h}=\bar{d}_p=\bar{d}_f=0$

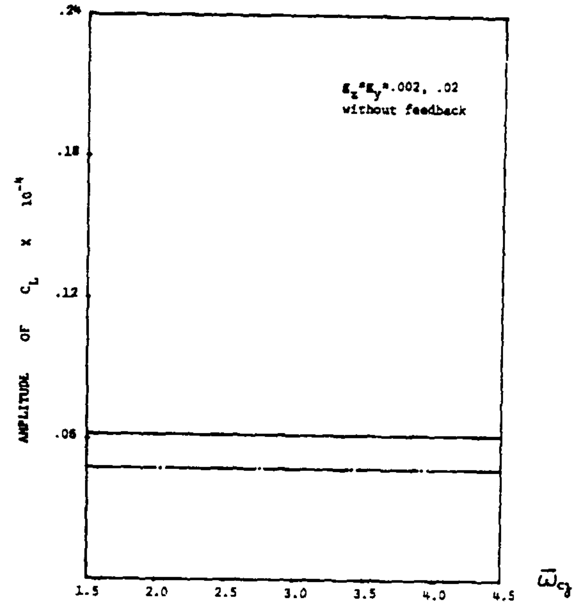

Fig. 12 4/rev roll loads as a function of fuselage vertical constrained frequency $\omega_\zeta=1.4, \bar{h}=\bar{d}_p=\bar{d}_f=0$

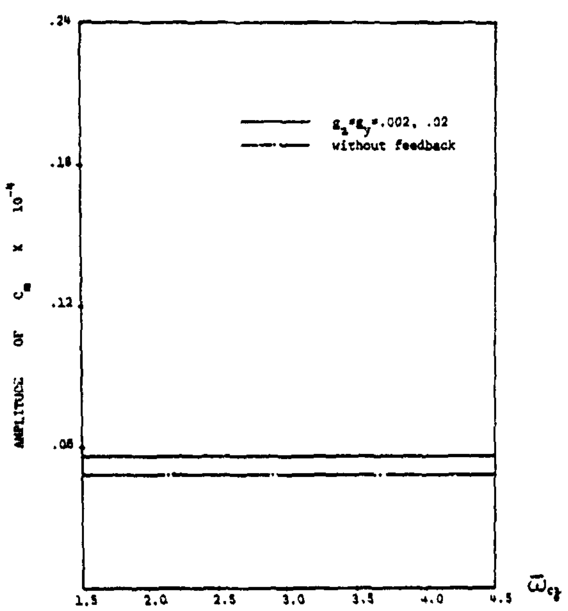

Fig. 11 4/rev pitch loads as a function of fuselage vertical constrained frequency $\omega_\zeta=1.4, \bar{h}=\bar{d}_p=\bar{d}_f=0$

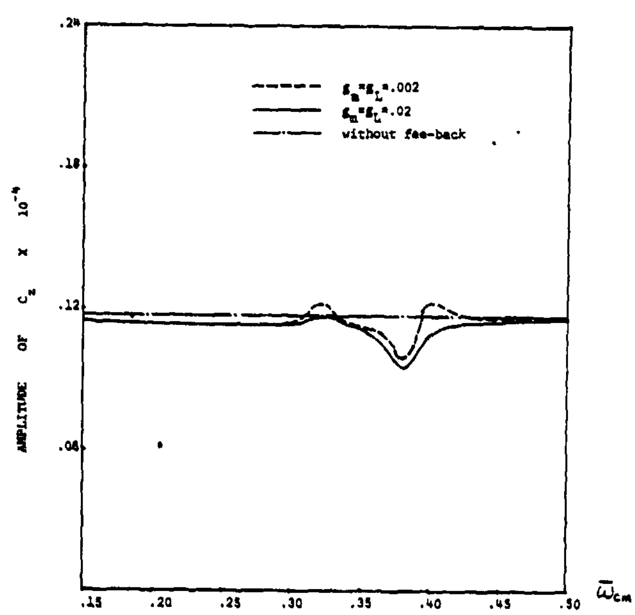

Fig. 13 4/rev vertical loads as a function of fuselage pitch constrained frequency $\omega_\zeta=.7, \bar{h}=\bar{d}_p=\bar{d}_f=0$

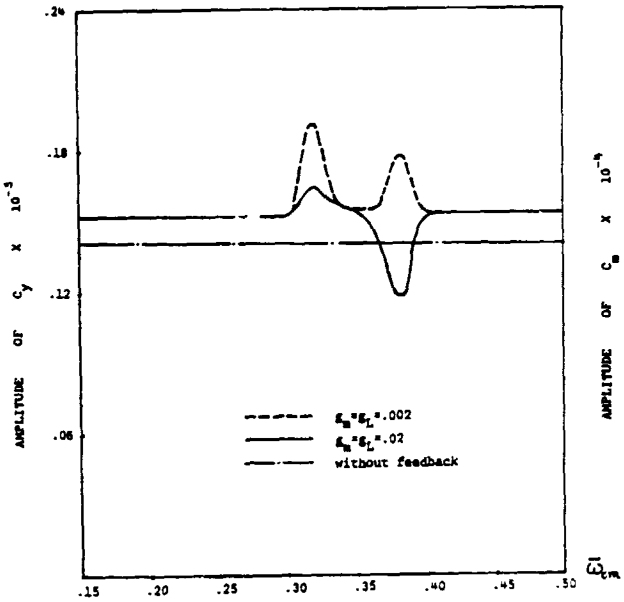

Fig. 14 4/rev lateral loads as a function of fuselage pitch constrained frequency
$\omega_\xi = .7$, $\bar{h} = \bar{d}_p = \bar{d}_f = 0$

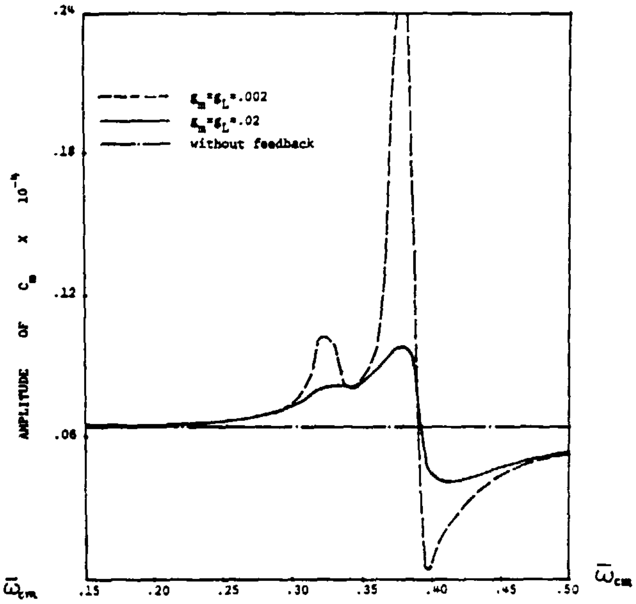

Fig. 16 4/rev pitch loads as a function of fuselage pitch constrained frequency
$\omega_\xi = .7$, $\bar{h} = \bar{d}_p = \bar{d}_f = 0$

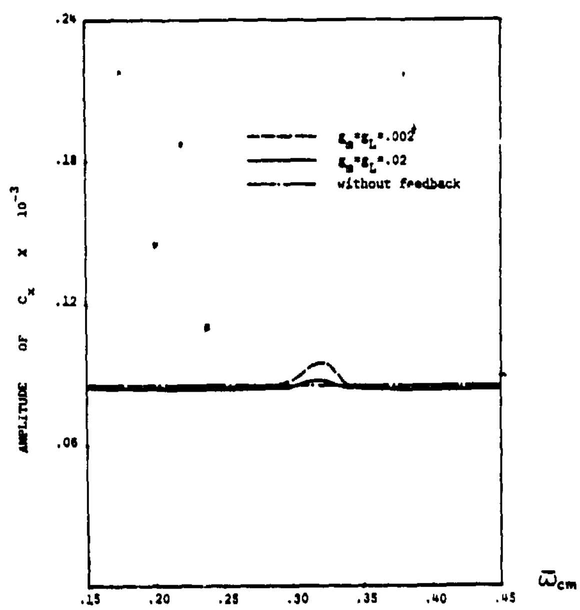

Fig. 15 4/rev longitudinal loads as a function of fuselage pitch constrained frequency
$\omega_\xi = .7$, $\bar{h} = \bar{d}_p = \bar{d}_f = 0$

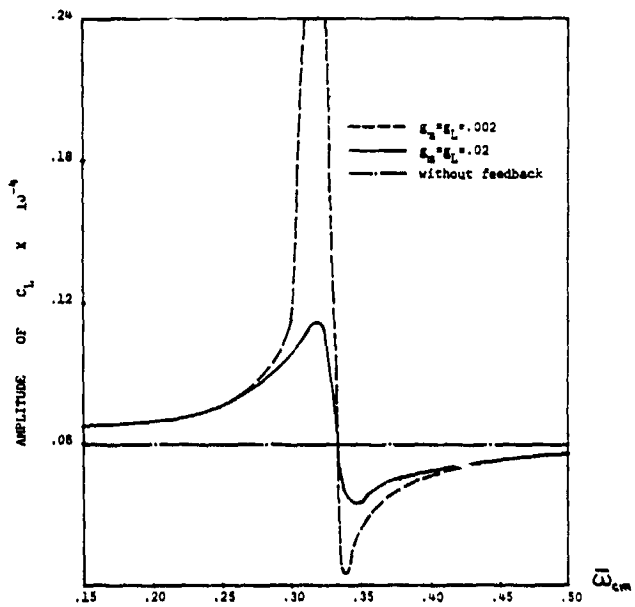

Fig. 17 4/rev roll loads as a function of fuselage pitch constrained frequency
$\omega_\xi = .7$, $\bar{h} = \bar{d}_p = \bar{d}_f = 0$

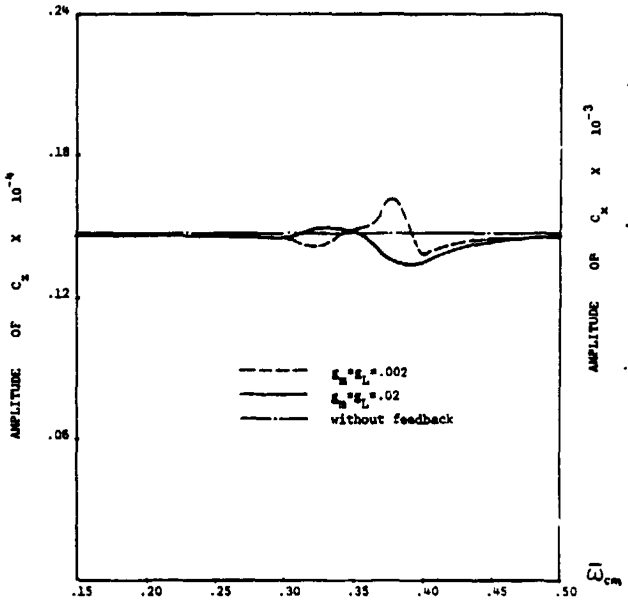

Fig. 18 4/rev vertical loads as a function of fuselage pitch constrained frequency
$\omega_f = 1.4$, $\bar{h} = \bar{d}_p = \bar{d}_f = 0$

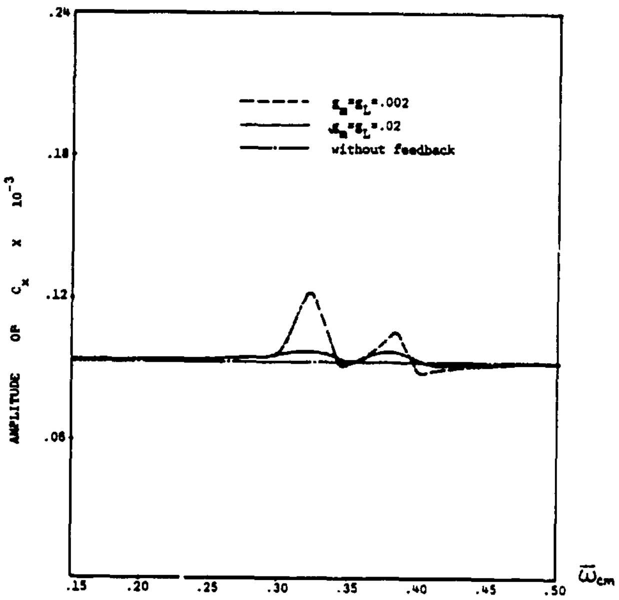

Fig. 20 4/rev longitudinal loads as a function of fuselage pitch constrained frequency
$\omega_f = 1.4$, $\bar{h} = \bar{d}_p = \bar{d}_f = 0$

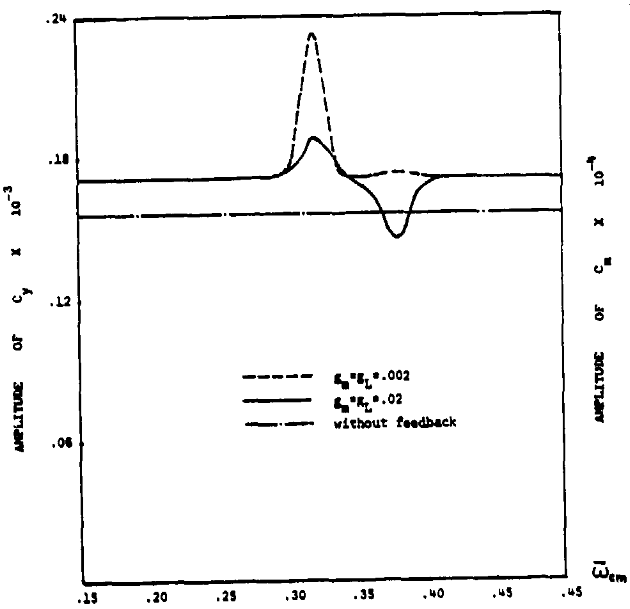

Fig. 19 4/rev lateral loads as a function of fuselage pitch constrained frequency
$\omega_f = 1.4$, $\bar{h} = \bar{d}_p = \bar{d}_f = 0$

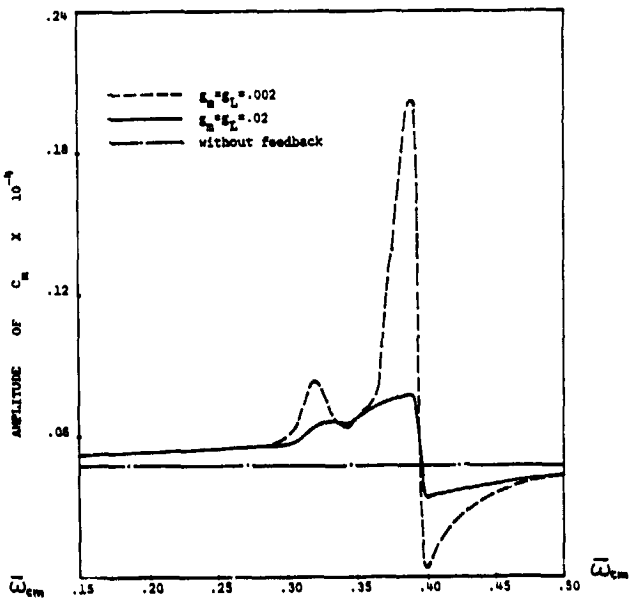

Fig. 21 4/rev pitch loads as a function of fuselage pitch constrained frequency
$\omega_f = 1.4$, $\bar{h} = \bar{d}_p = \bar{d}_f = 0$

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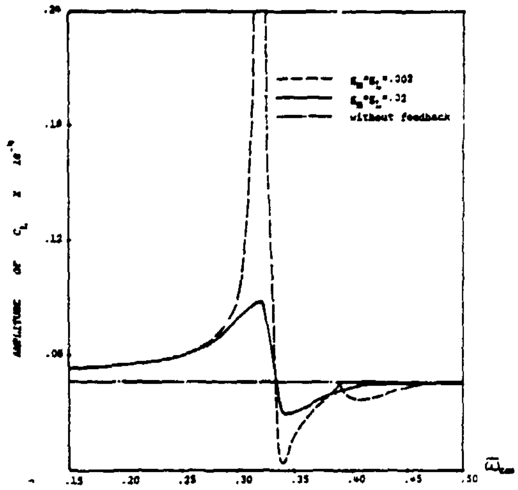

Fig. 22 4/rev roll loads as a function of fuselage pitch constrained frequency
$\omega_\psi = 1.4$, $\bar{h} = \bar{d}_p = \bar{d}_f = 0$

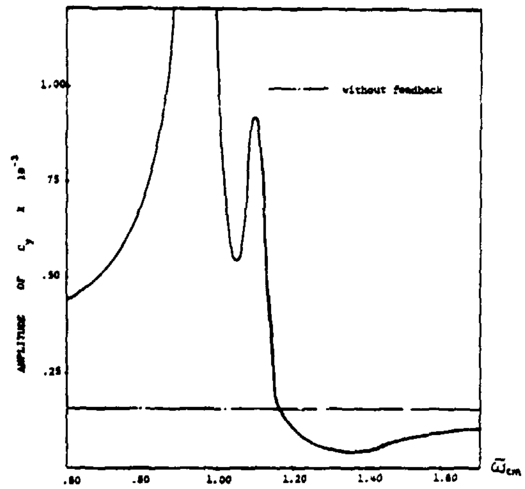

Fig. 25 4/rev lateral loads as a function of fuselage pitch constrained frequency
$\omega_\psi = 1.4$, $\bar{h} = .4$, $\bar{d}_p = 0$, $\bar{d}_f = .2$, $g_m = g_L = .002$

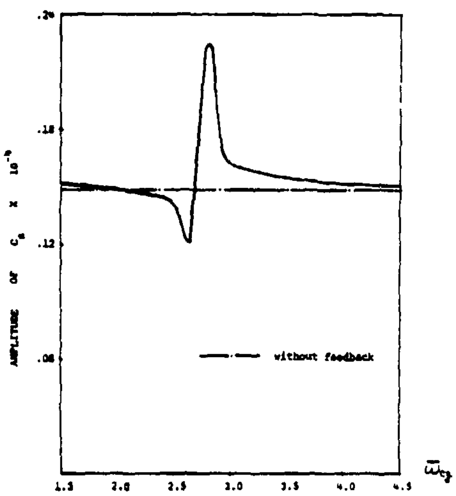

Fig. 23 4/rev vertical loads as a function of fuselage vertical constrained frequency
$\omega_\psi = 1.4$, $\bar{h} = .4$, $\bar{d}_p = 0$, $\bar{d}_f = .2$, $g_z = g_y = .002$

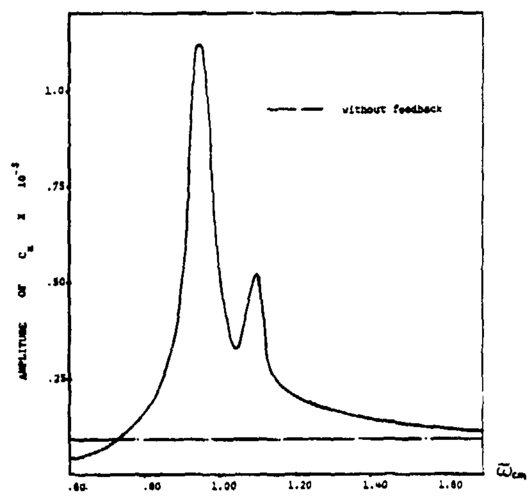

Fig. 26 4/rev longitudinal loads as a function of fuselage pitch constrained frequency
$\omega_\psi = 1.4$, $\bar{h} = .4$, $\bar{d}_p = 0$, $\bar{d}_f = .2$, $g_m = g_L = .002$

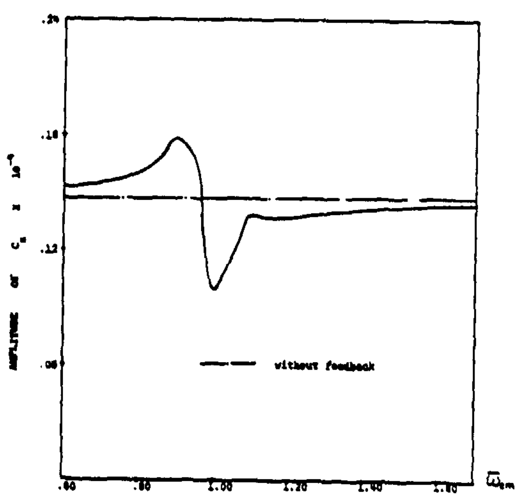

Fig. 24 4/rev vertical loads as a function of fuselage pitch constrained frequency
$\omega_\psi = 1.4$, $\bar{h} = .4$, $\bar{d}_p = 0$, $\bar{d}_f = .2$, $g_m = g_L = .002$

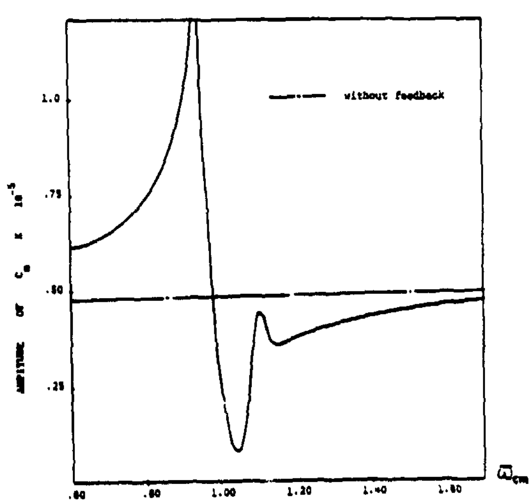

Fig. 27 4/rev pitch loads as a function of fuselage pitch constrained frequency
$\omega_\psi = 1.4$, $\bar{h} = .4$, $\bar{d}_p = 0$, $\bar{d}_f = .2$, $g_m = g_L = .002$

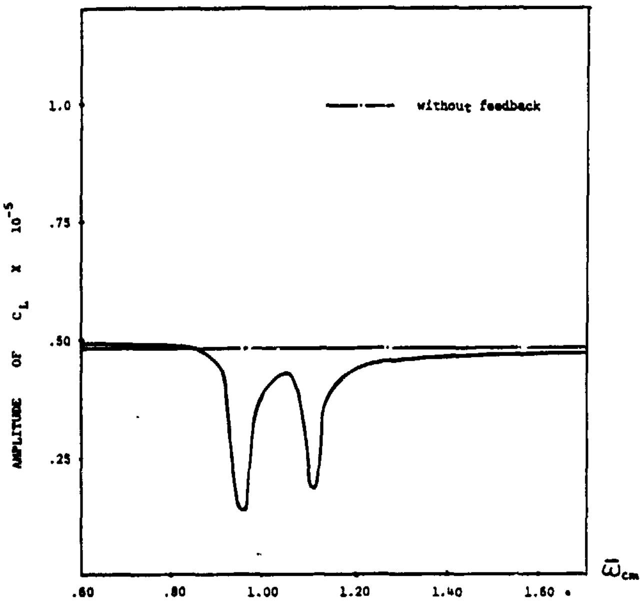

Fig. 28 4/rev roll loads as a function of fuselage pitch constrained frequency
$\bar{\omega}_{cy}$=1.4, $\bar{h}$=.3, $\bar{d}_p$=0, $\bar{d}_f$=.2, g_m=g_L=.002

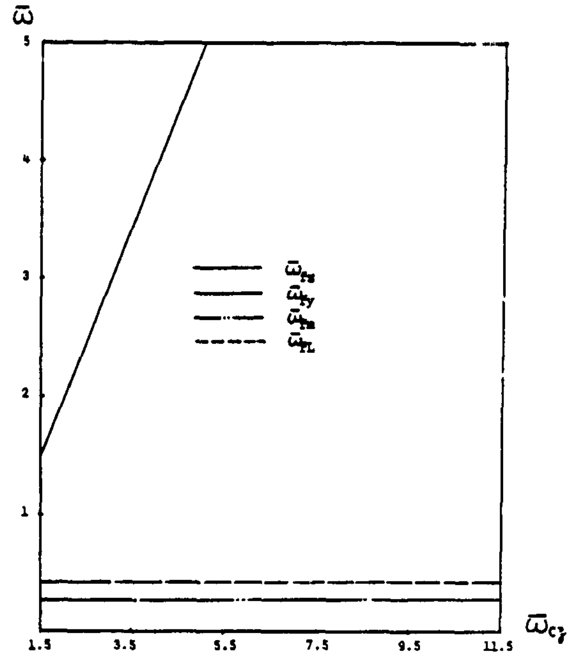

Fig. 30 fuselage natural frequency without rotor vs fuselage constrained vertical frequency

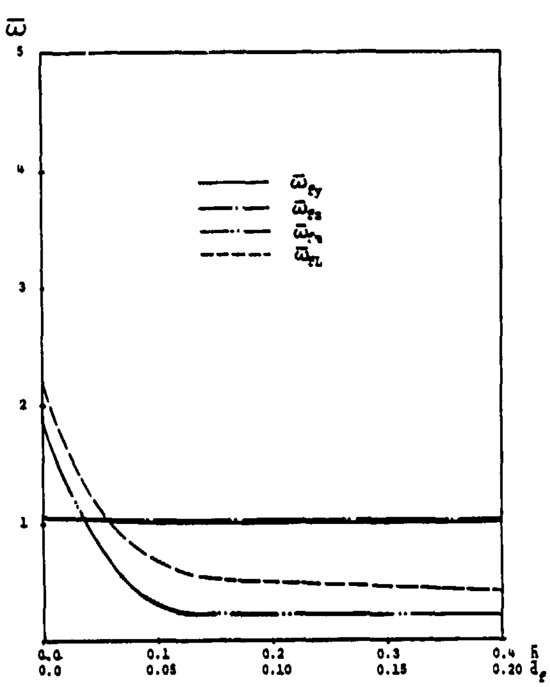

Fig. 29 fuselage natural frequency without rotor vs offsets $\bar{h}$, $\bar{d}_f$

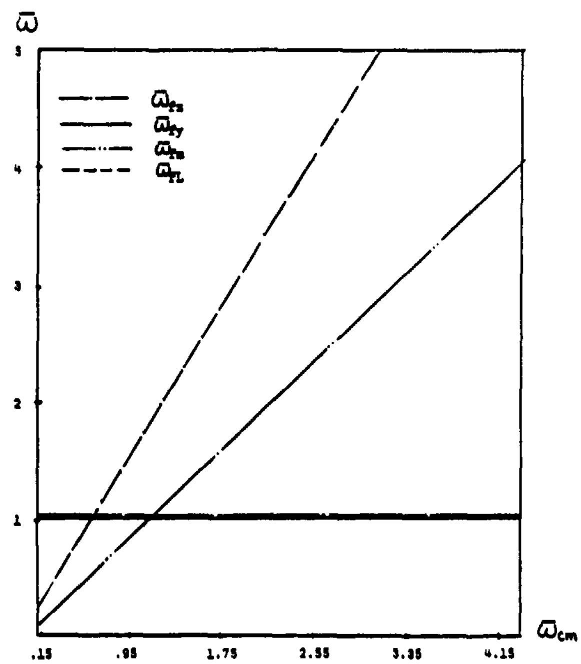

Fig. 31 fuselage natural frequency without rotor vs fuselage constrained pitch frequency

DISCUSSION
Paper No. 21

COUPLED ROTOR-BODY VIBRATIONS WITH INPLANE DEGREES OF FREEDOM
Huang Ming-Sheng
and
David A. Peters

Dev Banerjee, Hughes Helicopters: Dave, I'm glad to see a concerted effort at doing impedance matching at the hub and coupling the rotor with the fuselage. I think that's an important contribution to determining hub loads and hence fuselage vibrations. I'd like to go back to the 1964 paper of Gerstenberger and Wood. I think the displacement formulation approach that you've taken would require adding additional hub motion as degrees of freedom. However, if you take the mixed formulation approach as taken by Gerstenberger and Wood, that'll all come out as part of the solution. In other words your 6X6 complex hub-impedance matrix which is the exact hub coupling of the rotor with the fuselage would be included in the solution of the problem.

Peters: It would solve the whole problem at once.

Banerjee: Exactly.

Peters: There's nothing wrong with that, except you lose the advantage of making small changes to the fuselage at a very cheap computational cost [since] you have to do the whole problem. Another thing, remember the rotor impedance now is more complicated than normal rotor impedance because of the periodic coefficients. Now you have four per rev due to 4 per rev, and four per rev due to 8 per rev. If you had read Tom Hshu's original paper, he's got a whole section dedicated to figuring out how all these sines and cosines and phases come together. It's a big job.

Bob Loewy, Rensselaer Polytechnic Institute: Dave, I want to add my voice raised in praise for your work here. I think it's excellent and you're making a major contribution to helicopter vibrations in this. Maybe I should stop there, but I can't resist the urge to play "Trivial Pursuit." Just sort of really as a historical curiosity: the first time I ever saw a rotor impedance derivation, it was in the work of Alexander Flax--some of you may remember--and this was dated in the late 40s.

Peters: Oh, I'd love to have a copy of that or get the reference.

Loewy. It was never published as far as I know, and I wouldn't want you to think I was there, but I found it in some of the old Piasecki Helicopter Company literature. What he did was, he was solving a drive system vibration problem, and he derived the polar moment of inertia impedance of a rotor. It's interesting that John Burkham, as far as I know, was the first one to do an inplane impedance with a rigid hinged blade, and if you took his impedance expression and put it on a mass on a spring and then ran the equations out, you found that you got the ground resonance equations. As a third point of this kind, Bob Yntema then took blades which were flexible and derived impedances in all directions, for twisted blades as well as untwisted blades. And I remember being amazed to see that in those expressions, even though you shook inplane, you got flapping deflections of the blades, of course, because they were twisted. None of those included aerodynamics, but they were very early efforts in rotor impedance calculation.

Peters: Oh, I'd love to have those. Why don't you write them down on a piece of paper for me and let me go run them down?

Loewy: Sure will.

Don Kunz, U.S. Army Aeromechanics Laboratory: Dave, when you were doing your presentation, I was wondering if you were linearizing your equations. At the end you said you did--would you explain what you did?

Peters: Yes, on the very first slide where I showed the blade equations, those were already linearized. Since we're running a trimmed condition, that means there's no β_s and no β_c, we linearized about a steady coning angle. So the very first flapping equations up there are linearized, and that's why β, that steady coning angle appears as a forcing function. Now, if we weren't trimmed, then we'd have to linearize about a periodic equilibrium including the β_s and β_c.

Bob Wood, Hughes Helicopters: Dave, I just wanted to comment--I thought it was particularly interesting, your fuselage model and the fact that you could study the parameters and move that on. I wanted to add just one point to it, and that is what a number of us are looking at right now, which ties your paper really together somewhat with Dick Gabel's [paper]. If you think about it, if you're interested purely in getting the forced response in detail for a [production] helicopter, with dynamic NASTRAN now it's extremely simple to calculate that hub impedance matrix, just by putting in the three-unit loads and the three-unit moments. [You can then] solve the combined problem and then [combine] by superposition the appropriate NASTRAN responses.

Peters: And just match that to your rotor impedance and see what happens.

Wood: So in other words, a full dynamic NASTRAN model, such as Dick has, can be treated relatively easily.

Bob Taylor, Boeing Vertol: I'd just like to comment that I wouldn't want to use that in a preliminary design study. I'd much rather depend upon something like Dave has here; but your point is well taken, Bob.