Computation of the Radiation Characteristics of a Generalized Phased Array

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SUMMARY

With the advent of monolithic microwave integrated circuit (MMIC) technology, the phased array has become a key component in the design of advanced antenna systems. Array-fed antennas are used extensively in today's multiple beam satellite antennas. In this report, a computer program based on a very efficient numerical technique for calculating the radiated power (Romberg integration), directivity and radiation pattern of a phased array is described. The formulation developed is very general, and takes into account arbitrary element polarization, E and H-plane element pattern, element location, and complex element excitation. For comparison purposes sample cases have been presented. Excellent agreement has been obtained for all cases. Also included in appendixes A and B are a user guide and a copy of the computer program.

INTRODUCTION

One of the most important radiation characteristics of an antenna system is the directivity. Accurate determination of this parameter is essential for the analysis and design of advanced antenna systems. In general, for most commonly used array elements such as open-ended waveguides, horns, or microstrip patch antennas, an analytical expression of the \((\cos \theta)^q\)-type can be used to properly tailor actual element patterns (ref. 1). Experimentally measured element patterns could include mutual coupling, which may be significant in large arrays. Expressions for the directivity and array radiation pattern of a single element and a rectangular array using \((\cos \theta)^q\)-type element patterns have been reported (refs. 2 to 11).

It is the purpose of this work to generalize these results and obtain an efficient numerical technique for computing the directivity and the antenna radiation pattern of the generalized array. The generalized array characteristics used in this report includes arbitrary element location, element pattern \((\cos \theta)^q\)-type, other analytically describable functions or experimentally measured), element polarization and element excitation.

ARRAY RADIATION PATTERN

The geometry of the generalized array is shown in figure 1. Given the array geometry and element characteristics, the generalized array problem can be defined as: (1) to determine the power radiated and directivity at a given observation point (this is usually taken in the far-field zone), (2) to determine the co-polarization and cross-polarization component of the electric field (using Ludwig's criterion (ref. 12)). In solving this problem, two sets of coordinate system are used. Figure 2 shows a typical element coordinate system and the reference coordinate system, with the \(z\)-axis in the same
direction. For an array of $M$ elements located arbitrarily in the reference coordinate system (fig. 1), the $m$th element radiated field is given by equation A1.1:

$$
\vec{E}_m(\vec{r}_m) = I_m \left[ \frac{e^{-jkr_m}}{r_m} \hat{r}' U_{Em}(\theta') \left( a_m e^{j\psi_m} \cos \varphi' + b_m \sin \varphi' \right) \right.
\left. + \hat{\varphi}' U_{Hm}(\theta') \left( -a_m e^{-j\psi_m} \sin \varphi' + b_m \cos \varphi' \right) \right]
$$  \hspace{1cm} (A1.1)

for $0 \leq \theta' < \pi/2$, where

- $I_m$ $m$th element complex excitation coefficient
- $U_{Em}, U_{Hm}$ $m$th element $E$ and $H$ plane pattern
- $\hat{a}_m, b_m, \psi_m$ $m$th element polarization parameters (see table I)
- $K$ wave number $2\pi/\lambda$
- $r_m, \theta', \varphi'$ spherical coordinates in the element coordinate system

The element pattern $U_{Em}, U_{Hm}$ can be described with an analytical expression $((\cos \theta)q$-type or other functions) or with experimentally measured data (discrete). If measured data are used, the pattern may include mutual coupling effects. The polarization parameters in table I are subject to the normalization described by

$$a_m + b_m = 1 \hspace{1cm} (A1.2)$$

The electric field described by equation (A1.1) is in the element coordinate system. The total electric field due to all $M$ elements is the superposition of the electric field of each element of the array. The total electric field is given by

$$\vec{E}(\vec{r}) = \sum_{m=1}^{M} \vec{E}_m(\vec{r}) \hspace{1cm} (A1.3)$$

where the vector fields $\vec{E}(\vec{r})$ and $\vec{E}_m(\vec{r})$ are defined in the reference coordinate system. A transformation of $\vec{E}_m(\vec{r}_m)$ (eq. (A1.1)) in the element coordinate system into $\vec{E}_m(\vec{r})$ in the reference coordinate system is described next. Figure 3 shows a detailed description of these coordinate systems. The transformation of coordinates for this problem only involves a translation. The transformation procedure is outlined as follows. Knowing the observation coordinates $(r, \theta, \varphi)$ and $m$th element location $(x_m, y_m, z_m)$, the observation point in the primed coordinate system is found by using:

$$
x = r \sin \theta \cos \varphi \\
y = r \sin \theta \sin \varphi \\
z = r \cos \theta \hspace{1cm} (A1.4a)$$

$$
x' = x - y_m \\
y' = y - y_m \\
z' = z - z_m \hspace{1cm} (A1.4b)$$
\[
\begin{align*}
    r_m &= \sqrt{x^2 + y^2 + z^2} \\
    \theta' &= \cos^{-1} \frac{z'}{r_m} \\
    \phi' &= \tan^{-1} \frac{y'}{x'}
\end{align*}
\] (A1.4c)

With equation (A1.4c) computed, all parameters on equation (A1.1) can be calculated. The vector transformation is obtained by using:

\[
\begin{bmatrix}
    E_{Rm} \\
    E_{\Theta m} \\
    E_{\Phi m}
\end{bmatrix}
= \begin{bmatrix}
    \sin \theta \cos \phi & \sin \theta \sin \phi & \cos \theta \\
    \cos \theta \cos \phi & \cos \theta \sin \phi & -\sin \theta \\
    -\sin \phi & \cos \phi & 0
\end{bmatrix}
\begin{bmatrix}
    \sin \theta' \cos \phi' & \cos \theta' \cos \phi' & -\sin \phi' \\
    \sin \theta' \sin \phi' & \cos \theta' \sin \phi' & \cos \phi' \\
    \cos \theta' & -\sin \theta' & 0
\end{bmatrix}
\begin{bmatrix}
    0 \\
    E_{\Theta m} \\
    E_{\Phi m}
\end{bmatrix}
\] (A1.5)

where \( I \) is the identity matrix (3 x 3).

Equation (A1.5) transformed \( \vec{E}_m(r_m) \) into \( \vec{E}_m(r) \) in the reference coordinate system. This process is repeated for each array element. Notice that no constraints have been put into equation (A1.3) regarding the observation distance. This expression (eq. (A1.3)) is valid everywhere except at the location of the source itself. This formulation assumes that each pattern is boresighted in the +z direction. However, the identity matrix \( I \) in equation (A1.5) could be replaced by a rotation matrix (Euler matrix of transformation) to account for arbitrary pointing.

The array radiation pattern is usually divided into two orthogonal polarizations. Equation (A1.3) may be written:

\[
\vec{E}(r) = \sum_{m=1}^{M} \vec{E}_m(r) = \sum_{m=1}^{M} E_{\Theta m}(r) \vec{\theta} + \sum_{m=1}^{M} E_{\Phi m}(r) \vec{\phi}
\] (A1.5a)

which can be expressed as:

\[
\vec{E}(r) = E_{\theta} \vec{\theta} + E_{\phi} \vec{\phi}
\] (A1.5b)

The orthogonal components described in equation (A1.5b) are the usual spherical components. Another way of dividing the electric field into two orthogonal polarization is by using Ludvig's definition 3 (ref. 12). The following polarization vectors are introduced:
\[
\hat{R} = \hat{\theta} \left( a e^{j\psi} \cos \varphi + b \sin \varphi \right) + \varphi \left( -a e^{-j\psi} \sin \varphi + b \cos \varphi \right) \tag{A1.5c}
\]
\[
\hat{C} = \hat{\theta} \left( a e^{-j\psi} \sin \varphi - b \cos \varphi \right) + \varphi \left( -a e^{j\psi} \cos \varphi + b \sin \varphi \right) \tag{A1.5d}
\]

The reference polarization and cross polarization expressions of \( E(r) \) are:

Reference polarization of \( \hat{E} \): \( E_R = \hat{E} \cdot \hat{R}^* \) \tag{A1.5e}

Cross polarization of \( \hat{E} \): \( E_C = \hat{E} \cdot \hat{C}^* \) \tag{A1.5f}

With these expressions equation (A1.5a) can be rewritten as:

\[
\bar{E}(\overline{r}) = E_R \hat{R} + E_C \hat{C} \tag{A1.5g}
\]

The parameters \( a, b, \) and \( \psi \) can be obtained from table I.

**POWER RADIATED**

The total power radiated (time-averaged) by the array is given by:

\[
P_{\text{rad}} = \oint_S \text{Re}(\overline{E} \times \overline{H}^*) \cdot ds \tag{B.1}
\]

where

\( \nabla \times \overline{E} = -j \omega \mu_0 \overline{H} \) Maxwell equation

\( ds = \hat{r} r^2 \sin \theta \, d\theta \, d\phi \) differential surface area

\( S \) a sphere of radius \( r \)

In the far-field of the array (usually taken at \( 2\Delta^2/\lambda, \Delta \) is maximum array dimension), the power radiated given in equation (B.1) can be simplified to:

\[
P_{\text{rad}} = \oint_S \frac{\bar{E}(\overline{r}) \cdot \bar{E}^*(\overline{r})}{2 \sin \theta} r^2 \sin \theta \, d\theta \, d\phi \tag{B.2}
\]

(\( Z_0 \) is free space impedance)

Substituting equation (A1.3) into equation (B.2) gives:

\[
P_{\text{rad}} = \int_0^{2\pi} \int_0^{\pi/2} \left( \sum_{m=1}^{M} \bar{E}_m(\overline{r}) \right) \cdot \left( \sum_{n=1}^{M} \bar{E}_n(\overline{r}) \right)^* \frac{r^2}{Z_0} \sin \theta \, d\theta \, d\phi \tag{B.3}
\]

In general, the above expression does not have a closed form solution and is evaluated numerically using a Romberg integration algorithm (ref. 13).
Using far-field approximations, equation (A1.1) can be simplified as follows: (This will restrict the observation distance to be only in the far-field of the array.)

\[
\hat{\mathbf{E}}(\mathbf{r}) = \sum_{m=1}^{M} \hat{\mathbf{e}}_m \cdot \hat{\mathbf{e}}_m^* \quad (B.4)
\]

where

- \( \hat{\mathbf{e}}_m \) is the unit vector,
- \( \sin \theta \cos \phi \hat{x} + \sin \theta \sin \phi \hat{y} + \cos \theta \hat{z} \) is the position vector,
- \( \mathbf{r} \) is the position vector, \( x \hat{x} + y \hat{y} + z \hat{z} \),
- \( \hat{\mathbf{e}}_m \) is the array factor component,
- \( \hat{\mathbf{e}}_m^* \) is the conjugate of the array factor component,
- \( \mathbf{r} \) is the observation distance.

Substituting equation (B.4) into equation (B.2) produces:

\[
P_{\text{rad}} = \sum_{m=1}^{M} \sum_{n=1}^{M} I_m^* I_n \left( \frac{1}{2 \pi} \int_0^{2\pi} \int_0^{\pi/2} A(\theta, \phi) e^{j \mathbf{k} \cdot \left( \mathbf{r}_m - \mathbf{r}_n \right)} \sin \theta \, d\theta \, d\phi \right)
\]

where

- \( A(\theta, \phi) = \left[ \left( U_{Em}^* U_{En} a_m a_n \right) e^{j (\psi_m - \psi_n)} + U_{Hm}^* U_{Hn} b_m b_n \cos^2 \phi + \left( U_{Em}^* U_{En} b_m b_n \right) \right] \)
- \( \left( U_{Em}^* U_{En} - U_{Hm}^* U_{Hn} \right) a_m b_n e^{j \psi_m - \psi_n} \sin^2 \phi + \left( U_{Em}^* U_{En} \right) a_m b_n \sin \phi \cos \phi \)

By defining

\[
R_{mm} = \frac{1}{2 \pi} \int_0^{2\pi} \int_0^{\pi/2} A(\theta, \phi) e^{j \mathbf{k} \cdot \left( \mathbf{r}_m - \mathbf{r}_n \right)} \sin \theta \, d\theta \, d\phi \quad (B.7)
\]
the equation (B.6) can be expressed in the matrix form:

\[ P_{\text{rad}} = \sum_{m=1}^{M} \sum_{n=1}^{M} R_{mn} I_m I_n^* \]  

(B.8)

The coefficient \( R_{mn} \) in equation (B.7) is a \( M \times M \) matrix. The evaluation of \( R_{mn} \) is time consuming and it takes the most computer time in the analysis. Reference 10 shows a closed form solution to equation (B.7) for special case of the array element located in the x-y plane having identical polarization parameters.

DIRECTIVITY

The directivity is defined by

\[ D(\theta, \phi) = \frac{4\pi E(\hat{r}) \cdot E^*(\hat{r})}{Z_0} \frac{r^2}{\rho_{\text{rad}}} \]  

(C.1)

\( D(\theta, \phi) \) is known as total directivity. Also the reference directivity and the cross directivity can be easily obtained (ref. 14).

Reference directivity:

\[ D_R(\theta, \phi) = \frac{4\pi |E \cdot R|^2}{Z_0} \frac{r^2}{\rho_{\text{rad}}} \]  

(C.1a)

Cross directivity:

\[ D_C(\theta, \phi) = \frac{4\pi |E \cdot C|^2}{Z_0} \frac{r^2}{\rho_{\text{rad}}} \]  

(C.1b)

NUMERICAL RESULTS AND DISCUSSIONS

This section presents some numerical results to demonstrate the applications of the computer program. In order to substantiate the accuracy of the generalized array formulation and computer program, detailed comparisons were made with the results presented by King and Wong (ref. 2) and experimental data obtained at NASA Lewis (ref. 15). King and Wong reported on an \( N \times N \) planar array configuration with symmetrical element patterns of the \( \cos (\theta)q \)-type. They used direct integration to compute the radiated power. The examples considered were an array of \( 2 \times 2 \) elements for various element spacings and a \( 3 \times 3 \) array for which element pattern and frequency were varied. In the NASA Lewis experimental case, a \( 2 \times 2 \) array of rectangular
horns was used. In this case element pattern were measured in the array environment (to account for mutual coupling).

Very good agreement was obtained in all cases. The 2 x 2 array reported by King and Wong assumed a symmetrical E-H-plane patterns with a $q = 3.54$. Figure 4 shows a graphical description of the directivity as a function of element spacing for this array. The 3 x 3 array example used a symmetrical pattern but the performance is described as a function of element pattern, $(\cos (\theta)q$, varying $q$) and operating frequency. Table II shows the results from both approaches. In the NASA Lewis experimental case the 2 x 2 array was investigated relative to far-field patterns. These patterns were measured at three different scan angles (boresight, 3° and 5°). The element spacing and frequency were fixed ($S = 2.5 \lambda$, frequency = 30 GHz). These results are presented in figures 4(a) to (e).

A user guide for the programs developed is presented in appendix A. The implementation of equations (A1.3), (B.8), and (C.1) (antenna pattern, power, and directivity, respectively) with a computer program is given in appendix B. This program (appendix B) can be easily interfaced with available plotting routines for displaying the far-field antenna patterns. The numerical technique used to solve for equation (B.8) is not unique in any sense, but it was found to be faster than just using direct integration. Many other techniques can also be used, and easily implemented in the computer program (appendix B).

CONCLUDING REMARKS

One of the advantages of this generalized array formulation is that it does not break-down for special cases as might occur in approximations using closed forms. Also the formulation developed does not limit the pattern observation to the far-field zone. This can be very useful if the generalized formulation is going to be used with analysis programs for dual reflector configurations. The program developed can be easily modified to be implemented in the analysis of reflectors with phased array feeds.

This computer program is one of the key research tools at the NASA Lewis for analyzing advanced space communication antenna systems. The generalized formulation and computer program provides complete flexibility in analysis of array configurations and in the accurate analysis of experimental data.
Program Description

Given an angle \( \phi \) and an array of sources, each with its current magnitude and relative phase, this program calculates the cross polarization, reference polarization, and the far field magnitude for a series of angles \( \theta \).

Input (FT05)
- \( X,Y,Z \): coordinates of each source (in meters).
- \( \text{AMPL} \): current amplitude for each source (in amperes).
- \( \text{APHA} \): relative phase for each source (in radians).
- \( H \): total number of sources.
- \( \text{POL} \): denotes polarization
  - 1: linear polarization (x-direction)
  - 2: linear polarization (y-direction)
  - 3: right-hand circular polarization
  - 4: left-hand circular polarization
- \( \text{QE}, \text{QH} \): exponent of the cosine function that is used to approximate the element pattern in the analytic form.
- \( \text{INZT} \): interval between each elevation angle \( \theta \).
- \( I \): \( 1+\phi \), parameter to set for desired cut.

Extraneous Input: (to be changed accordingly)
- \( \text{line 2000 A\text{WAVE}=3E8/(frequency)} \)
- \( \text{line 2500 ZETA=}-(\text{total range of angles})+(j-1)\times\text{INZT}\times\text{PI/180} \)

Output:
- (FT06)
  - \( \text{AZETA} \): value of \( \theta \) at which the field values are taken (units degrees).
  - \( \text{ERAD} \): far field magnitude.
- (FT07)
  - \( \text{AZETA} \): (see FT06)
  - \( \text{AREF} \): reference polarization magnitude
- (FT06)
  - \( \text{AZETA} \): (see FT06)
  - \( \text{ACR} \): cross polarization magnitude

Using the Program

Create an input file assigned to FT05 and output files assigned to FT06, FT07, and FT08. Running the program will fill FT06, FT07, and FT08 with data that can be used in the program ZPLOT1 to plot the appropriate graphs, while also printing the number of points calculated.
To run:

DDEF FT05F001,VS,IN1
DDEF FT06F001,VS,PAOUT3
DDEF FT07F001,VS,PAOUT2
DDEF FT08F001,VS,PAOUT1
PACAL1

Example of IN1:

CINPUT  X=.015,-.015,-.015,.015  Y=.015,.015,-.015,-.015
AMPL=1,1,1,1  APHA=0,0,0,0  H=4,QE=22,QH=16,INZT=.1,I=1  CEND

Program Description

This program takes a series of sources with different amplitudes and phases, and determines the power via two methods.

Input (FT09):
M: number of sources  XX,YY,ZZ: x,y and z coordinates of source (meters)  A: amplitude of source  PHI: relative phase of source.
IPOL: denotex polarity
1: X-linear polarized
2: Y-linear polarized
3: circularly polarized
QE,QH: exponents of cosine functions.

Output:
(FT15) YPOWER: the power using direct integration method. This serves as an input to PADTV.

Program Description

This program requires the input file to PAA1 as well as the output file and then calculates the peak directivity.

Input:
(FT09) Input to PAA1
(FT15): Output to PAA1

Output (FT06)
DIR: peak directivity
Program Description

This program is designed to plot either the far-field, reference polarization, or cross polarization as calculated in PACAL1.

Input:
(FT36)
X(I),Y(J) (meters)

These values are the output of the previous program PACAL1. If the plot of the far field is desired, the file which was assigned to FT08 in PACAL1 should now be assigned to unit 6. Similarly, if the plot of the reference or cross polarization is desired, the file assigned to FT07 or FT06, respectively, should now be assigned to FT06.

Extraneous Input:

lines 1200,1250
IVARS=NP=total number of points-1

(This can be obtained from the output of PACAL1)

lines 3500,3600

VARS(4)=lower boundary of angles to be plotted.
VARS(5)=upper boundary of angles to be plotted.

Using the Program

After running PACAL1, there will be data in the files assigned to FT08,FT07, and FT06, which in this case will be denoted as A,B, and C. Assign either A,B, or C, to FT06 (after releasing FT08, FT07, and FT06), depending on whether the far field, reference, or cross polarization plots are desired. Then, run the program with the appropriate plotting device (sidecar 4015) and the plot will appear.

To run:

RELEASE FT08
RELEASE FT07
RELEASE FT06
DDEF FT06F001,VS,PAOUT1
ZPLOT1

If this routine is executed after that shown in the PACAL1 section, the far field will be plotted.
PROCDEF PAPLOT1

This procdef runs the programs PACAL1 and ZPLOT1 in succession while defining the necessary input and output devices.

As it is listed, file PA114 should contain the input to PACAL1. After assigning the devices FT08, FT07, and FT06 to files PAOUT1, PAOUT2, and PAOUT3, respectively, running PACAL1 will fill the respective file with the far field, reference polarization, and cross polarization magnitudes. After running PACAL1, setting the device FT06 to PAOUT1 will cause ZPLOT1 to plot the far field magnitude.

The Procdef:

```
ERASE PAOUT1 ERASE PAOUT2 ERASE PAOUT3
DDEF FT08F001,VS,PAOUT1 DDEF FT07F001,VS,PAOUT2
DDEF FT06F001,VS,PAOUT3
DDEF FT05F001,VS,PA114
PACAL1
RELEASE FT08 RELEASE FT07 RELEASE FT06
RELEASE FT05
GRAPH2D DDEF FT06F001,VS,PAOUT1
ZPLOT1
```
APPENDIX B

C****THIS PROGRAM WILL CALCULATE THE TRANVERSE ELECTRIC FIELD**********
C****AND THE REFERENCE/CROSS POLARIZATION COMPONENTS OF THE FIELD**********

CDIMENSION STAMENTS******************************************************************************
REAL INZT DIMENSION X(100),Y(100),AMPL(100),APHAC(100)

CINPUT DATA: X(I) ;X-COORDINATE OF HORN I,Y(I) ;Y-COORDINATE OF****
C Horn I.AMPL(I) ;AMPLITUDE OF I HORN, APHA(I) ; PHASE OF THE I ***
C Horn,M ; NUMBER OF HORN,S POL ; 1=X-POL,2=Y-POL,3-RHCP,4=LHCP.****
CQH ; EXPONENT H PLANE, QE ;EXPONENT OG E-PLANE******************************************************************************

NAMELIST/INPUT/X,Y,Z,AMPL,APHA,M,POL,QH,QE,INZT,I
READ(5,INPUT)
INPH=1
J=0
AWE=3E8/10E9
PI=4.*ATAN(1.)
AKO=2.*PI/AWE

SET THE ANGLES FOR THE PLOT******************************************************************************
CONTINUE
J=J+1
ZETA=(-88.+(J-1)*INZT)*PI/180.
APHI=(I-1)*INPH*PI/180.
AZETA=ZETA*180./PI
AAPHI=APHI*180./PI
IF(AZETA.GT.88.)GO To.
IF(AAPHI.GT.180.)GO TO 999
RAZP=O.
AIMZP=O.
DO 20 K=1,M
CSTART THE ARRAY FACTOR SUMATION******************************************************************************
ANG=APHA(K)*X(K)*SINCZETA)*COSCAPHI)+Y(K)*SINH(ZETA)*SINCAPHI)
ANG=AKO*ANG
RAZP=RAZP+AMPL(K)*COS(ANG)
AIMZP=AIMZP+AMPL(K)*SIN(ANG)
20 CONTINUE
SET POLARIZATION PARAMETERS******************************************************************************
IF(POL.EQ.1.)GO TO 500
IF(POL.EQ.2.)GO TO 550
IF(POL.EQ.3.)GO TO 600
IF(POL.EQ.4.) GO TO 650
GO TO 999
CONTINUE
0006800 \textbf{CONTINUE}
0006900
0007000
0007100 \textbf{C}*****\textbf{START THE ELEMENT PATTERN CALCULATION}**************************
0007200 \textbf{AUE}=(\text{COS}(ZETA))**QE
0007300 \textbf{AUH}=(\text{COS}(ZETA))**QH
0007400 IF(AUE.LT.1E-3)AUE=1E-3
0007500 IF(AUH.LT.1E-3)AUH=1E-3
0007600 \textbf{REEZL}=A1*\text{COS(SI)}*\text{COS(APHI)}+A2*\text{SIN(APHI)}
0007700 \textbf{REEZL}=AUE*REEZL
0007800 \textbf{AIMEZL}=A1*\text{SIN(SI)}*\text{COS(APHI)}
0007900 \textbf{AIMEZL}=AUE*AIMEZL
0008000 \textbf{REEPH}=A2*\text{COS(APHI)}-A1*\text{SIN(APHI)}*\text{COS(SI)}
0008100 \textbf{REEPH}=AUH*REEPH
0008200 \textbf{AIMEPH}=-1*A1*\text{SIN(APHI)}*\text{SIN(SI)}
0008300 \textbf{AIMEPH}=AUH*AIMEPH
0008400
0008500
0008600
0008700 \textbf{C}*****\textbf{START THE TOTAL FIELD CALCULATION AT ZETA PHI OBSERVATION}**********
0008800
0008900 \textbf{AREZE}=RAZP*REEZL-AIMZP*AIMEZL
0009000 \textbf{AIMZE}=AIMZP*REEZL+RAZP*AIMEZL
0009100 \textbf{AREPH}=RAZP*REEPH-AIMZP*AIMEPH
0009200 \textbf{AIMPH}=RAZP*AIMEPH+AIMZP*REEPH
0009300 \textbf{ARE}=AREZE*AREZE+AIMZE*AIMZE
0009400 \textbf{ARE}=AREPH*AREPH+AIMPH*AIMPH
0009500 \textbf{ERAD}=\text{AREZ}+\text{ARE}
0009600
0009700 \textbf{C}*****\textbf{START THE CALCULATION FOR CROSS POL AND THE REFERENCE POL}**********
0009800 \textbf{AB1}=A1*\text{COS(SI)}*\text{COS(APHI)}+A2*\text{SIN(APHI)}
0009900 \textbf{AB2}=\text{A1}+\text{SIN(SI)}*\text{COS(APHI)}
0010000 \textbf{AB3}=\text{A1}+\text{SIN(SI)}*\text{SIN(APHI)}+A2*\text{COS(APHI)}
0010100 \textbf{AB4}=\text{A1}+\text{SIN(SI)}*\text{SIN(APHI)}
0010200 \textbf{AB5}=\text{A1}+\text{COS(SI)}*\text{SIN(APHI)}-A2*\text{COS(APHI)}
0010300 \textbf{AB6}=\text{A1}+\text{SIN(SI)}*\text{SIN(APHI)}
0010400 \textbf{AB7}=\text{A1}+\text{COS(SI)}*\text{COS(APHI)}+A2*\text{SIN(APHI)}
0010500 \textbf{AB8}=\text{A1}+\text{SIN(SI)}*\text{SIN(APHI)}
0010600
0010700
0010800
0010900 \textbf{C}*****\textbf{START THE CROSS AND REFERENCE COMPUTATION}******************************
0011000 \textbf{REER}=(\text{AREZ}+\text{AB1}+\text{AIMZ}+\text{AB2})+(\text{AREPH}+\text{AB3}+\text{AIMPH}+\text{AB4})
0011100 \textbf{AIMER}=(\text{AREZ}+\text{AB2}+\text{AIMZ}+\text{AB1})+(\text{AREPH}+\text{AB4}+\text{AIMPH}+\text{AB3})
0011200 \textbf{REC}=\text{AREZ}+\text{AB5}+\text{AIMZ}+\text{AB6})+(\text{AREPH}+\text{AB7}+\text{AIMPH}+\text{AB8})
0011300 \textbf{AIMCR}=(\text{AREZ}+\text{AB8}+\text{AIMZ}+\text{AB5})+(\text{AREPH}+\text{AB8}+\text{AIMPH}+\text{AB7})
0011400 \textbf{ARE}=\text{REER}+\text{REER}+\text{AIMER}+\text{AIMER}
0011500 \textbf{ACR}=\text{REC}+\text{REC}+\text{AIMCR}+\text{AIMCR}
0011600 \textbf{WRITE}(8,400)AZETA,ERAD
0011700 \textbf{WRITE}(7,400)AZETA,AREF
0011800 \textbf{WRITE}(6,400)AZETA,ACR
0011900 \textbf{FORMAT}(5X,E15.5,5X,E15.5)
0012000
0012100
0012200 \textbf{GO TO 310}
0012300 \textbf{WRITE}(12,888)J
0012400 \textbf{FORMAT}(5X,'TOTAL NUMBER OF POINTS',I5)
0012500
0012600
0012700
0012800
0012900
0013000 \textbf{STOP}
0013100 \textbf{END}
C
DIMENSION XXCI00),YYCIOO)
DIMENSION AMATCI00,100),YAMATCIOO,100)
DIMENSION GGXHCIOO),GGXECIOO)
DOUBLE PRECISION BECIOO,lOJ),BHCIOO,lOO),CECIOO,lOO),CHCIOO,lOO)
DIMENSION CDE CIOO,100),CDHCIOO,100),PHICIOO),ACIOO),THETA(lOO,100)
DIMENSION ARCIOO),AICIOO) ,
DOUBLE PRECISION QE,QH,PCIOO,lOO),XK,XXE,XXH
DOUBLE PRECISION BESECIOO,100),BESH(100,lOO)
DOUBLE PRECISION ARHO
REAL LAMDA
DOUBLE PRECISION
DOUBLE PRECISION
DOUBLE PRECISION
DOUBLE PRECISION
DOUBLE PRECISION
YBEC100,lOO),YBHCIOO,100),YCECIOO,lOO),YCHCIOO,lOO)
ZACIOO),ZB
XTHETA,DTHETA
AP1,AP2
AF11, AF12
*****THIS ARE THE INPUTS TO POWER CALCULATION*************************
NAMELIST/INPUT/M,XX,YY,ZZ,A,PHI,IPOL,QE,QH
READ C9,INPUT)
FREQ=30E9 \FREQ,
PI=4.*(ATANCl.)
XK=2*PI/LAMDA
C*****THIS SECTION WILL CALCULATE THE DISTANCE AND ANGLE M,N***********
DO 40 JA=l,M
DO 50 IA=l,M
IF((XXCIA).EQ.XXCJA).AND.(YYCIA).EQ.YYCJA)GO TO 55
ALl=CXXCIA)-XXCJA»*CXXCIA)-XXCJA»
AL2=CYYCIA)-YYCJA»*CYYCIA)-YYCJA»
ABl=SQRTCALl+AL2)
AAl=CXXCIA)-XXCJA»/ABl
AA2=CYYCIA)-YYCJA»/ABl
THETACIA,JA)=AAl*AAl-AA2*AA2
PCIA,JA)=ABI
GO TO 89
PCIA,JA)=O.
THETACIA,JA)=l.
CONTINUE
50 CONTINUE
40 CONTINUE
C*****THIS SECTION WILL COMPUTE THE BESSEL AND GAMMA FUNCTIONS*********
XXE=QE+.5
XXH=QH+.5
CALL GMMMAXXE,GXXE,IER)
CALL GMMMAXXH,GXXH,IER)
AFll=XXE+l
AF12=XXH+l
CALL GMMMA(AFll,AREl,IER)
CALL GMMMA(AF12,ARHl,IER)
RATE=GXXE/AREl
RATH=GXXH/ARHl
DO 60 JB=l,M
DO 70 IB=l,M
ARHO=XK*PI/IB,JB)
CONTINUE
**Before I begin:**

This code snippet calculates the BH(i,j) and BE(i,j) coefficients. It involves several nested loops and calculations of trigonometric functions and other mathematical operations. The code is structured to compute values for specific sections and conditions. The comments and variable definitions are crucial for understanding the flow and purpose of each part of the code. The overall structure is well-organized, with clear demarcation between different sections of calculations.
C POWER OF SINGLE ELEMENT

SINGLE=(QE+QH+1)/(60*(2*QE+1)*(2*QH+1))

RATIO=POWER/SINGLE

YRATIO=YPower/SINGLE

WRITE(6,500)SINGLE

FORMAT(5X,'POWER OF SINGLE ELEMENT=',1X,E15.5)

WRITE(6,501)RATIO,YRATIO

FORMAT(5X,'CLOSED FORM RATIO=',1X,E15.5,'DIRECT INT RATIO=',1X,E15.5)

WRITE(6,502)POWER,YPower

FORMAT(5X,'CLOSE FORM POWER=',1X,E15.5,'DIRECT INT POWER=',1X,E15.5)

WRITE(15,600)YPower

WRITE(16,600)POWER

C*****SPECIAL CASES FOR CHECKING RESULTS PREVIOUSLY CALCULATED***********

C*****LARGE SPACING CASE 6 LAMDA OR GREATER**********************************

POWER1=0.

DO 550 I=1,M

POWER1=POWER1+A(I)

CONTINUE

POWER1=SINGLE*POWER1

IF(CPOL.GE.3) GO TO 810

GO TO 811

POWER1=2.*POWER1

CONTINUE

WRITE(6,503)POWER1

FORMAT(5X,'LARGE SPACING POWER=',1X,E15.5)

C*****SYMMETRIC PATTERN QH=QE POWER CALCULATION**************************

RECF=0.

REDI=0.

RIMCF=0.

RIMDI=0.

DO 560 J=1,M

DO 561 I=1,M

CA22=(AR(I))*AR(J)+AI(I)*AI(J)*BE(I,J)

CA33=(AR(I))*AR(J)+AI(I)*AI(J)*YBE(I,J)

CA44=(AR(I))*AI(J)-AR(J)*AI(I)*BE(I,J)

CA55=(AR(J))*AI(I)-AR(J)*AI(I)*YBE(I,J)

RECF=RECF+CA22

REDI=REDI+CA33

RIMCF=RIMCF+CA44

RIMDI=RIMDI+CA55

CONTINUE

CONTINUE

POLPCF=SQRT(RECF**2+RIMCF**2)*(1/60.)

POLPDI=SQRT(REDI**2+RIMDI**2)*CI/60.)

IF(CPOL.GE.3) GO TO 710

GO TO 711

POLPCF=2.*POLPCF

POLPDI=2.*POLPDI

CONTINUE

WRITE(6,503)POLPCF,POLPDI

FORMAT(5X,'POWER CLOSE FORM SYM=',1X,E15.5,'POWER D I SYM=',1X,E15.5)

STOP

END
DIMENSION THETA(100), PHI(100), ETHETA(100, 100), XX(100)
DIMENSION YY(100), AMP(100), PHASE(100)
DIMENSION AR(100, 100), AI(100, 100), AA(100, 100)
DIMENSION XRADI(100, 100), EPHI(100, 100)
DIMENSION UE(100), UH(100), XIL(100, 100)
DIMENSION A(100)

C READING VARIABLES
NAMELIST/INPUT/M, XX, YY, ZZ, QE, QH, A, PHI, IPOL
READ (9, INPUT)
DO 130 I = 1, M
AMP(I) = A(I)
PHASE(I) = PHI(I)
CONTINUE
READ (15, 131) POWER
PI = 4. * ATAN(1.)
FREQ = 29.5E9
XLAMDA = 3. E8 / FREQ
XK = 2 * PI / XLAMDA
ZO = 377
C TESTING POLARITY
IF (IPOL .NE. 1) GO TO 102
C = 1
B = 0
PSI = 0
GO TO 104
IF (IPOL .NE. 2) GO TO 103
C = 0
B = 1
PSI = 0
GO TO 104
C SOLVING EQUATIONS
DO 100 I = 1, 100
DO 101 J = 1, 100
THETAC(I,J) = (FLOAT(I-1)/99) * PI/2
PHIC(J) = (FLOAT(J-1)/99) * PI/2
UEC(I) = COS(THETAC(I,J)) * QE
UHC(I) = COS(THETAC(I,J)) * PHIC(J)
ETHETAC(I,J) = A1 (AA2 * AA3 + AA4)
BB1 = UH(I) * UH(I)
BB2 = CXC(SC(PHI(J))) * SIN(PHI(J))
BB3 = CXC(SC(PHI(J))) * COS(PHI(J))
BB4 = AA4
EPHI(I, J) = BB1 * (BB2 + BB3 - BB4)
DO 110 K=1,M
CC1=XX*K*(XX*K)*SIN(THETA(I))*COS(PHI(J))
CC2=YY*K*(YY*K)*COS(THETA(I))*SIN(PHI(J))
EEXP=CC1+CC2
DREAL=AMP(K)*COS(PHASE(K)+EEXP)
AIMG=AMP(K)*SIN(PHASE(K)+EEXP)
AR(I,J)=AR(I,J)+DREAL
A(I,J)=AI(I,J)+AIMG

DO 120 IA=1,100
DO 121 JA=2,100
IF (XRADICIA,JA) .GE. XRADICIA,JA) GO TO 121
XA=XRADICIA,JA
XRADICIA,JA=XRADICIA,JA+1
XRADICIA,JA=XB
WRITEC8,973)XRADICIA,JA)
FORMATC5X, 'THIS IS THE PEAK VALUE', 3X, E15.5)
DIR=4.*PI*XRAOIC1,1)/CPOWER*ZO)
OIR =10.*ALOG10CDIR)
WRITE (6,123) DIR
FORMATC5X,'DIRECTIVITY=', E15.5)
STOP
END
C***/ THIS PROGRAM CAN BE USED TO PLOT THE ANTENNA
C***/ FAR-FIELD PATTERN (E-PLANE OR H-PLANE CUTS)
C***BY R.J. ACOSTA
DIMENSION X(5000), Y(5000), IVARS(20), VARS(20), RTNARR(2)
DIMENSION XTITLE(5), YTITLE(5)
LOGICAL IAXIS
INTEGER N1
DATA XTITLE/'ELEV', 'ATIO', 'AN', 'GLE', 'DEG'/
DATA YTITLE/'REL', 'IVE', 'AMP', 'ITU', 'DE'/
CALL TITLE(4, 20, 15, XTITLE)
CALL TITLE(3, 17, 15, YTITLE)
IVARS(1) = 2
IVARS(2) = 1760
NP = 1760
N1 = NP
DO 15 J = 1, NP
READ(6, 300) X(J), Y(J)
300 FORMAT(5X, E15.5, 5X, E15.5)
CONTINUE
CALL SCLBAK(.FALSE., N1, Y, RTNARR).
VMAX = RTNARR(2)
DO 16 J = 1, NP
Y(J) = Y(J) / VMAX
IF(Y(J) .LT. 1E-8) Y(J) = 1E-8
Y(J) = 10. * ALOG10(Y(J))
CONTINUE
VARS(1) = 9.
VARS(2) = 8.
VARS(3) = 0.
VARS(4) = -90.
VARS(5) = 90.
VARS(6) = 6.
VARS(7) = 2.
VARS(8) = -1.
VARS(9) = 0.
CALL XAXIS(-1., -1., VARS)
VARS(2) = 9.
VARS(3) = 90.
VARS(5) = 8.
VARS(6) = 8.
VARS(4) = -80.
CALL YAXIS(-1., -1., VARS)
CALL GPLLOT(X, Y, IVARS)
CALL DISPLA(I)
READ(9, 993) XYZ
993 FORMAT(I1)
CALL TERM
STOP
END
REFERENCES


15. Smetana, J; and Acosta, R.: Preliminary Evaluation of MMIC Array Antenna Performance. Presented at the 1985 Antenna Applications Symposium, Sept. 18-20, 1985, Monticello, IL. (Cosponsored by the Univ. of IL and Rome Air Development Center.)
### TABLE I. - POLARIZATION PARAMETERS

<table>
<thead>
<tr>
<th>Polarization Type</th>
<th>( a_m )</th>
<th>( b_m )</th>
<th>( \psi_m )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear-X</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Linear-Y</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>RHCP(^a)</td>
<td>1/2</td>
<td>1/2</td>
<td>0.5( \pi )</td>
</tr>
<tr>
<td>LHCP(^b)</td>
<td>1/2</td>
<td>1/2</td>
<td>-0.5( \pi )</td>
</tr>
</tbody>
</table>

\(^a\)RHCP (right-hand circular polarized).
\(^b\)LHCP (left-hand circular polarized).

### TABLE II. - COMPARISON OF DIRECTIVITY RESULTS WITH THOSE OBTAINED BY KING AND WONG (ref. 2)

<table>
<thead>
<tr>
<th>Frequency, MHz</th>
<th>( S/\lambda )</th>
<th>Element Pattern, HPBW (deg)</th>
<th>Pattern, ( \cos \theta(\theta) ) ( \varphi_E = \varphi_H )</th>
<th>King-Wong, measured directivity, ( \text{dB} )</th>
<th>King-Wong, ( \text{dB calculated} )</th>
<th>NASA Lewis, ( \text{dB calculated} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>450</td>
<td>0.687</td>
<td>86.0</td>
<td>1.11</td>
<td>17.1</td>
<td>17.3</td>
<td>17.10</td>
</tr>
<tr>
<td>500</td>
<td>0.763</td>
<td>92.0</td>
<td>0.96</td>
<td>17.9</td>
<td>18.0</td>
<td>17.83</td>
</tr>
<tr>
<td>600</td>
<td>0.916</td>
<td>89.4</td>
<td>1.02</td>
<td>18.8</td>
<td>18.5</td>
<td>18.45</td>
</tr>
<tr>
<td>700</td>
<td>1.068</td>
<td>94.0</td>
<td>0.91</td>
<td>18.0</td>
<td>17.4</td>
<td>17.69</td>
</tr>
<tr>
<td>800</td>
<td>1.220</td>
<td>94.0</td>
<td>0.91</td>
<td>17.4</td>
<td>16.9</td>
<td>17.25</td>
</tr>
</tbody>
</table>
Figure 1. - Geometry of the generalized phased array.

Figure 2. - A typical element coordinate system $(X', Y', Z')$ and the reference coordinate system $(X, Y, Z)$.
Figure 3. - Geometrical picture of the coordinate transformation between element coordinate system and the reference coordinate system.

Figure 4. - Compared directivities for a 2x2 array as function of element spacing.
Figure 5.

(a) Boresight, reference polarization far-field antenna pattern. (E-plane cut).

(b) Scan case ($\phi_0 = 3.2^\circ$), reference polarization far-field antenna pattern. (E-plane cut).

Figure 5. - Continued.
Experimental
Theoretical

(c) Scan case (θ₀ = 5.2°), reference polarization far-field antenna pattern (E-plane cut).

Figure 5, - Concluded.
With the advent of monolithic microwave integrated circuit (MMIC) technology, the phased array has become a key component in the design of advanced antenna systems. Array-fed antennas are used extensively in today's multiple beam satellite antennas. In this report, a computer program based on a very efficient numerical technique for calculating the radiated power (Romberg integration), directivity and radiation pattern of a phased array is described. The formulation developed is very general, and takes into account arbitrary element polarization, E- and H-plane element pattern, element location, and complex element excitation. For comparison purposes sample cases have been presented. Excellent agreement has been obtained for all cases. Also included in appendixes A and B are a user guide and a copy of the computer program.
End of Document