LOSSES IN CHANNELS WITH INCREASED EXTERNAL TURBULENCE

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An approximate method for determining the effect of the level of turbulence on the aerodynamic characteristics of convergent and diffuser channels is examined. Utilized for this purpose is a pulse equation for the boundary layer, in which the turbulence of the external flow is introduced on the basis of experimental data, which represent the coefficient of friction \( c_f \) and the form-parameter \( H_0 \) on the degree of turbulence \( E \). It is found that, at significant levels of external turbulence, losses must be considered not only in the boundary layer, but also in the central region of the channel.
Examined herein is an approximate method for calculating the effect of the level of turbulence on the aerodynamic characteristics of convergent and diffuser channels. Utilized for this purpose is a pulse equation for the boundary layer, into which the turbulence of the external flow is introduced, based on experimental data which represent the coefficient of friction $c_f$ and the form-parameter $H_0$ on the degree of turbulence $E$.

The effect of external turbulence on the aerodynamic characteristics of channels has been studied little, although, in a number of cases, the effect of the turbulence of an approach stream has proved decisive.

In particular, by examining the determination of the losses in turbine cascades, pipes and diffusers, it is entirely necessary to introduce the degree of turbulence of the external flow into the calculation, and to consider this magnitude one of the decisive parametric criteria of similitude in the experiment.

The following considerations graphically emphasize what has been said. Let a flow with an increased degree of turbulence $E$ and initial parameters $p_0$ and $t_0$ (Fig. 1,a), with a pressure behind the cascade $p_2$, flow into a turbine cascade. We will select a current line $0a$ in the channel between the vanes, for which the local coefficient of losses $\zeta_i$ will have a minimal value. For the other current lines $0b$, $0c$, and so on, $\zeta_i \geq \zeta_i^{\text{min}}$. The process of expansion in the thermal diagram, which corresponds to the selected lines $0a$, $0b$, $0c$, is presented in Figure 1,b. By expressing the local loss coefficient $\zeta_i$ in standard form, in parts of the available energy in the outlet section, we obtain

$$\zeta_i = 1 - \frac{H_i}{H_0} = 1 - c_{f_i}^2/c_{f_0}^2.$$
Fig. 1. Flow process in the cascade with increased turbulence of the external flow (a) and its depiction in the $\bar{e}$-diagram (b).

here, $c_{2i}$ is the outlet velocity of the flow at the examined point, and $c_{2t}$ is the theoretical velocity, determined by the isoentropic differential $H_0$.

We will average the magnitudes of $\xi_i$ according to mass

$$\bar{\xi_i} = \frac{\rho_{2t} c_{2i} F_2}{m} \int_0^1 \frac{\rho_{2t} c_{2i}}{\rho_{2t} c_{2i}} \left(1 - \frac{c_{2i}}{c_{2i}}\right) dF_2.$$  (1)

The product $\rho_{2t} c_{2t} F_2 = m_t$ corresponds to the theoretical flow rate with the absence of losses. We will use $\Delta m$ to designate the difference in the flow rates

$$\Delta m = m_t - m = \rho_{2t} c_{2i} F_2 \left[1 - \int_0^1 \left(1 - \frac{\rho_{2t} c_{2i}}{\rho_{2t} c_{2i}}\right) dF_2\right].$$  (2)

Then

$$m = m_t - \Delta m = \rho_{2t} c_{2i} F_2 \left[1 - \int_0^1 \left(1 - \frac{\rho_{2t} c_{2i}}{\rho_{2t} c_{2i}}\right) dF_2\right].$$

The integrals in relationships (1) and (2) coincide in structure with those expressed for a relative area of energy losses $\bar{\xi}_2^{***}$, and a relative area of displacement $\bar{\xi}_2^*$, but the dimensionless velocities $\bar{c}_{2i}$ are expressed in parts of the theoretical velocity $c_{2t}$, rather than parts of the velocity $c_{2i}^{\text{max}}$ in the examined section.

By analogy with the magnitudes $\bar{\xi}^*$ and $\bar{\xi}^{***}$, we will consider the integrals
\[
\overline{\Delta}_2 = \int_0^1 \left( \frac{1 - \frac{c_{2t} c_{2t}'}{c_{2t}}}{\rho_{2t} c_{2t}'} \right) dF,
\] (3a)

\[
\overline{\Delta}^{**} = \int_0^1 \frac{\rho_{2t} c_{2t}'}{\rho_{2t} c_{2t}} \left( 1 - \frac{c_{2t}^2}{c_{2t}'} \right) dF
\] (3b)

as the generalized areas of displacement and energy loss.

Actually, with the absence of losses on the zero current line 0a, \(c_{2t} = c_{2\text{max}}\) and \(\overline{\Delta}^* = \overline{\delta}_2^*\), while \(\overline{\Delta}^{**} = \overline{\delta}_2^{**}\). If one introduces the coefficient of velocity for the zero current line \(\varphi_0\), equal to \(\varphi_0 = c_{2\text{max}} / c_{2t}'\) into the examination, then one may establish the following simple relationships between the generalized and standard areas \(\overline{\delta}_2^*\) and \(\overline{\delta}_2^{**}\):

\[
\overline{\Delta}^* = 1 - \varphi_0 (1 - \overline{\delta}_2^*); \overline{\Delta}^{**} = \varphi_0 (1 - \overline{\delta}_2^*) - \varphi_0^* (1 - \overline{\delta}_2^* - \overline{\delta}_2^{**}).
\] (4)

By substituting expression (2) into formula (1), with regard for the introduced designations (3) and (4), we obtain

\[
\zeta_\varepsilon = 1 - \varphi_0 \left[ 1 - \frac{\overline{\delta}_2^{**}}{1 - \overline{\delta}_2^*} \right].
\] (5)

With \(E = 0\), \(\varphi_0 = 1\) and

\[
\zeta_{\varepsilon = 0} = \overline{\delta}_2^{**} / (1 - \overline{\delta}_2^*).
\]

Thus,

\[
\zeta_\varepsilon = 1 - \varphi_0 (1 - \zeta_{\varepsilon = 0}).
\] (6)

Relationships (5) and (6) are common for evaluation of the losses in random channels. We would note that, during their derivation, we put nature aside in decreasing the coefficient of velocity on the line of minimal losses. This may be increased turbulence of the approach stream, non-uniformity, periodic nonstationariness, and so on.
How strongly the coefficient of losses varies, as a function of the causes which evoke a reduction in $\phi$, is evident from Figure 2, where the coefficients of losses $\zeta_E = 0$, with the presence of a potential focus, are plotted along the x-axis and the value of the losses $\zeta_E$ which corresponds to the coefficient $\phi$. If one assumes that the integral areas of the boundary layer do not depend on $E$, then, even with $\phi = 0.98$, the losses differ substantially from the losses found with a zero or small degree of turbulence of the approach stream.

In actuality, $\overline{\delta}^*$ and $\overline{\delta}^{**}$ vary appreciably with a change in turbulence, or its increase leads to an increase in the completeness of the profile, the Reynolds stresses, and an increase in the physical thickness of the boundary layer.

Thus, the problem of evaluating the losses in the channels amounts to the determination of the coefficient $\phi$, which characterizes the dissociation of energy in the nucleus, and the calculation of the minimum areas of the boundary layer, which develop under conditions of increased turbulence in this nucleus.

The question of evaluation of the coefficient $\phi$ presents an independent problem, the solution of which is the subject of studies [1,2].

Specifically, based on the statistical theory of turbulence in [1], it is shown that, for a pipe, the magnitude of $\phi$ may be found according to the relationship

$$\phi = 1 - 9.55E^{0.6}.$$ 

The experimental determination of this coefficient in turbine cascades is given in [2].

The second part of the problem, which consists of evaluating the magnitudes of $\overline{\delta}^*$ and $\overline{\delta}^{**}$, is more complex and, in this stage, may be solved with a great deal of approximation. One of the potential means of this evaluation comes from the properties of the solution of Carman's integral relationship.
For simplicity, we will examine the case of a flat boundary layer, and we will limit ourselves to a comparatively low level of turbulence \((E < 7-8\%)\), when one may still disregard the additional terms in the integral equation, and utilize its common form for analysis

\[
\frac{d\delta^{**}}{dx} + \frac{u_1'\delta^{**}}{u_1}(H+2) = \frac{\tau_w}{\rho u_1^2} = \frac{c_f}{2},
\]

(7)

where \(u_1\) is the velocity of the external flow and \(u_1' = du_1/dx\).

We will introduce the dimensionless magnitudes \(\overline{c_f}\) and \(\overline{H}\) \((c_f = c_{f0} \overline{c_f}, \overline{H} = H_0 \overline{H})\) into the examination, taking the magnitudes \(c_{f0}\) and \(H_0\) as the norming factors, which correspond to a gradientless flow, for which \(H_0 = 1.4\) and

\[c_{f0}/2 = \xi_0/Re^{**}.\]

With regard for what has been said, equation (7) takes on the form

\[
Re^{**} \frac{d\delta^{**}}{dx} + \frac{u_1'\delta^{**}}{u_1} Re^{**}(2+H_0 H) = \frac{\xi_0}{\xi_0}. \]

(8)

Simple transformations lead to the relationship

\[
\frac{d}{dx} \left(Re^{**} \delta^{**}\right) = (m+1) \xi_0 \xi_0' + [m - (m+1)(2+H_0 H)] \xi_0',
\]

(9)

where

\[\Gamma = \frac{u_1'\delta^{**}}{u_1} Re^{**}.\]

In examining the question of the change in the normed magnitudes of \(\overline{c_f}\) and \(\overline{H}\), one may note that \(\overline{c_f}\) hardly changes at first, according to the available test data in the diffuser region, i.e., it remains close to one, and then drops sharply to zero at the point of break-away of the boundary layer. In turn, the parameter \(\overline{H}\) increases continuously right up to break-away of the flow.

By utilizing the nature of the change in the indicated magnitudes from the complex \(\Gamma\), we will approximate \(\overline{c_f}\) and \(\overline{H}\) with the following relationships:

\[
\xi_0 = 1 - k_1 \Gamma^2; \quad H = 1 - k_2 \Gamma + k_3 \Gamma^2.
\]

(10)
We will show that the accuracy of the proposed approximation (numerical values of the magnitudes $k_1, k_2, k_3$) does not affect the quadrature of Carman's equation. Actually, by substituting (10) into (9), we obtain

$$\frac{d}{dx} (\delta^{**} \text{Re}^{**}) = a + b \Gamma + c \Gamma^2 + d \Gamma^3. \quad (11)$$

Here, $a = (m+1) \zeta_0$; $b = -(m+1)(1+H_0)(1+H)+1$; $c = (m+1)(k_2 H_0 - \zeta_0 k)$; $d = (m+1)H_0 k_3$.

The polynomial in the righthand part of equation (10) differs very little from the linear function, since the coefficients $c$ and $d$, which are small per se, stay factors at high levels of the parameter $\Gamma$, the magnitude of which does not exceed 0.04-0.06 at the breakaway point. In other words, without committing a large error, one may linearize relationship (11) and, as a result, obtain the well-known quadrature

$$\delta^{**} = \frac{a_1}{L \text{Re}^{**}} \frac{a_2}{a_3 + a_4} \left[ \int \delta_1^{**} dx \right]^{a_5}. \quad (12)$$

The coefficients of equation (12) are equal to

- $a_1 = [(m+1) \zeta_0]^{1/(m+1)}$;
- $a_2 = m^{1/(m+1)}$;
- $a_3 = 1 + (m+1)(H_0+1)$;
- $a_4 = 1/(m+1)$.

The obtained conclusion makes it possible to reveal an important property of the solution of (12), the essence of which consists of the fact that all of the constant coefficients in (12) are completely determined by the magnitudes which characterize the flow on a plate.

From here, it follows that the accuracy of calculation of the boundary layer in a gradient flow, based on the equation of T. Carman, depends on how accurately the dependences $c_{f_0}$ and $H_0$ are determined for a plate.¹

¹This method is utilized, in essence, in the known layer-calculating method of K. K. Fedyaevskiy, A. V. Kolesnikov and A. N. Smolyaninova, where the increased accuracy is achieved because of the introduction of more specific data on a gradientless flow [7].
By utilizing this very method and the examined property of equation (1), it is comparatively easy to introduce the level of turbulence of the external flow into the calculation. For this purpose, it is sufficient, as has already been noted, to carry out the detailed study of the boundary layer on a plate with an increased value of the magnitude $E$, and to utilize the obtained data on the coefficients $c_f$ and $H_0$ for calculation of the values of $a_1$ in equation (12).

![Graph](image)

**Fig. 3.** Dependence of the magnitude of $\delta_{E=0}^*/\delta_{E=0}^*$ on the level of turbulence with convergent $(u=0.3 + 0.7x)$ (1) and diffuser $(u=1-0.7x)$ (2) flows.

The still evidently insufficient available experimental data on the dependences of $c_f$ and $H_0$ on the level of turbulence $E$ [3-6] may be roughly approximated in the following manner:

\[
\begin{align*}
  c_f & = c_{f0} (1+0.01E^2), \\
  H_0 & = H_0 \left(0.75 + \frac{0.25}{1+0.1E} \right),
\end{align*}
\]

(13)

where $E$ is the level of turbulence in percent.

With regard for the listed expressions for the coefficients $a_1$ and $a_3$, we obtain the following dependences:

\[
\begin{align*}
  a_1 & = [(m+1)\zeta_0 (1+0.01E^2)], \\
  a_3 & = 1 + (m+1) \left[ H_0 \left(0.75 + \frac{0.25}{1+0.1E} \right) + 1 \right].
\end{align*}
\]

(14)
Their substitution into formula (12) gives

\[
\delta^{**} = \frac{\frac{1}{1} \frac{1}{1 + \frac{2+m}{m+1} + H_s \left( 0.75 + \frac{0.25}{1 + 0.1E^2} \right)} \times \frac{0.25}{1 + 0.1E^2}}{(Re_T)^{m+1} (a_m)^{m+1} + \frac{0.25}{1 + 0.1E^2}} \times \int_0^\infty \frac{1}{d_i} \left[ 0.75 + \frac{0.25}{1 + 0.1E^2} \right] \frac{1}{d_i^{m+1}}. \quad (15)
\]

The results of the calculation, according to relationship (15), of the integral depth of the loss of the pulse \( \delta^{**} \) in the outlet section of the convergent and diffuser channels are represented as a function of the level of turbulence in Figure 3.

We would note that the possibilities of the examined procedure are not limited to the case of increased turbulence of the external flow, but are considerably broader. Specifically, the very same method may be utilized to solve the problem of calculating the turbulent boundary layer in a flow of compressible gas. What is more, after the accumulation of data on the resistance of a plate and the integral characteristics of the layer by analogous means, one may correct the solution for calculation of moisture during the movement of a moist vapor flow in the cascades of turbines.

Conclusions

1. With considerable external turbulence of the flow, it is not necessary to take into account the losses, not only within the limits of the boundary layer, but also in the central part of the channel.

2. Based on the integral relationship of Carman, we obtained an analytical expression for calculating the depth of displacement with an increased external turbulence of the flow.
REFERENCES


