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AN APPROACH TO THE CALCULATION OF THE PRESSURE FIELD PRODUCED BY RIGID WIDE CHORD DUAL ROTATION PROPELLERS OF HIGH SOLIDITY IN COMPRESSIBLE FLOW

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SUMMARY

An unsteady lifting surface theory for the counter-rotating propeller is presented using the linearized governing equations for the acceleration potential and representing the blades by a surface distribution of pulsating acoustic dipoles distributed according to a modified Birnbaum series. The Birnbaum series coefficients are determined by satisfying the surface tangency boundary conditions on the front and rear propeller blades. Expressions for the combined acoustic resonance modes of the front prop, the rear prop and the combination are also given.

LIST OF IMPORTANT SYMBOLS

h_1, h_2	(hub/tip) radius ratio of front and rear propeller
$L_b^{(a)}$	Laguerre function of order a and degree b
$\vec{r}(r,\theta,z)$	position vector of a point in space
R_{12}	(rear propeller/front propeller) tip radius ratio
W_a	axial velocity of free stream
Z_1, Z_2	number of blades of front and rear propeller
α_1, α_2	blade angles of the front and rear propeller
$\bar{\alpha}_2$	trailing edge angle of the front blade
δ_1, δ_2	interblade azimuthal pitch of front and rear propeller

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λ_1, λ_2 advance ratios of the front and rear propeller

$\lambda_{k\ell}, \upsilon_{\tilde{\ell}k\ell}$ radial eigenvalues of the front and rear propeller

ω_1', ω_2 free stream fluctuation frequencies of the front and rear propeller

Ω_1, Ω_2 angular velocity of the front and rear propeller

$\bar{\sigma}_1, \bar{\sigma}_2$ interblade phase angle factor of front and rear propeller

$\bar{\rho}(\rho, \varphi, \zeta)$ position vector of a point on the blade

$\Phi_{k\ell}, \Psi_{\tilde{\ell}k\ell}$ normalized eigenfunctions of the front and rear propeller

INTRODUCTION

Interest in propeller propulsion has been revived recently by the need to economize on fuel consumption and fuel costs. Besides, with the development of transonic airfoil sections, even with airscrew propulsion, it is now possible to attain cruise speeds comparable to those of current turbofan transport aircraft at similar operating altitudes. Preliminary studies appear to indicate possibilities of realizing a clear 15 to 20 percent superiority (see Mikkelson et al.) of propulsive efficiency for the advanced turboprops over that of turbofan propulsion. For commercial acceptability it is important to keep the noise levels of the propeller system low. Consequently, NASA has mounted a program for advanced turbopropeller development to realize these objectives.

It has been known for a long time that both the power absorption characteristics and the efficiency of propellers at high forward speeds may be improved by increasing their solidity. Solidity can be increased by increasing the blade chord and the number of blades. In addition, the use of a counter-rotating propeller mounted closely behind on a common axis provides great increases in the efficiency since the energy losses due to slipstream rotation can be eliminated. When the two propellers are close together both of them will have nearly equal inflow velocity. But, due to the rotational component imparted by the front propeller, the rear propeller operates at an effectively higher rotational velocity or equivalently lower advanced ratio than the front one.

An extensive survey of recent NASA theoretical and experimental research on propellers has been published by Mikkelson (ref. 2), Bober and Mitchell (ref. 3). Methods available for the aerodynamic analysis of propellers may be classified broadly into Euler equation methods (ref. 4), vortex lifting line theory (refs. 5 to 11), lifting surface methods (refs. 11 to 18) and the Ffowcs Williams-Hawkings equation solution method (ref. 19). It will be seen that a calculation of the acoustic intensity distribution in the flow field of the propeller requires a knowledge of the aerodynamic loading on the propeller blades. Hence it is necessary to determine the pressure distribution on the blade surfaces prior to a noise field calculation.

A lifting surface theory will be presented here for a subsonic counter-rotating propeller (fig. 1(b)) using the acceleration potential formulation. Unlike the velocity potential perturbation, this method has the advantage that it does not have to deal with the discontinuities arising from the potential vortex sheets emanating from the front and rear propeller and their interaction with the blades. It will enable us to determine both the aerodynamic load distribution on the blade surface and the acoustic pressures in the flow field concurrently. We shall adopt a coordinate system in which the front propeller is stationary so that the free stream approaches it with an axial velocity W_a and a rotational velocity $\Omega_1 r$. In this coordinate system, the rear propeller rotates with an angular velocity, $\Omega_1 \pm \Omega_2$ (+ sign for counter-rotation and - sign for corotation). The axial velocity is assumed to be the same as for the front propeller but the airflow acquires a circumferential velocity $(\Omega_1 r - W_a \tan \bar{\alpha}_2)$ in passing through the front propeller. The rear propeller removes this rotation and converts it into useful thrust. The surface boundary conditions on the front propeller blade are satisfied in a rotating coordinate system while on the rear propeller they are satisfied in the same moving coordinate system but with the blade surfaces nonstationary.

This method gives a unified treatment of both the front and the rear propeller of a dual rotation propeller system taking into account their mutual interference in unsteady subsonic axial flow. Thus, it is possible to calculate the resultant aerodynamic loading on the blade, the performance of the dual rotation propeller and its acoustic field at any point in the flow for blades of arbitrary geometry. Expressions will be given for the blade surface loading, the acoustic pressure at a point in the flow and the acoustic resonance characteristics for the front propeller, the rear propeller and the combination.

FORMULATION

We now consider a counter-rotating propeller with the angular velocities Ω_1 and Ω_2 for the leading and the trailing propellers of equal hub and tip diameters, W the axial velocity of the airflow into the leading propeller. The leading propeller develops a thrust T_1 by accelerating the free stream by the induced velocity w_1 in the wake far downstream. In addition, the propeller also imparts a rotational velocity to the airflow in the same direction as Ω_1. When the number of blades Z_1 in the propeller is large, the air passing between the blades is guided closely so that we may assume the flow through it is similar to that through an axial compressor cascade except for the different boundary conditions at the tip. The air angle $\bar{\alpha}_2$ at the trailing edge nearly equals that of the mean line of the passage between the two blades. Assuming that the two propellers are close together, the axial velocity for both of them may be regarded as nearly equal. The free stream velocity components for the front and the rear propellers are given in cylindrical coordinates by

$$(0,\Omega_1 r, W_a) \quad \text{and} \quad (0, \Omega_1 r - W_a \tan \bar{\alpha}_2, W_a)$$

In this event, the linearized differential equations for the pressure field of flow singularities on the blades of the counter rotating propeller are the same as those for the blade rows of a stage of an axial compressor.

We shall represent the propeller blade surface by pressure singularities. Since the rear propeller lies downstream of the front propeller, it experiences a periodic wake which would cause the pressure pole on the rear propeller blade to fluctuate in strength with time. When the free stream has periodic fluctuations, of angular frequencies ω_1 and ω_2 for the front and rear propellers, the nondimensional linearized differential equations for the perturbation field of an oscillating unit pressure monopole on a blade of the front and rear propeller situated at $\vec{\rho}_1$ $(\rho_1, \varphi_1, \zeta_1)$ may be written respectively as

$$\frac{\partial^2 p_1}{\partial r_1^2} + \frac{1}{r_1}\frac{\partial p_1}{\partial r_1} + \left(\frac{1}{r_1^2} - \frac{M^2}{\lambda_1^2}\right)\frac{\partial^2 p_1}{\partial \theta^2} + \beta^2 \frac{\partial^2 p_1}{\partial z_1^2} - M^2 \frac{\partial^2 p_1}{\partial t_1^2}$$

$$2M^2\left[\frac{\partial^2 p_1}{\partial z_1 \partial t_1} + \frac{1}{\lambda_1}\left(\frac{\partial^2 p_1}{\partial \theta \partial t_1} + \frac{\partial^2 p_1}{\partial \theta \partial z_1}\right)\right] = -\frac{\delta(\vec{r}_1 - \vec{\rho}_1)}{r_1} e^{i\omega_1 t_1/\lambda_1}$$

$$\frac{\partial^2 p_2}{\partial r_1^2} + \frac{1}{r_1}\frac{\partial p_2}{\partial r_1} + \frac{1}{r_1^2}\left(\beta_2^2 + \frac{2r_1 M M_2}{\lambda_1} - \frac{M^2 r_1^2}{\lambda_1^2}\right)\frac{\partial^2 p_2}{\partial \theta^2} + \beta^2 \frac{\partial^2 p_2}{\partial z_1^2} - M^2 \frac{\partial^2 p_2}{\partial t_1^2}$$

$$-2M^2\left\{\frac{\partial^2 p_2}{\partial z_1 \partial t_1} + \frac{(r_1 - \lambda_1 \tan \alpha_2)}{\lambda_1 r_1}\left(\frac{\partial^2 p_2}{\partial \theta \partial t_1} + \frac{\partial^2 p_2}{\partial \theta \partial z_1}\right)\right\} = -\frac{\delta(\vec{r}_1 - \vec{\rho}_1)}{r_1} e^{i\bar{\omega} t_1/\lambda_1}$$

(2.1)

in which we have defined the dimensionless parameters

$$
\begin{array}{llll}
r_1 = r/R & z_1 \equiv z/R & \lambda_1 = W_a/\Omega_1 R & M_2 = M \tan \alpha_2 \\
\rho_1 = \rho/R & \zeta_1 = \zeta/R & \lambda_2 = W_a/\Omega_2 R & \beta = \sqrt{1 - M^2} \\
t_1 = t/t_o & t_o = R/W_a & \bar{\Omega} = \Omega_r/\Omega_1 & \beta_2 = \sqrt{1 - M_2^2} \\
\omega_1 = \omega_1'/\Omega_1 & \bar{\omega} = \omega_2/\Omega_1 & \Omega_r = \Omega_1 + \Omega_2 &
\end{array}
\tag{2.2}
$$

Since the rear propeller is rotating with an angular velocity $(\Omega_1 + \Omega_2)$ relative to the front propeller, the azimuth angle φ of the pressure pole is also changing with time as $(\varphi + \Omega_r t)$. We express the perturbation pressures p_1 and p_2 in terms of the Green's functions $P_1(r_1, \rho_1; k_1, a_1)$ and $P_2(r_1, \rho_1; k_2, a_2)$ as

$$
p_1(\vec{r}_1, \vec{\rho}_1, t_1) = \frac{1}{4\pi^2} \sum_{k_1} \int_0^{\infty} P_1(r_1, \rho_1; k_1, a_1) e^{i[k_1(\theta-\varphi)+a_1(z_1-\zeta_1)+\omega_1 t_1/\lambda_1]} \, da_1 \tag{2.3}
$$

$$
p_2(\vec{r}_1, \vec{\rho}_1, t_1) = \frac{1}{4\pi^2} \sum_{k_2} \int_0^{\infty} P_2(r_1, \rho_1; k_2, a_2) e^{i[k_2(\theta-\varphi)+a_2(z_1-\zeta_1)+\bar{\omega} t_1/\lambda_1]} \, da_2 \tag{2.4}
$$

Combining equations (2.1), (2.3), and (2.4) and equating the corresponding terms, we get the differential equation for the front and rear propeller monopole Green's functions P_1 and P_2 given by

$$
\frac{d^2 P_1}{dr_1^2} + \frac{1}{r_1} \frac{dP_1}{dr_1} + \left[M^2 \left(a_1 + \frac{1 + k_1}{\lambda_1} \right)^2 - a_1^2 - \frac{k_1^2}{r_1^2} \right] P_1 = - \frac{\delta(r_1 - \rho_1)}{r_1}
$$

$$
\frac{d^2 \tilde{P}_2}{dr_2^2} + \left(- \frac{1}{4} + \frac{x}{r_2} + \frac{\frac{1}{4} - \bar{\alpha}^2}{r_2^2} \right) \tilde{P}_2 = - \frac{\delta(r_2 - \rho_2)}{\upsilon^{3/2} r_2^{1/2}} \tag{2.5}
$$

where

$$\rho_2 = \nu\rho_1,\ r_2 = \nu r_1 \qquad \nu^2 = 4\beta^2\left[\left(a_2 - \frac{M^2\bar{\omega}_k}{\beta^2}\right)^2 - \frac{M^2\bar{\omega}_k}{\beta^4}\right]$$

$$P_2 = \tilde{P}_2\, r_1^{1/2},\ \bar{\alpha} = \beta_2 k_2 \qquad \chi = -\frac{MM_2k_2(a_2 + \bar{\omega}_k)}{\beta\left[\left(a_2 - \frac{M^2\bar{\omega}_k}{\beta_2}\right)^2 - \frac{M^2\bar{\omega}_k}{\beta_4}\right]}$$

$$\bar{\omega}_k = \frac{\bar{\omega} + k_2(1 + \bar{\Omega})}{\lambda_1} \tag{2.6}$$

We can expand the delta functions $\delta(\theta - \varphi + \omega t_1/\lambda_1)$ and $\delta(z_1 - \zeta_1)$ in Fourier series-integral form as

$$\delta(\theta - \varphi)\delta(z_1 - \zeta_1) = \frac{1}{4\pi^2}\sum_{k_1}\int_{-\infty}^{\infty} e^{ik_1(\theta-\varphi)+ia_1(z_1-\zeta_1)}\, da_1$$

$$\delta(\theta - \varphi + \bar{\omega}t_1/\lambda_1)\delta(z_1 - \zeta_1) = \frac{1}{4\pi^2}\sum_{k_2}\int_{-\infty}^{\infty} e^{ik_2(\theta-\varphi)+ia_2(z_1-\zeta_1)}\, da_2 \tag{2.7}$$

which can be used to determine the Green's functions P_1 and P_2 for the monopoles on the front and rear propellers governed by the nonhomogeneous differential equations (2.5).

FLOW SINGULARITIES FOR THE TWO PROPELLERS

Equation (2.5a) for the front propeller is a nonhomogeneous Bessel's differential equation while equation (2.5b) is the nonhomogeneous Whittaker's differential equation. We can write the complementary solutions for these two equations as

$$P_1(r_1,\rho_1) = A_* J_{k_1}(\lambda_{k_1} r_1) + B_* Y_{k_1}(\lambda_{k_1} r_1)$$

$$P_2(r_1,\rho_1) = \left\{ C_* N_{x_{k_2},\bar{\alpha}}(v_{k_2} r_1) + D_* N_{x_{k_2},-\bar{\alpha}}(v_{k_2} r_1) \right\} v_{k_2}^2 r_1^{1/2} \tag{3.1}$$

These solutions must satisfy the mixed type boundary conditions given by

$$P_1 = 0 \quad \text{at} \quad r_1 = 1 \qquad P_2 = 0 \quad \text{at} \quad r_1 = R_{12}$$

$$\frac{dP_1}{dr_1} = 0 \quad \text{at} \quad r_1 = h_1 \qquad \frac{dP_2}{dr_1} = 0 \quad \text{at} \quad r_1 = R_{12}h_2 \tag{3.2}$$

where A_*, B_*, C_*, and D_* are arbitrary constants. We shall assume here R_{12} = ratio of tip diameters of the front and rear props = 1. We look for solutions such that there is no radial flow at the hub and the pressure perturbation vanishes at the blade tip. From these boundary conditions we can obtain a transcendental equation

$$\frac{J_{k_1-1}(\lambda_{k_1} h_1) - J_{k_1+1}(\lambda_{k_1} h_1)}{Y_{k_2-1}(\lambda_{k_1} h_1) - Y_{k_1+1}(\lambda_{k_1} h_1)} = \frac{J_{k_1}(\lambda_{k_1})}{Y_{k_1}(\lambda_{k_1})} \tag{3.3}$$

giving the eigenvalues λ_{k_1},ℓ for the front propeller. The corresponding Green's function for the oscillating unit pressure monopole on the front propeller is given by

$$P_1(r_1) = -\frac{Y_{k_1}(\lambda_{k_1})}{J_{k_1}(\lambda_{k_1})} J_{k_1}(\lambda_{k_1} r_1) + Y_{k_1}(\lambda_{k_1} r_1) \tag{3.4}$$

The Erdelyi's form of Whittaker function used in equation (3.1) can be related to generalized Laguerre functions through Kummer's function by the relations

$$N_{\chi,\alpha}(z) = \frac{z^{\alpha-1/2}}{\Gamma(1+2\alpha)} M_{\chi,\alpha}(z) = \frac{e^{-z/2}z^{2\alpha}}{\Gamma(1+2\alpha)} {}_1F_1(\alpha + \tfrac{1}{2} - \chi;\ 1 + 2\alpha;\ z)$$

$$= \frac{e^{-z/2}z^{2\alpha}}{\Gamma(1+2\alpha)} \begin{pmatrix} \chi + \alpha - \frac{1}{2} \\ \chi - \alpha - \frac{1}{2} \end{pmatrix}^{-1} L^{(2\alpha)}_{\chi-\alpha-\frac{1}{2}}(z) \qquad (3.5)$$

where $M_{\chi,\alpha}$ is Whittaker's first function, ${}_1F_1$ is Kummer's confluent hypergeometric function. In general there are three distance cases giving transcendental equations for the eigenvalues of the rear propeller. We shall consider here only the case $(1 \pm 2\bar{\alpha}) \neq -m$, $m = 0, 1, 2, 3, \ldots$ for which the eigenvalues are given by

$$\frac{b_1 - \frac{5}{2} h_2 y\, b_{10} L^{(2\bar{\alpha})}_{\frac{2}{5} b_{10}}(h_2 y) - b_3 b_4 L^{(2\bar{\alpha})}_{\frac{2}{5} b_8}(h_2 y)}{b_2 - \frac{5}{4} h_2 y\, b_3 L^{(-2\bar{\alpha})}_{\frac{2}{5} b_3}(h_2 y) - b_4 b_6 L^{(-2\bar{\alpha})}_{\frac{2}{5} b_7}(h_2 y)} = \frac{b_{10} L^{(2\bar{\alpha})}_{\frac{2}{5} b_{10}}(y)}{b_3 h_2^{4\bar{\alpha}} L^{(-2\bar{\alpha})}_{\frac{2}{5} b_3}} \qquad (3.6)$$

in which

$$b_1 = \tilde{\ell} + \frac{7}{2}\bar{\alpha} - 2 \qquad b_2 = \tilde{\ell} - \frac{7}{2}\bar{\alpha} - 2 \qquad b_3 = \tilde{\ell} + \frac{3}{2}\bar{\alpha} - \frac{3}{4}$$

$$b_4 = \tilde{\ell} + \frac{7}{2}\bar{\alpha} - \frac{3}{4} \qquad b_6 = \tilde{\ell} - \frac{7}{2}\bar{\alpha} - \frac{3}{4} \qquad b_7 = \tilde{\ell} + \frac{3}{2}\bar{\alpha} - \frac{13}{4}$$

$$b_8 = \tilde{\ell} - \frac{3}{2}\bar{\alpha} - \frac{13}{4} \qquad b_{10} = \tilde{\ell} - \frac{3}{2}\bar{\alpha} - \frac{3}{4} \qquad b_{13} = \tilde{\ell} - \bar{\alpha} + \frac{7}{4}$$

$$b_{12} = \tilde{\ell} + \frac{7}{2}\bar{\alpha} + \frac{7}{4} \qquad y = v_{\tilde{\ell}k_2} R_{12} \qquad (3.7)$$

The Green's function for the pressure monopole on the rear propeller is therefore given by

$$P_2 = \left[C \left(\frac{r_2}{n}\right)^{4\bar{\alpha}} L^{(2\bar{\alpha})}_{\frac{2}{5}b_{10}}(r_2) + D\, L^{(-2\bar{\alpha})}_{\frac{2}{5}b_4}(r_2) \right] e^{-\frac{1}{2}r_2}\, r_2^{-\left(2\bar{\alpha}+\frac{1}{2}\right)} \tag{3.8}$$

where the coefficients (C,D) are given by

$$C = -\frac{\Gamma\left(\frac{2}{5}b_{12}\right)}{\Gamma\left(\frac{2}{5}b_{13}\right)} \frac{L^{(-2\bar{\alpha})}_{\frac{2}{5}b_4}(n)}{L^{(2\bar{\alpha})}_{\frac{2}{5}b_{10}}(n)} \; ; \qquad D = \frac{\Gamma\left(\frac{2}{5}b_{12}\right)}{\Gamma\left(\frac{2}{5}b_{13}\right)} \tag{3.9}$$

It is seen that the pressure monopole on the propeller has a characteristic Bessel function variation radially. The pressure monopole on the rear propeller has a characteristic radial variation described by the Whittaker's function. In the presence of an external forcing function, the differential equations (2.5) are nonhomogeneous and the pressure variation due to the monopole may be expressed by expanding in terms of appropriate orthonormal eigenfunctions. We shall choose the eigenfunction for the front propeller as the function $\Phi_{k_1\ell}(r_1)$ given by a linear combination of Bessel functions of the first and second king, $J_n(r_1)$ and $Y_n(r_1)$ normalized to satisfy the orthogonality condition

$$\int_{h_1}^{1} r_1 P_1(\lambda_{k\ell} r_1) P_1(\lambda_{km} r_1)\, dr_1 = \delta_{\ell m} m_{k\ell}^2$$

$$\Phi_{k\ell}(r_1) = P_1(\lambda_{k\ell} r_1)/m_{k\ell} \tag{3.10}$$

For the second propeller, since the Whittaker functions do not possess orthogonality properties, it is convenient to relate them to generalized Laguerre polynomials using the relations of equation (3.5). The normalized generalized Laguerre polynomial $L_{\tilde{\ell}}^{(2\bar{\alpha})}(r_2)$ satisfies the orthogonality condition

$$\int_{R_{12}h_2}^{R_{12}} e^{-z} z^{2\bar{\alpha}} L_{\tilde{\ell}}^{(2\bar{\alpha})}(r_2) L_{\tilde{m}}^{(2\bar{\alpha})}(r_2)\, dr_2 = \delta_{\tilde{\ell}\tilde{m}}\, n^2_{\tilde{\ell}k\ell}$$

$$\Psi_{\tilde{\ell}k\ell}(r_1) = L_{\tilde{\ell}}^{(2\bar{\alpha})}(r_2) \Big/ n_{\tilde{\ell}k\ell} \tag{3.11}$$

The solutions of the nonhomogeneous equations (2.5) may be expressed as the eigenfunction expansions given by

$$P_1^{(k_1)}(r_1,\rho_1;a_1) = \sum_{\ell} g_{\ell}^{(k_1)}(a_1)\Phi_{k_1\ell}(r_1)\Phi_{k_1\ell}(\rho_1)$$

$$P_2^{(k_2)}(r_1,\rho_1;a_2) = \sum_{\tilde{\ell}}\sum_{\ell} h_{\tilde{\ell}\ell}^{(k_2)}(a_2)\Psi_{\tilde{\ell}k_2\ell}(r_1)\Psi_{\tilde{\ell}k\ell}(\rho_1)(r_2\rho_2)^{\alpha+\frac{1}{2}}$$

$$e^{-\frac{1}{2}(r_2+\rho_2)} r_2^{-1/2} \upsilon^{1/2}_{\tilde{\ell}k\,\ell_1} \tag{3.12}$$

where $g_{\ell}^{(k_1)}(a\ )$ and $h_{\tilde{\ell}\ell}^{(k_2)}(a_2)$ are coefficients of the expansion to be determined. We shall now expand the Dirac delta functions $\delta(r_1 - \rho_1)$ in equations (2.5) in terms of the eigen functions corresponding to the front and rear propellers as

$$\delta(r_1 - \rho_1) = \sqrt{r_1\rho_1}\sum_{\ell}\Phi_{k_1\ell}(\rho_1)\Phi_{k_1\ell}(r_1)$$

$$\delta(r_1 - \rho_1) = \sum_{\tilde{\ell}}\sum_{\ell} e^{-\frac{1}{2}(r_2+\rho_2)}(r_2\rho_2)^{\bar{\alpha}}\Psi_{\tilde{\ell}k_2\ell}(\rho_1) \tag{3.13}$$

Then, the eigenfunctions $\Phi_{k_1\ell}(r_1)$ and $\Psi_{\tilde{\ell}k_2\ell}(r_1)$ satisfy respectively the Bessel and Laguerre differential equations

$$\frac{d^2\Phi_{k_1\ell}}{dr_1^2} + \frac{1}{r_1}\frac{d\Phi_{k_1\ell}}{dr_1} + \left(\lambda^2_{k_1\ell} - \frac{k_1^2}{r_1^2}\right)\Phi_{k_1\ell} = 0$$

$$\frac{d^2\Psi_{\tilde{\ell}k_2\ell}}{dr_2^2} + \left(\frac{2\bar{\alpha}+1}{r_2} - 1\right)\frac{d\Psi_{\tilde{\ell}k_2\ell}}{dr_2} + \frac{\tilde{p}}{r_2}\Psi_{\tilde{\ell}k_2\ell} = 0 \tag{3.14}$$

where $\tilde{\ell}$ is the degree of the Laguerre function defined by the relation

$$\tilde{\ell} = \chi_{\tilde{\ell}k_2\ell} + \frac{\upsilon^{-3/2}_{\tilde{\ell}k_2\ell}}{h^{(k_2)}_{\tilde{\ell}\ell}} - \left(\bar{\alpha} + \frac{1}{2}\right) \tag{3.15}$$

in terms of its eigenvalues $\upsilon_{\tilde{\ell}k_2\ell}$. The independent variables ρ_2 and r_2 are defined in terms of r_1 by the relation given in equation (2.6). In the above $\lambda_{k_1\ell}$ are the radial eigenvalues of the front and rear propeller, respectively. The eigensolutions of the Laguerre differential equation exist if the degree $\tilde{\ell}$ is an integer. Considering the oscillating unit pressure pole on the front propeller, the expansion coefficient $g_\ell^{(k_1)}$ is obtained by substituting for P_1 and $\delta(r_1 - \rho_1)$ from equations (3.12a) and (3.13a) into equation (2.5) and equating the corresponding terms. Thus,

$$g_\ell^{(k_1)}(a_1) = \left[\Lambda^2_{k_1\ell} + \beta^2\left(a_1 - \frac{M\hat{M}}{\beta^2}\right)^2\right]^{-1} \tag{3.16}$$

where

$$\Lambda^2_{k_1\ell} = \lambda^2_{k_1\ell} - \frac{\hat{M}^2}{\beta^2}, \quad \hat{M} = M(1 + k_1)/\lambda_1 \tag{3.17}$$

Both $\hat{M}$ and $\lambda_{k_1\ell}$ are always real. When the axial flow is subsonic, $M < 1$, $\beta^2 > 0$ and the sign of $\Lambda^2_{k_1\ell}$ depends on the relative magnitudes of $\lambda_{k_1\ell}$ and $\hat{M}/\beta$. But, for supersonic axial flow $M > 1$, $\beta^2 < 0$ and $\Lambda^2_{k_1\ell}$ is always positive.

The perturbation pressure p_1 in equation (2.3a) may be expressed in terms of the eigenfunction $\Phi_{k_1\ell}$ using the Fourier-Bessel expansion (3.12a) as

$$p_1(\vec{r}_1,\vec{\rho}_1,t) = \frac{1}{4\pi^2} \sum_{k_1} \sum_{\ell} \Phi_{k_1\ell}(r_1)\Phi_{k_1\ell}(\rho_1) I_\ell^{(k_1)}(\hat{Z}_1) e^{i[k_1(\theta-\varphi)+\omega_1 t_1/\lambda_1]} \tag{3.18}$$

where

$$I_\ell^{(k_1)}(\hat{Z}_1) = \int_0^\infty e^{ia_1\hat{Z}_1} g_\ell^{(k_1)}(a_1)da_1, \quad \hat{Z}_1 = z_1 - \zeta_1 \tag{3.19}$$

The integral may be evaluated by contour integration methods and written as

$$I_\ell^{(k_2)}(\hat{Z}_1) = \frac{\pi}{\Lambda_{k_1\ell}\beta} \left\{ e^{\hat{Z}_1\left(i\, M\hat{M}/\beta_2 - \Lambda_{k_1\ell}/\beta\right)} - e^{\hat{Z}_1\left(i\, M\hat{M}/\beta_2 + \Lambda_{k_1\ell}/\beta\right)} \right\} \tag{3.20}$$

If $\Lambda^2_{k_1\ell} > 0$, the second term in equation (3.20) is rejected since the integral diverges with $\hat{Z}_1$. We therefore put

$$I_\ell^{(k_1)}\hat{Z}_1) = \frac{\pi}{\Lambda_{k_1\ell}\beta} e^{if_o\hat{Z}_1}, \quad f_o = \frac{M\hat{M}}{\beta_2} + i\,\frac{\Lambda_{k_1\ell}}{\beta} \operatorname{sgn} Z_1 \tag{3.21}$$

Thus, the solution is always bounded and the sign of $\Lambda_{k_1\ell}$ is assigned as follows:

$$
\begin{array}{llll}
\text{if} & \Lambda^2_{k_1\ell} > 0 & \text{put} & \Lambda_{k_1\ell} = |\Lambda_{k_2\ell}| \\
\text{if} & \Lambda^2_{k_1\ell} < 0\ ,\ \hat{M} > 0 & & \Lambda_{k_1\ell} = i|\Lambda_{k_1\ell}| \\
\text{if} & \Lambda^2_{k_1\ell} < 0\ ,\ \hat{M} < 0 & & \Lambda_{k_1\ell} = i|\Lambda_{k_1\ell}|\,\text{sgn}\ \hat{Z}_1
\end{array}
\tag{3.22}
$$

Thus, the pressure field p_1 of a pulsating unit pressure pole situated at $(\rho_1,\ \varphi,\ \zeta_1)$ on a propeller blade becomes

$$
p_1(\vec{r}_1,\vec{\rho}_1,t_1) = \frac{1}{4\pi\beta}\sum_{k_1}\sum_{\ell}\Phi_{k_1\ell}(r_1)\Phi_{k_1\ell}(\rho_1)\Lambda^{-1}_{k_1\ell}e^{i[k_1(\theta-\varphi)+f_o\hat{Z}_1+\omega_1 t_1/\lambda_1]} \tag{3.23}
$$

The field of a pressure pole situated at $(\rho_1,\ \bar{\varphi} + \varphi + m_1\delta_1,\ \zeta_1)$ on the m_1 -th blade of the front prop is given by

$$
p_1(\vec{r}_1,\vec{\rho}_1,t_1) = \frac{1}{4\pi\beta}\sum_{k_1}\sum_{\ell}\Phi_{k_1\ell}(r_1)\Phi_{k_1\ell}(\rho_1)\Lambda^{-1}_{k_1\ell}
$$

$$
\times\ e^{i[k_1(\theta-\bar{\varphi}_1-\varphi-m_1\delta_1)+m_1\bar{\sigma}_1\delta_1+f_o\hat{Z}_1+\omega_1 t_1/\lambda_1]} \tag{3.24}
$$

where $\bar{\sigma}_1$ is the uniform interblade phase angle factor of the front prop, $\bar{\sigma}_1 = 0, 1, 2, 3, \ldots, (Z_1 - 1)$, and δ_1 is the interblade azimuth pitch angle $2\pi/Z_1$. The pressure field of a pulsating unit pressure dipole situated at $\rho_1,\ \bar{\varphi} + \varphi_1 + m_1\delta_1,\ \zeta_1)$ is obtained from that of the pole by differentiating p_1 along the normal to the local camber line giving

$$p_D^{(m_1)} = -\frac{i\cos\alpha_1}{4\pi\beta}\sum_{k_1}\sum_{\ell}\Phi_{k_1\ell}(r_1)\Phi_{k_1\ell}(\rho_1)\Lambda_{k_1\ell}^{-1}$$

$$\times f_2 e^{i[k_1(\theta-\bar{\varphi}_1-\varphi)+m_1\delta_1(\bar{\sigma}_1-k_1)+f_o\hat{Z}_1+\omega_1 t_1/\lambda_1]}$$

$$f_2 = \frac{k_1}{\rho_1} - f_o \tan\alpha_1 \tag{3.25}$$

Putting

$$k_1 = \upsilon_1 Z_1 + \bar{\sigma}_1 \tag{3.26}$$

the pressure field of unit dipoles situated on all the Z_1, blades of the front propeller is obtained by summing over m_1 and we get

$$p_{D_1} = \sum_{m=1}^{Z_1-1} p_D^{(m_1)} = -\frac{iZ_1\cos\alpha_1}{4\pi\beta}\sum_{k_1}\sum_{\ell}\Phi_{k_1\ell}(r_1)\Phi_{k_1\ell}(\rho_1)\Lambda_{k_1\ell}^{-1}$$

$$\times f_2 e^{i[k_1(\theta-\bar{\varphi}_1-\varphi)+f_0\hat{Z}_1+\omega_1 t_1/\lambda_1]} \tag{3.27}$$

Again considering an oscillating unit pressure pole on the rear propeller, the expansion coefficient $h_{\tilde{\ell}\ell}^{(k_2)}(a_2)$ is obtained by substituting for P_2 and $\delta(r_1 - \rho_1)$ from equations (3.12b) and (3.13b) into equation (2.5) and equating the corresponding terms as

$$h_{\tilde{\ell}\ell}^{(k_2)}(a_2) = \frac{1}{\delta\beta^3\left[\ell_1 - 2MM_2k_2(a_2+\bar{\omega}_k)\right]\left[a_2 - a_o^2\beta^2)^2 - a_o^2\right]^{3/2}}$$

$$a_o^2 = \frac{M^2\bar{\omega}_k^2}{\beta^4}\ ; \quad \ell_1 = \tilde{\ell} + \bar{\alpha} + \frac{1}{2} \tag{3.28}$$

The perturbation pressure p_2 in equation (2.3b) may be expressed in terms of the eigenfunction $\Psi_{\tilde{\ell}K_2\ell}(r_1)$ using the Fourier-Laguerre expansion equation (3.12b) as

$$p_2(\vec{r}_1,\vec{\rho}_1,t_1) = \frac{1}{4\pi^2}\sum_{\tilde{\ell}}\sum_{k_2}\sum_{\ell} \upsilon^{1/2}_{\tilde{\ell}k_2\ell}\Psi_{\tilde{\ell}k_2\ell}(r_1)\Psi_{\tilde{\ell}k_2\ell}(\rho_1)r_2^{\bar{\alpha}}\rho_2^{\bar{\alpha}+\frac{1}{2}}e^{-\frac{1}{2}(r_2+\rho_2)}$$

$$\times\mathcal{J}^{(k_2)}_{\tilde{\ell}\ell}(\hat{Z}_2)\exp[ik_2(\theta-\varphi)+i(\bar{\omega}+k_2\bar{\Omega})t_1/\lambda_1] \qquad (3.29)$$

where

$$\mathcal{J}^{(k_2)}_{\tilde{\ell}\ell}(\hat{Z}_2) = \int_{-\infty}^{\infty} e^{ia_2\hat{Z}_2}h^{(k_2)}_{\tilde{\ell}\ell}(a_2)\,da_2 \qquad \hat{Z}_2 = z_1 - \zeta_2 \qquad (3.30)$$

This integral may be evaluated by contour integration methods and written as

$$\mathcal{J}^{(k_2)}_{\tilde{\ell}\ell}(\hat{Z}_2) = \frac{\pi i}{\delta\beta^3 M}e^{if_4\hat{Z}_2}S_* \qquad (3.31)$$

where

$$S_* = \frac{1}{M_2k_2\left(\Delta_o^2 - a_o^2\right)^{3/2}} \qquad \Delta_o = \frac{\ell_1}{2MM_2k_2} - \frac{\bar{\omega}_k^2}{\beta^2}$$

$$f_4 = \frac{M^2\bar{\omega}_k^2}{\beta^2}\left(1 + \frac{\beta}{M\bar{\omega}_k}\,\mathrm{sgn}\,\hat{Z}\right)$$

$$(3.32)$$

We can then write p_2 as

$$p_2(\vec{r}_1,\vec{\rho}_1,t_1) = \frac{i}{32\pi\beta^3 M}\sum_{\tilde{\ell}}\sum_{k_2}\sum_{\ell} \upsilon^{1/2}_{\tilde{\ell}k_2\ell}\Psi_{\tilde{\ell}k_2\ell}(r_1)\Psi_{\tilde{\ell}k_2\ell}(\rho_1)r_2^{\bar{\alpha}}\rho_2^{\bar{\alpha}+\frac{1}{2}}e^{-\frac{1}{2}(r_2+\rho_2)}$$

$$S_*e^{if_4\hat{Z}_2+ik_2(\theta-\varphi)+i(\bar{\omega}+k_2\bar{\Omega})t_1/\lambda_1} \qquad (3.33)$$

giving the field of a pulsating unit pressure pole situated at the point $(\rho_1, \varphi, \zeta_2)$ on the rear propeller. When the pole is located at the point

$(\rho_1, \bar{\varphi}_2 + \varphi_2 + m_2\delta_2, \zeta_2)$ with a uniform interblade phase angle $\bar{\sigma}_2$, the pressure field is given by

$$p_2(\vec{r}_1,\vec{\rho}_1,t_1) = \frac{i}{32\pi\beta^3 M} \sum_{\tilde{\ell}} \sum_{k_2} \sum_{\ell} \nu^{1/2}_{\tilde{\ell}k_2\ell} \Psi_{\tilde{\ell}k_2\ell}(r_1)\Psi_{\tilde{\ell}k_2\ell}(\rho_1) r_2^{\bar{\alpha}} \rho_2^{\bar{\alpha}+\frac{1}{2}} e^{-\frac{1}{2}(r_2+\rho_2)}$$

$$S_* \exp i \; f_4\hat{Z}_2 + k_2(\theta - \bar{\varphi}_2 - \varphi - m_2\delta_2) + m_2\bar{\sigma}_2\delta_2 + (\bar{\omega} + k_2\bar{\Omega})t_1/\lambda_1 \tag{3.34}$$

Again, the pressure field of a pulsating unit pressure dipole at the same point on the blade is obtained by differentiating p_2 normal to the blade mean camber line and is given by

$$p_{D_2}^{(m_2)}(\vec{r}_1,\vec{\rho}_1,t_1) = \frac{-\cos\alpha_2}{32\pi\beta^3 M} \sum_{\tilde{\ell}} \sum_{k_2} \sum_{\ell} \nu^{1/2}_{\tilde{\ell}k_2\ell} \Psi_{\tilde{\ell}k_2\ell}(r_1)\Psi_{\tilde{\ell}k_2\ell}(\rho_1) r_2^{\bar{\alpha}} \rho_2^{\bar{\alpha}+\frac{1}{2}} e^{-\frac{1}{2}(r_2+\rho_2)}$$

$$f_5 \exp i\left[f_4\hat{Z}_2 + k_2(\theta - \bar{\varphi}_2 - \varphi) - m_2\delta_2(k_2 - \bar{\sigma}_2) + (\bar{\omega} + k_2\bar{\Omega})t_1/\lambda_1\right] \tag{3.35}$$

where

$$f_5 = \left(-\frac{k_2}{\rho_1} + f_4 \tan\alpha_2\right)S_* \tag{3.36}$$

$\delta_2 = 2\pi/Z_2$ = interblade azimuth pitch angle of the rear propeller and σ_2 = interblade phase angle factor of the rear propeller, $\sigma_2 = 0, 1, 2, 3, \ldots.$ $(Z_2 - 1)$. For the Z_2 blades, the perturbation pressure of the unit dipole situated on all the blades of the rear propeller is obtained by summation over m_2 and is given by

$$p_{D_2}(\vec{r}_1,\vec{\rho}_1,t_1) = \sum_{m_2=1}^{Z_2-1} p_{D_2}^{(m_2)}(\vec{r}_1,\vec{\rho}_1,t_1)$$

$$= \frac{Z_2 \cos \alpha_2}{32\pi\beta^3 M} \sum_{\tilde{\ell}} \sum_{k_2} \sum_{\ell} v^{1/2}_{\tilde{\ell}k_2\ell} \Psi_{\tilde{\ell}k_2\ell}(r_1)\Psi_{\tilde{\ell}k_2\ell}(\rho_1) r_2^{\bar{\alpha}} \rho_2^{\bar{\alpha}+\frac{1}{2}} e^{-\frac{1}{2}(r_2+\rho_2)}$$

$$f_5 \exp i\left[f_4\hat{Z}_2 + k_2(\theta - \bar{\varphi}_2 - \varphi) + (\bar{\omega} + k_2\bar{\Omega})t_1/\lambda_1\right] \tag{3.37}$$

where

$$k_2 = v_2 Z_2 + \bar{\sigma}_2 \tag{3.38}$$

The dipole solution for the second prop consists of an axial component which is wavelike in $\hat{Z}$, the axial distance from the rear propeller.

When $(\Delta_0^2 - M_*^2) \to 0$, $S_* \to \infty$ and $p_{D_2} \to \infty$ and is termed acoustic resonance. At resonance, $\Delta_0 = \pm M_*$. The eigen-numbers $\tilde{\ell}$ and k_2 at resonance satisfy the relation

$$\tilde{\ell} = -\frac{1}{2} - \beta_2 k_2 + \frac{2M^2 M_2 k_2}{\beta^2} \bar{\omega}_k(\bar{\omega}_k \pm 1) \tag{3.39}$$

the + and - sign corresponding to the advancing and retreating waves respectively.

In equation (3.27) if $\Lambda_{k_1\ell} = 0$, $p_{D_1} \to \infty$ and we have acoustic resonance of the front propeller. The corresponding circumferential mode number and the eigenvalue $\lambda_{k_1\ell}$ satisfy the relation

$$\lambda_{k_1\ell} = \pm \frac{M(1 + k_1)}{\beta\lambda_1} \tag{3.40}$$

The resonance condition for the counter-rotating propeller system is obtained by setting the azimuthal eigen-numbers k_1 and k_2 of the front and rear propeller equal. We then obtain a relation between $\tilde{\ell}$ and $\lambda_{k_1\ell}$ given by

$$\tilde{\ell} = -\frac{1}{2} - \beta_2\left(-1 \pm \frac{\lambda_{k\ell}\beta\lambda_1}{M}\right) + \frac{2M^2M_2}{\beta^2}\left(-1 \pm \frac{\lambda_{k\ell}\beta\lambda_1}{M}\right)\bar{\omega}_k(\bar{\omega}_k \pm 1) \qquad (3.41)$$

The condition $k_1 = k_2$ leads to the relation

$$\bar{\sigma}_2 - \bar{\sigma}_1 = \nu_1 Z_1 - \nu_2 Z_2 \qquad (3.42)$$

connecting the interblade phase angles of the front and rear props.

Equations (3.27) and (3.37) give the pressure field of a pulsating unit pressure dipole situated at a point of the front and rear propeller blade. If Δp_1 and Δp_2 be the upward acting pressures at the point $(\rho_1, \varphi, \zeta_1)$ and $(\rho_1, \varphi, \zeta_2)$ of the front and rear prop blades due to both the camber and thickness distribution and α_1, α_2 are the blade angles of the front and rear propellers at radius ρ_1, considering an element of area $(d\rho_1\, d\zeta_1/\cos\alpha_1)$ and $(d\rho_1\, d\zeta_1/\cos\alpha_2)$ at these points, the strength of the pressure dipole at these points would be

$$\text{front:}\quad \Delta p_1\, d\rho_1\, d\zeta_1/\cos\alpha_1 \qquad \text{rear:}\quad \Delta p_2\, d\rho_1\, d\zeta_1/\cos\alpha_2 \qquad (3.43)$$

The perturbation pressure field P_1 and P_2 for the whole blade is given by

$$P_1 = -\frac{iZ_1}{4\pi\beta}\sum_{k_1}\sum_{\ell}\int_{h_1}^{1}\int_{z_{1LE}}^{z_{1TE}} \Delta p_1(\rho_1,\varphi,\zeta_1)\Phi_*(\vec{r}_1,\vec{\rho}_1,t_1)\, d\rho_1\, d\zeta_1$$

$$P_2 = \frac{Z_2}{32\pi\beta^3 M}\sum_{\tilde{\ell}}\sum_{k_2}\sum_{\ell}\int_{R_{12}h_2}^{R_{12}}\int_{z_{1LE}}^{z_{2TE}} \Delta p_2(\rho_1,\varphi,\zeta_2)\Psi_*(\vec{r}_1,\vec{\rho}_1,t_1)\, d\rho_1\, d\zeta_1$$

(3.44)

where (Z_{1LE}, Z_{1TE}) and (Z_{2LE}, Z_{2TE}) denote the axial positions of the leading and trailing edge of the front and rear propeller blade at any radius ρ_1 and Φ_* and Ψ_* are the kernel functions defined by

$$\Phi_*(\vec{r}_1,\vec{\rho}_1,t_1) = \Phi_{k_1\ell}(r_1)\Phi_{k_1\ell}(\rho_1)\Lambda^{-1}_{k_1\ell}f_2 e^{i[k_1(\theta-\bar{\varphi}_1-\varphi)+f_0\tilde{Z}_1+\omega_1 t_1/\lambda_1]}$$

$$\Psi_*(\vec{r}_1,\vec{\rho}_1,t_1) = \Psi_{\tilde{\ell}k_2\ell}(r_1)\Psi_{\tilde{\ell}k_2\ell}(\rho_1)r_2^{\tilde{\alpha}}\,\rho_2^{\bar{\alpha}+\frac{1}{2}}\,e^{-\frac{1}{2}(r_2+\rho_2)}\,\upsilon^{1/2}_{\tilde{\ell}k_2\ell}$$

$$f_5 e^{i[f_4\hat{Z}_2+k_2(\theta-\bar{\varphi}_2-\varphi)+(\bar{\omega}+k_2\bar{\Omega})t_1/\lambda_1]}$$

(3.45)

If the blade loadings Δp_1 and Δp_2 are known, the resultant acoustic pressure at an arbitrary point in the flow at time t_1 is given by

$$P(\vec{r}_1,t_1) = P_1(\vec{r}_1,t_1) + P_2(\vec{r}_1,t_1) \qquad (3.46)$$

Since P_1 is a function of the mode numbers (k, ℓ) and P_2 is a function of the mode numbers $(\tilde{\ell}, k, \ell)$, the resultant acoustic pressure P at a point is a function of the mode numbers $(\tilde{\ell}, k, \ell)$. Summation over $\tilde{\ell}$ $(\tilde{\ell} = 1, 2, 3 \ldots)$, k $(k = 0, 1, 2, 3 \ldots)$ and ℓ $(\ell = 0, 1, 2, 3 \ldots.)$ of equation (3.46) gives the total sound pressure level at any point for all modes together.

PERTURBATION VELOCITIES

We shall split the surface pressure distribution Δp_1 and Δp_2 on the blades into thickness and camber components for blade. The thickness and camber contributions will be separated into radial and chordwise variables as

$$\Delta p_{1t} = F_1(\tilde{\omega}_1)\Phi_{k\ell}(\rho_1) \qquad \Delta p_{1c} = F_2(\tilde{\omega}_1)\Phi_{k\ell}(\rho_1)$$

$$\Delta p_{2t} = G_1(\tilde{\omega}_2)\Psi_{\tilde{\ell}k\ell}(\rho_1) \qquad \Delta p_{2c} = G_2(\tilde{\omega}_2)\Psi_{\ell\tilde{k}\ell}(\rho_1) \tag{4.1}$$

where $\tilde{\omega}$ is a Glauert angle defined by

$$y_1' = -\, c_1 \cos \tilde{\omega}_1 \qquad -\, c_1 \le y_1' \le +\, c_1 \qquad o \le \tilde{\omega}_1,\ \tilde{\omega}_2 \le \pi$$

$$y_2' = -\, c_2 \cos \tilde{\omega}_2 \qquad -\, c_2 \le y_2' \le +\, c_2 \tag{4.2}$$

The functions F_1, F_2, G_1, G_2 will be expressed in a Birnbaum-Glauert series expansion as

$$F_1(\tilde{\omega}) = A_o \cot \frac{\tilde{\omega}}{2} + \sum_{m=1}^{\infty} A_m \sin m\tilde{\omega} \qquad F_2(\tilde{\omega}) = B_o\left(\cot \frac{\tilde{\omega}}{2} - 2 \sin \tilde{\omega}\right) + \sum_{m=1}^{\infty} B_m \sin m\tilde{\omega}$$

$$G_1(\tilde{\omega}) = C_o \cot \frac{\tilde{\omega}}{2} + \sum_{m=1}^{\infty} C_m \sin m\tilde{\omega} \qquad G_2(\tilde{\omega}) = D_o\left(\cot \frac{\tilde{\omega}}{2} - 2 \sin \tilde{\omega}\right) + \sum_{m=1}^{\infty} D_m \sin m\tilde{\omega} \tag{4.3}$$

assuming the coefficients A_m, B_m, C_m, D_m to be independent of $\tilde{\ell}$, k, and ℓ. Combining equations (4.1) to (4.3), we have

$$H_1(\tilde{\omega}_1,\rho_1;k,\ell) = \Delta p_1 = \left(\mathscr{A}_o \cot \frac{\tilde{\omega}_1}{2} + \sum_{m=1}^{\infty} \mathscr{A}_m \sin m\tilde{\omega}_1\right)\Phi_{k\ell}(\rho_1)$$

$$H_2(\tilde{\omega}_2,\rho_1;\tilde{\ell},k,\ell) = \Delta p_2 = \left(\mathscr{B}_o \cot \frac{\tilde{\omega}_2}{2} + \sum_{m=1}^{\infty} \mathscr{B}_m \sin m\tilde{\omega}_2\right)\Psi_{\tilde{\ell}k\ell}(\rho_1) \tag{4.4}$$

The coefficients $\mathscr{A}_m$ and $\mathscr{B}_m$ are determined by satisfying the surface tangency boundary conditions on the blades. To satisfy the boundary conditions of flow tangency on the blades, we have to obtain the perturbation velocities $\vec{\tilde{u}}_1$ and $\vec{\tilde{u}}_2$ from the perturbation pressures P_1 and P_2 by integration of the unsteady Euler equation of motion. For this purpose we introduce the helical coordinate system (r_1, σ_1, τ_1) and (r_1, σ_2, τ_2) for the front and rear prop based on the undisturbed free stream for each (fig. 2(c)). We define the helical angles θ_{h_1} and θ_{h_2} defined by

$$\tan \theta_{h_1} = r_1/\lambda_1 \qquad \tan \theta_{h_2} = \frac{r_1}{\lambda_1} - \tan \bar{\alpha}_2 = \tan \theta_{h_1} - \tan \bar{\alpha}_2 \tag{4.5}$$

In addition, we also introduce the blade section coordinates (r_1, y_1', z_1') and (r_1, y_2', z_2') (fig. 28) which are related to the cylindrical coordinates (r_1, θ, z_1) by the transformation

$$\begin{pmatrix} dr_1 \\ dy_1' \\ dz_1' \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \sin \alpha_1 & \cos \alpha_1 \\ 0 & \cos \alpha_1 & \sin \alpha_1 \end{pmatrix} \begin{pmatrix} dr_1 \\ r_1\, d\theta \\ dz_1 \end{pmatrix} \; ; \; \begin{pmatrix} dr_1 \\ dy_2' \\ dz_2' \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \sin \alpha_2 & \cos \alpha_2 \\ 0 & \cos \alpha_2 & \sin \alpha_2 \end{pmatrix} \begin{pmatrix} dr_1 \\ r_1\, d\theta \\ dz_1 \end{pmatrix} \tag{4.6}$$

We also have the transformations between the $(\tilde{u}_{1r}, \tilde{u}_y, \tilde{u}_z)$, $(\tilde{u}_{2r}, \tilde{u}_{2y}, \tilde{u}_{2z})$ to the $(\tilde{u}_{r1}, \tilde{u}_{\sigma 1}, \tilde{u}_{\tau 1})$, and $(\tilde{u}_{r2}, \tilde{u}_{\sigma 2}, \tilde{u}_{\tau 2})$ coordinates

$$
\begin{pmatrix} \tilde{u}_{1r} \\ \tilde{y}_{1y'} \\ \tilde{u}_{1z'} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_1 & \sin\theta_1 \\ 1 & \sin\theta_1 & \cos\theta_1 \end{pmatrix} \begin{pmatrix} \tilde{u}_{r1} \\ \tilde{u}_{\sigma 1} \\ \tilde{u}_{t1} \end{pmatrix} ; \begin{pmatrix} \tilde{u}_{2r} \\ \tilde{u}_{2y'} \\ \tilde{u}_{2z'} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_4 & \sin\theta_4 \\ 0 & \sin\theta_4 & \cos\theta_4 \end{pmatrix} \begin{pmatrix} \tilde{u}_{r2} \\ \tilde{u}_{\sigma 2} \\ \tilde{u}_{t2} \end{pmatrix}
$$

$$
\begin{pmatrix} \tilde{u}_{1r} \\ \tilde{y}_{1y'} \\ \tilde{u}_{1z'} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_2 & \sin\theta_2 \\ 1 & \sin\theta_2 & \cos\theta_2 \end{pmatrix} \begin{pmatrix} \tilde{u}_{r2} \\ \tilde{u}_{\sigma 2} \\ \tilde{u}_{t2} \end{pmatrix} ; \begin{pmatrix} \tilde{u}_{2r} \\ \tilde{u}_{2y'} \\ \tilde{u}_{2z'} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_3 & \sin\theta_3 \\ 0 & \sin\theta_3 & \cos\theta_3 \end{pmatrix} \begin{pmatrix} \tilde{u}_{r1} \\ \tilde{u}_{\sigma 1} \\ \tilde{u}_{t1} \end{pmatrix} \tag{4.7}
$$

where

$$
\begin{aligned}
\theta_1 &= \theta_{h_1} - \alpha_1 & \theta_2 &= \theta_{h_2} - \alpha_1 \\
\theta_3 &= \theta_{h_1} - \alpha_2 & \theta_4 &= \theta_{h_2} - \alpha_2
\end{aligned} \tag{4.8}
$$

The first matrix transforms from the (r_1, σ_1, τ_1) to (r_1, y_1', z_1') coordinates; the second matrix transforms from (r_1, σ_2, τ_2) to (r_1, y_2', z_2'); the third matrix transforms from the (r_1, σ_2, τ_2) to the (r_1, y_1', z_1'); and the fourth matrix transforms from (r_1, σ_1, τ_1) to (r_1, y_2', z_2') coordinates. The resultant velocity including the respective free stream components are given by

$$\begin{pmatrix} U_{1r} \\ U_{1y'} \\ U_{1z'} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \sin\alpha_1 & \cos\alpha_1 \\ 0 & -\cos\alpha_1 & \sin\alpha_1 \end{pmatrix} \begin{pmatrix} 0 \\ r_1/\lambda_1 \\ 1 \end{pmatrix} + \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_1 & \sin\theta_1 \\ 0 & \sin\theta_1 & \cos\theta_1 \end{pmatrix} \begin{pmatrix} \tilde{u}_{r1} \\ \tilde{u}_{\sigma 1} \\ \tilde{u}_{\tau 1} \end{pmatrix}$$

$$+ \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_2 & \sin\theta_2 \\ 0 & \sin\theta_2 & \cos\theta_2 \end{pmatrix} \begin{pmatrix} \tilde{u}_{r2} \\ \tilde{u}_{\sigma 2} \\ \tilde{u}_{\tau 2} \end{pmatrix}$$

$$\begin{pmatrix} U_{2r} \\ U_{2y'} \\ U_{1z'} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \sin\alpha_2 & \cos\alpha_2 \\ 0 & \cos\alpha_1 & \sin\alpha_1 \end{pmatrix} \begin{pmatrix} 0 \\ \frac{r_1}{\lambda_1} - \tan\bar{\alpha}_2 \\ 1 \end{pmatrix}$$

$$+ \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_3 & \sin\theta_3 \\ 0 & \sin\theta_3 & \cos\theta_3 \end{pmatrix} \begin{pmatrix} \tilde{u}_{r1} \\ \tilde{u}_{\sigma 1} \\ \tilde{u}_{\tau 1} \end{pmatrix} + \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_2 & \sin\theta_2 \\ 0 & \sin\theta_2 & \cos\theta_2 \end{pmatrix} \begin{pmatrix} \tilde{u}_{r2} \\ \tilde{u}_{\sigma 2} \\ \tilde{u}_{\tau 2} \end{pmatrix} \tag{4.9}$$

The dimensionless equations of motion in the corresponding helical coordinates (r, σ, τ) may be written in the form

$$\frac{\partial \vec{\tilde{u}}_1}{\partial t_1} + \tilde{u}_{01} \frac{\partial \vec{\tilde{u}}_1}{\partial \sigma_1} = -\frac{1}{\gamma M^2} \left(\frac{\partial P_1}{\partial r_1}, \frac{\partial P_1}{\partial \sigma_1}, \frac{\partial P_1}{\partial \tau_1} \right)$$

$$\frac{\partial \vec{\tilde{u}}_2}{\partial t_1} + \tilde{u}_{02} \frac{\partial \vec{\tilde{u}}_2}{\partial \sigma_2} = -\frac{1}{\gamma M^2} \left(\frac{\partial P_2}{\partial r_1}, \frac{\partial P_2}{\partial \sigma_2}, \frac{\partial P_2}{\partial \tau_2} \right) \tag{4.10}$$

where

$$\tilde{u}_{01} = \frac{\left(W_a^2 + \Omega_1^2 r_1^2\right)^{1/2}}{W_a} = \left(1 + \frac{r_1^2}{\lambda_1^2}\right)^{1/2} = \sec\theta_{h_1}$$

$$\tilde{u}_{02} = \frac{\left\{W_a^2 - (r\Omega_r - W_a \tan\bar{\alpha}_2)^2\right\}^{1/2}}{W_a} = \left[1 + \left(\frac{\bar{\Omega} r_1}{\lambda_1} - \tan\bar{\sigma}_2\right)^2\right]^{1/2} = \sec\theta_{h_2}$$

$$\vec{\tilde{u}}_1 = \vec{u}_1/W_a \qquad \vec{\tilde{u}}_2 = \vec{u}_2/W_a \tag{4.11}$$

so that we have

$$\frac{\partial\vec{\tilde{u}}_1}{\partial t_1} + \sec\theta_{h_1}\frac{\partial\vec{\tilde{u}}_1}{\partial\sigma_1} = -\frac{1}{\gamma M^2}\left(\frac{\partial P_1}{\partial r_1}, \frac{\partial P_1}{\partial\sigma_1}, \frac{\partial P_1}{\partial\tau_1}\right)$$

$$\frac{\partial\vec{\tilde{u}}_2}{\partial t_1} + \sec\theta_{h_2}\frac{\partial\vec{\tilde{u}}_2}{\partial\sigma_2} = -\frac{1}{\gamma M^2}\left(\frac{\partial P_2}{\partial r_1}, \frac{\partial P_2}{\partial\sigma_2}, \frac{\partial P_2}{\partial\tau_2}\right) \tag{4.12}$$

We shall assume that the perturbation velocities $\vec{\tilde{u}}_1$ and $\vec{\tilde{u}}_2$ corresponding pressures P_1 and P_2 vary harmonically as follows:

$$\vec{u}_1 = \vec{\tilde{u}}_1\, e^{i\omega_1 t_1/\lambda_1} \qquad \vec{u}_2 = \vec{\tilde{u}}_2\, e^{i(\bar{\omega}+k_2\bar{\Omega})t_1/\lambda_1}$$

$$P_1 = \tilde{P}_1\, e^{i\omega_1 t_1/\lambda_1} \qquad P_2 = \tilde{P}_2\, e^{i(\bar{\omega}+k_2\bar{\Omega})t_1/\lambda_1} \tag{4.13}$$

Substituting into equation (4.12) we obtain the equations

$$\frac{\partial\vec{\tilde{u}}_1}{\partial\sigma_1} + i\,\frac{\omega_1}{\lambda_1}\cos\theta_{h_1}\,\vec{\tilde{u}}_1 = -\frac{\cos\theta_{h_1}}{\gamma M^2}\left(\frac{\partial\tilde{P}_1}{\partial r_1}, \frac{\partial\tilde{P}_1}{\partial\sigma_1}, \frac{\partial\tilde{P}_1}{\partial\tau_1}\right)$$

$$\frac{\partial\vec{\tilde{u}}_2}{\partial\sigma_2} + i\,\frac{(\bar{\omega} + k_2\bar{\Omega})}{\lambda_1}\cos\theta_{h_2}\,\vec{\tilde{u}}_2 = -\frac{\cos\theta_{h_2}}{\gamma M^2}\left(\frac{\partial\tilde{P}_2}{\partial r_1}, \frac{\partial\tilde{P}_2}{\partial\sigma_2}, \frac{\partial\tilde{P}_2}{\partial\tau_2}\right) \tag{4.14}$$

which can be integrated with respect to σ_1 and σ_2 respectively giving the perturbation velocities $\vec{\tilde{u}}_1$ and $\vec{\tilde{u}}_2$ produced by the front and rear propellers each in isolation as

$$\vec{\tilde{u}}_1 e^{i\omega_1\sigma_1 \cos\theta_{h_1}/\lambda_1} = \vec{\tilde{u}}_{1\infty} - \frac{\cos\theta_{h_1}}{\gamma M^2}\int e^{i\omega_1\sigma_1 \cos\theta_{h_1}/\lambda_1}\left(\frac{\partial\tilde{P}_1}{\partial r_1}, \frac{\partial\tilde{P}_1}{\partial\sigma_1}, \frac{\partial\tilde{P}_1}{\partial\tau_1}\right)d\sigma_1$$

$$\vec{\tilde{u}}_2 e^{i(\bar{\omega}+k_2\bar{\Omega})/\lambda_1\ \sigma_1 \cos\theta_{h_2}} = \vec{\tilde{u}}_{2\infty} - \frac{\cos\theta_{h_2}}{\gamma M^2}$$

$$\int e^{i(\bar{\omega}+k_2\bar{\Omega})/\lambda_1\ \sigma_2 \cos\theta_{h_2}}\left(\frac{\partial\tilde{P}_2}{\partial r_1}, \frac{\partial\tilde{P}_2}{\partial\sigma_2}, \frac{\partial\tilde{P}_2}{\partial\tau_2}\right)d\sigma_2$$

(4.15)

where $\vec{\tilde{u}}_{1\infty}$ and $\vec{\tilde{u}}_{2\infty}$ are the free stream values of the perturbation velocities corresponding to $\sigma_1, \sigma_2 \to \infty$ which we shall assume to be zero so that we have

$$\vec{\tilde{u}}_1 = \frac{e^{-i\omega_1\sigma_1 \cos\theta_{h_1}/\lambda_1}\cos\theta_{h_1}}{\gamma M^2}\int e^{i\omega_1\sigma_1 \cos\theta_{h_1}/\lambda_1}\left(\frac{\partial P_1}{\partial r_1}, \frac{\partial P_1}{\partial\sigma_1}, \frac{\partial P_1}{\partial\tau_1}\right)d\sigma_1$$

$$\vec{\tilde{u}}_2 = \frac{e^{-i(\bar{\omega}+k_2\bar{\Omega})/\lambda_1)\sigma_2 \cos\theta_{h_2}}\cos\theta_{h_2}}{\gamma M^2}$$

$$\int e^{i(\bar{\omega}+k_2\bar{\Omega})/\lambda_1\ \sigma_2 \cos\theta_{h_2}}\left(\frac{\partial P_2}{\partial r_1}, \frac{\partial P_2}{\partial\sigma_2}, \frac{\partial P_2}{\partial\tau_2}\right)d\sigma_2$$

(4.16)

The helix angles θ_{h_1} and θ_{h_2} can be related to the variables θ and z_1 by the relations

$$\theta_{h_1} = \theta - \frac{z_1}{\lambda_1} \qquad \theta_{h_2} = \theta - \frac{z_1\Omega_r R}{W_a} = \theta - \frac{\bar{\Omega}z_1}{\lambda_1} \qquad (4.17)$$

the differential transformation from (r_1, θ, z_1) to the helical coordinates (r_1, σ_1, τ_1) and (r_1, σ_2, τ_2) is given by

$$\begin{pmatrix} dr_1 \\ r_1\, d\theta \\ dz_1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \sin\theta_{h_1} & -\cos\theta_{h_1} \\ 1 & \cos\theta_{h_1} & \sin\theta_{h_1} \end{pmatrix} \begin{pmatrix} dr_1 \\ d\sigma_1 \\ d\tau_1 \end{pmatrix} ; \quad \begin{pmatrix} dr_1 \\ r_1\, d\theta \\ dz_1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \sin\theta_{h_2} & -\cos\theta_{h_2} \\ 0 & \cos\theta_{h_2} & \sin\theta_{h_2} \end{pmatrix} \begin{pmatrix} dr_1 \\ d\sigma_2 \\ d\tau_2 \end{pmatrix} \tag{4.18}$$

When moving on a given helical stream line, θ_{h_1} = constant, $d\tau_1 = d\tau_2 = 0$ and we have

$$\text{front prop:} \quad d\sigma_1 = dz_1 \sec\theta_{h_1} \; ; \qquad \text{rear prop:} \quad d\sigma_2 = dz_1 \sec\theta_{h_2} \tag{4.19}$$

Integrating and putting $z = 0$ for $\sigma = 0$, we have

$$\sigma_1 = z_1 \sec\theta_{h_1} \qquad \sigma_2 = z_1 \sec\theta_{h_2} \tag{4.20}$$

Hence, performing the integrations in equation (4.16) along the helical streamlines of the respective free streams for the perturbation velocity vectors we obtain

$$\vec{\tilde{u}}_1 = -\frac{e^{-i\omega_1 z_1/\lambda_1}}{\gamma M^2} \int e^{i\omega_1 z_1/\lambda_1} \left(\frac{\partial \tilde{P}_1}{\partial r_1}, \frac{\partial \tilde{P}_1}{\partial \sigma_1}, \frac{\partial \tilde{P}_1}{\partial \tau_1} \right) dz_1$$

$$\vec{\tilde{u}}_2 = -\frac{e^{-i(\bar{\omega}+k_2\bar{\Omega}) z_1/\lambda_1}}{\gamma M^2} \int e^{i(\bar{\omega}+k_2\bar{\Omega}) z_1/\lambda_1} \left(\frac{\partial \tilde{P}_2}{\partial r_1}, \frac{\partial \tilde{P}_2}{\partial \sigma_2}, \frac{\partial \tilde{P}_2}{\partial \tau_2} \right) dz_1 \tag{4.21}$$

where the amplitudes of the perturbation pressures $\tilde{P}_1$ and $\tilde{P}_2$ are given by

$$\tilde{P}_1 = -\frac{iZ_1}{4\pi\beta} \sum_{k_1} \sum_{\ell} \int (\pi\mathcal{A}) \mathcal{F}_{20} \mathcal{U}_3 \exp i[k_1(\theta - \bar{\varphi}_1 - \varphi_1) + f_o \hat{Z}_1]\, d\zeta_1$$

$$\tilde{P}_2 = \frac{Z_2}{32\pi\beta^3 M} \sum_{k_2} \sum_{\tilde{\ell}} \sum_{\ell} \int (\pi\mathcal{B}) \mathcal{F}_{50} \mathcal{V}_3 \exp i[f_4 Z_2 + k_2(\theta - \varphi_2 - \varphi_2)]\, d\zeta_2 \tag{4.22}$$

with

$$\mathscr{A} = (\mathscr{A}_0, \mathscr{A}_1, \mathscr{A}_2, \dots \mathscr{A}_m, \dots)^T \qquad \boldsymbol{\pi} = \left(\cot\frac{\tilde{\omega}}{2}, \sin\tilde{\omega}, \sin 2\tilde{\omega}, \dots \sin m\tilde{\omega}, \dots\right)$$

$$\mathscr{B} = (\mathscr{B}_0, \mathscr{B}_2, \mathscr{B}_2, \dots \mathscr{B}_m, \dots)^T \qquad \mathscr{V}_3 = (k_2\mathscr{V}_1 - f_4\mathscr{V}_2)S_*$$

$$\mathscr{U}_1 = \int_{h_1}^{1} \frac{1}{\rho_1} \Phi^2_{k_1\ell}(\rho_1)\, d\rho_1 \qquad \mathscr{V}_1 = \int_{R_{12}h_2}^{R_{12}} \Psi^2_{\tilde{\ell}k_2\ell}(\rho_1) e^{-\frac{1}{2}\rho_2} \rho_2^{\bar{\alpha}-\frac{1}{2}} \Upsilon_{\tilde{\ell}k_2\ell}\, d\rho_2$$

$$\mathscr{U}_2 = \int_{h_1}^{1} \Phi^2_{k_1\ell}(\rho_1) \tan\alpha_1\, d\rho_1 \qquad \mathscr{V}_2 = \int_{R_{12}h_2}^{R_{12}} \Psi^2_{\tilde{\ell}k_2\ell}(\rho_1) e^{-\frac{1}{2}\rho_2} \rho_2^{\bar{\alpha}+\frac{1}{2}} \tan\alpha_2\, d\rho_2$$

$$\mathscr{U}_3 = k_1\mathscr{U}_1 - \mathscr{U}_2 f_o \qquad \mathscr{F}_{50} = \Psi_{\tilde{\ell}k_2\ell}(r_1) r_2^{\bar{\alpha}} e^{-\frac{1}{2}r_2} \upsilon^{-1/2}_{\tilde{\ell}k\ell}$$

$$\mathscr{F}_{20} = \Phi_{k_1\ell}(r_1)/\Lambda_{k_1\ell} \tag{4.23}$$

Introducing the expressions for $\tilde{P}_1$ and $\tilde{P}_2$ from equation (4.22) into equation (4.21) we can write the perturbation velocity components of $\vec{\tilde{u}}_1$ and $\vec{\tilde{u}}_2$ in concise form

$$\tilde{u}_{r1} = K_{r1}\mathscr{A} \qquad \tilde{u}_{\sigma 1} = K_{\sigma 1}\mathscr{A} \qquad \tilde{u}_{\tau 1} = K_{\tau 1}\mathscr{A}$$

$$\tilde{u}_{r2} = K_{r2}\mathscr{B} \qquad \tilde{u}_{\sigma 2} = K_{\sigma 2}\mathscr{B} \qquad \tilde{u}_{\tau 2} = K_{\tau 2}\mathscr{B} \tag{4.24}$$

where

$$K_{r1} = + \frac{i Z_1 e^{-i\omega_1 z_1/\lambda_1}}{4\pi\gamma\beta M^2} \sum_{k_1} \sum_{\ell} \iint \pi \frac{\partial}{\partial r_1} \left\{ \mathcal{F}_{20} \mathcal{U}_4 e^{i[k_1(\theta - \bar{\varphi}_1 - \varphi_1) + f_0 \hat{Z}_1]} \right\} e^{i\omega_1 z_1/\lambda_1} \, d\zeta_1 \, dz_1$$

$$K_{\sigma 1} = + \frac{i Z_1 e^{-i\omega_1 z_1/\lambda_1}}{4\pi\gamma\beta M^2} \sum_{k_1} \sum_{\ell} \iint \pi \frac{\partial}{\partial \sigma_1} \left\{ \mathcal{F}_{20} \mathcal{U}_4 e^{i[k_1(\theta - \bar{\varphi}_1 - \varphi_1) + \theta_0 \hat{Z}_1]} \right\} e^{i\omega_1 z_1/\lambda_1} \, d\zeta_1 \, dz_1$$

$$K_{\tau 1} = K_{\sigma 1} \cot 2\, \theta_{h1}$$

$$K_{2r} = \frac{-Z_2}{32\pi\gamma\beta^3 M^3} \sum_{\tilde{\ell}} \sum_{k_2} \sum_{\ell} \iint \pi \frac{\partial}{\partial r_1} \; \mathcal{F}_{50} \mathcal{V}_3 \exp i[f_4 \hat{Z}_2 + k_2(\theta - \varphi \; - \varphi\;)] \, d\zeta_2 \, dz_2$$

$$K_{\sigma 2} = - \frac{Z_2}{32\pi\gamma\beta^3 M^3} \sum_{\tilde{\ell}} \sum_{k_2} \sum_{\ell} \iint \pi \frac{\partial}{\partial \sigma_2} \; \mathcal{F}_{50} \mathcal{V}_3 \exp i[f_4 \hat{Z}_2 + k_2(\theta - \bar{\varphi}_2 - \varphi_2)] \, d\zeta_2 \, dz_2$$

$$K_{\tau 2} = \frac{\bar{\Omega} \tan \theta_{h_2} - \cot \theta_{h_2}}{2\bar{\Omega} - \tan \theta_{h_1} \tan \theta_{h_2}} K_{\sigma 2} \tag{4.25}$$

are row vectors. In the above, the integration over z_1 is an indefinite integral valid over the range $(-\infty, +\infty)$, the integral over ζ_1 is a definite integral extending over the domain $(\zeta_{1LE}, \zeta_{1TE})$ for the front propeller and over the limits $(\zeta_{2LE}, \zeta_{2TE})$ for the rear propeller.

BOUNDARY CONDITIONS AND BIRNBAUM COEFFICIENTS

The equations to the upper and lower surfaces of the front and rear propeller blades can be expressed in terms of the mean camber line and thickness and may be written

$$z'_{u1} = z'_{u1}(r_1, y'_1) \qquad z'_{L1} = z'_{L1}(r_1, y'_1)$$

$$z'_{u2} = z'_{u2}(r_1, y'_2, t_1) \qquad z'_{L2} = z'_{L2}(r_1, y'_2, t_1) \tag{5.1}$$

assuming that the profile of the blade is independent of the radius r. In the coordinate system chosen, the front rotor blades are stationary and the equation of the front blade surface is invariant with time. But, since the rear propeller moving relative to the front prop with an angular velocity Ω_r, the rear prop surface is a function of time. We can write the dimensionless equation for the m-th twisted, tapered and swept blade in the (r, θ, z) reference frame as front:

$$F = z_1 - r_1 \cot \alpha_1 \tan (\theta - \bar{\varphi}_1 - \delta_{1m}) - \zeta_{01} - Z_1' + Y_1' \cot \alpha_1 + z_1' c_1 \csc \alpha_1 = 0$$

$$F = z_1 - r_1 \cot \alpha_2 \tan \left(\theta - \bar{\varphi}_2 - \delta_{2m} + \frac{\bar{\Omega} t_1}{\lambda_1}\right) - \zeta_{02} - Z_2' + Y_2' \cot \alpha_2 + z_2' c_2 \csc \alpha_2 = 0 \tag{5.2}$$

where

ζ_{0i} the dimensionless axial displacement of the propeller from the origin (fig. 1(b)).

Y_i', Z_i' the dimensionless Y and Z coordinated of the blade half-chord line S (fig. 1(a)) such that $Y' = Y/R$ and $Z' = Z/R$.

C_i dimensionless blade half-chord at any radius (fig. 2(a)).

Z_i' dimensionless ordinate of the blade profile.

α_i blade angle distribution.

$\bar{\varphi}_i$ off-set angle of the first blade (fig. 1(a)).

the subscript $i = 1,2$ to designate the front and rear prop blades respectively. From these the upper and lower blade surfaces are given by the equations:

$$z_{1U} = \zeta_{01} + Z_1' - Y_1' \cot \alpha_1 - z_{1U}' c_1 \csc \alpha_1 + r_1 \cot \alpha_1 \tan (\theta - \bar{\varphi}_1 - \delta_{1m})$$

$$z_{1L} = \zeta_{01} + Z_1' - Y_1' \cot \alpha_1 - z_{1L}' c_1 \csc \alpha_1 + r_1 \cot \alpha_1 \tan (\theta - \bar{\varphi}_1 - \delta_{1m})$$

$$z_{2U} = \zeta_{02} + Z_2' - Y_2' \cot \alpha_2 - z_{2U}' c_2 \csc \alpha_2 + r_1 \cot \alpha_2 \tan (\theta - \bar{\varphi}_2 - \delta_{2m} + \bar{\Omega} t_1/\lambda_1)$$

$$z_{2L} = \zeta_{02} + Z_2' - Y_2' \cot \alpha_2 - z_{2L}' c_2 \csc \alpha_2 + r_1 \cot \alpha_2 \tan (\theta - \bar{\varphi}_2 - \delta_{2m} + \bar{\Omega} t_1/\lambda_1) \tag{5.3}$$

The flow tangency condition for the front prop may be expressed as

$$\tau_{U1} = \frac{dz_{1U}'}{dy_1'} = \left(\frac{U_{1z'}}{U_{1y'}}\right)_{z' = 0+} \qquad \tau_{L1} = \frac{dz_{1L}'}{dy_1'} = \left(\frac{U_{1z'}}{U_{1y'}}\right)_{z' = 0-} \tag{5.4}$$

where $U_{1y'}$ and $U_{1z'}$ are the y_1' and z_1' components of the resultant velocity at the front prop due to both the propellers. For the rear prop, the surface boundary condition is obtained by equating the substantial derivative $DF/Dt = 0$ where $F(r, \theta, z, t) = 0$ is the equation of the rear prop blade surface. For an arbitrarily chosen first blade, m = 1, this may be written as

$$U_{2r} \sin 2\left(\theta - \bar{\varphi}_2 + \frac{\bar{\Omega} t_1}{\lambda_1}\right) + 3U_{2o} - 2(\tau_2 c_2 + \tan \alpha_2) U_{2z} \cos^2\left(\theta - \bar{\varphi}_2 + \frac{\bar{\Omega} t_1}{\lambda_1}\right)$$

$$= 2 \tan \bar{\alpha}_2 + 2(\tau_2 c_2 + \tan \alpha_2) \cos^2\left(\theta - \varphi_2 + \frac{\bar{\Omega} t_1}{\lambda_1}\right) - 4\bar{\Omega} \tan \theta_{h_1} \tag{5.5}$$

Since the dimensionless free stream for the rear propeller has the velocity components

$$U_{2r} = \tilde{u}_{r2} \; ; \; U_{2\theta} = \tan \theta_{h_1} - \tan \bar{\alpha}_2 + \tilde{u}_{\theta 2} \; ; \; U_{2z} = 1 + \tilde{u}_{z2} \tag{5.6}$$

The boundary conditions on the rear blade will be satisfied at an arbitrary time $t_1 = 0$ on the upper and lower surface and can be written

$$U_{2r} \sin 2(\theta - \bar{\varphi}_2) + 2U_{2\theta} - 2(\tau_{U2}c_2 + \tan \bar{\alpha}_2)U_{2z} \cos^2(\theta - \bar{\varphi}_2)$$
$$= 2 \tan \bar{\alpha}_2 + 2(\tau_{U2}c_2 + \tan \bar{\alpha}_2) \cos^2(\theta - \bar{\varphi}_2) - 4\bar{\Omega} \tan \theta_{h_1}$$

$$U_{2r} \sin 2(\theta - \bar{\varphi}_2) + 2U_{2\theta} - 2(\tau_{L2}c_2 + \tan \bar{\alpha}_2)U_{2z} \cos^2(\theta - \bar{\varphi}_2)$$
$$= 2 \tan \bar{\alpha}_2 + 2(\tau_{L2}c_2 + \tan \bar{\alpha}_2) \cos^2(\theta - \bar{\varphi}_2) - 4\bar{\Omega} \tan \theta_{h_1} \tag{5.7}$$

The velocity components U_{1r}, $U_{1y'}$ and $U_{1z'}$ can be expressed in terms of the components $(\tilde{u}_{r1}, \tilde{u}_{\sigma1}, \tilde{u}_{\tau1})$ and $(\tilde{u}_{r2}, \tilde{u}_{\sigma2}, \tilde{u}_{\tau2})$ as

$$U_{1r} = \tilde{u}_{r1} + \tilde{u}_{r2}$$

$$U_{1y'} = \cos \alpha_1 + \tan \theta_{h_1} \sin \alpha_1 + \tilde{u}_{\sigma1} \cos \theta_1 + \tilde{u}_{\tau1} \sin \theta_1 + \tilde{u}_{\sigma2} \cos \theta_2 + u_{\tau2} \sin \theta_2$$

$$U_{1z'} = \sin \alpha_1 - \tan \theta_{h_1} \cos \alpha_1 - \tilde{u}_{\sigma1} \sin \theta_1 + \tilde{u}_{\tau1} \cos \theta_1 - \tilde{u}_{\sigma2} \sin \theta_2 + u_{\tau2} \cos \theta_2 \tag{5.8}$$

Substituting into equation (5.8) we can write the boundary conditions for the front prop as

$$\left(K_{\sigma1}\bar{A}_{11}^{(i)} + K_{\tau1}\bar{A}_{12}^{(i)}\right)\mathscr{A} + \left(K_{\sigma2}\bar{A}_{12}^{(i)} + K_{\tau2}\bar{A}_{14}^{(i)}\right)\mathscr{B} = \mathscr{C}_1^{(R_i)}$$
$$\left(K_{\sigma1}\bar{A}_{21}^{(i)} + K_{\tau1}\bar{A}_{22}^{(i)}\right)\mathscr{A} + \left(K_{\sigma2}\bar{A}_{23}^{(i)} + K_{\tau2}\bar{A}_{24}^{(i)}\right)\mathscr{B} = \mathscr{C}_2^{(R_i)} \tag{5.9}$$

where $\bar{A}_{\ell m}^{(i)}$ and $\mathscr{C}_{\ell}^{(R_1)}$ are defined by

$$\bar{A}_{11}^{(i)} = \sin\theta_1 + \tau_{U1}^{(i)}\cos\theta_1 \qquad \bar{A}_{12}^{(i)} = -(\cos\theta_1 - \tau_{U1}^{(i)}\sin\theta_1)$$

$$\bar{A}_{13}^{(i)} = \sin\theta_2 + \tau_{U1}^{(i)}\cos\theta_2 \qquad \bar{A}_{14}^{(i)} = -(\cos\theta_2 - \tau_{U1}^{(i)}\sin\theta_2)$$

$$\bar{A}_{21}^{(i)} = \sin\theta_1 + \tau_{L1}^{(i)}\cos\theta_1 \qquad \bar{A}_{22}^{(i)} = -(\cos\theta_1 - \tau_{L1}^{(i)}\sin\theta_1)$$

$$\bar{A}_{23}^{(i)} = \sin\theta_2 + \tau_{L1}^{(i)}\cos\theta_2 \qquad \bar{A}_{24}^{(i)} = -(\cos\theta_2 - \tau_{L1}^{(i)}\sin\theta_2)$$

$$C_1^{(i)} = \sin\alpha_r - \tau_{U1}^{(i)}\cos\alpha_r - (\cos\alpha_r + \tau_{U1}^{(i)}\sin\alpha_r)\tan\theta_{h_1}$$

$$C_2^{(i)} = \sin\alpha_r - \tau_{L1}^{(i)}\cos\alpha_r - (\cos\alpha_r + \tau_{L1}^{(i)}\sin\alpha_r)\tan\theta_{h_1}$$

(5.10)

Considering the rear propeller, we have on substituting for the velocity components ($\tilde{u}_{r2}$, $\tilde{u}_{\theta 2}$, $\tilde{u}_{z2}$) the following

$$U_{2r} = \tilde{u}_{r1} + \tilde{u}_{r2}$$

$$U_{2\theta} = \tilde{u}_{\sigma 1}\sin\theta_{h_1} - \tilde{u}_{\tau 1}\cos\theta_{h_1} + \tilde{u}_{\sigma 2}\sin\theta_{h_2} - \tilde{u}_{\tau 2}\cos\theta_{h_2}$$

$$U_{2z} = \tilde{u}_{\sigma 1}\cos\theta_{h_1} + \tilde{u}_{\tau 1}\sin\theta_{h_1} + \tilde{u}_{\sigma 2}\cos\theta_{h_2} + \tilde{u}_{\tau 2}\sin\theta_{h_2}$$

(5.11)

into equation (5.11), the equations for the boundary conditions on the upper and lower surface of the rear propeller become

$$\left(K_{r1}\bar{A}_{60}^{(i)} + K_{\sigma 1}\bar{A}_{61}^{(i)} + K_{\tau 1}\bar{A}_{62}^{(i)}\right)\mathscr{A} + \left(K_{r2}\bar{A}_{60}^{(i)} + K_{\sigma 2}\bar{A}_{63}^{(i)} + K_{\tau 2}\bar{A}_{64}^{(i)}\right)\mathscr{B} = \mathscr{C}_6^{(S_1)}$$

$$\left(K_{r1}\bar{A}_{70}^{(i)} + K_{\sigma 1}\bar{A}_{71}^{(i)} + K_{\tau 1}\bar{A}_{72}^{(i)}\right)\mathscr{A} + \left(K_{r2}\bar{A}_{70}^{(i)} + K_{\sigma 2}\bar{A}_{73}^{(i)} + K_{\tau 2}\bar{A}_{74}^{(i)}\right)\mathscr{B} = \mathscr{C}_7^{(S_1)}$$

(5.12)

where we have defined $\bar{A}_{\ell m}^{(i)}$ and $C_{\ell}^{(S_1)}$ as follows.

$$\bar{A}_{61} = 2[\sin \theta_{h_1} - (\tau_{U2}c_2 + \tan \alpha_s) \cos^2(\theta - \bar{\varphi}_2) \cos \theta_{h_1}]$$

$$\bar{A}_{62} = - 2[\cos \theta_{h_1} + (\tau_{U2}c_2 + \tan \alpha_2) \cos^2(\theta - \bar{\varphi}_2) \sin \theta_{h_1}]$$

$$\bar{A}_{71} = 2[\sin \theta_{h_1} - (\tau_{L2}c_2 + \tan \alpha_s) \cos^2(\theta - \bar{\varphi}_2) \cos \theta_{h_1}]$$

$$\bar{A}_{72} = - 2[\cos \theta_{h_1} + (\tau_{L2}c_2 + \tan \alpha_s) \cos^2(\theta - \bar{\varphi}_2) \sin \theta_{h_1}]$$

$$\bar{A}_{63} = 2[\sin \theta_{h_2} - (\tau_{U2}c_2 + \tan \alpha_s) \cos^2(\theta - \bar{\varphi}_2) \cos \theta_{h_2}]$$

$$\bar{A}_{64} = - 2[\cos \theta_{h_1} + (\tau_{U2}c_2 + \tan \alpha_2) \cos^2(\theta - \bar{\varphi}_2) \sin \theta_{h_2}]$$

$$\bar{A}_{73} = 2[\sin \theta_{h_2} - (\tau_{L2}c_2 + \tan \alpha_s) \cos^2(\theta - \bar{\varphi}_2) \cos \theta_{h_2}]$$

$$\bar{A}_{74} = - 2[\cos \theta_{h_1} + (\tau_{L2}c_2 + \tan \alpha_s) \cos^2(\theta - \bar{\varphi}_2) \sin \theta_{h_2}]$$

$$\mathscr{C}_6^{(S_1)} = 4 \tan \alpha_2 - 2(1 + 2\bar{\Omega}) \tan \theta_{h_1} + 4(\tau_{U2}c_2 + \tan \alpha_s) \cos^2(\theta - \bar{\varphi}_2)$$

$$\bar{A}_{60}^{(1)} = \sin 2(\theta - \bar{\varphi}_2)$$

$$\mathscr{C}_7^{(S_1)} = 4 \tan \alpha_2 - 2(1 + 2\bar{\Omega}) \tan \theta_{h_1} + 4(\tau_{L2}c_2 + \tan \alpha_s) \cos^2(\theta - \bar{\varphi}_2)$$

$$\bar{A}_{70}^{(1)} = \sin 2(\theta - \bar{\varphi}_2) \tag{5.13}$$

Therefore, the boundary conditions, equations (5.13) and (5.12) can be written in the compact form

$$K\mathscr{A} + L\mathscr{B} = \gamma_1 \qquad K = K_{\sigma 1}\bar{A}_{11}^{(1)} + K_{\tau 1}\bar{A}_{12}^{(1)} \qquad L = K_{\sigma 2}\bar{A}_{13}^{(1)} + K_{\tau 2}\bar{A}_{14}^{(1)}$$

$$M\mathscr{A} + N\mathscr{B} = \gamma_2 \qquad M = K_{\sigma 1}\bar{A}_{21}^{(1)} + K_{\tau 1}\bar{A}_{22}^{(1)} \qquad N = K_{\sigma 2}\bar{A}_{23}^{(1)} + K_{\tau 2}\bar{A}_{24}^{(1)}$$

$$P\mathscr{A} + Q\mathscr{B} = \gamma_3 \qquad P = k_{r1}\bar{A}_{60}^{(1)} + K_{\sigma 1}\bar{A}_{61}^{(1)} + K_{\tau 1}\bar{A}_{62}^{(1)} \qquad Q = K_{r2}\bar{A}_{60}^{(1)} + K_{\sigma 2}\bar{A}_{63}^{(1)} + K_{\tau 2}\bar{A}_{64}^{(1)}$$

$$R\mathscr{A} + S\mathscr{B} = \gamma_4 \qquad R = K_{r1}\bar{A}_{70}^{(1)} + K_{\sigma 1}\bar{A}_{71}^{(1)} + K_{\tau 1}\bar{A}_{72}^{(1)} \qquad S = K_{r2}\bar{A}_{70}^{(1)} + K_{\sigma 2}\bar{A}_{73}^{(1)} + K_{\tau 2}\bar{A}_{73}^{(1)}$$

(5.14)

The four equations may be written as the pair of matrix equations

$$K_M\mathscr{A} + L_N\mathscr{B} = \Gamma_1$$

$$P_R\mathscr{A} + Q_S\mathscr{B} = \Gamma_2$$

(5.15)

where the matrices K_M, L_N, P_R, Q_S, Γ_1 and Γ_2 are defined by

$$K_M = \begin{pmatrix} K \\ M \end{pmatrix} \qquad L_N = \begin{pmatrix} L \\ N \end{pmatrix} \qquad P_R = \begin{pmatrix} P \\ R \end{pmatrix}$$

$$Q_S = \begin{pmatrix} Q \\ S \end{pmatrix} \qquad \Gamma_1 = \begin{pmatrix} \gamma_1 \\ \gamma_2 \end{pmatrix} \qquad \Gamma_2 = \begin{pmatrix} \gamma_3 \\ \gamma_4 \end{pmatrix}$$

$$\gamma_1 = \mathscr{C}_1^{(R_1)} \qquad \gamma_2 = \mathscr{C}_2^{(R_1)} \qquad \gamma_3 = \mathscr{C}_6^{(S_1)} \qquad \gamma_4 = \mathscr{C}_7^{(S_1)}$$

(5.16)

Solving this pair of nonhomogeneous simultaneous equations for A and B we obtain the Birnbaum coefficient vectors A and B as

$$\mathscr{A} = \left(L_N^{-1}K_M - Q_S^{-1}P_R\right)^{-1}\left(L_N^{-1}\Gamma_1 - Q_S^{-1}\Gamma_2\right)$$

$$\mathscr{B} = \left(K_M^{-1}L_N - P_R^{-1}Q_S\right)^{-1}\left(K_M^{-1}\Gamma_1 - P_R^{-1}\Gamma_2\right)$$

(5.17)

The matrices K_M, L_N, P_R, and Q_S are primarily aerodynamic in nature whereas the matrices Γ_1 and Γ_2 are purely geometric representing the blade section and blade planform. Truncating the Birnbaum series at $m = M_*$ we have $(M_* + 1)$ coefficients for each of the front and rear propeller. Considering

P_* points on each side of the blade at which the boundary conditions are satisfied, we have $2P$ equations to determine the $2(M_* + 1)$ coefficients of the front and rear propellers. Hence, we must have $2P_* = 2(M_* + 1)$ giving the relation between the number of points on the blade and the number of coefficients of the Birnbaum series. The matrices K_M, L_N, P_R, and Q_S are of order ($2P_*$ by $2P_*$) and the matrices Γ_1 and Γ_2 are of order $2P_*$ by 1. Each of the vectors $\mathscr{A}$ and $\mathscr{B}$ is of order $2P*$ by 1.

The vectors $\mathscr{A}$ and $\mathscr{B}$ are functions of the off-set angles $\bar{\varphi}_1$ and $\bar{\varphi}_2$. For purposes of numerical study, we may set $\bar{\varphi}_1 = 0$ and $\bar{\varphi}_2 = \Omega_r t = \Omega t_1/\lambda_1$. Since the blade passing is periodic, it is only necessary to vary the time t_1 in the range $0 \leqslant t_1 \leqslant \delta_2\lambda_1/\bar{\Omega}$ and obtain the values of $\mathscr{A}$ and $\mathscr{B}$ at desired intervals.

From a knowldege of $\mathscr{A}$ and $\mathscr{B}$ the pressure loadings H_1 and H_2 on the front and rear propellers can be calculated using equations (4.4) from which the thrust and torque for each blade can be calculated for performance estimation of the counter-rotating propeller system. If the free stream were steady, $\omega_1' = 0$. The inflow into the rear propeller would experience the inviscid periodic wake of the front propeller with an angular frequency $\omega_2 = \Omega_r Z_1$.

The above solution is applicable for calculating the unsteady and steady aerodynamic behavior of the blades which are assumed to be rigid. However, this assumption may be relaxed by allowing the blade geometry parameters Y_i', Z_i', and α_i in equation (5.7) to be functions of the blade loading and stiffness characteristics for an aeroelastic analysis of the system. It has been mentioned earlier (cf. eq. 3.46) that the time dependent perturbation pressure P at any point in the flow equals the sum of the perturbation pressures P_1 and P_2 due to the front and the rear propellers. This can be calculated from eq. (3.44).

CONCLUSION

The lifting surface theory for the counter-rotating propeller has been presented above. It is possible to obtain the acoustic modes and the corresponding sound pressure levels of the combination using the Birnbaum coefficients $\mathscr{A}_m$ and $\mathscr{B}_m$ together with the Birnbaum series. Further, the aerodynamic performance of the dual rotation propeller system can also be calculated.

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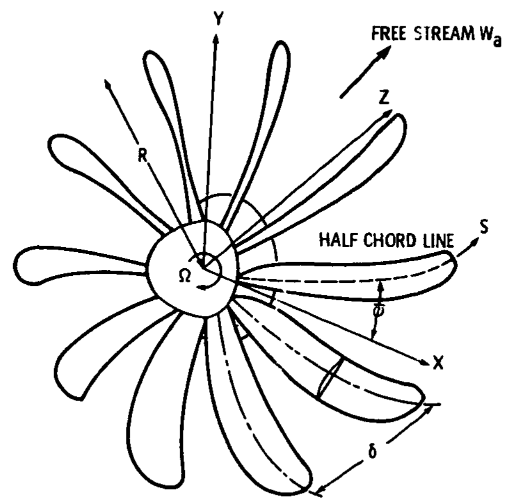
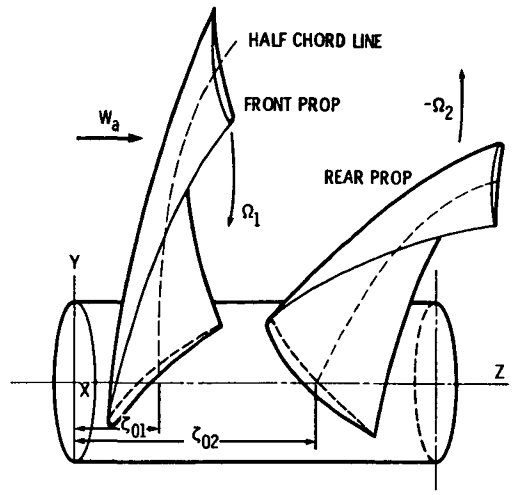


$\nu = \frac{2\pi}{Z_1}$

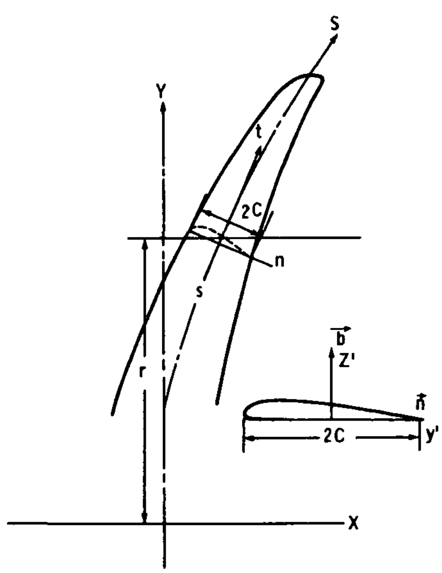
$\bar{\varphi}$ = OFF-SET ANGLE

(a) Single rotation propeller

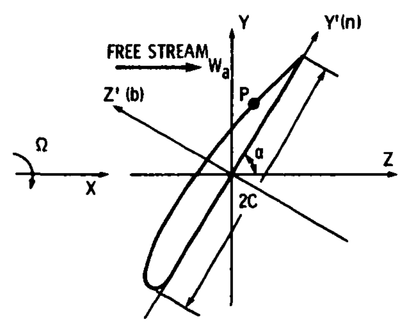


(b) Counter-rotating propeller blades.

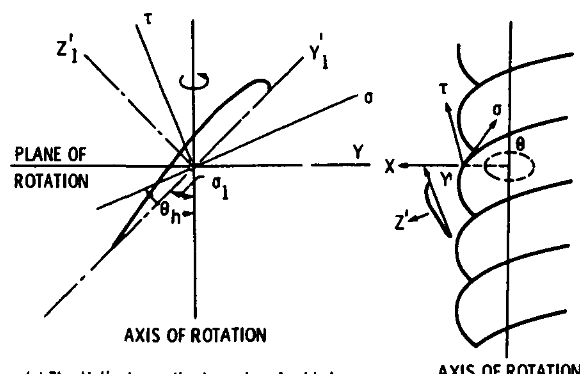
Figure 1.



(a) A view of a blade and it's cross section.



(b) The blade section coordinates.



(c) The Helical coordinate system for blade.

Figure 2.

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15 Supplementary Notes

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16 Abstract

An unsteady lifting surface theory for the counter-rotating propeller is presented using the linearized governing equations for the acceleration potential and representing the blades by a surface distribution of pulsating acoustic dipoles distributed according to a modified Birnbaum series. The Birnbaum series coefficients are determined by satisfying the surface tangency boundary conditions on the front and rear propeller blades. Expressions for the combined acoustic resonance modes of the front prop, the rear prop and the combination are also given.

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