DISTRIBUTED AUTOMATIC CONTROL OF TECHNOLOGICAL PROCESSES
IN CONDITIONS OF WEIGHTLESSNESS

A.I. Kukhtenko, V.I. Merkulov, Yu.I. Samoylenko, Yu. P.
Ladikov-Royev

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Some problems associated with the automatic control of liquid metal and plasma systems under conditions of weightlessness are examined, with particular reference to the problem of stability of liquid equilibrium configurations. The theoretical fundamentals of automatic control of processes in electrically conducting continuous media are outlined, and means of using electromagnetic fields for simulating technological processes in a space environment are discussed.
This paper examines several aspects relating to the problem of automatic control of liquid metal and plasma objects, i.e., electrical conductors, such as occur during the first stages of investigation of problems in space technology. This particular problem requires a comprehensive, systematic approach. There are a whole series of interrelated engineering and theoretical problems and subproblems. Among the theoretical questions we may point out the following.

1. Equilibrium and stability of equilibrium configurations of liquids in the state of weightlessness. In the realm of science, this problem has at its origin such ancient theories as the theory of Plato, which geometrically speaking comes down to the search for the minimum surface pulled into a given contour, and the theory of equilibrium and stability of equilibrium figures of a rotating liquid. Also similar to the above is the problem of equilibrium and stability of a liquid situated in cavities of vessels of a certain configuration, which have been studied in numerous works as relating to the problem of motion of rockets with liquid fuel and oxidizers.

2. The stability of state of a substance and the control of phase transitions. The problem of automatic control of the state of a substance and of its phase transitions derives from very
difficult and not yet fully answered questions of nonuniform statistical thermodynamics of electrical charged particles.

The studies in this field are general in kind and require additional theoretical and experimental research to account for specific features of the problem of stability of state of matter in conditions of weightlessness with presence of external electrostatic, electromagnetic and thermal control fields, when questions of the collective behavior of a system of many interacting particles become in the theoretical context highly individual and especially difficult.

3. Optimization of the control process. The limited fuel resource, the desire to minimize the time over which the processes occur, and other interpretations involving an extremum necessitate, from the very start of the investigation of a particular system of automatic control of liquid metal objects in conditions of weightlessness, a consideration not only of the stability of configurations and state of the substance, as already mentioned, but also the problem of optimization of the control process. This, again, is theoretically a very difficult problem in the field of optimal control of objects with distributed parameters. The difficulties are primary due to the complexity of the description of the control object itself; in addition, there is the necessity of developing and employing high specific methods of numerical solving of variational problems with especially complicated boundary conditions.

To a certain extent, the present paper illuminates only the first of the above three theoretical topics and presents a summary of current experimental laboratory research in this regard.

The stability of processes in a controlled continuum. In nature and in technology there are many examples when a process
reaching certain critical values gives rise to vibrations and waves of growing amplitude, which carry away the greater share of the energy and thereby greatly curtail the energy efficiency of the process. For example, plasma instabilities result in anomalously large transport coefficients perpendicular to the magnetic lines of force, so that it is not possible to heat the plasma to the thermonuclear temperature and satisfy Lawson's criterion. We know of many chemical reactions enabling a cheap and quick method of producing a needed article; but the reaction chain contains an unstable link, which forces the engineers to use different and safer reactions, but expensive and more time consuming. As already pointed out, instabilities concern the transfer of the energy of the fundamental state (the process) to perturbations. Instabilities in plasma concern the transfer of the potential energy of the heated plasma, as well as the energy of the magnetic containment field, to the perturbations. In order to produce metals of high purity, the melt is electromagnetically suspended in a vacuum, without touching the walls of the inductor. But it is not possible to suspend a large mass of liquid metal without using additional means of stabilization, since instability will occur. This instability is similar to Rayleigh-Taylor instability, which occurs when a heavier liquid is placed on top of a light one. The different is that the perturbations are disposed strictly along the magnetic lines of force. The cause of the instability is the tendency of the system to take up the position with least gravitational energy. In weightlessness conditions, such instability would not arise, which opens up prospects for producing large volumes of metal with very high purity. However, other kinds of instability may occur under weightlessness; for example, constrictions in fine jets, or instabilities associated with the use of electromagnetic fields with convex geometry for transportation or stabilization of the position of the molten volume.
Until recently, studies on suppression of instabilities have been conducted in two areas. The first concerns the variation of the parameters of the controlled object to improve its stability, e.g., the configuration and magnitude of the plasma containment magnetic fields. It is also possible to use additional windings with currents capable of creating a "magnetic pit" and so on. The second area is devoted to investigations of the influence of natural "passive" media, interacting with perturbations in the parameters of the process under investigation by means of the boundary conditions. This area includes the work of V. D. Shafranov, devoted to the influence of a highly conductive jacket on the stability of a plasma pinch. The first method of suppression of instabilities is energy-inefficient, since its stabilization requires an energy equaling that spent on formation of the primary process itself. For example, to stabilize helical perturbations in the plasma filament as per the Krushkal-Shafranov criterion it is necessary to produce a lengthwise magnetic field many times greater than the field of natural current required to guarantee equilibrium. Furthermore, many theoretical investigations and experiments have shown that full stability cannot be achieved in the majority of cases. All that is possible is to advance the critical values of the parameters or translate the instability to a different wave range.

The second method of suppression of instabilities is also ineffective, due to the limited class of operators produced from natural media, and the impossibility of amplifying the wave reflection. The Institute of Cybernetics of the Ukrainian SSR Academy of Sciences is developing spatially-distributed regulating systems for the stabilization of processes in continuous media, through the use of synthetic control media that interact with the perturbations in the object at the boundary conditions, according to the principle of feedback. We should remark that, for the stabilization of objects with localized parameters
described by ordinary differential equations, there are well-developed technical means of implementing virtually any given feedback operator. However, until recently, there was little progress in the realization of space-time control operators for objects with distributed parameters, such as the processes that occur in a continuum.

Fig. 1. Diagram of a linear filament control medium.

Problems of control of processes in a continuum impose specific demands on the regulators, most important of which are: high spatial resolution; ability of the regulator to interact with perturbations of different wavelength in different ways (property of spatial dispersion); the property of amplification. The first of these demands compels the use of only such regulating devices having a large number of parallel channels of interaction with the object. Such is the property of a continuum making contact with the object at a surface rather extensive in space. To meet the second demand, we must use the class of media with phenomenological laws of large capacity, so that the control input can achieve any required relationship between the frequency $\omega$ and the wave number $k$. Only media having internal energy sources or a link to external sources may have the property of amplification. All three of these properties are possessed by linear filament synthetic media. In the case of electromagnetic control, such a medium may be built as follows (Fig. 1). We take an arbitrary current-carrying circuit in space. The dimensions of the circuit may be quite large. Its ends may leave the volume under consideration. The ends of the
circuit may be connected to active two-pole networks, capable of amplifying the currents induced in the circuit. Performing the operation of translation in the direction of the coordinate axes by the law:

\[ x = x_0 + na; \quad y = y + ml; \quad z = z_0 + kl \quad (k, m, n = 0, \pm 1, \pm 2, \ldots), \]

we obtain a medium similar to a kind of crystal, with the aforementioned circuits situated at its lattice points. The important thing about this medium is the fact that the dimensions of the "molecule" in such "crystal" are much larger than the lattice interval. By virtue of nonlocal excitation of current, such medium possesses the property of spatial dispersion. The presence of the active two-pole networks within the circuit assures the required amplification. After averaging and discarding the insignificant terms, the equation of state (generalized Ohm's law) of the synthetic fiber medium may be written as:

\[ i' (t, \vec{x}) = \int_{-\infty}^{\infty} dt \int \sigma^{ij}(t, \vec{x}) E_i(t, \vec{x}) d\vec{x}. \]  

(1)

Here \( \vec{j}(t, \vec{x}) \) is the vector of the density of the volume current; \( E(t, \vec{x}) \) is the electric field; \( \sigma^{ij}(t, \vec{x}) \) is the generalized tensor of conductivity.

For an unbounded homogeneous medium, Ohm's law in the Laplace-Fourier transformation has the usual form:

\[ \tilde{i}' (\omega, \vec{k}) = \sigma^{ij}(\omega, \vec{k}) E_i(\omega, \vec{k}). \]

(2)

It is characteristic that the generalized tensor of conductivity \( \sigma^{ij}(\omega, \vec{k}) \) depends not only on the frequency \( \omega \), but also on the wave vector \( \vec{k} \). This relationship may be varied within wide limits by altering the configurations of the circuits and by hooking up various two-pole networks to them.
Let us find the expression for the tensor of conductivity [2] in the case when the controlling medium is a thin shell, formed by a system of windings, connected to active two-pole networks outside the volume in consideration. We introduce the vector-function of the winding density \( \gamma(x) \):

\[
\gamma(x) = \Omega_0^{-1} \int \int \delta(x - \xi) \frac{\rho(\xi)}{\rho(\xi)} d\xi d\xi,
\]

where \( \xi \) is the radius-vector of the point on the stabilizing shell \( s \); \( \rho(\xi) \) is the equation of the closed conducting loop, situated on the surface \( s \); \( \delta(\xi) \) is the two-dimensional Dirac function; \( \Omega_0 \) is the averaging region, which is small in relation to the wavelength of the perturbation, but large enough that quite a lot of conductors pass through it.

Using the function \( \gamma(x) \), the electromotive force may be expressed in terms of the integral of the surface \( s \):

\[
u(i\omega) = \int \int E(i\omega, \xi) \gamma(\xi) d\xi
\]

where \( u(i\omega) \) is the Laplace transform of the electromotive force; \( E(i\omega, \xi) \) is the transform of the electrical field strength.

According to Ohm's law, the current in the winding is determined by:

\[
l(i\omega) = u(i\omega) z^{-1}(i\omega),
\]

where \( z(i\omega) \) is the impedance of the circuit, including the attached active two-pole networks. The surface density of the current is:

\[
j(x) = l(i\omega) \gamma(x).
\]
Using equations (4)-(6), we find:

\[
J(i\omega, \vec{x}) = \int \int \gamma(x) \gamma(\vec{y}) \mu^{-1}(i\omega, \vec{y}) d\vec{y} = \int \int \gamma(i\omega, \vec{x}, \vec{y}) E(i\omega, \vec{y}) d\vec{y}.
\]

Expression (7) is a generalization of Ohm's law, similar to (1). If we take the basis:

\[
\gamma_{m,k}(\vec{y}) = \gamma_{m,k} e^{(i(m\theta + k\phi)}.
\]

where \( \gamma_{m,k} \) are constant vectors, \( r, \theta, \xi \) are a cylindrical system of coordinates, then if the controlling shell is a round cylinder of radius \( b \), it will follow from expressions (7)-(8) that a winding with numbers \( m,k \) will interact with the field harmonic having the same numbers, and will create the identical harmonic of the current density on the stabilizing shell. In the real form, it follows from equation (8) that, corresponding to each number \( m \) and \( k \), there should be two windings: sine and cosine (Fig. 2). Using expression (7), we may derive the boundary condition fulfilled on the surface of the shell:

\[
\phi_{1,m,k}(b) = \beta_{m,k} f_{m,k} \phi_{1,m,k}(b),
\]

where \( \phi_{1,m,k}(b) \) and \( \phi_{1,m,k}(b) \) are the amplitudes of the magnetic field incident on the shell and reflected from it by the potential; \( f_{m,k} \) is the coefficient of reflection:

\[
f_{m,k} = 2\pi \mu_0 k b \frac{R_{m,k}^2}{L_{m,k} - L_{m,k}^-}.
\]

Here \( \mu_0 \) is the magnetic constant; \( L_{m,k} \) is the inductance of the winding; \( L_{m,k}^- \) is the "negative" inductance of the active two-pole network connected in the circuit; \( \beta_{m,k} \) is nonzero and expressed
in terms of modified Bessel functions $I_m(kb), K_m(kb)$. The potentials of the incident and reflected field, as well as the density function of the winding, are assumed to be expanded in Fourier series:

$$
\varphi_0(r, \theta, \zeta) = \sum_{m,k} \varphi_{0,m,k}(r) \exp i(m\theta + u\zeta);
$$

$$
\varphi_1(r, \theta, \zeta) = \sum_{m,k} \varphi_{1,m,k}(r) \exp i(m\theta + k\zeta);
$$

$$
\Gamma(\theta, \zeta) = \sum_{m,k} \cos(m\theta + k\zeta); \quad \gamma^\theta = \frac{\partial \gamma}{\partial \zeta}; \quad \gamma^r = -\frac{1}{r} \frac{\partial \gamma}{\partial \theta}.
$$

The method of mutually-orthogonal windings enables a gradual increase in complexity of the regulatory system. First, the windings for stability of the most dangerous perturbations are fashioned; then, as needed, windings are created for the other modes.

![Fig. 2. Structure of control windings (m = 2, k = 1).](image)

The Kurchatov Institute of Atomic Energy has adopted the first stage of a system for stabilization of a toroidal plasma pinch, developed by the Institute of Cybernetics of the Ukrainian SSR AS, in a plasma facility of the Tokamak TO-I type. Adoption
of the regulation system permitted a substantial increase in the containment time of the plasma filament and enhanced parameters of the plasma. Fig. 3 shows oscillograms of the voltage drop $U$ and current $I$ in the plasma with the regulator on and off. The substantial decrease in voltage indicates an increased conductivity and, consequently, temperature.

![Oscillogram of the drop in voltage and current with the regulator on (a) and off (b). Key: c - μs.](image)

The theoretical aspects of automatic control in a continuum and their methods of technical implementation have been used in problems of containment and stabilization of volumes of molten metal by electromagnetic fields.

Simulation of weightlessness in earth conditions by means of a magnetic field. In order to realize technological processes in outer space, it is important to simulate conditions of weightlessness in earth conditions. In particular, it is necessary to suspend molten metal volumes in a vacuum.

Theoretically, such possibility is afforded by the interaction of the electric current induced in the metal with an external magnetic field. However, practical implementation of such technique requires the solving of two problems: induction
of the current and creation of a magnetic field sufficient to
equalize the force of gravity, and assurance of stability of the
resulting equilibrium.

The first problem may have different solutions, depending on
the purpose of the facility. The most rigorous demands occur
with the suspension of an insulated, single-bonded, molten metal
volume.

Such suspension may be achieved only through the interaction
of a high-frequency external magnetic field and high-frequency
eddy currents of the skin layer generated by this same field.

To gain an idea as to the lower power limit necessary for
this, consider the condition of equilibrium of a molten metal
layer with thickness \( h \), supported at the bottom by an alternating
magnetic field of strength \( H \) and frequency \( \omega \).

The minimum frequency is found from the condition that the depth of the skin layer is equal to the thickness of the liquid
layer:

\[
h = \sqrt[2]{\frac{\mu \omega}{\sigma}}
\]

where \( \sigma \) is the electrical conductance of the metal; \( \mu \) is its
magnetic permeability. The magnetic field strength is found
from the condition of equality of the mean magnetic and hydro-
static pressure \( H^2/2 = \rho gh \), where \( \rho \) is the metal density.

The results of a calculation for the magnetic field strength
and the joulan heat liberated in the skin layer are shown in
the table for various metals. It is evident that the equilibrium
temperature of the metal may, in certain cases, restrict the
management of the technological processes. A somewhat lesser
energy density occurs in the equilibrium of a volume of molten
<table>
<thead>
<tr>
<th>Material (a)</th>
<th>Thickness of molten metal layer, m (b)</th>
<th>Magnetic field strength, G (c)</th>
<th>Joulian heat, W/m^2 (d)</th>
<th>Equilibrium temperature of metal (e)</th>
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</thead>
<tbody>
<tr>
<td>Aluminum (f)</td>
<td>0.01</td>
<td>3.4 \times 10^4</td>
<td>17 \times 10^2</td>
<td>740/1300</td>
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<tr>
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<td>3.4 \times 10^5</td>
<td>17 \times 10^2</td>
<td></td>
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<tr>
<td>Iron (g)</td>
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<td>270 \times 10^2</td>
<td>1480/2600</td>
</tr>
<tr>
<td></td>
<td>0.1</td>
<td>1.3 \times 10^5</td>
<td>270 \times 10^2</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1.0</td>
<td>4.1 \times 10^5</td>
<td>270 \times 10^2</td>
<td></td>
</tr>
<tr>
<td>Lead (h)</td>
<td>0.01</td>
<td>4.9 \times 10^4</td>
<td>300 \times 10^3</td>
<td>1520/2700</td>
</tr>
<tr>
<td></td>
<td>0.1</td>
<td>1.55 \times 10^5</td>
<td>300 \times 10^3</td>
<td></td>
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<tr>
<td></td>
<td>1.0</td>
<td>4.9 \times 10^5</td>
<td>300 \times 10^3</td>
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<tr>
<td>Tungsten (i)</td>
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<td>19 \times 10^2</td>
<td>1800/3200</td>
</tr>
<tr>
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<td>19 \times 10^2</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1.0</td>
<td>6.3 \times 10^4</td>
<td>19 \times 10^2</td>
<td></td>
</tr>
<tr>
<td>Titanium (j)</td>
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<td>3.1 \times 10^4</td>
<td>185 \times 10^2</td>
<td>1350/2400</td>
</tr>
<tr>
<td></td>
<td>0.1</td>
<td>0.9 \times 10^5</td>
<td>185 \times 10^2</td>
<td></td>
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<tr>
<td></td>
<td>1.0</td>
<td>3.1 \times 10^4</td>
<td>185 \times 10^2</td>
<td></td>
</tr>
</tbody>
</table>

Key: a - material; b - thickness of molten metal layer, m; c - magnetic field strength, G; d - joulian heat, W/m^2; e - equilibrium temperature of metal; f - aluminum; g - iron; h - lead; i - tungsten; j - titanium.

NOTE. In the numerator, the equilibrium metal temperature corresponds to a coefficient of emission $\varepsilon = 0.1$, in the denominator $\varepsilon = 1$.

metal of toroidal geometry, when it is possible to induce a current that is constant over the cross section.

Of great practical interest for engineers are technological problems when the liquid phase is in immediate contact with a solid phase of the same metal. In this case, a conduction suspension of the molten metal is possible: electrodes, connected to the solid phase, produce the current in the metal. As for the
magnetic field, this may be the natural field of the current or an external magnetic field. In the latter case, the current density and, consequently, the joulian heat can be made insignificantly small. The field may be constant in this case.

After a particular scheme of equilibrium is chosen, it is necessary to guarantee its stability. In the general case, such possibility is provided by a control shell, as mentioned above. An important characteristic of a control shell is it reacting not
Fig. 6. General view of the toroidal inductor.

only to the rate of deviation, but also to the magnitude of deviation of the metal from the equilibrium position. This property lets us use the shell to compensate for the nominal and structural errors of the equilibrium-support system. Fig. 4 shows the layout for conduction containment of a cylindrical molten metal mass. A current of density \( j \) is passed across cooled electrodes 1 and 2 along a rod. This, in conjunction with the transverse magnetic field \( B \), creates a vertical mass force \( jB \), equal to the force of gravity \( pg \). The stabilizing shell (indicated by circles) assures stability of the resulting equilibrium.

In the event that design simplicity is more important than the power consumption, the stabilization may be achieved by a noncontrolled variable magnetic field, e.g., a circular polarization field. This field may also be used to achieve the equilibrium.
This principle has been used to create an active experimental facility for magnetic suspension of metal (Fig. 5). The facility, where experiments have been carried out, consists of two generators of frequency 66 kHz and a toroidal copper inductor (Fig. 6). Fig. 5 schematically shows the inductor 1 with toroidal transformer 2, creating the azimuth current, and cylindrical transformer 4, creating a lengthwise alternating current, shifted in phase by \( \pi/2 \), containing the metal 3. An aluminum ring (Fig. 7) with large diameter \( D = 95 \text{ mm} \) and small diameter \( d = 10 \text{ mm} \) (weight 60-70 g) was placed in the inductor and suspended by magnetic field. In the solid state, the ring experienced slight vibration; upon melting, the vibrations vanished and the ring remained immobile. Even when heated to a temperature on the order of 2000° there was no flow. In the absence of the lengthwise magnetic field, a constriction type instability occurred on the ring, as is evident in Fig. 7. When the field was turned on, the constriction disappeared. We should mention that this facility is more energy-efficient than the previous one.
Thus, the methods of distributed automatic control have great promise for the solving of various problems of space technology.

Bibliography

