IONOSPHERIC EFFECTS TO ANTENNA IMPEDANCE

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IONOSPHERIC EFFECTS TO ANTENNA IMPEDANCE

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The reciprocity between high power satellite antennas and the surrounding plasma are examined. The relevant plasma states for antenna impedance calculations are presented and plasma models, and hydrodynamic and kinetic theory, are discussed. A theory from which a variation in antenna impedance with regard to the radiated power can be calculated for a frequency range well above the plasma resonance frequency is given. The theory can include photo and secondary emission effects in antenna impedance calculations.
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1. Statement of the problem

This work had the objective of investigating the interactions which occur between a satellite antenna and the surrounding plasma, particularly with respect to the real, practically important problem of radiation of high transmitted power with sharply beamed antennas. The plasma influences in the immediate vicinity of the satellite were to be considered, as well as the largely undistorted plasma in the surrounding ionosphere or magnetosphere.

Because of the complex relations of the material- and medium-related interactions, the work was begun with simple linear radiators such as are also used for feed lines, in order to provide a starting base for later studies of plasma interactions with flat radiators. The investigations according to this plan were to be done under the aspect of communications technology applications such as exist particularly for communications and navigation satellites, and linked with plasma conditions which correspond to conditions in earth orbit or in a synchronous orbit.

In consideration of the limited time available, only one year (including the startup time), it was not possible to complete the original plan (intended for a considerably longer period). Here we can only mention the key areas of numerical simulation considering photoeffects, determination and measurement of the inherent and coupled impedance for a dipole and monopole system under the aspects of incorrect matching, depolarization losses, and on-board interference problems.

* Numbers in margin indicate foreign pagination
2. Status of Science and Technology

The effect of a medium, such as plasma, on the behavior of an antenna is expressed, among other things, in the deviation of the antenna input resistance from its free space value. This effect often has a bad effect for transmissions from satellites (incorrect matching) and, in the last twenty years, has led to an intensive study of this phenomenon, both from the theoretical and from the experimental side. The theoretical considerations are based on many different theories. Most impedance calculations refer to the cold plasma model with a quasistatic approximation [1]. Microscopic consideration of the plasma with the kinetic theory (Boltzmann-, Vlasov- Equation) is more exact but considerably more difficult to apply. Comparison of the results resulting from the kinetic theory with those form the hydrodynamic theory (macroscopic consideration) for the case of a short dipole in a warm plasma is given in [3] and [4].

A comparison with the results of the cold plasma theory is also given in [5]. The importance of the plasma sheath for the behavior of the input impedance, assuming both continuous and abrupt sheath models, has been investigated by, among others, [2], [4], and [6]. Experimental testing of the results found theoretically by simulations in plasma chambers is described in [1] and [7 - 9]. Measurements carried out directly in the ionosphere under real conditions, [6] and [10], are of special importance, as these experiments are not subjected to any limitations such as the maintenance of scale dimensions [11].

The strong interactions of satellite antennas with the plasma surrounding them, and the resultant reactions in the impedance behavior of the antennas have often been used, for instance, to determine important data about plasma parameters such as temperature or density, using the well-known plasma model [12, 13]. By far the greatest portion of the work on antennas in plasma is based on linearized models. The linearization leads to substantial simplification of formal derivations, and to good agreement with measurements, to the extent that the calculations apply for the case of low signal amplitudes. There are only a few works concerned with the phenomenon of nonlinear behavior of the antenna input resistance.
There has not been any pressing need for nonlinear considerations, because the satellite transmitting powers were low because of the technological limits. The first critical perturbations were observed, for instance, with Helios I on switching from a moderate-gain antenna to a sharply directional S-band antenna [14, 15].

Large signal amplitudes at low frequency applied to antennas in the ionosphere or magnetosphere can cause dramatic changes in the surrounding plasma, and, therefore, also in the antenna current-voltage characteristic. Extensive results from ionospheric measurements with large signal amplitudes, with theoretical interpretations, are presented by [16 - 18]. Consideration of nonlinear effects at high transmitter powers and also at high frequencies — far above the plasma resonance frequency — are described in [19, 20].

As yet, the effect of photo- and secondary electron emission on the impedance behavior of satellite antennas has attracted no interest, especially in the magnetosphere (synchronous orbit). There are, though, already a series of numerical investigations which have been done for the cases of metallic cylinders and spheres [21 - 23]. These consider both the case of incident plasma and the intensive solar radiations. Only in recent years have there been intensive attempts to understand these phenomena better. These attempts have been stimulated by increased observations of disturbances in satellite functioning due to uncontrolled charging or discharging of the satellite surfaces [24 - 26]. [27] has given a very good survey of this problem area. The results of these researches are also of great importance for the study of the impedance behavior of satellite antennas. The large amount of data provides an important contribution to a more complete view of the immediate vicinity of the antenna in the plasma, and important hints on theoretical models for calculation of the impedance behavior of satellite antennas in the plasma.
3. Assumptions under which the work was carried out

The radiation and propagation of electromagnetic waves in plasmas is part of the special work areas of the contracting office, especially in application to problems of space research and technology. The workers themselves, however, had no previous knowledge in the field of plasma physics or of wave radiation and propagation in plasmas. Therefore, they required some time to start up. Adequate literature was available at the university. The service of the Expert Information Center at Leopoldshaven was utilized. Some researches were carried on out of the country if special information was required. Information from military fields, especially foreign, was not included because it was not accessible.

The study was a purely theoretical work which was not to be supplemented by measurements, such as in a plasma simulation chamber. Use of the computer center was available for later numerical computations under the typical university working conditions. The study itself was a compromise between an application-oriented problem and a fundamental physical problem. The applications included in particular consideration of radiation problems at high powers for a planned Spacelab working load experiment in the region near earth, as well as for the TV-SAT in synchronous orbit. The physical problem was the nonlinear interaction of antennas with the surrounding plasma due to the radiation of high transmitted powers.

4. Structure and course of the work

The treatment of the radiation problems of antennas in plasmas is a very extensive problem. It has already been discussed in the literature from many different viewpoints. As the workers had no previous knowledge of this problem area, it was first necessary to become familiar with the fundamentals of plasma physics. An intensive literature study was also carried out, supported by the capabilities of the Leopoldshaven Expert Information Center, on the complex of the interactions of antennas with the surrounding plasma. The major thrust of these researches was a search for a reasonable theory which would make possible a description of the nonlinear behavior of the antenna input resistance due to radiation in a plasma at high transmitted power, and which would also allow
consideration of the effects of photo- or secondary electron emission. In particular, we found one work in which the numerical computations had been carried out from the viewpoint of the effect of the transmitting power on the antenna input resistance due to plasma reactions at frequencies above the plasma resonance frequency. With reference to this work, we attempted, by appropriate choice of parameters, to get initial estimates of the order of magnitude of the variations of the antenna input resistance with respect to the conditions specified for the planned TV-SAT in synchronous orbit as well as for a possible Spacelab full load experiment near the earth. As the theory of calculating the antenna-plasma system interactions was sketched only briefly in the work cited, but it seemed quite promising as a starting point for our own further calculations, we made an intensive study of the fundamentals of this theory. It appeared that the problem can be handled only with substantial numerical calculations (iterative computation procedures) and, as a result, at great cost in time. For these reasons we attempted to find a simplified route to a solution by means of reasonable approximations. The objective could not be attained, though, within the established period of just one year, because of the complexity of the problem posed.

As the results of the first preliminary estimates appeared alarming with respect to the unperturbed use of planned future high-power satellites, appropriate industrial authorities were informed (e.g., Prof. Edenhofer at MBB on 10 June 1980).

5. Results obtained and their effects

If effects from the external magnetic field, the thermal motion of electrons and ions, and the ionic or electronic shield surrounding the antenna are to be considered realistically, the exact computation of the antenna impedance is mathematically very complex. One major problem for calculating the behavior of a source in the plasma is the determination of valid boundary conditions. For instance, for the ionic or electronic shield should the normal component of the velocity of the charge carrier to the antenna surface be approximated with $\vec{V}_{\text{normal}} = 0$ (rigid boundary condition) or $\vec{V}_{\text{normal}} \neq 0$ (absorbative boundary condition), or should plasma shielding of the antenna be neglected entirely? The
plasma may be isotropic or anisotropic (including the magnetic field effects); it may be warm or cold (inclusion of thermal pressure gradients); and only the electron motions (1-fluid model) or the electron and ion motions (2-fluid model) may be considered. In the latter case, the motion of the ions in a HF field can be neglected because of the high mass of the ions in comparison to that of the electrons, as a general rule. Likewise, for the relatively thin ionospheric and magnetosperic plasmas we can neglect collisions of the charge carriers with each other or with neutral particles at high frequencies.

The physical assumptions in theoretical calculations as well as the complexity are essentially determined by the choice of the plasma model. Most theoretical works are based on the simpler hydrodynamic plasma model (macroscopic consideration). The parameters of the plasma can then be derived from consideration of the behavior of "average" particles which affect the plasma; or they can be determined from the equations which arise from averaging the Boltzmann equation (8) by integration across the velocity space [28, 29]. The essential equations of the hydrodynamic model, with consideration of the approximations mentioned above are described by the motion equation and the continuity equation:

\[
\frac{n \cdot \vec{\alpha}}{\alpha} = -ne\vec{E} - ne \cdot \vec{V}_p - ne \cdot \vec{V}_B \quad (1)
\]

\[
\vec{V} \cdot (ne\vec{V}) + \frac{\partial (ne\vec{V})}{\partial t} = 0 \quad (2)
\]

and by the Maxwell equations; particularly for the quasistatic approximation (usual for electrically short antennas):

\[
\vec{V} \cdot \vec{E} = \rho/\epsilon_0 \quad (3)
\]

or for dynamic consideration (generally valid):

\[
\vec{V} \times \vec{E} = -\mu_0 \frac{\partial \vec{H}}{\partial t} \quad (4)
\]

\[
\vec{V} \times \vec{H} = + \epsilon_0 \frac{\partial \vec{E}}{\partial t} - ne\vec{V} + \frac{\vec{J}}{q} \quad (5)
\]
Here n, m, and \( \vec{v} \) are the quantitative density, mass, and velocity of the electrons; \( p \) is the thermal pressure (determined from the equation of state); and \( \rho, \rho_q, \vec{j}_q \) describe the space charge density, source charge density, and source current density; \( \epsilon_0 \) is the dielectric constant of vacuum, and \( -e \) is the electron charge. The quantities \( n, \vec{E}, \vec{H} \) and \( p \) can be represented by the superimposition of an unperturbed quantity with a perturbed one.

The general route to calculation of the antenna impedance is the solution of the equations for the electrical field in order to obtain the reaction reacting on the source model. For not too great powers one can begin, for a good approximation, with the linearized plasma model; that is, the second order perturbed terms in Equations (1) and (2) can be neglected. But use of the hydrodynamic model has only limited applicability, especially of studies are to be done in the region of the plasma resonance frequency or below it. The phenomenon of Landau damping can no longer be described, nor can components of the longitudinal or transverse radiation resistance be determined. One decisive disadvantage can arise in consideration of the effects due to the plasma shielding of the antenna because no exact calculation of the space charge carrier zone in the near vicinity of the antenna can be done with the macroscopic approach. This is valid only if the effect of photo- or secondary electron emissions are to be included in the calculations. Auxiliary models such as that of a vacuum zone around the antenna or continuous charge carrier distributions, described, for example, by the Boltzmann factor law, are either not physically well founded, or they allow only a very crude approximation to the real conditions.

The kinetic gas theory (microscopic viewpoint) gives a more precise description of the plasma and the reactions on a source in the plasma. It is based on the concept that the particle distribution function \( f \)
describes the state of the electrons or ions in a plasma. Here the number of one type of particle in an element of the plasma space between \((r,v)\) and \((r + dr, v + dv)\) is given by
\[ f(r,v,t) \, dx \, dy \, dz \, dv_x \, dv_y \, dv_z \, \text{dxdydz} \]
indicates the spatial volume centered on \(r\) and \(dv_x \, dv_y \, dv_z\) characterizes the velocity interval centered on \(v\). The fundamental equation which the distribution function must fulfill is the Boltzmann equation or the Vlasov equation for the collision-free case
\[
\frac{\partial f}{\partial t} + v \cdot \nabla f + \frac{\mathbf{F}}{m} \cdot \nabla f = \left( \frac{\partial f}{\partial t} \right)_\text{coll}
\]
Here \(\mathbf{F}\) indicates the force of the electromagnetic fields acting on the particle, and the indices \(r\) and \(v\) characterize the gradients of the \((x,y,z)\) space or of the velocity space. The solution of the Boltzmann equation can be attained only numerically, and is explained in more detail later in connection with the impedance calculation.

Selection of the appropriate plasma model for impedance calculation depends on which natures of interactions of the antenna-plasma system are to be considered, and what possible limitations can arise from specification of the parameters (e.g., frequency). At frequencies far above the plasma resonance frequency, and neglecting the plasma shielding, both plasma models, the hydrodynamic as well as the kinetic theory, give agreeing results in the impedance calculation.

The following presents a theory of Laframboise et al. [19, 20]. It describes the effect of the plasma shielding on the antenna (also called a sheath), caused by radiation of high transmitter power, on the antenna impedance. The application of this theory allows nonlinear impedance behavior even for transmitter frequencies far above the critical plasma resonance frequency. It is shown that because of the high transmitter power the electrons are forced away from the antenna due to a nonlinear effect. By means of diagrams presented, it is attempted to obtain an estimate of the resulting sheath impedance and its reaction on the total impedance of a dipole antenna. The relevant
parameters are inserted, corresponding to those for the coming generation of communications satellites, in order to get initial predictions about potentially interfering plasma reactions on the antenna. By use of the kinetic theory to calculate the perturbed space charge distribution, the theory presented is basically suited to undertake an expansion by additionally calculating photo- or secondary electron emission.

We first studied the nonlinear behavior of the plasma in the immediate vicinity of an antenna driven at high transmitter power. Let us consider the motion of an electron in the oscillating $\hat{\mathbf{E}}$ and $\hat{\mathbf{B}}$ fields of a wave, neglecting the static $\mathbf{E}_0$ and $\mathbf{B}$ fields as well as pressure gradients. The momentary position of the electron is then described completely by the motion equation (1). The nonlinearity results from the second-order perturbing term product, partly from the $\mathbf{v} \times \mathbf{B}$ term and partly from the development of $\mathbf{E}$ at the instantaneous position $\mathbf{r}_0$ of the particle

$$\mathbf{E}(\mathbf{r}) = \mathbf{E}(\mathbf{r}_0) + (\mathbf{d} \cdot \nabla) \mathbf{E}/\mathbf{r} = \mathbf{E}_0 + \ldots$$

(9)

For an electric field of the form $\mathbf{E} = \mathbf{E}_1(\mathbf{r}) \cdot \cos \omega t$, where $\mathbf{E}_1(\mathbf{r})$ contains the spatial dependence, the shift can be described by means of the motion equation (1) purely through the force of the electric field:

$$\mathbf{d} \mathbf{r}_1 = -\left(\frac{q}{m_0^2}\right) \cdot \mathbf{E}_1(\mathbf{r}) \cdot \cos \omega t \cdot \mathbf{\dot{r}}_0$$

(10)

Along with Equations (9) and (10), and Equations (1) and (4), which link the $\mathbf{v} \times \mathbf{B}$ term with the electric field, one finally obtains a time-averaged force term of the form

$$\mathbf{F}_{NL} = -\frac{1}{4} \frac{q^2}{m_0^2} \mathbf{E}_1^2(\mathbf{r})$$

(11)

In plasma physics this nonlinear term is also known as the ponderomotive force. It is of particular interest for the propagation of lasers in plasmas, where it shows a focusing effect on the beam path; thus it is also effective even at extremely high frequencies. In relation to the antenna problem, this ponderomotive force forces the electrons away
from the antenna, thus perturbing the space charge distribution. As this force can be expressed by a gradient, its presence is equivalent to the effect of an additive potential on an electron:

$$\varphi_a = \left(\frac{q}{4m} \omega^2\right) E^2(r)$$

(12)

In the extension of the electrostatic probe theory according to [30] and [31], the above-described nonlinear interactions for an antenna with a surrounding (collision-free) plasma can now be included in the steady-state Vlasov-Poisson system, which is a nonlinear integro-differential equation system. The self-consistent solution of this Vlasov-Poisson system can be attained only by iterative numerical procedures, and is sketched briefly in the following.

A cylindrically symmetrical dipole antenna in a collision-free plasma is assumed. For symmetry, the particle paths can be represented by the invariants of the motion (angular momentum J, total energy E). Then the general solution of the Vlasov equation (8) is a function of these invariants of motion. Through analysis of all possible particle paths the distribution function f can be matched to the boundary value problem in phase space (E-J). This analysis of the possible particle paths is done most practically in the formulation of the effective potential

$$U(r) = q \cdot \varphi(r) + \left(\frac{J^2 + \delta^2}{2mr^2}\right)$$

(13)

and represents a kind of potential threshold for particles approaching the antenna. That is, a particle with energy E can reach a certain location r in front of the antenna only under the condition that

$$E - U(r) \geq 0.$$  

In other cases it is reflected again. In Equation (13) above, \(\varphi(r)\) corresponds to the spatial fall-off of the static antenna potential. The term \(\delta^2/2mr^2\) corresponds to \(\varphi_a\) from Equation (12). From the consideration that for antenna radius \(r_a \ll \) wavelength \(\lambda\) the instantaneous near field of an antenna exhibits the same location dependence as the static coulombic field of an infinitely long cylinder.
and can be represented by

\[ \dot{E}_{hf} = n_r \frac{c_i}{r} \cos \omega t = \dot{E}_1 \cos \omega t, \quad (14) \]

then because of \((14)\) the additive potential of Equation \((12)\) corresponds to the quadratic angular momentum \(J^2\) in the form

\[ \phi_a = \frac{q c_i^2}{4 \pi \omega^2} \frac{1}{r^2} = \frac{m}{2q} \frac{\delta^2}{ma^2} \quad (15) \]

so that it can easily be inserted into the probe theory. The particle densities for electrons or ions are calculated by integrating the distribution functions across the invariants of motion \((\phi_a)\) for not considered ion density. With these space charges, in the next step a new potential \(\phi(r)\) is calculated from the Poisson equation.

With this new potential, an improved space charge density can again be determined, etc. The convergence of this iteration procedure leads to the self-consistent solution of the Vlasov-Poisson system. Choice of specification of known values at the beginning of iteration remains open either for an initial potential \(\phi(r)\) or to a space charge density distribution \(n(r)\).

Solution of the Vlasov-Poisson system yields a complete density and potential profile for all radii \(r\) of the antenna radius \(r_A\) and also makes possible the most accurate possible calculation of the sheath impedance. The effect of the sheath impedance on the antenna impedance is described by a substitute circuit diagram and is a series circuit with the impedance outside the plasma sheath, so that the sum of both impedances results in the antenna impedance. This shows a path for calculations which includes consideration of nonlinear effects at high signal amplitudes for determining the antenna impedance. It can be utilized in the iterative computations of dimensionless variables.

Using the Gaussian law of the form

\[ Q = -2 \pi r_A \epsilon_0 \cdot \left( \phi / \phi_r \right)_A \]
in which \( Q \) is the static charge per meter of antenna length, and with the aid of the calculated potential profile it is possible to describe, according to [20], the differential sheath capacitance per unit length of a cylindrical antenna by

\[
C = \lim_{\Delta \phi \to 0} \frac{\Delta Q}{\Delta \phi} = -2\pi e_0 \cdot \lim_{x \to 1} \frac{\partial X}{\partial \chi} \cdot \left[ \frac{x_1 - x_1^o}{x_2 - x_1} \right]_{x=1}
\]

with the dimensionless quantities \( \chi = e\phi/kT \) (\( k = \) Boltzmann constant, \( T = \) temperature) and \( x = r/r_A \). The calculation process is characterized by the fact that first the dimensionless electrical field \((\partial \chi/\partial x)_A\) as a function of the normalized antenna potential \( \chi_A^* = e\phi_A^*/kT \) for given transmitter power, is rewritten by means of a capacity parameter

\[
G = e\phi_A^*/kT = \frac{q^2 c_i^2 / 4 \pi \varepsilon_0}{r_A^2 kT} \approx \frac{1}{c_i^2},
\]

and the antenna radius normalized to the Debye length \( \lambda_D = \) determined as \( r_A^* / \lambda_D \). The derivative of this quantity with respect to \( \chi_A^* \), multiplied by \( 2\pi e_0 \), then gives the sheath capacitance, \( C_s \). Figures 1 and 2 show the result of these considerations, valid for a cylindrical antenna in a collision-free plasma, for two values \( \chi_A^*/\lambda_D \), from Laframboise et al. [20]. The figures show the dimensionless static electrical field \((\partial \chi/\partial x)_A\) at the antenna surface as a function of the dimensionless antenna potential \( \chi_A^* \) for various power parameters, \( G \). The slopes of these curves give the differential sheath capacitance.

The significant result of these diagrams is the drop in the slopes of the curves, indicating a diminishing sheath capacitance for increasing transmitter powers. From the coupling of the sheath impedance with the sheath-free antenna impedance there results a variation of the reactive component of the antenna impedance (incorrect matching). With the aid of these diagrams, and on the basis of the theory described previously, simple estimations are now possible for the reactive components, even though only for a relatively limited range of parameters, for instance, \( 2 \approx r_A^*/\lambda_D \approx 0.5 \).
The active component (sheath admittance) can be calculated just as simply from diagrams 3 and 4 as was done for the reactive component. These diagrams show the dimensionless ion current contribution, $i_i$, per unit of antenna length plotted as a function of the dimensionless static antenna potential $\tau_A$ for various capacity parameters, $G$. The slopes of the curves give the differential sheath admittance per unit of antenna length. The current $i_i$ is normalized to the resting current $i_r = e n_w r_A (2\pi kT/m_i)^{1/2}$ ($n_w$ = unperturbed plasma density), which results from the thermal motions of the ions in the plasma without the influence of external fields if the antenna potential is equal to the plasma potential. However, the determination of the admittance from these diagrams is valid only for negative static antenna potentials and with the electron contribution neglected.
As in the previous diagrams, an increase in transmitted power causes a decrease in the sheath admittance. In contrast to the sheath capacitance, however, the effect of this admittance on the effective behavior of the antenna is very slight, as the following example will show. But it is still remarkable that additional, even though only slight, losses are to be expected for a radiating antenna in a plasma due to a finite admittance. The antenna impedance can be estimated from Diagrams 1 - 4 with consideration of the power, plasma density, and transmitter frequency.

In the following example the sheath impedance is calculated for a dipole antenna driven at a frequency of 5 GHz in a steady, homogeneous, isotropic and loss-free plasma. The transmitted frequency and power are oriented to the values for future communications satellites. All the calculations are carried out for the parameters \( r_A/\lambda_D = 2 \) or \( r_A/\lambda_D = 0.5 \) in order to get a comparison of the different results.\( \ast \)
arising from the ionospheric conditions or from the tendency to magneto-
spheric conditions. The quantities \( r_A = 0.66 \text{ cm} \) (antenna radius) and 
\( T = 3000 \text{ K} \) (plasma temperature) are established because of use of diagrams 
1 to 4. Similarly, the ratio of the antenna radius to the Debye length 
\( r_A/\lambda_D \) for the unperturbed plasma density is specified. The value of 
the sheath impedance thus obtained from diagrams 1 - 4 are summarized 
in the following table for a dipole antenna \( 25 \lambda \) long with an assumed 
transmitter power of 1000 W (corresponding to \( G = \cdot \)) and with the 
static antenna potential \( X_A = -10 \). The quantities referred per 
meter of antenna length are \( G/\Omega_m^{-1} \) the sheath admittance, \( c_s/pFm^{-1} \) 
the sheath capacitance, and \( c_r/pFm^{-1} \) the resulting antenna capacitance 
(Equation 18). The antenna free space capacitance is determined by

\[
c_o = \frac{\pi e_0}{\ln(h/r_A)}
\]

where \( e_0 \) is the dielectric constant of free space and \( 2h \) is the dipole 
length.

<table>
<thead>
<tr>
<th>( c_o/pFm^{-1} )</th>
<th>( G/\Omega_m^{-1} )</th>
<th>( c_s/pFm^{-1} )</th>
<th>( c_r/pFm^{-1} )</th>
<th>( r_A/\lambda_D )</th>
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<td>23</td>
<td>1.2 \times 10^{-5}</td>
<td>40</td>
<td>18.6</td>
<td>2</td>
</tr>
<tr>
<td>8.5 \times 10^{-7}</td>
<td>27.8</td>
<td>21.5</td>
<td>0.5</td>
<td></td>
</tr>
</tbody>
</table>

As already mentioned, the table shows only very small values for 
the sheath admittance, and these decrease still further with decreasing 
\( r_A/\lambda_D \). These are due to the reduction of the unperturbed plasma 
density toward the magnetosphere and by the simultaneous expansion of 
the sheath region. Thus, the losses due to a finite admittance remain 
egligibly small. The expansion of the sheath region on reduction of 
\( r_A/\lambda_D \) is also the cause for the decrease in sheath capacitance. The 
resultant total capacitance, \( C_r \), of the antenna is now obtained from 
the series connection of the sheath capacitance, \( C_s \), with the capacitance, 
\( C_o' \) of an auxiliary assumed sheath-free dipole antenna in homogeneous
plasma with a radius \( r'_A = r_S \):

\[
C_r = \frac{\varepsilon_r}{\varepsilon_r - 1},
\]

\( C'_r \) is calculated from Equation (17), in which \( \varepsilon_o \) transforms into \( \varepsilon_o \varepsilon_r \) (\( \varepsilon_r \) = relative dielectric constant of the plasma). The exact sheath radii can be determined only from the individual computation of the space charge distributions. They have been established at \( r_S/r_A = 1.5 \) for \( Z_A/\lambda_D = 2 \) and \( r_S/r_A = 2.5 \) for \( Z_A/\lambda_D = 0.5 \). The calculated values of the resultant capacitance are also shown in the table.

The principal result of this estimation is that the capacitance \( C_r \) resulting from sheath formation for both cases of \( Z_A/\lambda_D \) exhibits a distinct deviation from the original value, \( C_o \), of the order of 19% and 7%, respectively. It must be noted, though, that these estimates can be carried out only approximately for the high frequency region of 5 GHz, using this scheme (Diagrams 1 to 4). In particular, it is difficult to maintain the condition that the antenna radius is small with respect to the wavelength, and, therefore, the condition for Equation (14), at high frequencies. The radius has a specified, fixed value due to the ratio \( Z_A/\lambda_D \), independent of the frequency. Reduction of \( r_A \) is not very reasonable, as it forces a simultaneous reduction of \( \lambda_D \) and, therefore, also an increase in the plasma density, so that the plasma could no longer correspond to ionospheric conditions. For this reason the numerical results of the estimation are of less importance than the prediction of the results that even at high frequencies, high above the plasma frequency, critical interactions between antenna and plasma can cause detectable variations of the antenna impedance and change the radiation behavior because of incorrect matching.

More accurate quantitative results can only be attained through a computation method matched to the specific problem. The procedure of Laframboise et al. which has been presented appears particularly
suitable for this, because it provides a complete picture of the density and potential profiles; but it requires a substantial numerical cost. Attempts to avoid this cost and find a semianalytical empirical procedure by way of reasonable approximations and simplifications was unable to provide a generally satisfactory result. As an example, we may mention the formula for the spatial logarithmic potential decrease of a long cylindrical antenna

$$\phi(x) - \phi_0 \cdot (1 - A \ln(e^{-1}e^{-x^2}) \cdot B)$$

where A and B are unknown functions which depend on the plasma parameters as well as on the antenna characteristics. If the potential profile were known, the density profile would also be defined by the Poisson equation. But from the given boundary conditions, no unambiguous description can be found for the unknown functions A and B.

A numerical method such as suggested here is, to be sure, quite costly and time-intensive in the setting up and running of a computer program in which such a process along with the effects of photo- or secondary electron emission can be integrated. It is to be expected that especially the antenna systems of synchronous satellites in the magnetosphere, which are strongly exposed to intensive solar radiation and to the incidence of substorms can be sensitively disturbed. These effects have not yet been treated in the literature in connection with antennas radiating high power; but a series of works can be found (e. g. [32]) which have treated the problem for passive bodies.

In order to get an impression of how strongly the plasma conditions are affected by photoemission in the immediate vicinity of passive bodies in the magnetosphere, Diagram 5 finally gives the density profile for a spherical quartz sample (radius $R_o$), calculated from the photoelectron energy distribution in Diagram 6. Figure 5 shows that the electron density in the near vicinity of the body rises by a factor larger than $10^2$, so that it is far above the average density $N_e$. This fact in particular could have a critical effect on the feed zone of an antenna, similar to what was observed, for instance, with Helios I [13, 14].
**Figure 5**

- $N_e [\text{cm}^{-3} ] = 5$
- $T_e [10^5 \text{K}] = 1$
- a) $V_e (V) = 10$
- b) $\ldots = 5$
- c) $\ldots = 2$
- d) $\ldots = 1$

**Figure 6**

- $E [\text{eV}]$
- $2\pi \sqrt{2m_e^2} f_{2h} [\text{eV/}\text{cm}^3]$

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6. Summary

The plasma states to be considered in impedance determination are sketched briefly and the two most useful plasma models, the hydrodynamic and kinetic theories, are discussed. A theory is presented, from which the variation of the impedance of a dipole antenna in plasma can be determined even for frequencies far above the plasma resonance frequency. One publication on this subject served as a basis for an initial estimate of the magnitude of the variations. In one example, matched to the parameter range for future communications satellites, it is shown that the reactive component in particular of an antenna impedance can diverge distinctly from its value in the unperturbed case due to plasma sheath effect, even for high frequencies. The theory described is also suited to consider effects due to photo- and secondary electron emission. By means of results from the literature, an example of a passive body in a plasma is used to clarify how strongly the environmental relations of an antenna in a magnetospheric plasma can be changed by this effect.
7. LITERATURE


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