Ramp-Integration Technique for Capacitance-Type Blade-Tip Clearance Measurement

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ABSTRACT

The analysis of a proposed new technique for capacitance type blade tip clearance measurement is presented. The capacitance between the blade tip and a mounted capacitance electrode within a guard ring forms one of the feedback elements of a high speed operational amplifier. The differential equation governing the operational amplifier circuit is formulated and solved for two types of inputs to the amplifier - a constant voltage and a ramp. The resultant solution shows an output that contains a term that is proportional to the derivative of the product of the input voltage and the time constant of the feedback network. The blade tip clearance capacitance is obtained by subtracting the output of a balancing reference channel followed by integration. The proposed sampled data algorithm corrects for environmental effects and varying rotor speeds on-line, making the system suitable for turbine instrumentation. System requirements, block diagrams, and a typical application are included.

[Key words: Tip clearance measurement; Turbomachinery; Capacitance]

INTRODUCTION

Computational codes for predicting gas turbine characteristics generated through improved graphics, numerics, and physical realism require a thorough understanding of the aerodynamics that govern combustion and heat transfer processes in gas turbine engines. The accuracy and predictive capabilities of these codes must be verified by comparison with reliable experimental data over different operating conditions. High fidelity instrumentation techniques that can operate in hostile engine environments are therefore required. The clearance between a turbine blade tip and its shroud is one of the important factors that govern the efficiency of a turbine engine. Obtaining reliable data on such clearance therefore is an essential part of the above research objective. Earlier, different techniques were used by Barranger (1,2) to obtain such data. The present paper gives the details of the analysis of a proposed system that is expected to meet the objectives.

SYSTEM REQUIREMENTS

Figure 1 is a schematic of a rotor blade and the probe tip of a typical shroud mounted capacitance sensing probe. As the blade passes under the probe tip, the capacitance $C_p(t)$ between the inner electrode and the blade tip changes as a function of the geometry of the gap. If $C_0$ represents any static capacitance that is present or that is deliberately introduced across $C_p(t)$, then the capacitance at any moment is given by

$$C(t) = C_0 + C_p(t)$$

Figure 2 shows $C(t)$ as a function of time for the passing of two blades under the probe tip.

The probe tip diameter usually is made much smaller than the inner blade spacing, which means $C(t)$ will be equal to $C_0$ between the blades. The blade capacitance $C_p$ is the difference between the peak value of $C(t)$ and $C_0$. This difference increases with decreasing clearance. The instrumentation system should be capable of measuring $C_p$ accurately at all rotor speeds. Furthermore, it should also have static calibration capability, good dynamic response, and sufficient sensitivity. The proposed system is expected to meet these requirements and be suitable for turbine instrumentation.

PRINCIPLE OF OPERATION

The objective of the proposed system is to measure the capacitance between the blade tip and an electrode in a transducer mounted on the shroud. This electrode is surrounded by a guard ring and an earthed sleeve as shown in Figure 3. The transducer is connected to an operational amplifier circuit as shown in Figure 4. Doebelin (3) describes a similar capacitance feedback technique except that now a feedback resistor $R_2$ is introduced and a different type of input voltage is applied. In the present system, the total capacitance across the feedback resistor changes with tip clearance, effectively introducing a
change in the RC time constant of the circuit. This change in the time constant is measured to obtain the capacitance change due to tip clearance by the methods described below. Another feature of the circuit in Figure 4 is that the connecting cable and the dielectric material used in the transducer appear effectively across the two inputs of the operational amplifier.

In an ideal operational amplifier the potential difference between its inputs approaches zero. Hence, any variation in the dielectric material and cables should have minimum effect on the system performance. Further, because of the almost zero potential difference between the two inputs, the main electrode and the guard ring will be almost at the same potential, thus providing the required guarding. Various conditions under which the change in time constant can be measured are analyzed below.

ANALYSIS

To formulate the governing differential equation, the blade tip clearance capacitance \( C_b(t) \) is lumped with the fixed capacitance \( C_0 \) to form \( C(t) = C_0 + C_b(t) \) as shown in Figure 5. The operational amplifier is assumed to be ideal. Then, the current through \( R_2 \) is equal and opposite in sign to sum of the currents through \( R_2 \) and \( C(t) \). From Figure 5, the current through \( R_2 \) will be \( V_2(t)/R_2 \) and the current through \( C(t) \) will be \( d(V_2 \cdot C(t))/dt \). The resulting differential equation for the above circuit is

\[
\frac{dV_1(t)}{dt} = \frac{V_0 - V_2(t)}{R_2} \cdot dC(t) + C(t) \cdot \frac{dV_2(t)}{dt}
\]

or

\[
\frac{dV_2(t)}{dt} = \frac{1}{C(t)} \cdot \left[ \frac{V_1(t)}{R_1} \cdot V_2(t) \cdot \left( \frac{1}{R_2} + \frac{dC(t)}{dt} \right) \right]
\]

(2)

The objective of the analysis is to establish a relationship between \( V_2(t) \) and \( C_b(t) \) for a given type of input \( V_1 \). Different configurations can be visualized satisfying Equation (2) depending upon the choice of \( R_2 \) and type of \( V_1 \). However, only three cases are analyzed because of their direct application to the blade tip clearance capacitance measurement.

CASE 1: INTEGRATOR WITH AN INITIAL CONDITION

With \( R_2 \) removed from the circuit (\( R_2 = \infty \)), \( V_1 = 0 \) and \( V_2(0) \) as the initial voltage, the circuit becomes an integrator with an initial condition \( V_2(0) \). Equation (2) will then be

\[
\frac{dV_2(t)}{dt} = \frac{V_2(t)}{C(t)} \cdot \frac{dC(t)}{dt}
\]

As shown in the appendix, the solution of this equation is

\[
V_2(t) = C_0 \cdot \frac{\int V_2(0) dt}{C(t)}
\]

This particular case may be difficult to implement in practice since to apply this method the integrator must be brought to the same initial voltage condition between blade pairs. Furthermore, static calibration is not possible, because the amplifier will be driven to saturation by the amplifier offsets.

In cases 2 and 3, constraints are placed on the \( R_2C(t) \) time constant. As seen in Figure 2, \( C_b \) (corresponding to the clearance) occurs somewhere in the middle of the total blade passage time \( T \). The \( R_2C(t) \) time constant is chosen to be small compared to \( T \) in order that the transients resulting from a change in the input die down well ahead of this peak. For a good high frequency response, \( R_2C(t) \) is also required to be much less than the period of the highest significant frequency in the blade tip capacitance waveform.

CASE 2: \( V_1 \) IS A DC VOLTAGE

The approximate solution of Equation (2) when \( V_1 \) is a constant voltage is given by, (refer to the appendix for the derivation)

\[
-\frac{dV_2(t)}{dt} = \frac{V_1 - d}{R_1} \cdot \left( V_2(t) \cdot C(t) \right)
\]

or

\[
-\frac{dV_2(t)}{dt} = \frac{V_1 - d}{R_1} \left[ 1 - R_2 \cdot \frac{dC_b(t)}{dt} \right]
\]

(3)

In order to find \( C_b \), a solution of Equation (3) is required. Figure 6 is an implementation of that solution. For the purpose of deleting the constant term in Equation (3) and thereby obtaining a voltage proportional to \( dC_b(t)/dt \), a reference channel containing \( R_2 \) and \( C_0 \) identical with that of the signal channel is introduced. (A means for correcting the output when the signal and reference channels are not exactly identical is shown later.) The output of the signal channel is subtracted from the reference channel. As a result of this balancing scheme the \( V_1 \cdot R_2/R_1 \) term in Equation (3) is removed and the small quantity proportional to \( dC_b(t)/dt \) only is amplified by the differential amplifier. If \( G \) is the gain of the differential amplifier, then the output of the differential amplifier is given by

\[
-\frac{dV_2(t)}{dt} = \frac{GR_2V_1}{R_1} \cdot \frac{R_2 \cdot dC_b(t)}{dt}
\]

(4)

In Figure 6 the integrator with time constant \( R_4C_4 \) must be reset for each blade to obtain \( V_0(t) \) proportional to \( C_b(t) \). The required sensitivity can be obtained by appropriate choice of \( G, R_2, R_1, V_1, R_4, \) and \( C_4 \).
This configuration of the circuit has the advantage that it requires no initialization for each blade and the circuit is always ready to respond to blade tip clearance capacitance changes. The only disadvantage with this method is that there is no response to \( C_b \) for static calibration. As shown below, when the input is a ramp voltage instead of a constant voltage, a system that responds to static changes in \( C_b \) can be realized.

**CASE 3: \( V_1 \) IS A RAMP**

For this case, \( V_1 \) is proportional to time \( t \), where \( t \) starts between blade pairs and lasts the duration of the blade passing time \( T \). The approximate solution of Equation (2) when \( V_1 = Kt \) (\( K \) is the slope of the ramp in volts/second) is given by (refer to the appendix for the derivation),

\[
-V_2(t) = \frac{R_2}{R_1} \left[ Kt - \frac{d}{dt} (Kt R_2 C(t)) \right]
\]

Substituting \( C(t) = C_0 + C_b(t) \) and \( dC(t)/dt = dC_b(t)/dt \), we obtain

\[
-V_2(t) = \frac{KR_2}{R_1} \left[ t - R_2 C_0 - R_2 \frac{d}{dt} (tC_b(t)) \right]
\]

The schematic arrangement shown in Figure 6 is again used except that the input voltage is now a ramp. The output of the reference channel is changed to

\[
-V_2(t)_{\text{ref}} = \frac{KR_2}{R_1} \left[ t - R_2 C_0 \right]
\]

The differential amplifier output can now be written as, (again assuming balanced channels)

\[
-V_d(t) = G \left( \frac{KR_2}{R_1} \right) R_2 \frac{d}{dt} (tC_b(t))
\]

In a practical situation it is difficult to match the \( R_i C_i \)'s exactly, although \( R_i \) values can be matched accurately. For this case, \( V_d \) becomes,

\[
-V_d(t) = \frac{GKR_2}{R_1} \left[ R_2 \frac{d}{dt} (tC_b(t)) + R_2 C_0 \right]
\]

where \( R_2 C_0 \) is the difference in time constants \( R_2 C_1 \) of the channels. The integrator output will then be,

\[
V_0(t) = \frac{GKR_2}{R_1} \cdot \frac{tC_b(t)}{R_2 + R_4 C_0} + pt
\]

where

\[
p = \frac{GKR_2^2 C_0}{R_1 R_3^2 C_4}
\]

The signal \( C_b(t) \) can be obtained from the measured values of \( V_0(t) \) using the algorithm developed below.

**SIGNAL PROCESSING**

Equation (10) shows that the output voltage is proportional to the product of the instantaneous values of time and the blade tip clearance capacitance as well as \( p \), representing the difference in the initial time constants \( R_2 C_0 \) of the reference and the signal channels. When the measurements are taken at different speeds of the rotor, it is clear that the tip clearance can be obtained only after applying corrections for the instantaneous values of time. A means must also be found to correct for \( p \). In the proposed system these difficulties are circumvented by generating a voltage proportional to the rotor speed and by sampling \( V_0(t) \) between the blades to solve for \( p \).

The schematic arrangement for implementing this scheme is shown in Figure 7. The angle clock is a fast updating frequency synthesizer which among other things will produce:

1. A frequency proportional to the speed of the rotor. Let this frequency be \( f_r \).
2. A fixed number of pulses (preset) per blade on the rotor at all speeds. This signal can be used to accurately estimate the angular position of the rotor. Let this number of pulses (preset) be \( N \).

The voltage input to the integrator one whose time constant is \( R_3 C_3 \) will then be,

\[
-V_1 = mf_r
\]

In a practical situation it is difficult to match \( R_i C_i \)'s exactly, although \( R_i \) values can be matched accurately. For this case, \( V_1 \) becomes,

\[
V_1 = \frac{1}{R_3 C_3} \int m - f_r \cdot dt = \frac{mf_r}{R_3 C_3} \cdot t = Kt
\]

Substituting for \( K \) in Equation (10) and solving for \( C_b(t) \), we get,

\[
C_b(t) = \frac{R_2 R_4 C_0 C_3}{Gmf_r R_2^2} (V_0(t) - pt)
\]

The period \( T \) in seconds between the blades (Figure 2) can be related to the frequency \( f_r \) by,

\[
f_r = \frac{1}{MT}
\]
where \( M \) is the number of blades on the rotor. As mentioned before, the angle clock generates a fixed number \( N \) of preset pulses per blade in the duration \( T \). Therefore, at all speeds,

\[
T = \frac{N}{\Delta T}
\]

where \( \Delta T \) is the duration between the pulses. Hence,

\[
f_r = \frac{1}{MN \Delta T}
\]

Substituting Equation (15) in the Equation (12), we get

\[
C_b(t) = \frac{R_1 R_4 C_1 C_3 C_3}{GmR_2} \left( V_0(t) - pt \right)
\]

If \( V_0(t) \) is sampled \( N \) times per blade (i.e., a rate of \( MNf_r \)), then

\[
C_b(j) = \frac{R_1 R_4 C_1 C_3 C_3}{GmR_2} \left[ V_0(j) - pj \Delta T \right]
\]

\( j = 1 \) to \( N \).

From Figure 2 it can be seen that \( C_b = 0 \) at \( t = \Delta T, \) \( (C_b(1) = 0, j = 1) \) for large \( N \). Equation (17) will then be,

\[
V_0(1) = p \Delta T
\]

or,

\[
p = \frac{V_0(1)}{\Delta T}
\]

Substituting for \( p \) in Equation (17), we obtain,

\[
C_b(j) = S \left[ \frac{V_0(j) - jV_0(1)}{j} \right]
\]

\( j = 1 \) to \( N \) where,

\[
S = \frac{R_1 R_4 C_1 C_3 C_3}{GmR_2^2} = \text{a constant}
\]

Equation (19) is a normalized value for all blade passing frequencies, thereby eliminating the requirement for measurement of time \( t \). Thus any change in \( C_0 \) can be corrected continuously as long as \( C_0 \) remains relatively constant during \( T \). This will correct for drifts due to temperature changes, other environmental effects, and long term aging.

TYPICAL APPLICATION

Sample calculations, integrator reset tests, and computer simulations were performed for a typical application. The candidate engine has 50 blades on the rotor and a maximum speed of 60,000 rpm. The maximum blade capacitance \( C_b \) is expected to be approximately 1 pF. For high frequency response the measurement system is required to have an \( R_2 C_0 \) time constant of less than 1 us. By choosing \( R_2 = 1 \) k\( \Omega \) and \( C_0 = 10 \) pF, the time constant becomes 0.01 us which easily meets this requirement. Further, the time constant is much less than 20 us, the blade passage time \( T \) at the maximum rotor speed. This means that the transient which begins at the initiation of the ramp will disappear well ahead of the measured capacitance peak.

Values for other circuit components were chosen for the measurement system. A ramp slope of \( K = 0.5 \) V/us, and a differential amplifier gain of 100, the sensitivity becomes 100 mV for a 1 pF blade tip capacitance.

The dynamic response of the system is limited by the slew rate of the operational amplifiers chosen for the circuit. Currently available high speed amplifiers have slew rates greater than 100 V/us, which is more than adequate for the application. One of the most severe speed requirements is the resetting of the integrators between rotor blade pairs. A circuit of a 10 us period ramp generator integrator was tested under simulated conditions using a high speed operational amplifier. The oscilloscope photograph of the test signals (Figure 8), shows that the resetting time is less than 1 us, which is less than 5 percent of the blade passage time \( T \) (20 us).

An assessment was made of the validity of the approximate solution for \( V_2(t) \) as applied to the measurement system. The exact solution of Equation (2) was compared with the approximate solution of Equation (6). Equation (2) was solved numerically for the ramp input using Runge-Kutta fourth order method. The results tally with the solution given by Equation (6), thereby verifying the approximation for this typical application.

CONCLUSION

A proposed new technique for capacitance type blade tip clearance measurement has been described. An analysis was performed on an operational amplifier with a parallel combination of blade tip capacitance and a resistance in the feedback. When the operational amplifier is driven with a ramp input voltage, the tip clearance capacitance can be measured with an algorithm which can be easily implemented. The measuring system has static calibration capability, is independent of changes in rotor speed and is not affected by environmentally induced changes in probe and circuit parameters. In a typical application the system has a sensitivity of 100 mV/pF and a time constant of 0.01 us.

APPENDIX

Assuming that the operational amplifiers used are ideal, the governing differential equation that relates the output voltage \( V_2 \) of the operational amplifier to changes in the feedback capacitance as given by Equation (2) is repeated below.
\[-\frac{dV_2(t)}{dt} = \frac{1}{C(t)} \left[ \frac{V_1(t)}{R_1} + V_2(t) \cdot \left( \frac{1}{R_2} + \frac{dC(t)}{dt} \right) \right] \]  
(A1)

The circuit configuration for the above equation is shown in Figure 5. Let
\[ P = \frac{1}{R_2 C(t)} + \frac{1}{C(t)} \cdot \frac{dC(t)}{dt} \]  
(A2)

and
\[ V_1(t) = -Q = \frac{1}{R_1 C(t)} \]  
(A3)

The solution of Equation (A1) is known to be,
\[ V_2(t) = \text{const} \cdot e^{-Pdt} + e^{-Pdt} \int Q \cdot e^{Pdt} \cdot dt \]  
(A4)

where,
\[ e^{Pdt} = e^{\int \left( \frac{1}{R_2 C(t)} + \frac{1}{C(t)} \cdot \frac{dC(t)}{dt} \right) dt} \]
\[ = e^{\int \frac{1}{R_2 C(t)} \cdot dt} \cdot e^{\int \frac{dC(t)}{C(t)} \cdot dt} \]
\[ = C(t) \cdot e^{\int \frac{1}{R_2 C(t)} \cdot dt} \]  
(A5)

\[ e^{-Pdt} = \frac{1}{C(t)} \cdot e^{\int \frac{1}{R_2 C(t)} \cdot dt} \]  
(A6)

CASE 1

With \( R_2 \) removed from the feedback (\( R_2 = \infty \)), \( V_1 = 0 \), and \( V_2(0) \) as the initial voltage, Equation (A4) becomes,
\[ V_2(t) = \frac{C_0}{C(t)} \cdot V_2(0) \]

For the other two cases, let the time constant \( R_2 C(t) \) be small when compared to other periods of interest. In Equation (A4), this makes the initial transient term disappear quickly when compared to other terms. Then,
\[ V_2(t) = e^{-Pdt} \int Q \cdot e^{Pdt} \cdot dt \]  
(A7)

Substituting for \( Q \), \( e^{Pdt} \), \( e^{-Pdt} \), from the previous equations, we get,
\[-V_2(t) = \frac{e^{-Pdt}}{R_1 C(t)} \cdot \int e^{\int \frac{1}{R_2 C(t)} dt} \cdot V_1 \cdot dt \]  
(A8)

Rewriting Equation (A8) and integrating by parts,
\[-V_2(t) = e^{-Pdt} \int e^{\int \frac{1}{R_2 C(t)} dt} \cdot \frac{dC(t)}{dt} \cdot dt \]
\[ + \frac{d^2C(t)}{dt^2} \cdot \frac{V_1}{C(t)} \]  
(A9)

where,
\[ Z = \frac{R_2 e^{\int \frac{1}{R_2 C(t)} \cdot dt}}{e^{\int \frac{1}{R_2 C(t)} \cdot dt}} \cdot e^{\int \frac{dC(t)}{C(t)} \cdot dt} \]
\[ + \frac{dC(t)}{dt} \]  
(A10)

CASE 2: \( V_1 \) IS A CONSTANT

Let \( V_1 = V \). Substituting \( \frac{dC(t)}{dt} = \frac{dC_0(t)}{dt} \) in Equation (A11), we get
\[ \int \frac{dC(t)}{C(t)} \cdot \frac{dV_1(t)}{dt} \cdot dt \]
\[ - \int e^{\int \frac{1}{R_2 C(t)} \cdot dt} \cdot e^{\int \frac{dC(t)}{C(t)} \cdot dt} \cdot \frac{dV_1(t)}{dt} \cdot dt \]
\[ = e^{\int \frac{1}{R_2 C(t)} \cdot dt} \cdot \frac{dV_1(t)}{dt} \cdot dt \]  
(A11)

where,
\[ Z = \frac{R_2 e^{\int \frac{1}{R_2 C(t)} \cdot dt}}{e^{\int \frac{1}{R_2 C(t)} \cdot dt}} \cdot e^{\int \frac{dC(t)}{C(t)} \cdot dt} \cdot \frac{dV_1(t)}{dt} \cdot dt \]  
(A12)
It can be shown that the term containing the integral in Equation (A12) can be neglected when the $R_2C(t)$ is small compared to the period of the highest significant frequency contained in $C_b(t)$. Therefore, Equation (A12) becomes,

$$Z = \frac{R_2^2}{2} \frac{dC_b(t)}{dt}$$  \hspace{1cm} (A13)

Substituting for $Z$ in Equation (A10),

$$-V_2(t) = \frac{R_2^2}{R_1} \left[ 1 - R_2 \cdot \frac{dC_b(t)}{dt} \right]$$  \hspace{1cm} (A14)

CASE 3: $V_1$ IS A RAMP

Substituting $V_1 = Kt$ in Equation (A10),

$$-V_2(t) = \frac{1}{R_1} \left[ R_2Kt - Z \right]$$  \hspace{1cm} (A15)

where,

$$Z = \frac{R_2K}{C(t)} \cdot e^{-\frac{t}{R_2C(t)}} \cdot \int e^{\frac{t}{R_2C(t)}} \left( C(t) + t \frac{dC_b(t)}{dt} \right) dt$$  \hspace{1cm} (A16)

Rewriting $Z$ and again integrating by parts,

$$Z = \frac{R_2Ke}{C(t)} \cdot \int e^{\frac{t}{R_2C(t)}} \cdot \frac{dt}{R_2C(t)}$$

$$\cdot \left( C(t) + t \frac{dC_b(t)}{dt} \right) R_2C(t)$$

or,

$$Z = \frac{R_2K}{R_1} \left[ t - R_2C_0 - R_2C_b(t) - R_2t \frac{dC_b(t)}{dt} \right]$$

or,

$$-V_2(t) = \frac{R_2K}{R_1} \left[ t - R_2C_0 - R_2 \frac{dC_b(t)}{dt} \right]$$  \hspace{1cm} (A19)

REFERENCES


Figure 1. - Schematic of rotor blade and probe tip of typical shroud mounted capacitance sensing probe.

Figure 2. - Capacitance as a function of time for passing of two blades under probe tip.
Figure 3. - Cross-sectional view of the capacitance transducer.

Figure 4. - Arrangement of the transducer in the feedback of an operational amplifier circuit.
Figure 5. - Lumped parameter model for analysis.

Figure 6. - Schematic arrangement to obtain $V_0$ proportional to $C_b$. 
Figure 7. - Normalized instrumentation system.
Figure 8. - Resetting of the integrator.
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