CALCULATING FAR-FIELD RADIATED SOUND PRESSURE LEVELS FROM NASTRAN OUTPUT

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SUMMARY

FAFRAP is a computer program which calculates far-field radiated sound pressure levels from quantities computed by a NASTRAN direct frequency response analysis of an arbitrarily shaped structure. Fluid loading on the structure can be computed directly by NASTRAN or an added-mass approximation to fluid loading on the structure can be used. Output from FAFRAP includes tables of radiated sound pressure levels and several types of graphic output. FAFRAP results for monopole and dipole sources compare closely with an explicit calculation of the radiated sound pressure level for those sources.

INTRODUCTION

FAFRAP computes far-field radiated sound pressure levels using the Helmholtz exterior integral equation by numerically integrating fluid pressures and normal velocities over the fluid-structure interface of a finite element model. The numerical integration requires the XYZ coordinates, unit normal vector, tributary area, fluid pressure, and outward normal velocity for every grid point on the fluid-structure interface. ALTER statements in a NASTRAN direct frequency response analysis are used to obtain these quantities. Fluid pressures at the fluid-structure interface are computed directly by NASTRAN if an explicit fluid finite element mesh is used. Alternatively, FAFRAP will calculate fluid pressures if an added-mass approximation to fluid loading is used.

THEORY

Consider the arbitrarily shaped body in figure 1. Let \( \mathbf{z} \) be the position vector to an exterior fluid point \( P \), and \( \mathbf{z} = |\mathbf{z}| \). Let \( \mathbf{x} \) be the position vector to a point on the fluid-structure interface (with \( \mathbf{x} = |\mathbf{x}| \)), let \( \mathbf{r} = \mathbf{z} - \mathbf{x} \) (with \( r = |\mathbf{r}| \)), and let \( \mathbf{n} \) be the unit outward normal at the location \( \mathbf{x} \). The time-harmonic (\( e^{i\omega t} \)) pressure at \( \mathbf{z} \) is given by the Helmholtz integral (ref. 1)

\[
p(\mathbf{z}) = \int_S \left[ \mathbf{n} \cdot \mathbf{v} (\mathbf{x}) + ik + 1/r \right] p(\mathbf{x}) \cos \theta \left( e^{-ikr} / r^2 \right) dS
\]

(1)
Figure 1. Geometry for Far-Field Radiated Sound Pressure Level Computations

where \( v_n(x) \) and \( p(x) \) are the complex normal surface velocity and complex surface pressure, respectively, and \( k = \omega/c \), where \( \omega \) = circular frequency, and \( c \) = speed of sound in the fluid.

Equation (1) can be simplified if only far-field locations are of interest. As \( |z| \to \infty \), \( ik + 1/r + ik \), and from the law of cosines, \( r + z = x \cos \alpha \). Therefore, at far-field locations

\[
p(z) \approx (ik e^{-ikz/4\pi z}) \int_S \left[ p v_n(x) + p(x) \cos \beta \right] e^{ikx \cos \alpha} \, ds
\]

where \( \cos \beta = (z/|z|) \cdot n \).

One convention for presenting far-field pressures is as "sound pressure level (RMS) in dB relative to 1 \( \mu Pa \) at 1 yard." Sound pressure levels (SPL) due to an excitation force applied as amplitude rather than RMS is obtained from eq. (2) by substituting \( z = 36 \) inches and by multiplying by the conversion factor \( 1 \text{ psi} = 6.895 \times 10^9 \mu Pa \) to convert pressure \( p(z) \) from pounds per square inch (psi) to micropascals (\( \mu Pa \)). Therefore, for \( |z| = 36 \) inches

\[
\text{SPL} = 20 \log \left( (6.895 \times 10^9 \, p(z))/\sqrt{2} \right)
\]

**FLUID LOADING**

Fluid pressures on the fluid-structure interface can be computed by NASTRAN if an explicit fluid finite element mesh is used. The finite element method that models the exterior surrounding fluid out to a predetermined distance is described by Everstine (refs. 2,3). This method uses, as the fundamental unknowns, the structural displacements and a velocity potential in the fluid. The outer boundary is terminated with nonreflective (wave-absorbing) boundary conditions, which assume that the outgoing waves are locally planar.
This approach to fluid loading results in an accurate model of the fluid mass at the expense of a much larger model due to the increased number of degrees of freedom introduced in modeling the fluid region.

An alternative to using an explicit fluid finite element mesh is to use an appropriate added-mass approximation to fluid loading. The added-mass is applied to the grid points on the fluid-structure interface. For example, at low frequencies for a conical section, the effect of the fluid pressure is that of a mass-like impedance. Junger and Feit (ref. 4) show this impedance to be

\[ z = -i\omega m_a \]  

(4)

where the effective added mass per unit area is

\[ m_a = \rho_f R \left( \frac{n}{n^2 + 1} \right) \]  

(5)

where \( \rho_f \) = fluid mass density, \( R \) = radius of conical section, and \( n \) = circumferential harmonic number.

IMPLEMENTATION

The numerical integration of eq. (2) requires, for each grid point on the fluid-structure interface, the XYZ coordinates, unit normal vector, tributary area, fluid pressure, and outward normal velocity. All of these quantities can be obtained directly from NASTRAN using the OUTPUT2 utility module. The following ALTER statements will output the required data blocks on to the NASTRAN UTI file.

\$ FAFRAP ALTER STATEMENTS, RIGID FORMAT 8, APR 84 VERSION
ALTER 21,21 $
GP3 GEOM3,EQEXIN,GEOM2/SLT,GPTT/S,N,NOGRAV/NEVER=1 $
ALTER 55 $
SSG1 SLT,BGPDT,CSTM,SIL,EST,MPT,GPTT,EDT,,CASECC,DIT/PG/LUSET/NSkip
SDR2 CASECC,CSTM,MPT,DIT,EQEXIN,SIL,GPTT,EDT,BGPDT,,,EST,XYCDB,PG/OPG1,,,,/\#STATICS#/S,N,NOSORT2/-1/
S,N,STRNFLG $
ALTER 137 $
OUTPUT2 PG,BGPDT,EQEXIN,FRL,UPVC $  
ENDALTER $

These ALTER statements allow a static unit pressure load to be applied to the structure during the dynamic frequency response analysis. The unit pressure load is applied to the fluid-structure interface of the finite element model. The components of the load vector created by the static pressure load are used by FAFRAP to compute the unit normal vector and tributary area of the grid points on the fluid-structure interface. The following is a list of the
quantities in the data blocks written with the OUTPUT2 statement.

- PG - load vector components
- BGPDT - XYZ coordinates
- EQEXIN - internal to external grid point numbering equivalencing
- FRL - frequency response list
- UPVC - grid point displacements

The displacements are converted to velocities by the relationship \( v = i_\omega u \), where \( u \) is displacement and \( v \) is velocity. If an explicit fluid finite element mesh has been used, then the pressure at a fluid grid point on the fluid-structure interface is evaluated as the time derivative of the velocity potential (ref. 2). If an added-mass approach to fluid loading is used, FAFRAP calculates pressure from the displacement.

Several user-defined input parameters to FAFRAP control the number of far-field locations at which to calculate an SPL and the different types of output.

**OUTPUT**

Several types of output are available from FAFRAP. There are tables of computed values and three types of graphics output. Table 1 lists the SPL at far-field locations. The headings COLAT and LON refer to colatitudinal and longitudinal far-field locations, respectively. These tables are printed for each subcase and frequency. Table 2 lists phasor sum, RMS velocity, maximum SPL and where it occurs, maximum SPL in a horizontal plane and where it occurs, and radiated power for each subcase. Equations 6 and 7 define the phasor sum and RMS velocity, respectively,

\[
\text{phasor sum} = \frac{\sum^n_i (v_{n, i} A_i)}{A} \\
\text{RMS velocity} = \frac{\sqrt{\sum^n_i (v_{n, i}^2 A_i)}}{A}
\]

where the summation is for all \( i \) grid points on the fluid-structure interface. Radiated power represents a summation of all pressure intensities in the far-field.

A separate plotting program, FAFPLOT, was written to display SPL's in any of the three principal planes for any subcase or frequency. Figure 2 is an example of a polar plot of SPL generated by FAFPLOT. The two numbers in the lower left-hand corner refer to the subcase and frequency defined for that plot. The polar plot is useful in evaluating the sound pressure pattern generated by the structure.

Log-log plots can be generated for plots of pressure, velocity, or impedance at a grid point on the fluid-structure interface versus frequency. An example of this type of plot is shown in figure 3. The log-log plots are useful in evaluating the response of specific points on the fluid-structure interface for different load cases.
PATRAN (ref. 5) can also be used to display all of the SPL's in the far-field for one subcase and frequency as a color contour plot (fig. 4). This type of plot gives a good view of the overall radiated sound pressure pattern.

COMPARISON TO ANALYTICAL SOLUTION

Analytical solutions exist for the pressure fields produced by two simple radiators, the monopole and dipole sources. The equations defining the pressure fields generated by these sources were used to validate the results of FAFRAP. A NASTRAN analysis was performed for each of the sources to provide the necessary input for FAFRAP. The equations defining the pressure fields for a monopole and dipole source can be found in equations 4.15 and 4.75 respectively, of Ross (ref. 6). After converting the equations to provide results in the correct units, the difference between the far-field radiated SPL calculated by FAFRAP and the values obtained from the equations was less than one percent.
REFERENCES


Table 1. Sound Pressure Levels in the Far-Field
Table 2. Summary of FAFRAP Results

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Figure 2. Polar Plot of Sound Pressure Levels in the Far-Field
Figure 3. Log-log Plot of Pressure Versus Frequency at a Grid Point
Figure 4. Contour Plot of Sound Pressure Levels in the Far-Field