1.a. **Title:** Experimental Evaluation Criteria for Constitutive Models of Time Dependent Cyclic Plasticity

b. **Grant No.:** NAG 3-51

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2. **Abstract**

   Successful tests were performed on notched members at temperatures far above those recorded so far. Simulation of the notch root stress response was accomplished to establish notch stress-strain behavior. Cyclic stress-strain profiles across the net-section were recorded and on-line direct notch strain control was accomplished. Data were compared to three analysis techniques with good results.

3.a. **Objective**

   The main objective of this research project was to generate experimental data that can be used to evaluate the accuracy of constitutive models of time-dependent cyclic plasticity.

b. **Approach**

   Experimental isothermal data were generated on smooth uniaxial specimens and on center notched plates made of Hastelloy X. Notch root strains were measured with a noncontacting interferometric technique. Load histories were selected to demonstrate material memory, cyclic creep, cyclic relaxation, cyclic hardening, and time-dependent creep and relaxation.

4. **Background and Scope**

   Analysis techniques exist that are used to calculate the stresses and strains that exist at the hot sections in gas turbine engines. This calculated stress-strain response is then considered in conjunction with fatigue damage procedures to predict the fatigue lives of the critical components in the engine. To evaluate the accuracy of these techniques, an experimental program was devised that resulted in tests on laboratory type specimens that exhibit similar behavior to some of the actual components in turbine engines. This approach was taken because it is extremely difficult to obtain experimental data for comparison purposes at even the most accessible regions of the engine itself.

   All tests have been completed for this project. Constant temperature test data were generated on both smooth and notched specimens made of Hastelloy X, which is a high temperature alloy used for turbine disks. Notched specimens were center notched plates with either a circular or elliptical hole.
Smooth specimens established the material response to a variety of load patterns at different temperatures. This material showed both time-independent and time-dependent characteristics (cyclically hardened under cyclic loading, cyclic relaxation of mean stress when plastic strain was present, history dependent memory under variable loads, and creep and stress relaxation at elevated temperatures). Extensive amounts of data were generated at 70° and 1,200°F and limited data at 1,500° and 1,600°F. In some cases these tests were performed so as to isolate the individual phenomena as much as possible and in other cases these phenomena were combined.

Data were generated on the notched specimens to determine the notch root stress-strain response under various load patterns. A computer controlled, laser-based interferometric technique (interferometric strain gage, ISG) was successfully extended and employed to measure the strains in the region of the notch. This technique was employed because it could measure strains that resulted from variable loads at elevated temperatures over smaller gage lengths than any other techniques. The majority of the data were generated at 70° and 1,200°F and limited data at 1,500° and 1,550°F which was the upper temperature limit at which this test set-up could produce data. All these data were designed to be used as a criteria against which constitutive models of time-dependent cyclic plasticity could be compared.

In addition to producing data only at the notch root, strains were also determined across the net section of these specimens. This resulted in a stress-strain profile.

In an attempt to isolate the various time-dependent and time-independent phenomena, a technique was developed for on-line strain control with the ISG. This resulted in limited data. However, the fact that this can be done could influence future test programs and result in data that would lead to a better understanding of the behavior of notched components.

5. Examples of Results with Analysis Techniques

A variety of methods are available for the analysis of structures and components that are subjected to variable loads that result in inelastic strains. These analysis techniques usually can accommodate inelastic strains produced by both time-dependent creep and time-independent plastic strain. The finite element method is perhaps the most popular technique for these problems. However, other methods exist that show potential for much more economical analyses.

Regardless of the analysis procedure employed, an accurate set of constitutive relationships are required. If the uniaxial stress-strain response of the material is not adequately described, an analysis of any notch geometry will not be successful. Also, with all the possible variables associated with elevated temperature cyclic behavior, it is necessary to experimentally verify any analysis technique on simple notched laboratory specimens before attempting to analyze a complicated component such as a gas turbine engine.

Experimental data on smooth specimens and center notched plates have been generated. Smooth specimen data were generated at 70°, 1,200° and 1,600°F.
Notched specimen data include temperatures of 70°, 1,200° and 1,550°F. All of these data were generated for the purpose of establishing experimental evaluation criteria for constitutive models of time-dependent cyclic plasticity.

For comparison purposes, three analysis techniques were compared to some of these data. References 1-3 describe the analysis techniques and experimental procedures in detail. This paper presents a short summary of the three techniques and several examples of experimental versus analysis predictions.

5.a. Experimental Technique

All tests were performed on specimens machined from Hastelloy X, a nickel-based superalloy used in components requiring oxidation resistance. Smooth specimens used for room temperature testing were machined with straight gage sections; specimens tested at elevated temperatures contained hour glass shaped gaged sections. For these high temperatures tests, diametral strain and axial stress were converted to axial strain. Elevated temperatures were produced with an induction furnace.

Notch specimen data that are presented in this paper were produced on thin plates with a notch located at the center of the plate. This notch was a circular hole with a theoretical stress concentration factor of 2.37 based on net section nominal stress. Notch root strains were determined with an interferometric technique that is described in Ref. 1 and 4. With this technique, normal strains were measured over very short gage lengths. The physical part of the gage consisted of indentations on the flat surface of the specimen. These indentations were pyramidal in shape with inclined sides tilted 45° to a normal of the surface. The indentations used for this experiment were placed 100 microns apart and were 25 microns square. They were placed 50 microns from the edge of the notch.

A He-Ne laser was used to simultaneously illuminate both indentations. Due to the coherent and monochromatic nature of this light, two interference fringe patterns resulted that were 90° relative to each other and 45° relative to the laser light. Movement of the indentations resulted in proportional movement of the fringe patterns. Averaging the movement of both fringe patterns eliminated rigid body motion. By monitoring the motion of these fringe patterns, strain could be determined. The fringe patterns were electronically sensed and the analog signal of relative light intensity was relayed to a minicomputer system. Final output of this system was an analog equivalent of strain that ranged from 0 to 10 volts.

5.b. Smooth Specimen Simulation

The most direct approach to determine uniaxial constitutive behavior is to directly control a smooth specimen so as to produce the required stress-strain combination that is dictated by a mechanics analysis of the notch geometry. The Neuber relation is the result of such an analysis that has been extensively used for room temperature fatigue life predictions.
For this study it was assumed that smooth specimens could be used to supply the needed stress-strain (constitutive behavior) at both remote and local regions. Notched specimens were subjected to controlled loading rates and peaks. Remote strains were measured with the ISG. Smooth specimens were subjected to the same strain patterns that were recorded from the ISG (the same strain rate was also maintained). A smooth specimen was then controlled so that it would follow the pattern predicted by the Neuber relation. Figures 1 and 2 show the stress-strain behavior as predicted by the Neuber relation versus the experimentally determined notch root stress-strain simulation. Notch root stresses were simulated by subjecting a smooth specimen to the same strains as measured with the ISG.

All these data were generated with the material in the stable condition. As can be seen, the agreement is excellent. Similar elevated temperature data at 1,200°F are shown in Fig. 2. A 100 sec. hold time in both tension and compression was used for the elevated temperature tests. At both load levels the direct ISG-stress simulation data did not show stress relaxation from the hold periods, whereas the Neuber prediction showed a pronounced effect. The actual difference in the general trend of the stress-strain response would result in significant errors in the peak values, which are often used for damage analyses.

5.c. Model of Uniaxial Behavior

The direct use of smooth specimens for determining constitutive behavior is not practical for most design applications. An accurate mathematical model of a materials behavior that can be used with mechanics analyses would be beneficial. In an attempt to satisfy this need, a new constitutive modeling technique was developed that is capable of predicting typical uniaxial materials behavior at room and elevated temperatures. Simulation of the time-independent phenomena of cyclic hardening or softening, cyclic relaxation of mean stress and history dependent memory, and the time-dependent behavior of creep and stress relaxation was accomplished. This constitutive model is based on a generalized analysis of any configuration of classical rheological model elements and special purpose elements that were developed specifically for this constitutive modeling technique.

The modeling technique provides for the use of classical elements such as elastic springs, viscous dampers and frictional sliders. Special elements to simulate cyclic hardening and relaxation of mean stress were also added. All these elements could be readily arranged in any manner to predict the stress-strain response of materials under complex loading. The theory supporting this technique is based on the ability to formulate matrix representations of the model parameters so as to provide a set of equations that may be solved numerically to determine the model response. For the analysis of notched members, a numerical technique was created to expand the Neuber relation with the constitutive model to include time-dependent phenomena. This technique was used to form a specific constitutive model that was constructed from the material properties of Hastelloy X.

For comparison purposes this model was used to predict the response of a uniaxial specimen that was subjected to a complicated strain history at
1,600°F. Figure 3 shows this comparison. The maximum discrepancy between the two responses is about 5 ksi or 9% of the total stress range experienced. The major differences occur during the times of stress relaxation.

This constitutive model was combined with the Neuber relation to predict the notch root strain response of a circular notched specimen tested at 1,200°F. This specimen was subjected to completely reversed constant rate, cyclic loads with hold times at both the tension and compression peaks. Comparisons of the experimental and model prediction for this test is shown in Fig. 4. The results of the comparison are relatively good. The general form of the response was very close to the measured output. The model strain values were within 18% of the experimental values at all times.

5.d. Finite Element Analysis

The previous two analysis techniques employed the Neuber relation to relate remote and local behavior. Although these analysis techniques are relatively economical, their ability to deal with complicated geometries, without any experimental data on the stress concentrations, is limited. The most popular and versatile method of stress analysis is the finite element method. This method was used in a straight forward manner to calculate the notch root strains for two notch geometries.

The finite element analysis of the experimental data that were generated in this study used a large scale general purpose program, ANSYS. This program was utilized on a Prime 750 computer system that is linked to Tektronix interactive graphics terminals.

All materials behavior that were required for this program were obtained from uniaxial data. Only cyclic stable behavior was simulated under isothermal conditions. For the creep portion of the program, only secondary creep was accounted for even though ANSYS does allow for primary creep.

For notched members made of Hastelloy X, good correlation was obtained between the analysis and experimental data at room temperature. Figure 5 shows the notch root strains on an elliptical center notched plate as predicted by ANSYS and as measured by the ISG. This plate was subjected to a completely reversed, symmetric load pattern. This agreement is extremely good considering that the stress-strain behavior is simulated by only two straight line segments.

Figure 6 shows experimental data and predictions for a circular notch. This test was performed at 1,200°F. The load pattern was symmetric with hold times in both tension and compression. Correlation of experimental ISG results and analytical predictions are very poor compared to the room temperature results. At least a portion of this inaccuracy can be attributed to not including primary creep in the program.
6. Graduate Students Supported

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<thead>
<tr>
<th>Name</th>
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<th>Degree</th>
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<tbody>
<tr>
<td>L. J. Lucas</td>
<td>9/15/80 - 6/15/82</td>
<td>M.S.</td>
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<tr>
<td>B. L. Spletzer</td>
<td>9/15/81 - 12/31/83</td>
<td>Ph.D.</td>
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<tr>
<td>M. E. Melis</td>
<td>6/15/82 - 12/31/83</td>
<td>M.S.</td>
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(Undergraduates who have made significant contributions to the program are: M. A. McGaw, L. J. Melling and B. E. Schultz).

7. Publications and Presentations

FIGURE 1  NEUBER PREDICTION AND STRESS SIMULATION OF STABILIZED LOCAL BEHAVIOR
FIGURE 2 NEUBER PREDICTION AND STRESS SIMULATION AT 1,200°F
FIGURE 3 Comparison of Model and Experimental Response for 1600°F Test
FIGURE 4 Comparison of Model and Experimental Notch Root Response for 1200°F Test
FIGURE 5 ELLIPTICAL NOTCH STRAIN VERSUS TIME AT ROOM TEMPERATURE
FIGURE 6  STRAIN VERSUS TIME OF CIRCULAR NOTCHED SPECIMEN AT 1,200° F