Structural Analysis of Turbine Blades Using Unified Constitutive Models

Lewis Research Center
Cleveland, Ohio

Prepared for the
International Conference on Computers in Engine Technology
sponsored by the Institution of Mechanical Engineers
STRUCTURAL ANALYSIS OF TURBINE BLADES USING UNIFIED CONSTITUTIVE MODELS

SYNOPSIS This paper assesses the utility of advanced constitutive models and structural analysis methods in predicting the cyclic life of an air-cooled turbine blade. Five structural analysis methods were exercised in calculating the cyclic stress-strain response at the airfoil critical location. The methods studied were a cyclic elastic finite-element analysis, nonlinear finite-element analyses based on classical inelastic models and the unified models of Bodner and Walker, and a simplified inelastic procedure. These analyses were compared in terms of computing times and of predicted crack initiation lives using the Strainrange Partitioning method.

1 INTRODUCTION

Hot section components of gas turbine engines are subject to severe thermomechanical loads during each mission cycle. Inelastic deformation can be induced in localized regions leading to eventual fatigue cracking. Assessment of durability requires reasonably accurate calculation of the structural response at the critical location for crack initiation.

In recent years, nonlinear finite-element computer codes have become available for calculating inelastic structural response under cyclic loading. The plasticity computations in these codes have been based on classical incremental theory using a hardening rule to define the motion of the yield surface under cycling, a yield criterion and a flow rule. Generally the von Mises yield criterion and the normality flow rule have been used. Creep analyses are based on a separate constitutive model that is not directly coupled to the plasticity model. However, analytical studies of hot section components such as turbine blades (1) and combustor liners (2) have demonstrated that existing nonlinear finite-element computer codes based on classical methods do not always predict the cyclic response of the structure accurately because of the lack of interaction between the plasticity and creep deformation response.

Under the Hot Section Technology Project (HOST), the NASA Lewis Research Center has been sponsoring the development of unified constitutive material models and their implementation in nonlinear finite-element computer codes for the structural analysis of hot section components (3-6). The unified constitutive theories are designed to encompass all time-dependent and time-independent aspects of inelasticity including plasticity, creep, stress relaxation, and creep recovery. These theories avoid the noninteractive summation of inelastic strain into plastic and creep components and most of them avoid specifying yield surfaces to partition stress space into elastic and elastoplastic regions. In eliminating these overly simplified assumptions of the classical theory, unified models can more realistically represent the behavior of materials under cyclic loading conditions and high temperature environments.

A major problem with nonlinear, finite-element computer codes is that they are generally too costly in computing times and resources to use in the early design stages for engine hot section components. A program has been underway at Lewis to develop simplified and more economical procedures for performing nonlinear structural analyses. A simplified inelastic analysis method developed at Lewis is based on the assumption that the inelastic regions in the structure are local and constrained by the surrounding elastic material. This implies that the total strain history can be defined by elastic analyses and can use elastic finite element solutions or local strain measurements as input (7,8). Neuber-type corrections have been incorporated in the method to account for strain redistribution under applied mechanical loading. This procedure was implemented in a computer program to predict the stress-strain history at the fatigue crack initiation location of a thermomechanically cycled structure from elastic input data. Classical plasticity models and creep constitutive equations were incorporated into the procedure. This simplified inelastic analysis has been exercised on a wide variety of problems including multiaxial loadings, nonisothermal conditions, different materials and constitutive models, and dwell times at various points in the cycles. Comparisons of the results of the simplified analyses with nonlinear finite-element solutions for these problems have shown reasonably good agreement.

Over 30 methods for predicting low-cycle fatigue life have been identified in a recent review article by Halford (9). These methods differ somewhat in the structural analysis parameters used for life prediction. Basic structural/material response information required by various life prediction methods includes the total and inelastic strain ranges, inelastic strain rate, proportion of time-dependent and time-independent inelastic...
deformation, peak tensile and mean stresses, and cycle frequency.

The purpose of this study was to evaluate several structural analysis methods of different levels of sophistication with regard to their effect on the life prediction of a hot section component. Analytical methods selected for evaluation were (1) an elastic finite-element analysis, (2) a nonlinear finite-element analysis based on classical theory, (3) nonlinear finite-element analysis based on unified theories and (4) the simplified nonlinear procedure. The two unified theories which were considered were those of Bodner and Walker (5,6). The evaluation points up the basic requirement of cyclic constitutive models; to provide with sufficient accuracy, through tractable structural analysis schemes, the pertinent input to life prediction methods.

The structural under consideration was an airfoil of a generic air-cooled turbine blade being studied for use in the high pressure stage turbine of a commercial aircraft engine. This airfoil was utilized for a demonstration of a finite-element analysis incorporating a Walker unified model by Pratt and Whitney (6), under contract to NASA as part of the HOST Program. A typical mission representative of advanced engines was used for the heat transfer and structural analyses. Several cyclic structural analyses of this airfoil problem were performed at Lewis on a CRAY-XMP computer using the MARC nonlinear finite-element computer code (10). These analyses were conducted with the unified constitutive models of Bodner and Walker, as well as with classical inelastic methods. A simplified inelastic analysis for the critical location was also performed at Lewis. Comparisons were made of CPU (Central Processor Unit) times and of calculated crack initiation lives for the different structural analysis methods using the Total Strain version of the Strain Rate Partitioning (TS-SRP) life prediction method (11).

2 ANALYTICAL PROCEDURE

2.1 Problem description

The turbine blade under study is a Pratt and Whitney generic design for use in the high-pressure stage turbine of a commercial aircraft engine. The airfoil is about 6 cm in span, 2.5 cm in chord width and has a tip-to-hub radius ratio of 1.15. Material properties for a cast nickel-base superalloy, B1900 + Hf (5), were assumed for the analyses.

The three-dimensional finite-element model created by Pratt and Whitney for the MARC analyses of the turbine blade airfoil is shown in Fig. 1. A total of 173 solid elements with 418 nodes and 1086 unsuppressed degrees of freedom was used to model the airfoil shell. This model included 24 twenty-node elements around the expected high strain region of the leading edge and 149 eight-node elements for the remainder of the airfoil. Displacements were tied at the interfaces of the two types of elements to prevent separation around midsize nodes. Boundary conditions were applied to constrain all nodes at the base of the model to lie on the base plane of the airfoil. Additional boundary conditions were applied to prevent rigid body motion.

Figure 2 illustrates the flight mission originally selected by Pratt and Whitney and used for these analyses. This type of cycle is representative of a transatlantic flight for an advanced commercial aircraft engine. High transient thermal stresses and inelastic strains are induced during the engine takeoff, climb and descent parts of the cycle. Creep occurs during the maximum takeoff, climb and cruise steady-state hold times. On shutdown at the end of each cycle, a uniform airfoil temperature of 429 °C and a rotational speed of 200 rpm were assumed.

Metal temperatures were calculated from MARC transient and steady-state three-dimensional heat transfer analyses. The input for these heat transfer analyses are proprietary Pratt and Whitney information. The calculated metal-temperature, cycle-time profiles for the midspan leading edge, trailing edge and cold spot locations are shown in Fig. 3. Figure 4 shows the temperature distribution at the maximum takeoff condition where the highest temperatures occurred.

2.2 Finite element analysis - Classical inelastic theory

The MARC code was also used to perform the structural, as well as heat transfer, analyses. The input for these analyses was the unified constitutive models of Bodner and Walker, as well as with classical inelastic methods. A simplified inelastic analysis for the critical location was also performed at Lewis. Comparisons were made of CPU (Central Processor Unit) times and of calculated crack initiation lives for the different structural analysis methods using the Total Strain version of the Strain Rate Partitioning (TS-SRP) life prediction method (11).

The mission cycle was subdivided into 81 load-time increments. Structural analyses were performed for 2 complete flight cycles. Plasticity analyses were performed for the transient parts of the cycle and creep analyses during the steady-state maximum takeoff, maximum climb and cruise hold times.

2.3 Finite element analysis - Unified inelastic theory

Most unified models can be described by a set of constitutive equations that have the basic form

\[ \dot{\sigma} = f \left( \frac{\dot{\varepsilon} - \sigma}{K} \right) \] (1)

\[ K = h_1(K) \left( \dot{\varepsilon}^* \right) \right] \right) - r_1(K) \] (2)

\[ \dot{\sigma} = h_2(\sigma) \left( \dot{\varepsilon}^* \right) \sigma^* \right) \right) - \tau_2(\sigma) \] (3)

Equation (1) is a flow law relating the inelastic strain rate and the stresses where \( \dot{\varepsilon} \) and \( \dot{\varepsilon}^* \) are deviatoric stress and inelastic strain rate tensors. The tensor \( \sigma^* \) defines the kinematic or directional
hardening (the Bauschinger effect) and is frequently referred to as a back stress. \( K \) is a scalar internal variable, commonly called a drag stress, which defines the isotropic hardening. Temperature effects on the inelastic strain rate are generally taken into account by considering some of the material constants of the constitutive model to be temperature dependent. Equations (2) and (3) are evolutionary equations describing the growth laws for the internal variables. Most evolutionary equations for the back stress-drag stress type of model include both hardening and recovery terms where \( h_1 \) and \( h_2 \) are functions describing the hardening, \( d \) is a dynamic recovery function, and \( r_1 \) and \( r_2 \) are static thermal recovery functions. Almost all back stress-drag stress models use the inelastic strain rate, \( \dot{\varepsilon}_i \), as the hardening criterion. About a dozen unified theories which have been proposed in the literature were considered in the HOST program. Differences among the models occurred primarily in the functional relationships used in the constitutive equations.

The major exception to the basic form of Eqs. (1) to (3) is the Bodner model. The flow law for this model is of the form

\[
\varepsilon_i^* = \frac{D}{n} \exp \left[ \frac{1}{2} \frac{\varepsilon_i^*}{2 \langle 3 \rangle} \right] \frac{\varepsilon_i}{\langle 2 \rangle} \tag{4}
\]

where \( D \) and \( n \) are material constants with \( D \) representing the limiting strain rate in shear and \( \varepsilon_i \) is the total strain rate in the \( i \)th direction.

A major difference between Eqs. (1) and (4) is that the back stress models assume the direction of the inelastic strain rate vector to be coincident with the direction of \( \varepsilon \), whereas the Bodner model assumes it to be coincident with the direction of \( \dot{\varepsilon}_i \).

Isotropic and directional hardening are distinguished by a partitioning of the internal variable \( Z \) into components \( Z_1 \) and \( Z_0 \) rather than by a back stress.

\[
Z = Z_1 + Z_0 \tag{5}
\]

The evolutionary equations for the internal variable are of the form

\[
Z_1 = h_1(Z_1 - Z_1) \dot{\varepsilon}_i - r_1 \tag{6}
\]

\[
Z_0 = h_2(Z_2) \dot{\varepsilon}_i - d(Z_2) \dot{\varepsilon}_i - r_2 \tag{7}
\]

The constant \( Z_1 \) in Eq. (6) is the saturation value of \( Z_1 \). Cyclic hardening or softening is controlled by the \( Z_1 \) component and depends on whether the initial value of \( Z_1 \) is less than or greater than \( Z_1 \). Another difference between the evolutionary equations for the Bodner model and the back stress-drag stress type of model is that the former uses the plastic work rate, \( \dot{W}_p \), as the measure of hardening rather than the magnitude of the inelastic strain rate. There are essentially nine material constants to be determined for the Bodner model, of which only three have been found to be temperature-dependent for most materials studied.

Because of its simplicity compared to the more common back stress models, the Bodner model was selected for more detailed study and development under NASA contractual efforts. Of the back stress-drag stress type of theory, the Walker model has undergone the most development for finite-element analysis. The Walker model has 14 temperature-dependent material constants to be experimentally determined.

In the course of the NASA contract studies, Bodner and Walker incorporated additional terms in their models to account for cyclic hardening during nonproportional loading. As the measure of nonproportionality, Bodner used the angle between the stress and stress rate directions while Walker used the angle between the strain and strain rate directions. Walker also revised his model to use an exponential law in place of the previous power law for the functional form of the term, \( f(Z - \varepsilon) K \), in Eq. (1).

Under a NASA sponsored effort with Southwest Research Institute and Pratt and Whitney Aircraft, the unified constitutive theories of Bodner and Walker were evaluated and further developed to model the high-temperature cyclic behavior of B1900 + HF alloy. A detailed discussion of these unified constitutive models, as well as the material constants for both models, are presented in the contractor's annual status reports (5,6). The models were implemented into the MARC code through a user subroutine, HYPELA. The model constitutive equations were incorporated using an explicit Euler technique and a self-adaptive solution scheme.

2.4 Simplified analysis - Classical inelastic theory

The basic assumption of the simplified procedure is that the material cyclic response can be calculated using the total strain history obtained from elastic analyses. Classical incremental plasticity methods involving a yield criterion and a hardening rule are used to characterize the material. As in the MARC finite-element analyses, a bilinear kinematic hardening rule was used to represent the effect of cycling on the yield condition. Since the simplified procedure is one-dimensional and the results have to be related to uniaxial fatigue data, the elastically-calculated strains used as input are correlated in terms of von Mises effective strains. To compute cyclic stress-strain loops, the input effective strains are assigned signs on the basis of the dominant principal strains.

Only elastic finite-element analyses for key points in the cycle were required; these were for three peak strain conditions (startup, maximum takeoff and shutdown). The elastic solutions for the critical location were then linearly subdivided into a further 60 increments to define the stress-strain cycle. These increments are analyzed sequentially to obtain the cumulative plastic and creep strains and to track the yield surface. Creep computations are performed for increments involving dwell times using the creep characteristics incorporated in the code. For the airfoil problem, the creep
effects were determined on the basis of a combination of stress relaxation and creep strain accumulation.

A FORTRAN IV computer program (ANSYMP) was created to automatically implement the simplified inelastic procedure. A description of the calculational scheme is presented in previous papers (7,8) on the development of this method.

3. DISCUSSION OF RESULTS

The entire discussion of the structural and life analyses results for the airfoil presented herein will be based on the critical location at the leading edge at midspan, which was the hot spot as shown in Fig. 4. This location contained the element and Gaussian integration point which exhibited the largest total strain change during a mission cycle.

The calculated stress-strain hysteresis loops at the critical location for the first two mission cycles are shown in Figs. 5 and 6 for MARC finite-element analyses using classical creep-plasticity models and the Walker model, respectively, and in Fig. 7 for the simplified analysis. Figures 5 to 7 are plotted in terms of von Mises effective stress and strain with a sign criterion based on the sign of the dominant normal stress. Comparison of Figs. 5 and 6 shows that the maximum compressive strain, which occurs at the hot end of the cycle, was about the same for the creep-plasticity and Walker model finite-element analyses. This result is to be expected since the problem was largely thermally driven and indicates that the thermal strain calculations were consistent between the two methods from startup at room temperature to maximum takeoff. However, the reason for the differences shown in Figs. 5 and 6 for the peak strains during the cold part of the cycle in descending to shutdown has not been resolved as yet. These differences result in a smaller cyclic strain range for the unified analysis than for the classical creep-plasticity analysis. The maximum compressive strain shown in Fig. 7 for the simplified analysis was somewhat smaller than for the finite-element analyses because the maximum compressive strain did not quite occur at maximum takeoff. Therefore the selection of the maximum takeoff condition as one of the mission points for an elastic finite-element analysis resulted in a slight truncation in the calculated peak strain and strain range. In all cases, the inelastic strain effects on the stress-strain hysteresis loops after the first cycle thermally driven stress-strain response had essentially stabilized by the end of the second cycle.

At the time of writing, the analysis using the Bodner model had only been carried out to the end of the first cycle. Computational instability problems were encountered during the steady-state hold times. Figure 8 shows a comparison of the radial stress-strain hysteresis loops calculated by the Bodner and Walker models for the first cycle. The stress-strain response predicted by the two unified models was similar except near the tensile peak of the cycle. It is believed that the reversal in the Bodner stress-strain loop between -1100 and -1400 microstrain during unloading was an artifact due to computational instability and does not represent real material behavior. Without this anomaly, the predicted cyclic responses from the two unified models would have been very close.

The results from these structural analyses (elastic, elastic-plastic-creep, Walker unified and simplified) are summarized in Table 1 in terms of the total strain range and mean stress for the second cycle. CPU (Central Processor Unit) times for 2 complete analytical cycles (except for the elastic cycle) are indicated in the first column. The CPU time for the simplified analysis, including 81 sec to perform the elastic finite-element analyses for the startup, maximum takeoff and shutdown which was the hottest part of the cycle, was 50 times faster than for the MARC classical finite-element analysis. The MARC analysis using the Walker model was somewhat more economical in CPU time than with the creep-plasticity models. For the first analytical cycle, the Bodner model used about the same CPU time as the classical models.

Also presented in Table 1 are predicted cyclic lives to crack initiation using the TS-SRP method. These predictions were based on unpublished NASA data for out-of-phase bithermal behavior of 81900 + Hf alloy at maximum and minimum temperatures of 871 and 483 °C, respectively. Comparison of the calculated strain ranges and lives shown in Table 1 for the different structural analysis methods demonstrate the sensitivity of life predictions to the constitutive models and analytical methodologies employed. In the present case, the lowest cyclic life prediction was obtained using the classical nonlinear finite-element analysis and the largest using the Walker unified model. It is probable that the simplified procedure would have given the most conservative life prediction if the maximum compressive strain used for the input total strain history had been more accurately defined.

4 SUMMARY OF RESULTS

This paper evaluates the utility of advanced constitutive models and structural analysis methods in predicting the cyclic life of an air-cooled turbine blade for a gas turbine aircraft engine. Structural analysis methods of different levels of sophistication were exercised in obtaining the cyclic stress-strain response at the critical airfoil location. Calculated strain ranges over conditions and the stress-strain cycles were used in predicting crack initiation lives using the TS-SRP life prediction method. The major results of this study were as follows:

1. Both the strain range and predicted life were sensitive to the type of constitutive model used. However, the maximum compressive strain at the hot end of the cycle was not significantly affected by the constitutive model, which was to be expected since this was a largely thermally driven problem. The differences shown in the calculated strain ranges between the Walker and classical models were mainly due to differences in the peak strains computed at the
cold end of cycle. The reason for these differences in the peak strains have not been resolved as yet.

2. The stress-strain responses calculated using the Bodner and Walker unified models were very similar. Computational instabilities were encountered with the Bodner model during the steady-state hold times, indicating that improvements are required in the integration procedure.

3. Due to the differences in the calculated strain ranges, the lowest cyclic life was predicted using the creep-plasticity models in the nonlinear finite-element analysis and the highest using the Walker unified model. It is probable that the simplified procedure would have given the most conservative life prediction if the input total strain peak at the hot end of the cycle had been more accurately defined.

4. The simplified procedure, including the computing times for the initial elastic finite-element analyses, was about 50 times faster than the cyclic finite-element analyses and about 4000 times faster for just the cyclic inelastic computations. The CPU time for the MARC finite-element analyses was somewhat less using the Walker unified model than the classical creep-plasticity models. Preliminary analytical results using the Bodner model indicate that it would use about the same CPU time as the creep-plasticity finite-element analysis.

5 REFERENCES


### TABLE 1. - SUMMARY OF STRUCTURAL ANALYSIS RESULTS

<table>
<thead>
<tr>
<th>Analytical method, CPU time, sec</th>
<th>Strain range, micro-strain</th>
<th>Mean stress, MPa</th>
<th>Predicted cyclic life</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elastic (1793)</td>
<td>2822</td>
<td>-189</td>
<td>69 100</td>
</tr>
<tr>
<td>Elastic-plastic creep (4038)</td>
<td>2886</td>
<td>2</td>
<td>42 400</td>
</tr>
<tr>
<td>Unified (walker) (3660)</td>
<td>2270</td>
<td>90</td>
<td>123 600</td>
</tr>
<tr>
<td>Simplified (81 + 1)</td>
<td>2771</td>
<td>-5</td>
<td>50 700</td>
</tr>
</tbody>
</table>

**Figure 1. - Airfoil Finite-Element Model.**
Figure 2. - Mission cycle used for analysis.

Figure 3. - Airfoil temperature cycle.
FIGURE 4. - AIRFOIL TEMPERATURE DISTRIBUTION AT MAXIMUM TAKEOFF.

FIGURE 5. - MARC FINITE-ELEMENT ANALYSIS STRESS-STRAIN CYCLE USING CLASSICAL CREEP-PLASTICITY MODELS.
Figure 6. - MARC finite-element analysis stress-strain cycle using Walker unified model.

Figure 7. - Simplified analysis stress-strain cycle.
Figure 8. - Comparison of stress-strain cycles using Bodner and Walker models.
This paper assesses the utility of advanced constitutive models and structural analysis methods in predicting the cyclic life of an air-cooled turbine blade. Five structural analysis methods were exercised in calculating the cyclic stress-strain response at the airfoil critical location. The methods studied were a cyclic elastic finite-element analysis, nonlinear finite-element analyses based on classical inelastic models and the unified models of Bodner and Walker, and a simplified inelastic procedure. These analyses were compared in terms of computing times and of predicted crack initiation lives using the Strainrange Partitioning method.

**Key Words (Suggested by Author(s))**
- Hot section technology
- Nonlinear structural technology
- Constitutive models

**Distribution Statement**
- Unclassified - unlimited
- STAR Category 39

---

*For sale by the National Technical Information Service, Springfield, Virginia 22161*
This paper assesses the utility of advanced constitutive models and structural analysis methods in predicting the cyclic life of an air-cooled turbine blade. Five structural analysis methods were exercised in calculating the cyclic stress-strain response at the airfoil critical location. The methods studied were a cyclic elastic finite-element analysis, nonlinear finite-element analyses based on classical inelastic models and the unified models of Bodner and Walker, and a simplified inelastic procedure. These analyses were compared in terms of computing times and of predicted crack initiation lives using the Strainrange Partitioning method.
