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ANALYTICAL OBSERVATIONS ON THE AERODYNAMICS OF A DELTA WING WITH LEADING EDGE FLAPS

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ABSTRACT

The effect of a leading edge flap on the aerodynamics of a low aspect ratio delta wing is studied analytically. The separated flow field about the wing is represented by a simple vortex model composed of a conical straight vortex sheet and a concentrated vortex. The analysis is carried out in the cross-flow plane by mapping the wing trace, by means of the Schwarz-Christoffel transformation into the real axis of the transformed plane. Particular attention is given to the influence of angle of attack and flap deflection angle on lift and drag forces. Both lift and drag decrease with flap deflection, while the lift-to-drag ratio increases. A simple coordinate transformation is used to obtain a closed form expression for the lift-to-drag ratio as a function of flap deflection. The main effect of leading edge flap deflection is a partial suppression of the separated flow on the leeside of the wing. Qualitative comparison with experiments is presented, showing agreement in the general trends.
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NOMENCLATURE

\( \phi \)  perturbation velocity potential
\( \Gamma \)  vortex strength
\( \alpha_t \)  exponent in the Schwarz-Christoffel transformation
\( \alpha \)  angle of attack
\( \epsilon \)  half apex angle of a main delta wing
\( \delta \)  flap deflection angle
\( \sigma \)  coordinate of the physical plane
\( \sigma_v \)  vortex position in physical plane
\( \eta \)  coordinate of the transformed plane
\( \eta_v \)  vortex position in the transformed plane
\( \eta_i \)  transformation coefficients
\( U \)  uniform velocity
\( F \)  complex potential
\( k \)  the ratio of the span length of the main wing to the total span length
\( U \)  uniform velocity
\( \alpha \)  normal vector of the flap surface
\( R \)  resultant force
\( R_w \)  resultant force on the main wing
\( R_f \)  resultant force on the flap surface
\( u, v, w \)  velocity components
\( C_L \)  lift coefficient
\( C_D \)  drag coefficient
\( C_p \)  pressure coefficient
\( C_N \)  normal force coefficient
\( \Delta C_L \)  lift increment; \( C_{L\text{flap}} - C_{L\text{withoutflap}} \)
\( N \)  normal force
1. INTRODUCTION

It is a well known phenomenon that above the leeside of a low aspect ratio delta wing at incidence there occurs a vortical flow originating from leading edges, and that this vortical flow is stable up to certain angle of attack. This vortical flow causes an enhancement of lift, called vortex lift, especially needed during maneuvering as well as in take-off and landing. Leading edge flaps have been suggested as a control device of this vortical flow to retain the necessary aircraft stability and control. This application of the leading edge flap is suitable for subsonic-transonic tactical delta wing aircraft, as well as aircraft having both supersonic cruise efficiency and high maneuverability.1

Coe and Weston2 made experimental investigations on the effects of leading edge flaps. From their flow visualization it was found that leading edge deflection was effective in partly suppressing the formation of leading edge vortices and promoting attached flow condition. Hoffler and Rao3 conducted experiments and theoretical studies using the vortex panel method, including the effect of tabbed flaps. Their results indicated an increase of lift-to-drag ratio. From the experimental results known so far we conclude that during flap deflection the drag is reduced considerably and the vortical flow is partially suppressed. The former contributes to an increase of lift-to-drag ratio and the latter causes a detrimental effect on lift. The second effect has been overcome by the use of flap tabs.4,5

Although many theoretical studies on the delta wing configuration have been conducted,6,7,8,9 the flapped delta wing has not been analyzed outside the panel method context.3 Moreover, the panel method does not provide a satisfactory explanation for the experimentally observed changes in the lift-to-drag ratio. In the present study we attempt to assess the trends of the effect of a leading edge flap on the aerodynamic characteristics of a slender delta wing. For mathematical convenience the conical geometry of the delta wing and the flap are chosen as shown in Fig. 1. The analysis is conducted in the cross-flow plane normal to the wing with the assumption that the flow field is also conical. For this purpose the simplest model for the separated flow9 is chosen. This model is composed of a concentrated vortex and a straight vortex sheet, as shown in Fig. 2. In this model the analysis is carried out in a plane normal to the wing surface, which is
shown in Fig. 1. By means of a simple coordinate transformation the aerodynamic forces are resolved into lift and drag. A closed-form expression for the lift-to-drag ratio is found, which indicates that this ratio must increase with flap deflection. Comparison between theory and experiment shows generally similar trends for the aerodynamic coefficients.
2. MATHEMATICAL MODEL

2.1 Methodology

To study the qualitative trends of the effect of a leading edge flap on the aerodynamic characteristics of a low aspect ratio delta wing, the simplest model for the vortex sheet, suggested by Brown and Michael, is chosen. This model, as shown in Fig 2, simply assumes the rolled-up shear layer to consist of a straight vortex sheet and a singular vortex core and requires this singularity system to be force free in a global sense.

The trace of the wing in the cross-flow plane is transformed into the horizontal axis of the transformed plane by the Schwarz-Christoffel transformation. The advantage of this transformation is that the boundary condition on the wing surface is satisfied automatically in the mapping plane.

In this chapter the following effects of flap deflection are calculated and compared with experiments:

- The influence of pressure forces acting on the flap surface on the lift-to-drag ratio.
- The drag on the main wing and the flap surface.
- The lift increment on the main wing and the flap surface.
- The influence of flap deflection angle on the vortex position and the force normal to the wing.

2.2 Analogy between 2-D unsteady flow and 3-D constant flow

In this section a comparison between the three dimensional vortex sheet growing conically in space and the two dimensional vortex sheet growing in time is made. For the detailed mathematical procedure, refer to the paper by Küchmann and Weber.

For an incompressible inviscid and irrotational flow the governing equation is

\[ \nabla^2 \Phi = 0 \]  

(1)

where \( \Phi \) is the perturbation of the velocity potential due to the wing and the associated vortex sheets originating at the leading edges.
For a conical flow the perturbation of the velocity potential and the geometry of the vortex sheet can be written

\[ \Phi(x, r, \theta) = z \phi(\tilde{r}, \theta) \]
\[ S(x, r, \theta) = r - z f(\theta) \]

where \( \tilde{r} = r/z \) is the conical parameter.

In conical coordinates (1) becomes

\[ \frac{1}{\tilde{r}} \frac{\partial \phi}{\partial \tilde{r}} + \left( 1 + \tilde{r}^2 \right) \frac{\partial^2 \phi}{\partial \tilde{r}^2} + \frac{1}{\tilde{r}^2} \frac{\partial^2 \phi}{\partial \theta^2} = 0. \]

The condition for the vortex sheet to be a streamline is

\[ \mathbf{V} \cdot \nabla S = 0, \]

Combining (5) with (2) and (3) gives

\[ (U + \phi - \tilde{r} \frac{\partial \phi}{\partial \tilde{r}}) f - \frac{\partial \phi}{\partial \tilde{r}} + \frac{f' \partial \phi}{f \partial \theta} = 0 \]

where \( \phi - \tilde{r} \frac{\partial \phi}{\partial \tilde{r}} = v_x \).

The Bernoulli equation on the vortex sheet, postulating that no pressure difference across the vortex sheet exists, has the form

\[ -\frac{p - p_0}{\rho/2} = 2U \left( \phi - \tilde{r} \frac{\partial \phi}{\partial \tilde{r}} \right) + \left( \phi - \tilde{r} \frac{\partial \phi}{\partial \tilde{r}} \right)^2 + \left( \frac{\partial \phi}{\partial \tilde{r}} \right)^2 + \frac{1}{\tilde{r}^2} \left( \frac{\partial \phi}{\partial \theta} \right)^2. \]

For an unsteady two dimensional flow, the potential function and the geometry of the vortex sheet can be written

\[ \Phi(t, r, \theta) = \frac{R^2}{T} \left( \frac{t}{T} \right)^{2m-1} \phi(\tilde{r}, \theta) \]
\[ S(t, r, \theta) = r - R \left( \frac{t}{T} \right)^m f(\theta) \]
where \( r = \left( \frac{t}{R} \right)^m \) is Prandtl’s unsteady similarity variable, and \( T \) and \( R \) are characteristic time and length.\(^6\)

For the unsteady case we finally have the velocity potential equation as

\[
\frac{1}{r} \frac{\partial \phi}{\partial r} + \frac{\partial^2 \phi}{\partial r^2} + \frac{1}{r^2} \frac{\partial^2 \phi}{\partial \theta^2} = 0 .
\]  

(10)

Boundary condition for the vortex sheet

\[
m f - \frac{\partial \phi}{\partial r} + \frac{f' \partial \phi}{f \partial \theta} = 0 .
\]  

(11)

Bernoulli equation for the vortex sheet

\[-\frac{p}{\rho/2} = \left( \frac{R}{T} \right)^2 \left( \frac{t}{T} \right)^{2(m-1)} \left( 2(2m-1)\phi - 2m^2 \frac{\partial \phi}{\partial r} + \left( \frac{\partial \phi}{\partial \theta} \right)^2 + \frac{1}{r^2} \left( \frac{\partial \phi}{\partial \theta} \right)^2 \right) .
\]  

(12)

Let’s compare the velocity potential equation, the boundary condition and the Bernoulli equation on the vortex sheet for the unsteady case ((10),(11),(12)) with those of the conical case ((4),(5),(7)). First, the equations for the velocity potential in the two cases become identical if \( F \ll 1 \). Next, the boundary condition and Bernoulli equation on the vortex sheet become equal if \( m = 1 \) and \( u_x = \phi - f(\partial \phi/\partial r) \ll 1 \). Finally we must get the relationship between \( x \) and \( t \). This relationship can be obtained by returning the potential equation in both cases to the original variables. This gives

\[
x \left( \frac{x^2}{r^2} \phi_{xx} + \frac{1}{r} \phi_x - 2 \frac{x}{r} \phi_{xr} + \phi_{rr} + \frac{1}{r^2} \phi_r + \frac{1}{r^2} \phi_{\theta \theta} \right) = 0
\]  

(13)

\[
t \left( \frac{t^2}{m^2} \phi_{tt} - \frac{2 - 3m}{rm} \phi_x + \frac{1}{r^2} \frac{m^2 - 3m + 1}{m^2} \phi + \frac{t}{r^2} \frac{2 - 3m}{m^2} \phi_t - 2 \frac{t}{r} \phi_{tr} + \frac{1}{r^2} \phi_{\theta \theta} + \frac{1}{r} \phi_r + \phi_{rr} \right) = 0
\]  

(14)
With \( m = 1 \) and \( z = Ut \) these two equations become identical.

Now we see that the three dimensional conical flow could be calculated from the
two dimensional unsteady flow if the following criteria were satisfied

(1) \( \bar{\rho} \ll 1 \), meaning that the flow is geometrically slender.
(2) \( \nu \ll 1 \), meaning that the flow is aerodynamically slender

If these two conditions are satisfied, we can solve the conical flow problem from a
2-D unsteady problem with the substitution of \( z = Ut \) and \( m = 1 \).

The flow around a low aspect ratio delta wing satisfies these two conditions
except near the vortex core, where \( \nu \) is close to \( U \). Then, this analogy can
be applied to slender delta wings with caution. Moreover, in concentrated vortex
models, the axial velocity becomes singular at the core location, making the analogy
more restrictive.

2.3 Vortex modelling

In the present model, composed of a straight vortex sheet and a singular vortex
core as shown in Fig. 2, the vortex sheet and the vortex core do not follow stream-
lines; that is to say the vortex sheet and the vortex core are not independently force
free. Thus, Joukowski forces act both on the vortex sheet and the vortex core. In
Brown and Michael's method these two forces are required to balance each other
out, meaning that the whole system is force free.\(^9\) Using the unsteady analogy
derived in 2.2 and from Fig. 2c, we can find a force free condition for the whole
vortex system. In the unsteady flow, the force free condition is

\[
\rho U \Gamma_i \left( \frac{d\sigma_{vi}}{dt} - V_{\sigma_{vi}} \right) + \rho \frac{d(\Gamma_i)}{dt} (\sigma_{vi} - \sigma_{oi}) = 0 ,
\]

where \( \Gamma_i \) is the \( i \)th vortex strength,

\( \sigma_{vi} \) is the position of \( i \)th vortex,

\( V_{\sigma_{vi}} \) is the velocity at the \( i \)th vortex position excluding its own induced velocity,

\( \sigma_{oi} \) is the origin of the straight vortex sheet.

The first term is the force acting on the vortex core, the second is that acting
on the vortex sheet. The term \( d\sigma_{vi}/dt - V_{\sigma_{vi}} \) is the relative velocity causing the
Joukowsky force on the concentrated vortex core. The rate of change of the $i^{th}$ vortex strength is $d(i\Gamma_i)/dt$ and this amount of vorticity is supplied through the straight vortex sheet having uniform strength, which connects the points from $\sigma_{oi}$ to $\sigma_{vi}$.

From 2.2, the relationship between the 3-D conical flow and the 2-D unsteady flow is

$$\frac{d}{dx} = U \frac{d}{dz}$$

and from the geometry of a delta wing, the conical relationship gives

$$k = z\epsilon$$

$$\frac{d\Sigma}{dz} = \frac{\Sigma}{z} = \frac{\epsilon}{k}$$

where $\Sigma$ stands for any quantity associated with the kinematics of the flow field, and $k$ is defined in Fig. 3 and $\epsilon$ is the half apex angle of the main delta wing.

Then the force free condition on the $i^{th}$ vortex system can be rewritten

$$\frac{U\epsilon}{k}(2\sigma_{vi} - \sigma_{oi}) = \nabla\sigma_{vi}$$

$$\nabla\sigma_{vi} = \frac{d}{d\sigma}(F - i\Gamma_i \log(\sigma - \sigma_{vi}))$$

where $F$ is the total complex potential of the flow field.

In our case we have a system of four vortices fed by vortex sheets emanating from the leading edges and from the flap junctions. Since only the symmetrical configurations are treated, the analysis considers one half of the cross flow plane. Thus $\sigma_{o1} = i(k + (1 - k)e^{i\delta})$ for the leading edge vortex and $\sigma_{o2} = ik$ for the flap junction vortex. Then the force free conditions for both vortex systems are

$$\frac{U\epsilon}{k}(2\sigma_{v1} + i(k + (1 - k)e^{-i\delta})) =$$

$$\frac{d}{d\sigma}(F - i\Gamma_1 \log(\sigma - \sigma_{v1}))|_{\sigma = \sigma_{v1}}$$

7
The derivation of the complex potential is facilitated by transforming the trace of the wing, together with its line of symmetry, into the real axis of a transformed plane. To perform this transformation the Schwarz-Christoffel transformation is used. The mapping procedure is shown in Fig. 3.

The general form of Schwarz-Christoffel transformation is

\[ \frac{d\sigma}{d\eta} = A \prod_{i=1}^{n} (\eta - \eta_i)^{\alpha_i}, \]

(21)

In integral form

\[ \sigma = C + A \int_{\eta_0}^{\eta} \prod_{i=1}^{n} (\eta - \eta_i)^{\alpha_i} d\eta, \]

(22)

where \( \alpha_i = k_i/\pi - 1 \), \( k_i \) is the angle at the \( i^{th} \) vertex of the polygon, and \( \eta_i \) is the corresponding point of \( \sigma \) in the mapping plane. \( A \) and \( C \) are complex constants.

From Fig. 3, we get

\[ k_1 = \pi, \ k_2 = \pi/2, \ k_3 = \pi - \delta, \ k_4 = 2\pi, \ k_5 = \pi + \delta, \ k_6 = \pi/2, \]

(23)

then

\[ \alpha_1 = 0, \ \alpha_2 = -1/2, \ \alpha_3 = -\delta/\pi, \ \alpha_4 = 1, \ \alpha_5 = \delta/\pi, \ \alpha_6 = -1/2. \]

(24)

From Riemann's mapping theorem we can arbitrarily choose two points. Let these two points be

\[ \eta_4 = 0 \]  
\[ \eta_6 = \eta_2 \]  
\[ \eta_0 = \eta_2 \]  
\[ \eta_4 = 0 \]  
\[ \eta_6 = \eta_2 \]  
\[ \eta_0 = \eta_2 \]  
\[ \eta_4 = 0 \]  
\[ \eta_6 = \eta_2 \]  
\[ \eta_0 = \eta_2 \]
These conditions imply that the leading edge is transformed to the origin and the initial point of the integration is taken arbitrarily at \( \eta_2 \). Requiring that the velocity in both planes be equal at infinity, we have

\[
\frac{d \sigma}{d \eta} \bigg|_{\sigma \to \infty} = 1 .
\]  

(26)

With (25) and (26), the transformation function becomes

\[
\sigma = \int_{\eta_2}^{\eta} \frac{\eta}{\sqrt{(\eta - \eta_2)(\eta - \eta_3)}} \left( \frac{\eta - \eta_4}{\eta - \eta_3} \right)^{\delta/\pi} \, d\eta .
\]  

(27)

The constants \( \eta_2, \eta_3, \eta_4, \eta_5 \) can be obtained by matching the four corresponding points in the physical and the transformed plane

\[
\begin{align*}
\sigma_3 &= ik \rightarrow \eta_3 & (28a) \\
\sigma_5 &= ik\left(1 + (1 - k)d^2\right) \rightarrow 0 & (28b) \\
\sigma_5 &= ik \rightarrow \eta_5 & (28c) \\
\sigma_6 &= 0 \rightarrow \eta_6 & (28d)
\end{align*}
\]

Here lengths are normalized with the half span length \( a(x) \).

Applying (28),

\[
\begin{align*}
1 - k \int_{\eta_2}^{\eta_4} \frac{\xi}{\sqrt{(\eta_5 - \xi)(\xi - \eta_2)}} \left( \frac{\eta_5 - \xi}{\eta_3 - \xi} \right)^{\delta/\pi} \, d\xi &= 0 & (29a) \\
1 - k \int_{\eta_2}^{\eta_6} \frac{\xi}{\sqrt{(\eta_6 - \xi)(\xi - \eta_2)}} \left( \frac{\eta_5 - \xi}{\xi - \eta_3} \right)^{\delta/\pi} \, d\xi &= 0 & (29b) \\
1 - k \int_{0}^{\eta_4} \frac{\xi}{\sqrt{(\eta_6 - \xi)(\xi - \eta_2)}} \left( \frac{\eta_5 - \xi}{\xi - \eta_3} \right)^{\delta/\pi} \, d\xi &= 0 & (29c) \\
k \int_{\eta_5}^{\eta_6} \frac{\xi}{\sqrt{(\eta_6 - \xi)(\xi - \eta_2)}} \left( \frac{\xi - \eta_5}{\xi - \eta_3} \right)^{\delta/\pi} \, d\xi &= 0 . & (29d)
\end{align*}
\]
This system is solved numerically for \( \eta_2, \eta_3, \eta_5, \eta_6 \) as shown in Appendix 1.

From Fig. 3 we see that the complex potential in the transformed plane is the sum of the potentials of a uniform velocity and that of four vortices, two of which are the image vortex systems needed to satisfy the boundary condition on the wing surface. Here we neglect the influence of the vortex sheet on the flow field.

Then the complex potential in mapping plane is

\[
F(\eta) = U \alpha \eta + \frac{i \Gamma_1}{2\pi} \log \left( \frac{\eta - \eta_{v1}}{\eta - \eta_{\bar{v}}_1} \right) - \frac{i \Gamma_2}{2\pi} \log \left( \frac{\eta - \eta_{v2}}{\eta - \eta_{\bar{v}}_2} \right),
\]

and the complex velocity in the physical plane is

\[
\frac{dF}{d\sigma} = \frac{dF}{d\eta} \frac{d\eta}{d\sigma} = \left[ U \alpha + \frac{i \Gamma_1}{2\pi} \left( \frac{1}{\eta - \eta_{v1}} - \frac{1}{\eta - \eta_{\bar{v}}_1} \right) - \frac{i \Gamma_2}{2\pi} \left( \frac{1}{\eta - \eta_{v2}} - \frac{1}{\eta - \eta_{\bar{v}}_2} \right) \right] \frac{d\eta}{d\sigma}.
\]

where

\[
\frac{d\eta}{d\sigma} = \frac{\sqrt{(\eta - \eta_2)(\eta - \eta_3)}}{\eta} \frac{d\eta}{d\sigma} = \eta^{1/4}.
\]

According to (32) the velocity becomes singular at the leading edge, \( \eta = 0 \), and at the flap junction, \( \eta = \eta_5 \). To avoid this singularity we apply the Kutta condition at these two points. The two equations for the Kutta condition are

\[
U \alpha \cdot \frac{i \Gamma_1}{2\pi} \left( \frac{1}{\eta_{v1}} - \frac{1}{\eta_{\bar{v}}_1} \right) - \frac{i \Gamma_2}{2\pi} \left( \frac{1}{\eta_{v2}} - \frac{1}{\eta_{\bar{v}}_2} \right) = 0,
\]

\[
U \alpha + \frac{i \Gamma_1}{2\pi} \left( \frac{1}{\eta_5 - \eta_{v1}} - \frac{1}{\eta_5 - \eta_{\bar{v}}_1} \right) - \frac{i \Gamma_2}{2\pi} \left( \frac{1}{\eta_5 - \eta_{v2}} - \frac{1}{\eta_5 - \eta_{\bar{v}}_2} \right) = 0.
\]

With the complex potential \( F \) the force free conditions given by (19) and (20)
Combining (33) and (34) with (35) and (36), we get the following two complex equations for the vortex positions.

\[
\frac{\varepsilon}{k \alpha} \left( 2 \pi \sigma_1 + i(1 - k)e^{-i \delta} \right) = \left[ U - \frac{i \Gamma_1}{2 \pi \eta_{v1} - \eta_1} - \frac{i \Gamma_2}{2 \pi \eta_{v1} - \eta_{v2} - \eta_1} \right] \times \left( \frac{\sqrt{(\eta_{v1} - \eta_0)(\eta_{v1} - \eta_0)}(\eta_{v1} - \eta_0)}{\eta_{v1}} \right)^{\delta/\pi} \\
+ \frac{i \Gamma_1}{2 \pi \eta_{v1}} \int \left[ \log(\eta - \eta_{v1}) - \log(\sigma - \sigma_{v1}) \right] d\sigma_{v1} \\
\frac{U \varepsilon}{k} \left( 2 \pi \sigma_2 + ik \right) = \left[ U - \frac{i \Gamma_1}{2 \pi \eta_{v2} - \eta_1} - \frac{i \Gamma_2}{2 \pi \eta_{v2} - \eta_1} \right] \times \left( \frac{\sqrt{(\eta_{v2} - \eta_0)(\eta_{v2} - \eta_0)}(\eta_{v2} - \eta_0)}{\eta_{v2}} \right)^{\delta/\pi} \\
- \frac{i \Gamma_2}{2 \pi \eta_{v2}} \int \left[ \log(\eta - \eta_{v2}) - \log(\sigma - \sigma_{v2}) \right] d\sigma_{v2} \\
\]
\[\frac{\epsilon}{k\alpha} (2\bar{v}_2 + ik) = \left[ 1 + \frac{D - B}{CB - AD} \left( \frac{1}{\eta v_2 - \eta v_1} - \frac{1}{\eta v_2 - \eta v_1} \right) + \frac{C - A}{CB - AD} \frac{1}{\eta v_2 - \eta v_2} \right] x, \quad (38)\]

\[\left( \frac{dc}{d\eta} \right)_{\sigma = \sigma_2} + \frac{A - C}{CB - AD} \left. \frac{d}{dc} \left( \log(\eta - \eta v_2) - \log(\sigma - \sigma v_2) \right) \right|_{\sigma = \sigma_2},\]

where

\[A = \frac{1}{\eta v_1} - \frac{1}{\eta v_1}, \quad (39a)\]

\[B = \frac{1}{\eta v_2} - \frac{1}{\eta v_2}, \quad (39b)\]

\[C = \frac{1}{\eta 5 - \eta v_1} - \frac{1}{\eta 5 - \eta v_1}, \quad (39c)\]

\[D = \frac{1}{\eta 5 - \eta v_2} - \frac{1}{\eta 5 - \eta v_2}. \quad (39d)\]

In the absence of flaps the only governing parameter would be the ratio of \(\epsilon/\alpha\). In the present case two additional parameters, \(\delta\) and \(k\), are included in the group \(\epsilon/(k\alpha)\). The flap deflection angle \(\delta\) does not appear explicitly but through the transformation parameters \(\eta_2, \eta_5, \eta_5\) and \(\eta_6\), which are functions of \(\delta\) and \(k\).

A Newton-Raphson method is used to solve the above two equations, the detailed numerical procedure is given in Appendix 2.

With the vortex positions known we get the vortex strengths from

\[\frac{i \Gamma_1}{2\pi} = \frac{D - B}{CB - AD} U\alpha, \quad (40a)\]

\[\frac{i \Gamma_2}{2\pi} = \frac{C - A}{CB - AD} U\alpha. \quad (40b)\]

2.4 Aerodynamic forces
2.4.1 Resultant force

To calculate the resultant force on each surface we consider the pressure coefficient.

\[ C_p = 1 + \alpha^2 - (u/U)^2 - (v/U)^2 - (w/U)^2 \]  

where the \( v \) and \( w \) components are given by

\[ v = \frac{\partial \Phi}{\partial y} \]  
\[ w = \frac{\partial \Phi}{\partial z} \]

\[ \frac{v}{w} = \text{Re} \left( \frac{\partial F}{\partial y} \right) \]  
\[ \frac{w}{v} = \text{Re} \left( \frac{\partial F}{\partial z} \right) \]  
\[ \frac{v}{w} = \text{Re} \left( \frac{\partial \sigma}{\partial y} \right) \]

\[ v = \frac{\partial \Phi}{\partial y} \]

\[ w = \frac{\partial \Phi}{\partial z} \]

\[ \frac{v}{w} = \text{Re} \left( \frac{\partial F}{\partial y} \right) \]

\[ \frac{w}{v} = \text{Re} \left( \frac{\partial F}{\partial z} \right) \]

Here \( \sigma = y + i z \) and \( V = (\partial F/\partial y)(\partial y/\partial \sigma) \) i.e. the complex velocity in the cross-flow plane.

The \( u \) component is best obtained from the conical flow condition

\[ u = U \frac{1}{x} \left( \Phi - y \frac{\partial \Phi}{\partial y} - z \frac{\partial \Phi}{\partial z} \right) \]

\[ = U + \frac{1}{x} \frac{\partial \Phi}{\partial x} \]

\[ = U + \frac{1}{x} \text{Re} \left[ U \alpha + \frac{i \Gamma_1}{2 \pi} \log \left( \frac{\eta - \eta_{v1}}{\eta - \eta_{v2}} \right) \right] \]  
\[ \quad - \frac{i \Gamma_2}{2 \pi} \log \left( \frac{\eta - \eta_{v1}}{\eta - \eta_{v2}} \right) - yv - zw \]
The presence of the feeding sheet induces a discontinuity in the $u$ component of the velocity, as demonstrated in Appendix 3.

The resultant force is given by

$$ R = \int_{\text{each surface}} p \, d\sigma $$

(45)

This integration is carried out numerically in the physical plane.

2.6.2 Lift-to-Drag ratio

Most experiments show an increase of lift-to-drag ratio with flap deflection. Before we calculate the lift-to-drag ratio we must consider the normal vector on the flap surface. The rotation of the leading edge flap about the flap hinge axis causes the normal vector to have a component opposite to the free stream. Associated with this component there is a force tending to decrease the drag, which is responsible for the increase in $C_L/C_D$. This component can be obtained by performing the following transformations:

Let's take the longitudinal axis of the wing as $x$, the axis normal to the wing surface as $y$ and the starboard direction as $-z$, as shown in Fig. 1. $e_x, e_y, e_z$ denote the unit vectors along axes $x, y, z$. With the rotations shown in Fig. 4, these vectors are expressed as

$$
\begin{pmatrix}
    e_x' \\
    e_y' \\
    e_z'
\end{pmatrix} =
\begin{pmatrix}
    \cos \epsilon & 0 & \sin \epsilon \\
    0 & 1 & 0 \\
    -\sin \epsilon & 0 & \cos \epsilon
\end{pmatrix}
\begin{pmatrix}
    e_x \\
    e_y \\
    e_z
\end{pmatrix}
$$

(46)

$$
\begin{pmatrix}
    e_x'' \\
    e_y'' \\
    e_z''
\end{pmatrix} =
\begin{pmatrix}
    1 & 0 & 0 \\
    0 & \cos \delta & \sin \delta \\
    0 & -\sin \delta & \cos \delta
\end{pmatrix}
\begin{pmatrix}
    e_x' \\
    e_y' \\
    e_z'
\end{pmatrix}
$$

(47)

Identifying $n$ with the normal vector to the flap surface we have

$$ n = e_y'' $$

(48)
Then

\[ n = \cos \delta \, c_y + \sin \delta \, c_z \]
\[ = \cos \delta \, c_y + \sin \delta (-\sin \epsilon \, c_x + \cos \epsilon \, c_z) \]  \hspace{1cm} (49)
\[ \approx -\epsilon \sin \delta \, c_x + \cos \delta \, c_y + \sin \delta \, c_z \]

The first term in the last line shows the forward component of the normal vector. This term contributes to the increase of the lift-to-drag ratio when the flap is deflected. The first component, \(-\epsilon \sin \delta\), gives the thrust force and is of higher order than the other two. As the flap is deflected the vortex comes closer to the wing surface, causing a considerable change in drag.

On the main wing the resultant force is normal to the wing surface. Then the lift and the drag components are

\[ L_w = R_w \]  \hspace{1cm} (50a)
\[ D_w = R_w \alpha \]  \hspace{1cm} (50b)

where \( R_w \) is the resultant force on the main wing.

On the flap surface there are all three components of force, as indicated by (49)

\[ R_{Fx} = -R_F \epsilon \sin \delta \]  \hspace{1cm} (51a)
\[ R_{Fy} = R_F \cos \delta \]  \hspace{1cm} (51b)
\[ R_{Fz} = R_F \epsilon \sin \delta \]  \hspace{1cm} (51c)

where \( R_F \) is the resultant force on the flap surface.

Since the \( R_{Fz} \) component is cancelled by symmetry, only \( R_{Fx} \) and \( R_{Fy} \) contribute to lift and drag

\[ L_F = R_{Fy} - R_{Fx} \alpha \]  \hspace{1cm} (52a)
\[ D_F = R_{Fy} \alpha + R_{Fx} \cos \delta \]  \hspace{1cm} (52b)

From (51a) and (51b)

\[ R_{Fx} = -\epsilon \tan \delta \, R_{Fy} \]  \hspace{1cm} (53)
From (51), (52) and (53) we get the drag and lift on the flap

\[
L_F = \alpha R_F u \left(1 + \frac{\varepsilon}{\alpha \tan \delta}\right) \tag{54a}
\]

\[
D_F = \alpha \left(1 - \frac{\varepsilon}{\alpha \tan \delta}\right) \tag{54b}
\]

The total lift and drag are

\[
L = R_w + R_F u \left(1 + \frac{\varepsilon}{\alpha \tan \delta}\right) \tag{55}
\]

\[
D = \alpha \left(R_w + R_F u \left(1 - \frac{\varepsilon}{\alpha \tan \delta}\right)\right) \tag{56}
\]

(55) and (56) show the influence of the forward component of force on the lift and the drag. Due to this thrust component the lift increases by an amount of order \( \alpha^2 \), while the drag is reduced by almost the same order as the drag itself. This increases the lift-to-drag ratio during flap deflection.

For moderate deflection angle the lift-to-drag ratio is

\[
\frac{L}{D} = \frac{1}{\alpha} \left[1 + \frac{R_F u \varepsilon}{R_w + R_F u \alpha \tan \delta} + O(\alpha^2)\right]. \tag{57}
\]

Numerical results show that the lift itself also decreases with flap deflection while the lift-to-drag ratio increases.

2.4.3 Normal force

As mentioned in the previous section the lift decreases with flap deflection. By considering the normal force we can infer the reason for this reduction.

The normal force can be calculated by considering the momentum change in the cross-flow plane

\[
N = -\rho U \int \left(\frac{\partial \phi}{\partial y} - U \alpha\right) dy dz \tag{58}
\]

Applying Stokes' theorem

\[
N = -\rho U \int_c \phi \ dz \tag{59}
\]
In terms of the complex potential and the real stream function

\[ N = -\rho U \int \text{Im} (F \, d\sigma - i \, \theta \, d\sigma) \]  

(60)

Since the contour is a streamline

\[ N = -\rho U \int \text{Im} (F \, d\sigma) \]  

(61)

Substituting for \( F \)

\[ N = -\rho U \text{Im} \int_c \left( U \alpha \eta + \frac{i \Gamma_1}{2\pi} \log \left( \frac{\eta - \eta_{v1}}{\eta - \eta_{c1}} \right) - \frac{i \Gamma_2}{2\pi} \log \left( \frac{\eta - \eta_{v2}}{\eta - \eta_{c2}} \right) \right) \frac{d\sigma}{d\eta} \, d\eta \]  

(62)

To perform this integral we first express the part containing the logarithmic terms on a circle of infinite radius, where \( d\sigma/d\eta = 1 \)

\[ \int_c \left( \frac{i \Gamma_1}{2\pi} \log \left( \frac{\eta - \eta_{v1}}{\eta - \eta_{c1}} \right) - \frac{i \Gamma_2}{2\pi} \log \left( \frac{\eta - \eta_{v2}}{\eta - \eta_{c2}} \right) \right) \frac{d\sigma}{d\eta} \, d\eta \]  

(63)

We see that in the transformed plane the normal force arising from the logarithmic terms does not depend explicitly on the wing trace, but implicitly through the vortices positions and strengths. This equation shows that the normal force is just the sum of the rate change of impulse of two pairs of vortices. Then the result could be simply obtained by considering the impulse of a pair vortices, or by integrating around the vortex sheet as shown in Fig. 5a. Following this second approach gives

\[ \int_{r=\infty} \frac{d\Gamma_1}{2\pi} \left( \log(\eta - \eta_{v1}) - \log(\eta - \eta_{c1}) \right) \, d\eta \]  

\[ = \frac{i \Gamma_1}{2\pi} \left[ \int_{\eta_{v1}}^{\eta_{c1}} \log(\eta - \eta_{v1}) |p| - \log(\eta - \eta_{c1}) |p| - i \, 2\pi \, d\eta \right] \]  

(64)

\[ + \int_{\eta_{v1}}^{\eta_{c1}} \log(\eta - \eta_{c1}) |p| - \log(\eta - \eta_{v1}) |p| \, d\eta \]  

\[ = \Gamma_1 \, (\eta_{v1} - \eta_{c1}) \]
Then the normal force originating from the logarithmic terms is

\[ N_c = 2\pi a\rho U [\Gamma_1(\eta v_1 - \eta v_1) - \Gamma_2(\eta v_2 - \eta v_2)] \]  \hspace{1cm} (63)

This force is associated with the separated flow field about the wing.

The component of normal force from the first term in (63) has a singularity at infinity. This component could be identified with the force produced by the wing if the flow were attached. Integrating along the wing trace, as shown in Fig. 5b, we find

\[
\int_c \frac{\eta^2}{\sqrt{(\eta - \eta_1)(\eta - \eta_2)}} (\eta - \eta_5)^{\delta/\pi} \, d\eta
\]

\[
= \int_{\eta_3}^{\eta_5} \frac{\eta^2}{\sqrt{(\eta - \eta_0)(\eta - \eta_2)}} \left| \frac{\eta - \eta_5}{\eta - \eta_3} \right|^{\delta/\pi} \, d\eta
\]

\[
+ \int_{\eta_3}^{\eta_5} \frac{\eta^2}{\sqrt{(\eta - \eta_0)(\eta - \eta_2)}} \left| \frac{\eta - \eta_5}{\eta - \eta_3} \right|^{\delta/\pi} \frac{1}{\rho} \, e^{i\theta} \, d\eta
\]

\[
+ \int_{\eta_3}^{\eta_5} \frac{\eta^2}{\sqrt{(\eta - \eta_0)(\eta - \eta_2)}} \left| \frac{\eta - \eta_5}{\eta - \eta_3} \right|^{\delta/\pi} \frac{1}{\rho} \, e^{i\theta} \, d\eta
\]

\[
+ \int_{\eta_3}^{\eta_5} \frac{\eta^2}{\sqrt{(\eta - \eta_0)(\eta - \eta_2)}} \left| \frac{\eta - \eta_5}{\eta - \eta_3} \right|^{\delta/\pi} \frac{1}{\rho} \, e^{i\theta} \, d\eta
\]

\[
= 2\left( \int_{\eta_3}^{\eta_5} + \int_{\eta_5}^{\eta_6} + \cos \alpha \int_{\eta_5}^{\eta_6} \right) \frac{\eta^2}{\sqrt{(\eta - \eta_0)(\eta - \eta_2)}} \left| \frac{\eta - \eta_5}{\eta - \eta_3} \right|^{\delta/\pi} \, d\eta
\]

where subscript \( p \) represents the principal value that means all values must be calculated in the same Riemann surface.
Then the total normal force is

\[
N = \text{Im} \left( \rho U \Gamma_1(\eta v_1 - \eta v_1) - \rho U \Gamma_2(\eta v_2 - \eta v_2) \right) - \rho U^2 \alpha \text{Im} \left[ \frac{\eta^2}{\sqrt{(\eta - \eta_0)(\eta - \eta_2)}} \left( \frac{\eta - \eta_2}{\eta - \eta_3} \right)^{1/2} \right] d\eta
\]

The normal force is composed of two parts; the normal force from equivalent attached flow and that from the separated flow.

Dividing this total normal force by the dynamic pressure \( \rho U^2/2 \) and the projected area \( k a(z) (k + (1 - k)\cos \delta)/\alpha \), we get the normal force coefficient

\[
C_N = \frac{\epsilon}{\alpha} \frac{1}{k + (1 - k)\cos \delta} \times \text{Im} \left[ 2\Gamma_1(\eta v_1 - \eta v_1) - 2\Gamma_2(\eta v_2 - \eta v_2) - 4\alpha \left( \left( \int_{\eta_2}^{\eta_3} + \int_{\eta_3}^{\eta_5} + \cos 2\delta \int_{\eta_2}^{\eta_5} \right) \frac{\eta^2}{\sqrt{(\eta - \eta_0)(\eta - \eta_2)}} \left( \frac{\eta - \eta_2}{\eta - \eta_3} \right)^{1/2} \right] d\eta \right]
\]

where the vortex intensity and \( \eta \) have been normalised with \( U \alpha(z) \) and \( a(\alpha) \) respectively.

Here all the values of the integrand are the principal values. This shows explicitly the effect of the flap angle on the normal force. The region from \( \eta_3 \) to \( \eta_5 \) is the flap surface.

For flap deflection angles much larger than the angle of attack, it is conceivable that the vortex may be located underneath the wing surface. In this case the dominant component in the cross flow plane is not \( U \alpha \) but the projection of \( U \) on a plane normal to the flap surface. This fact imposes a limit to the flap angles for which this theory is expected to apply.
2. RESULTS AND CONCLUSIONS

3.1 Results

3.1.1 Conformal mapping

The following table shows the transformation coefficients $a_2$, $a_3$, $a_4$ and $a_5$ for varying flap deflection angle $\delta$, with $\epsilon = 0.5$ and $\zeta = 15.95^\circ$.

<table>
<thead>
<tr>
<th>$\delta$(deg.)</th>
<th>$a_2$</th>
<th>$a_3$</th>
<th>$a_4$</th>
<th>$a_5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>40.</td>
<td>-0.6369</td>
<td>-0.5160</td>
<td>0.9785</td>
<td>1.301</td>
</tr>
<tr>
<td>36.</td>
<td>-0.5729</td>
<td>-0.5411</td>
<td>0.9559</td>
<td>1.277</td>
</tr>
<tr>
<td>32.</td>
<td>-0.7051</td>
<td>-0.5722</td>
<td>0.9513</td>
<td>1.251</td>
</tr>
<tr>
<td>28.</td>
<td>-0.7455</td>
<td>-0.5984</td>
<td>0.9355</td>
<td>1.222</td>
</tr>
<tr>
<td>24.</td>
<td>-0.7624</td>
<td>-0.6204</td>
<td>0.9193</td>
<td>1.195</td>
</tr>
<tr>
<td>20.</td>
<td>-0.8161</td>
<td>-0.6562</td>
<td>0.9037</td>
<td>1.165</td>
</tr>
<tr>
<td>16.</td>
<td>-0.8553</td>
<td>-0.6903</td>
<td>0.8884</td>
<td>1.136</td>
</tr>
<tr>
<td>12.</td>
<td>-0.8924</td>
<td>-0.7205</td>
<td>0.8738</td>
<td>1.108</td>
</tr>
<tr>
<td>8.</td>
<td>-0.9236</td>
<td>-0.7471</td>
<td>0.8592</td>
<td>1.083</td>
</tr>
<tr>
<td>4.</td>
<td>-0.9544</td>
<td>-0.7722</td>
<td>0.8132</td>
<td>1.055</td>
</tr>
<tr>
<td>2.</td>
<td>-0.9822</td>
<td>-0.7955</td>
<td>0.7010</td>
<td>1.016</td>
</tr>
</tbody>
</table>

Table I

We see that as $\delta \to 0$, $a_2 \to -1$ and $a_5 \to 1$, which corresponds to the Brown and Michael case.

3.1.2 Aerodynamic forces

To compare with the experimental results of ref. [3] a wing with $\epsilon = 15.95^\circ$ and $\zeta = 0.6$ is chosen. The angle of attack is varied from $0^\circ$ to $20^\circ$.

Fig. 6 shows the contribution to the drag of the main wing. It can be seen that the model overestimates the drag, particularly at higher angles of attack.

Fig. 7 shows the contribution to the drag of the flap. Notice that this component of drag can reach negative values. This is due to the forward force component discussed above.
Fig. 8 and 9 show the increment of lift on the main wing and flap respectively. The abrupt departure from experiments observed at about 12° appears to suggest vortex breakdown in the experiments.

Fig. 10 shows the evolution of the resultant drag. The negative values of drag arise from the conical assumptions, where no trailing vortices and associated drag are considered.

Fig. 11 depicts the total lift change. Again, the sudden departure from the experimental trends may indicate that those measurements were affected by vortex breakdown.

Fig. 12 shows the change in the $C_L/C_D$ ratio. No experimental values were available. The no-deflection case is included for comparison.

Fig. 13 shows the positions of vortices for varying flap deflection. For flap deflections less than 10° the secondary vortex almost disappears. The vortex positions in the physical plane are shown in Fig. 16. We notice that flap deflection causes the secondary vortex to move away from the surface and the leading edge vortex to approach the surface.

Fig. 15 describes the strength of both vortices as function of flap deflection. Flap deflection strengthens the secondary vortex and weakens the leading edge vortex. The strength of leading edge vortex reduces by a half in 40° flap deflection.

Fig. 16 shows the different components of normal force. We notice that flap deflection acts primarily on the leading edge vortex component of the normal force. The secondary vortex has a negligible effect on the total force.

3.2 Conclusions

A conical delta wing with leading edge flaps was analyzed using a simple vortex-producing-sheet model consisting of two pairs of vortices with corresponding vortex sheets. One pair originates at the leading edges and the other at the flap junctions. The leading edge vortex pair is much stronger and responsible for most of the lift and drag changes that take place during flap deflection. In agreement with experimental evidence, it was found that flap deflection causes a decrease of lift and drag and an increase of the lift-to-drag ratio. A closed form expression exhibiting the lift-to-drag ratio increase was derived. The rapid reduction in drag is attributable to a thrust
force component developed by the flaps, while the main reason for the lift reduction is the partial suppression of the vortical flow. There is agreement with experiments in the general trends.

The two main sources of limitation of the theory are the crudeness of the vortex system representation, and the assumption that a cross-flow plane can be suitably defined. The latter aspect becomes relevant in highly warped wings, which in this case amounts to flap deflections that are large and comparable to the angle of attack, where the notion of a plane normal to the wing becomes meaningless.
References


Fig. 1. Overview of the assumed flow field.
Fig. 2. Brown and Michaele vortex model.
Fig. 3. Mapping procedure
(a) Integration contour for logarithmic terms

(b) Integration contour for (68)

Fig. 5. Integration contour for normal force
Fig. 3. Lift increment on main wing

Fig. 9. Lift increment on flap
Fig. 12. Lift-to-drag ratio
Fig. 13. Vortex position for $\alpha = 10^\circ$
Fig. 14. Vortex loci

- direction of loci as \( \delta \) increases

Fig. 15. Vortex strength for \( \alpha = 10^\circ \)
Fig. 16. Normal force components for $\alpha = 10^\circ$
APPENDIX

A 1. Numerical procedure for conformal mapping

A 1.1 Wolfe method

Wolfe's method is just a variation on the secant method for systems of nonlinear equations. This method has the advantage that as the solution becomes closer to the root, the convergence speed is much faster than that of Newton-Raphson methods. For a single equation the secant method is equivalent to solving the following two equations, with two initial guesses

\[
\begin{align*}
    p_1 + p_2 &= 1 \\
    p_1 f(x_n) + p_2 f(x_{n-1}) &= 0
\end{align*}
\]

(A1.1a) (A1.1b)

The solution is updated as

\[
x_{n+1} = p_1 x_{n-1} + p_2 x_n
\]

(A1.1c)

For a system of \( n \) equations a set of \( n+1 \) equations must be solved with \( n+1 \) vectors of initial guesses,

\[
\sum_{j=1}^{n+1} p_j = 1
\]

(A1.2a)

\[
\sum_{j=1}^{n+1} p_j f_i(x_1^j, x_2^j, \ldots, x_n^j) = 0 \quad i = 1, 2, \ldots, n
\]

(A1.2b)

For the solution update we ignore the vector \( x^j \) which gives the maximum residue.

\[
x^j|_{k+1^{th \ step}} = \sum_{j=1}^{n+1} p_j x^j|_{k^{th \ step}},
\]

(A1.3)

where \( x^j = (x_1^j, x_2^j, \ldots, x_n^j) \).
This procedure is repeated until the maximum residue becomes smaller than a given value.

**A 1.2 Integration at singular points**

The integrand of the mapping function is singular at the points $\xi = \eta_2, \eta_3, \eta_5$. To perform the numerical integration, the integrand is expanded to 1st order in Taylor series about those points. For example, at $\xi = \eta_2$,

$$
\int_{\eta_2}^{\eta_2+\varepsilon} \frac{\xi}{\sqrt{(\eta_5 - \xi)(\eta - \eta_2)}} \left(\frac{\eta_5 - \xi}{\eta_3 - \xi}\right)^{\delta/\pi} d\xi
$$

$$
= \int_{0}^{\varepsilon} \frac{\xi + \eta_2}{\sqrt{\eta_5 - \xi - \eta_2}} \left(\frac{\eta_5 - \xi - \eta_2}{\eta_3 - \xi - \eta_2}\right)^{\delta/\pi} d\xi
$$

$$
= \frac{\eta_2}{\sqrt{\eta_5 - \eta_2}} \left(\frac{\eta_5 - \eta_2}{\eta_3 - \eta_2}\right)^{\delta/\pi} \int_{0}^{\varepsilon} \frac{1}{\sqrt{\delta}} d\xi
$$

$$
= \frac{2\eta_2}{\sqrt{\eta_5 - \eta_2}} \left(\frac{\eta_5 - \eta_2}{\eta_3 - \eta_2}\right)^{\delta/\pi} \sqrt{\varepsilon}
$$

where $\varepsilon = \xi - \eta_2$.

Romberg integration is used on the rest of the range.

**A 2. Numerical procedure for the vertex positions**

**A 2.1 Newton- Raphson method for complex functions**

Consider the system of real equations

$$
f(x) = 0 \quad (A.2.1)
$$

The general form of the Newton-Raphson method is

$$
x_{n+1} = x_n - J^{-1}[f(x_n)] f(x_n) \quad (A.2.2)
$$

where $J[f(x_n)]$ is the Jacobian of $f(x)$ at $x_n$.

In the case of a system of complex equations

$$
x = (x_1 + iy_1, x_2 + iy_2, \ldots, x_n + iy_n) \quad (A.2.3)
$$
Applying the scheme to the real and imaginary parts of \( f \) we get

\[
\begin{pmatrix}
  z_1 \\
  y_1 \\
  z_2 \\
  y_2 \\
  \vdots \\
  z_n \\
  y_n \\
\end{pmatrix}_{n+1} = \begin{pmatrix}
  z_1 \\
  y_1 \\
  z_2 \\
  y_2 \\
  \vdots \\
  z_n \\
  y_n \\
\end{pmatrix}_n - J^{-1} \begin{pmatrix}
  \text{Re} f_1 \\
  \text{Im} f_1 \\
  \text{Re} f_2 \\
  \text{Im} f_2 \\
  \vdots \\
  \text{Re} f_n \\
  \text{Im} f_n \\
\end{pmatrix}_n
\]

(A.2.4)

with

\[
J = \begin{pmatrix}
  \text{Re} \left( \frac{\partial f_1}{\partial x_1} \right) & \text{Re} \left( \frac{\partial f_1}{\partial y_1} \right) & \cdots & \text{Re} \left( \frac{\partial f_n}{\partial x_1} \right) & \text{Re} \left( \frac{\partial f_n}{\partial y_n} \right) \\
  \text{Im} \left( \frac{\partial f_1}{\partial x_1} \right) & \text{Im} \left( \frac{\partial f_1}{\partial y_1} \right) & \cdots & \text{Im} \left( \frac{\partial f_n}{\partial x_n} \right) & \text{Im} \left( \frac{\partial f_n}{\partial y_n} \right) \\
  \vdots & \vdots & \ddots & \vdots & \vdots \\
  \text{Re} \left( \frac{\partial f_1}{\partial x_1} \right) & \text{Re} \left( \frac{\partial f_1}{\partial y_1} \right) & \cdots & \text{Re} \left( \frac{\partial f_n}{\partial x_n} \right) & \text{Re} \left( \frac{\partial f_n}{\partial y_n} \right) \\
  \text{Im} \left( \frac{\partial f_1}{\partial x_1} \right) & \text{Im} \left( \frac{\partial f_1}{\partial y_1} \right) & \cdots & \text{Im} \left( \frac{\partial f_n}{\partial x_n} \right) & \text{Im} \left( \frac{\partial f_n}{\partial y_n} \right)
\end{pmatrix}
\]

(A.2.5)

For the present problem we have two equations for the vortex positions, or a 4 \( \times \) 4 matrix. The reason why the Newton - Raphson method is used here is that for Wolfe methods we need one more initial guess than for the Newton - Raphson method, which gives 6 \( \times \) 6 matrix. A more important reason is that the Wolfe method needs very accurate initial guesses for good convergence.

A 2.2 Integration path for the mapping function

In using Newton - Raphson method to solve the equations for the vortex positions we need to calculate the corresponding points between the physical plane and the transformed plane by carrying out the transformation

\[
\sigma = \int_{\eta_2}^{\eta} \frac{\eta}{\sqrt{(\eta - \eta_2)(\eta - \eta_3)}} \frac{(\eta - \eta_5)^{5/2}}{(\eta - \eta_5)} d\eta .
\]

(A2.6)

If we take the integration path as a straight line connecting \( \eta_2 \) with the assumed vortex position, we encounter the following problem: when the vortex position is
very close to the real axis, which is the case when \( \alpha \) is small, the integration path almost passes through the singular points \( \eta_3 \) and \( \eta_4 \). To avoid this possible numerical singularity, the integration path is taken in the following way:

\[
\eta_2 \rightarrow \eta_2 + i \Delta \eta_2 \rightarrow \text{assumed Re } \eta_c + i \Delta \eta_2 \rightarrow \text{assumed } \eta_v \quad (A2.7)
\]

The maximum imaginary value is taken as 2, which is large enough to calculate the vortex positions.

A 2.3 Velocity at the vortex location

(37) and (38) in chapter 2 contain terms of the form

\[
\frac{d}{d\sigma} \left[ \log(\eta - \eta_v) - \log(\sigma - \sigma_v) \right]_{\sigma = \sigma_v},
\]

which can be written as

\[
\frac{d}{d\eta} \left[ \log(\eta - \eta_v) - \log(\sigma - \sigma_v) \right]_{\eta = \eta_v} \frac{d\eta}{d\sigma} \left|_{\sigma = \sigma_v} \right.
\]

In terms of finite differences this quantity can be approximated by

\[
\left[ \left( \log \frac{\eta_v}{\eta_v + \Delta \eta_v} - \log \frac{\sigma_v}{\sigma_v + \Delta \sigma_v} \right) \right] \left( \frac{d\eta}{d\sigma} \right)\left|_{\sigma = \sigma_v} \right.
\]

This is done in the transformed plane to avoid inverting (A 2.6).

A 3. Derivation of the axial velocity component

In 2.4 we derived the axial velocity component under conical flow assumptions

\[
u = U + \frac{1}{x} \left( \phi - y \frac{\partial \phi}{\partial y} - z \frac{\partial \phi}{\partial z} \right)
\]

which is

\[
u = \frac{c}{k} \text{Re} \left( U \eta + \frac{r_1}{2\pi} \log \frac{\eta - \eta_v}{\eta - \eta_v} - \frac{r_2}{2\pi} \log \frac{\eta - \eta_v}{\eta - \eta_v} - \eta \right)(A3.1a)
\]
Here we have logarithmic branch points at \( \eta_{v1}, \eta_{v2}, \eta_{v1}, \eta_{v2} \). To numerically evaluate functions across branch cuts care must be exercised since computers usually give the argument of a complex variable between \(-\pi\) and \(\pi\) while the value of interest here is between 0 and \(2\pi\). As an example we consider two branch points at \( \eta_{v1} \) and \( \eta_{v1} \) as shown in Fig. 5a.

\[
\log(\eta - \eta_{v1}) = \begin{cases} 
\log(\eta - \eta_{v1})_p = \log(\eta - \eta_{v1})_c + i 2\pi & \text{in I,II,III,IV} \\
\log(\eta - \eta_{v1})_p + i 2\pi = \log(\eta - \eta_{v1})_c + i 4\pi & \text{in V}
\end{cases}
\]

(A3.2a)

\[
\log(\eta - \eta_{v1}) = \begin{cases} 
\log(\eta - \eta_{v1})_p = \log(\eta - \eta_{v1})_c + i 2\pi & \text{in I} \\
\log(\eta - \eta_{v1})_p + i 2\pi = \log(\eta - \eta_{v1})_c + i 4\pi & \text{in II,III,IV,V}
\end{cases}
\]

(A3.2b)

where the subscript \( p \) means the principal value and \( c \) means the value calculated from the computer. Actually the values are calculated only in region IV,V and half of III due to the symmetry of the flow.
A Fortran program for the Schwarz-Christoffel transformation

IMPLICIT REAL*8(A-H,O-Z)

DIMENSION SS(10,10), OA(10,10), Z(10), IPVT(10), XF(10), 
NORM(10), X(10,10), XZ(10), B(10), XX(10), VX(10), SX(10)

DATA IMAX, JMAX/4.5 /
DATA PI/3.141592654 /
DATA ERR, EPS/1.D-8, 1.D-2 /
DATA XRTO / .6153846154 /

C700 FORMAT (5X, 'FLAP ANGLE = ', D15.6)

DO 1 J=1, JMAX
1 READ(35, *) (X(I,J), I=1, IMAX)
WRITE(30, 121)
111 READ(5, *) DLTA
DLTA = DLTA * PI/180.

N=0
DO 5 J=1, JMAX
5 DO 3 I=1, IMAX
3 VX(I)=X(I, J)
CALL MATEL (VX, IMAX, JMAX, PI, XRTO, DLTA, ERR, EPS, SX)
DO 4 I=1, IMAX
4 SS(I, J)=SX(I)
SS(JMAX, J)=1.
CONTINUE
DO 6 I=1, IMAX
6 B(I)=O.
B(JMAX)=1
CALL DECOMP (10, JMAX, SS, COND, IPVT, OA, Z, ZNORM)
CALL SOLVE (10, JMAX, SS, B, IPVT, XZ)
DO 7 I=1, IMAX
XX(I)=O.
7 DO 7 J=1, JMAX
7 XX(I)=XX(I)+X(I, J) * XZ(J)
CALL NORM (OA, IMAX, JMAX, FM, FNRM, XLOG, JC)
IF (XLOG.LT. -10.) GO TO 6
IF (N.GT.50) GO TO 9
DO 10 I=1, IMAX
10 X(I, JC)=XX(I)
N=N+1
WRITE (6, 500) N, (X(I, JC), I=1, IMAX)
X(1, JC)=-DABS(X(1, JC))
X(2, JC)=-DABS(X(2, JC))
X(3, JC)=DABS(X(3, JC))
X(4, JC)=DABS(X(4, JC))
IF (X(2, JC).LT.X(3, JC)) GO TO 300
CX=X(2, JC)
X(2, JC)=X(3, JC)
SUBROUTINE MATEL(X, IMAX, JMAX, PI, XRTO, DLTA, ERR, EPS, Y)
IMPLICIT REAL*(A-H,O-Z)
DIMENSION X(10), Y(10)
A2=X(1)
A3=X(2)
A5=X(3)
A6=X(4)
A2P=A2+EPS
A3H=A3-EPS
A3P=A3+EPS
A6T=A6-EPS
CALL ROMBERG(1, A2, A3, A5, A6, A2P, A3H, XINT1)
CALL ROMBERG(2, A2, A3, A5, A6, A3P, O., ERR, DLTA, PI, XINT2)
CALL ROMBERG(2, A2, A3, A5, A6, O., A5, ERR, DLTA, PI, XINT3)
CALL ROMBERG(1, A2, A3, A5, A6, A6T, ERR, DLTA, PI, XINT4)
Y(1)=XRTO+XINT1+2.0*(A5-A2)/(A3-A2)**(DLTA/PI)
Y(2)=1.0-XRTO*
XINT2+*(A5-A3)**(DLTA/PI)*EPS**((1.0-DLTA/PI)/
(1.0-DLTA/PI)*DSQRT((A6-A3)*(A3-A2)))
Y(3)=1.0-XRTO-XINT3
Y(4)=XRTO-XINT4-
RETURN
END
SUBROUTINE NCRM(X, IM, JM, XMAX, XNRM, XLOC, N)
IMPLICIT REAL*8(A-H, O-Z)
DIMENSION X(10,10), XNRM(10)
XMAX=0.
N=1
DO 2 J=1, JM
XNRM(J)=0.
1 DO 1 I=1, IM
XNRM(J)=XNRM(J)*X(I, J)**2
IF(XMAX.GT.XNRM(J)) GO TO 2
XMAX=XNRM(J)
N=J
2 CONTINUE
XLOC=DLOC(XMAX)
RETURN
END

SUBROUTINE DECOMP (NDIM, N, A, COND, IPVT, WORK, CA, Z, ZNORM)
IMPLICIT REAL*S(A-H, O-Z)
DIMENSION A(NDIM, N), WORK(N), CA(NDIM, N), Z(N), IPVT(N)
INTEGER NDIM, I, J, K, KP1, KB, KM, M

DO 1 I=1, N
DO 1 J=1, N
CA(I, J)=A(I, J)
1 CONTINUE
IPVT(N)=1
IF(N.EQ.1) GO TO 80
NM1=N-1
ANORM=0.
DO 10 J=1, N
T=0.
DO 5 I=1, N
T=T+ABS(A(I, J))
5 CONTINUE
IF(T.GT.ANORM) ANORM=T
10 CONTINUE
DO 35 K=1, NM1
KP1=K+1
M=K
DO 15 I=KP1, N
IF(ABS(A(I, K)).GT.ABS(A(M, K))) M=I
15 CONTINUE
IPVT(K)=M
IF(N.ME.K) IPVT(N)=IPVT(N)
T=A(M, K)
A(I, K)=A(K, K)
A(K, K)=T
IF(T.EQ.0.) GO TO 35
DO 20 I=KP1, N
A(I, K)=-A(I, K)/T
20 CONTINUE
20 CONTINUE
DO 30 J=KP1,N
T=A(M,J)
A(M,J)=A(K,J)
A(K,J)=T
IF(T.EQ.0.) GO TO 30
DO 25 I=KP1,N
A(I,J)=A(I,J)+A(I,K)*T
CONTINUE
25 CONTINUE
30 CONTINUE
35 CONTINUE
DO 50 K=1,N
T=0.
IF(K.EQ.1) GO TO 45
K41=K-1.
DO 40 I=1,KM1
T=T*A(I,K)*WORK(I)
CONTINUE
40 EK=1.
IF(T.LT.0.) EK=-1.
IF(A(K,K).EQ.0.) GO TO 90
WORK(K)=-(EK+T)/A(K,K)
CONTINUE
50 DO 60 KB=1, NM1
K=N-KB
T=0.
KP1=K+1.
DO 55 I=1,KP1,N
T=T+A(I,K)*WORK(K)
CONTINUE
55 WORK(K)=T
M=IPVT(K)
IF(M.EQ.K) GO TO 60
T=WORK(M)
WORK(M)=WORK(K)
WORK(K)=T
60 CONTINUE
YNORM=0.
DO 65 I=1,N
YNORM=YNORM+ABS(WORK(I))
CONTINUE
CALL SOLVE(NDIM,N,A,WORK,IPVT,Z)
ZNORM=0.
DO 70 I=1,N
ZNORM=ZNORM+ABS(Z(I))
CONTINUE
COND=ANORM*ZNORM/YNORM
IF(COND.LT.1.) COND=1.
RETURN
80  COND=1.
    IF (A(1,1).NE.0.) RETURN
90  COND=1.E+32
    RETURN
END

SUBROUTINE SOLVE (NDIM,N,A,B,IPVT,X)
    IMPLICIT REAL*8 (A-H,O-Z)
    DIMENSION IPVT(N),A(NDIM,N),B(N),X(N)
    DO 1 I=1,N
         X(I)=B(I)
         CONTINUE
    IF (N.EQ.1) GO TO 50
1    N1=N-1
    DO 20 K=1,N1
         KP1=K+1
9     M=IPVT(K)
         T=X(M)
         X(M)=X(K)
         X(K)=T
         DO 10 I=KP1,N
            X(I)=X(I)+A(I,K)*T
         CONTINUE
10     CONTINUE
    DO 20 K=1,N1
         KM1=N-K3
98     K=KM1+1
         X(K)=X(K)/A(K,K)
         T=T-X(K)
         DO 30 I=1,KM1
            X(I)=X(I)+A(I,K)*T
98     CONTINUE
30     CONTINUE
    X(I)=X(I)/A(I,I)
    RETURN
END

SUBROUTINE ROMERG (N,A2,A3,A5,A6,A,B,ERR,DLTA,PI,RES)
    IMPLICIT REAL*8 (A-H,O-Z)
    DIMENSION Z(10,10)
    I=1
    DEL=B-A
    IF (N-2).NE.0,50
30     Z(1,1)=5*DEL*(FUN1(A,A2,A3,A5,A6,DLTA,PI)
         +FUN1(B,A2,A3,A5,A6,DLTA,PI))
    GO TO 10
40  \( Z(1, 1) = 0.5 \times \text{DEL} \times (\text{FUN2}(A, A2, A3, A5, A6, DLTA, PI) + \text{FUN2}(B, A2, A3, A5, A6, DLTA, PI)) \)
GO TO 10
50  \( Z(1, 1) = 0.5 \times \text{DEL} \times (\text{FUN3}(A, A2, A3, A5, A6, DLTA, PI) + \text{FUN3}(B, A2, A3, A5, A6, DLTA, PI)) \)
GO TO 10
10  \( J = \text{IFIX}(2.0 \times (I - 1)) \)
\( \text{DEL} = \text{DEL} / 2. \)
\( I = I + 1 \)
\( Z(I, 1) = 0.5 \times Z(I - 1, 1) \)
DO 1 \( K = 1, J \)
\( X = A^* (2.0 \times \text{FLOAT}(K) - 1.0) \times \text{DEL} \)
IF (N - 2) \( GOTO 70, 60 \)
60  \( Z(I, 1) = Z(I - 1, 1) + \text{DEL} \times \text{FUN1}(X, A2, A3, A5, A6, DLTA, PI) \)
GO TO 1
70  \( Z(I, 1) = Z(I - 1, 1) + \text{DEL} \times \text{FUN2}(X, A2, A3, A5, A6, DLTA, PI) \)
GO TO 1
80  \( Z(I, 1) = Z(I - 1, 1) + \text{DEL} \times \text{FUN3}(X, A2, A3, A5, A6, DLTA, PI) \)
GO TO 1
1  CONTINUE
DO 2 \( K = 1, I \)
\( Z(I, K) = (4.0 \times (K - 1) \times Z(I, K - 1) - Z(I - 1, K - 1)) / (4.0 \times (K - 1) - 1.0) \)
2  CONTINUE
\( \text{DIFF} = \text{DABS}(Z(I, I)) - Z(I, I - 1) \)
IF (DIFF .LT. ERR) \( GOTO 20 \)
GO TO 10
20  \( \text{RES} = Z(I, I) \)
RETURN
END

FUNCTION FUN1(X, A2, A3, A5, A6, DLTA, PI)
IMPLICIT REAL*8(A-H, O-Z)
FUN1 = X * ((A5-X) / (X3-X)) ** (DLTA/PI) / DSQRT((A6-X) * (X-A2))
RETURN
END

FUNCTION FUN2(X, A2, A3, A5, A6, DLTA, PI)
IMPLICIT REAL*9(A-H, O-Z)
FUN2 = X * ((A5-X) / (X-A3)) ** (DLTA/PI) / DSQRT((A6-X) * (X-A2))
RETURN
END

FUNCTION FUN3(X, A2, A3, A5, A6, DLTA, PI)
IMPLICIT REAL*8(A-H, O-Z)
FUN3 = X * ((X-A5) / (X-A3)) ** (DLTA/PI) / DSQRT((A6-X) * (X-A2))
RETURN
END
A 6. Fortran program for the aerodynamic forces

******************************************************************************

DELTA WING WITH FLAP PROBLEM
WITH ONLY ONE VORTEX FROM LEADING EDGE
USING NEWTON-RAPHSON METHODS
WITH DIFFERENT INTEGRATING PATH

******************************************************************************

IMPLICIT INTEGER*4(I-N)
COMPLEX*16 XI,X2,H
REAL*8 ERR,PI,ALP,AFX,XX1,XX2,DLT,A2,A3,A5,A6,GAM,ALFD
* CL3,CLW,CDW,CLF,CDF,CLT,CDT,CLL,TCL,TCD,RTOR,RTO
DIMENSION Z(10),W(10),GAM(10)
COMMON /DATA1/ IMAX,JMAX
COMMON /DATA2/ XI,ERR,PI
COMMON /DATA3/ ALP,APX
COMMON /DATA4/ X(10)
COMMON /DATA5/ XX1,XX2,DLT
COMMON /DATA6/ A2,A3,A5,A6

DO 100 I=1,10
CALL DATA(ALPD)

CALL NEWTON(Z,W)

CALL VORTEX(W,GAM)

C CALL PRESSURE(W,GAM)

CALL LIFT(GAM,W,CL3)

C CALL LIFT1(GAM,W,CLN,CDN,CLF,CDF,CLT,CDT,CLL)
C * TCL,TCD,RTOR,RTO)
C WRITE(5C,300)ALFD,CL3,CLW,CDW,CLF,CDF,CLT,CDT,CLL
C * TCL,TCD,RTOR,RTO

100 CONTINUE
300 FORMAT(1X,FS.2,1X,D11.4/5X.5(1X,D11.4)/5X.5(1X,D11.4))
200 FORMAT(2X,"DLT",5X,"VORTEX POSITION",9X,"GAM",9X,
* "CLL",9X,"CLV",9X,"CLT")
STOP
END

47
SUBROUTINE DATA(ALPD)

C INPUT OF DATA USED IN THE WHOLE PROGRAM

C IMAX : RANK OF MATRIX 1.e. THE NUMBER OF Unknowns
JMAX : 5 FOR NEWTON-RAPHSON METHODS (I-1.J), (I+1.J), (I,J-1)
       ,(I,J+1), (I.J)

C XI : SQRT(-1)

C ERR : CONVERGENCE OF ROMBERG INTEGRATION
PI : 3.14.....

C ALP : ANGLE OF ATTACK

C APX : HALF OF APEX ANGLE OF MAIN DELTA WING

C X(I) : INITIAL GUESs

C XK : RATIO OF MAIN DELTA WING TO TOTAL SPAN LENGTH (=1)

C DLT : FLAP DEFORMATION ANGLE


C IMPLICIT INTEGER*4(I-N)

C COMPLEX*16 XI.X

C REAL*8 ERR,PI,ALP,APX.XK1.XK2.DLT.A2,A3.A5.A6,ALPD.DLTD

C COMMON /DATA1/ IMAX,JMAX

C COMMON /DATA2/ XI.ERR,PI

C COMMON /DATA3/ ALP,APX

C COMMON /DATA4/ X(10)

C COMMON /DATA5/ XK1.XK2.DLT


C DATA ERR,PI / 1.D-6, 3.141592654 /
C DATA XI / (0.,1.)/
C DATA IMAX,JMAX / 1.5 /
C DATA ALP,APX / .17453278 , .5205121025 /
C DATA XK1.XK2 / .6153846154 , .8846153846 /

C WRITE(6,1)
C READ(5,*) DLT
C READ(5,*) ALPD
D LT= DLT*PI/180.
C WRITE(6,2)
C READ(5,*) A2,A3.A5.A6
C WRITE(34,100) ALP,APX.XK1.XK2.DLT.A2,A3.A5.A6
C WRITE(6,600)
C READ(5,*) (X(I),I=1.IMAX)

1 FORMAT(1X.'DLT')
2 FORMAT(1X.'TRANS. COEFF.' )
100 FORMAT(/5X,'1. GIVEN DATA '/
       * 8X,'ANGLE OF ATTACK = ',D11.4/
       * 8X,'APEX ANGLE  = ',D11.4/
       * 8X,'FLPA RATIO = ',D11.4/
       * 8X,'TAP RATIO  = ',D11.4/
       * 8X,'FLAP ANGLE = ',D11.4/
       * 8X,'TRAN. COEFF. = ',4(2X,D11.4)/

48
SUBROUTINE NEWTON(VZ, VW)

C CALCULATION OF VORTEX POSITION

IMPLICIT INTEGER*4(I-N)
IMPLICIT*16 XI, X, W, WN, Z, ZM, DER, DERM, VZ, VH
REAL*8 ERR, PI, ALP, APX, XK1, XK2, DLT, A2, A3, A5, A6, SS, B, XZ, OA

DIMENSION W(10,10),
* WN(10),Z(10),DER(10),ZM(10,10),DERM(10,10),
* SS(10,10),B(10),
* IPV(10),OA(10,10),XZ(10),
* VZ(10),VW(10)

COMMON /DATA1/ IMAX, JMAX
COMMON /DATA2/ XI, ERR, PI
COMMON /DATA3/ ALP, APX
COMMON /DATA4/ X(10)
COMMON /DATA5/ XK1, XK2, DLT
COMMON /DATA6/ A2, A3, A5, A6

N=0

100 CALL NEWTON(X, W)

DO 4 I=1, IMAX

5 VN(J)=W(I, J)

CALL ZCALC(WN, Z, DER)

DO 6 J=1, JMAX

ZM(I, J)=Z(J)

6 DER(J, J)=DER(J)

4 CONTINUE

CALL MATEL(V, ZM, DERM, SS, B)

CALL NORM(X, XLCC)

IF(N.GT.50) GO TO 11

IF(XLCC.LT.7.) GO TO 11

CALL DECOMP(SS, IPV, OA)

CALL SOLVE(SS, OA, IPV, SS)

CALL NEXPNT(X, XZ)

WRITE(6,112) N, XLCC, (ZM(I, 5), I=1, IMAX), (W(I, 5), I=1, IMAX)

N=N+1

GO TO 100

11 WRITE(6,113) N, XLCC, (ZM(I, 5), I=1, IMAX), (W(I, 5), I=1, IMAX)

DO 12 I=1, IMAX

VZ(I)=ZM(I, 5)

12 VW(I)=V(I, 5)

WRITE(34,200) VZ(1), VW(1)
SUBROUTINE NEWPNT(XN, YN)
IMPLICIT INTEGER*4(I-N)
COMPLEX*16 XN, YN, XI
REAL*8 ERR, PI, XRE, XIM
DIMENSION XN(10), YN(10,10)
COMMON /DATA1/ IMAX, JMAX
COMMON /DATA2/ XI, ERR, PI
DO 1 I=1, IMAX
   XRE=REAL(XN(I))/100.
   XIM=IMAG(XN(I))/100.
   YN(I,1)=XN(I)-XRE
   YN(I,2)=XN(I)*XRE
   YN(I,3)=XN(I)-XIM
   YN(I,4)=XN(I)*XIM
1 RETURN
END

SUBROUTINE MATLAB(X, Z, DER, S, BB)
IMPLICIT INTEGER*4(I-N)
COMPLEX*16 X, Z, DER, XI, A, B, O, FUN2
REAL*8 S, BB, ERR, PI, ALP, APX, XK1, XK2, DLT, A2, A3, A5, A6
DIMENSION X(10,10), Z(10,10), DER(10,10), S(10,10), BB(10)
COMMON /DATA2/ XI, ERR, PI
COMMON /DATA3/ ALP, APX
COMMON /DATA4/ XK1, XK2, DLT
COMMON /DATA6/ A2, A3, A5, A6
A=(FUN2(X(1,2),Z(1,2),DER(1,2))-
   FUN2(X(1,1),Z(1,1),DER(1,1)))/
   (X(1,2)-X(1,1))
B=(FUN2(X(1,4),Z(1,4),DER(1,4))-
   FUN2(X(1,3),Z(1,3),DER(1,3)))/
   ((X(1,4)-X(1,3))*(-XI))
O=FUN2(X(1,5),Z(1,5),DER(1,5))
S(1, 1) = DREAL(A)
S(1, 2) = DREAL(B)
S(2, 1) = DIMAG(A)
S(2, 2) = DIMAG(B)
EE(1) = - DREAL(O)
BB(2) = - DIMAG(O)
RETURN
END

SUBROUTINE NORM(X, XLOG)
IMPLICIT INTEGER*4(I-N)
IMPLICIT REAL*8 (A-H,O-Z)
DIMENSION X(10)
COMMON /DATA1/ IMAX, JMAX
IM = IMAX*2
XNORM = 0.
DO 2 J = 1, IM
XNORM = XNORM + (DABS(X(J)))**2
XLOG = DLOG10(XNORM)
RETURN
END

SUBROUTINE NEXPMT(X, Y)
IMPLICIT INTEGER*4(I-N)
COMPLEX*16 X, XI
REAL*8 Y, ERR, PI
DIMENSION X(10), Y(10)
COMMON /DATA2/ XI, ERR, PI
X(1) = X(1) + Y(1) * XI**Y(2)
X(1) = DREAL(X(1)) * XI**DABS(DIMAG(X(1))))
RETURN
END

SUBROUTINE BOXERG(IS, A, B, RES)
IMPLICIT INTEGER*4(I-N)
COMPLEX*16 A, B, RES, XI, Z, DEL, FUN1, X, DIF, FUNS
REAL*8 ERR, PI
DIMENSION Z(10, 10)
COMMON /DATA2/ XI, ERR, PI
I = 1
DEL = B - A
IF(IS.EQ.1) GO TO 111
Z(1, 1) = 5*DEL*(FUNS(A) * FUNS(B))
GO TO 10
111 Z(1, 1) = 5*DEL*(FUN1(A) * FUN1(B))
10 J = IFIX(2.***(I-1))
DEl=DEl/2.
I=I+1
Z(I,1)=5*Z(I-1,1)
DO 1 K=1,J
  X=A+(2.*FLOAT(K)-1.)*DEl
  IF(IS.EQ.1) GO TO 3
  Z(I,1)=Z(I,1)+DEl*FUN5(X)
  GO TO 1
3  Z(I,1)=Z(I,1)+DEl*FUN1(X)
1 CONTINUE
DO 2 K=2,J
  Z(I,K)=(4.*K**2-Z(I.K-1)-Z(I-1.K-1))/(4.*K**2-1.)
2 CONTINUE
DIFF=Z(I,J)-Z(I,J-1)
IF(DABS(Diff).LT.ERR) GO TO 20
GO TO 10
20 RES=Z(I,1)
RETURN
END

FUNCTION FUN1(X)
IMPLICIT INTEGER*4(I-N)
COMPLEX*16 XI.XI.FUN1
COMMON /DATA1/ XI.ERR.PI
COMMON /DATA2/ XK1.XK2.DLT
FUN1=X**((X-A5)**(DLT/PI))*CDSQRT(X-A2)*CDSQRT(X-A6)
RETURN
END

SUBROUTINE ZCALC(V,Z,DER)
IMPLICIT INTEGER*4(I-N)
COMPLEX*16 V,Z,DER.XI,EPS,SING,B,EB,EM,A2P,RP,RPB,
*  REM,P2,P3P,P3M,XINT,XINT1,XINT2,XINT1P,XINT2P,
*  XINT1M,XINT2M,ZM,PI,C1,C2,C3
DIMENSION V(10),Z(10),DER(10)
COMMON /DATA1/ IMAX,JMAX
COMMON /DATA2/ XI.ERR.PI
COMMON /DATA3/ XK1,XK2.DLT
COMMON /DATA4/ A2,A3,A5.A6
EPS=0.01*XI
C1=A2-A5
C2=A2-A6
C3=A2-A3
SING=2.*A2*C1***(DLT/PI)**CDSQRT(EPS)/
* (CDSQRT(C2)**C3***(DLT/PI))
DO 1 I=1,JMAX
   B=V(I)
   DB=B/100.
   BP=B*DB
   BN=B-DB
   A2P=A2*EPS
   RB=REAL(B)
   RBP=REAL(BP)
   RBN=REAL(BN)
   P2=A2+2.*XI
   P3=RB+2.*XI
   P3P=RBP+2.*XI
   P3M=RBN+2.*XI
   CALL RCMERG(1,A2P,P2,XINT)
   CALL RCMERG(1,P2,P3,XINT1)
   CALL RCMERG(1,P3,B,XINT2)
   CALL RCMERG(1,P2,P3P,XINT1P)
   CALL RCMERG(1,P3P,BP,XINT2P)
   CALL RCMERG(1,P2,P3M,XINTM)
   CALL RCMERG(1,P3M,BM,XINT2M)
   Z2=XINT+XINT1+XINT2+XINTM+XINT2M+SING
   Z(I)=XINT+XINT1+XINT2+SING
   ZP=XINT+XINT1P+XINT2P+SING
   DER(I)=((CDSLOC((Z(I)-Z2)/(ZP-Z(I))))/2.**DIS)**
* (CDSQRT(B-A6)**CDSQRT(B-A2)**(B-A3)**(DLT/PI))/
* (B**(B-A5)**(DLT/PI))
1 CONTINUE
RETURN
END

FUNCTION FUN2(X1,Z,DER)
IMPLICIT INTEGER*4(I-N)
COMPLEX*16 X1,Z,DER,XI,FUN2
REAL*8 ERR,PI,ALP,APX,XX1,XX2,DLT,A2,A3,A5,A6
COMMON /DATA3/ X1,ERR,PI
COMMON /DATA3/ ALP,APX
COMMON /DATA5/ XX1,XX2,DLT
COMMON /DATA6/ A2,A3,A5,A6
FUN2=APX*(2.*DCONJG(Z)*XI*(XX1*1.0-XK1)*CDEXP(-XI*DLT))/
* (XX1*ALP)
* -(1.**((1./X1-1.)/DCONJG(X1))*(X1-DCONJG(X1)))/
* (CDSQRT(X1-A6)**CDSQRT(X1-A2)**(X1-A3)**(DLT/PI))/
* (X1**(X1-A5)**(DLT/PI))
* -DER/(1./X1-1.)/DCONJG(X1)
RETURN
END
SUBROUTINE VORTEX (H, GAM)
IMPLICIT INTEGER*4 (I-N)
COMPLEX*16 W, XI
REAL*8 ERR, PI, GAM, ALP, APX
DIMENSION W(10), GAM(10)
COMMON /DATA2/ XI, ERR, PI
COMMON /DATA3/ ALP, APX
GAM(1) = 2. * PI * ALP / REAL (XI * (1.0 / W(1) - 1.0 / DCCNJG(W(1))))
WRITE (34, 100) GAM(1)
FORMAT (/5X, '3. VORTEX STRENGTH'/
* 8X, 'VORTEX STRENGTH = ', D11.4/) RETURN
END

SUBROUTINE LIFT1 (GAM, VW, CLW, CDW, CLF, CDF, CLT, CDT, CLL
* TCL, TCD, RTO, RTO)
IMPLICIT INTEGER*4 (I-N)
COMPLEX*16 VW, X1, X2, X3, X4, ES, A2P, A2M, A3P, A6M,
* ALM, ASP, SING5, SING6, SING7, SING8,
* CI, C2, C3, C4, C5, C6, C7, C8, C9, ZZ, Z,
* XI, SING1, SING2, SING3, SING4, Z1, Z2, Z3, Z4,
* CI0, CI1, CI2, CI3, CI4, CI5, CI6, X5, X6, Z5, Z6
REAL*8 GAM, A2, A3, A5, A6, U, V, H, CD, ERR, PI, X2, X1, DLT,
* ALP, APX, CL1, CL2, CL3, CL4, CLW, CDW, CLF, CDF, CLT, CDT,
* CDF, CCL, RTOR, RTO, TCL, TCD, CLS, CLG
DIMENSION VW(10), GAM(10), Z(700), CP(700)
COMMON /DATA2/ XI, ERR, PI
COMMON /DATA3/ ALP, APX
COMMON /DATA4/ X1, X2, DLT
COMMON /DATA5/ A2, A3, A5, A6
ES = (A6 - A2) / 200.
A2P = A2 + ES
A3M = A3 - ES
A3P = A3 + ES
A6M = A6 - ES
A5M = A5 - ES
A5P = A5 + ES
C1 = A2 - A5
C2 = A2 - A6
C3 = A2 - A3
C4 = A3 - A5
C5 = A3 - A6
C6 = A3 - A2
C7 = A6 - A5
C8 = A6 - A2
C9 = A6 - A3
C10 = A6
C11 = -A2
C12 = -A5
C13=-A3
C14=A5-A6
C15=A5-A2
C16=A5-A3
SING1=2.*A2*CDSQRT(ES)*C13*(DLT/PI)/
* (CDSQRT(C2)*C3*(DLT/PI))
SING2=-PI*A3*C4*(DLT/PI)*(-ES)**((PI-DLT)/PI)/
* (PI-DLT)*CDSQRT(C5)*CDSQRT(C6))
SING3=PI*A3*C4*(DLT/PI)*ES**((PI-DLT)/PI)/
* (PI-DLT)*CDSQRT(C5)*CDSQRT(C6))
SING4=-2.*A6*C7**(DLT/PI)**CDSQRT(-ES)/
* (CDSQRT(C8)*C9**(DLT/PI))
SING5=-C12**(DLT/PI)*(-ES)**2/
* (CDSQRT(C10)*CDSQRT(C11)*C13**(DLT/PI)**2)
SING6=C12**(DLT/PI)*ES**2/
* (CDSQRT(C10)*CDSQRT(C11)*C13**(DLT/PI)**2)
SING7=-A5**1*ES**2(PI-DLT)/PI)/
* (CDSQRT(C14)*CDSQRT(C15)*C16**(DLT/PI)*PI-DLT))
SING8=A5**1*ES**2((PI-DLT)/PI)/
* (CDSQRT(C14)*CDSQRT(C15)*C16**(DLT/PI)**(PI-DLT))
DO 10 1=1,101
10 XI=A2P+(A3M-A2P)*FLOAT(I-1)/100.
CALL RMBRG(1,A2P,X1,Z1)
Z(I)=Z1+SING1
ZZ=Z(I)
CALL VEL(VW,GAM,X1,ZZ,U,V,W)
DO 20 I=102,202
20 CP(I)=1.+ALP**2-U**2-V**2-W**2
DO 30 I=203,303
30 XI=-357-A3P)*FLOAT(I-102)/100.
CALL RMBRG(1,A3P,X2,ZZ)
Z(I)=XI**XX1+SING3+2
ZZ=Z(I)
CALL VEL(VW,GAM,X2,ZZ,U,V,W)
DO 55 I=204,404
55 XI=-357-ES+(.357-ES)*FLOAT(I-203)/100.
CALL RMBRG(1,ES,X3,Z3)
Z(I)=XI**XX1+SING3+3
ZZ=Z(I)
CALL VEL(VW,GAM,X3,ZZ,U,V,W)
DO 80 I=304,604
80 XI=ES+(.475-ES)*FLOAT(I-304)/100.
CALL RMBRG(1,ES,X4,Z4)
Z(I)=XI**XX1+(1.-XX1)*CDSQRT(XI**DLY))**SING6+Z4
ZZ=Z(I)
CALL VEL(VW,GAM,X4,ZZ,U,V,W)
DO 100 I=405,905
100 XI=.475+(ASH-.475)*FLOAT(I-405)/100.
CALL RMBRG(1,ES,X5,Z5)
Z(I)=XI°·(XK1+(1.-XK1)°·CDEXP(XI°·DLT))·SING6+Z5
Z2=Z(I)
CALL VEL(VW,GAM,X5,ZZ,U,V,H)
DO 50 I=SO5,605
X6=ASP·(A6M-ASP)·FLOAT(I-505)/100.
CALL ROMRG(1,ASP,X6,Z6)
Z(I)=XI°·XK1°·SING8·Z6
Z2=Z(I)
CALL VEL(VW,GAM,X6,ZZ,U,V,H)
50 CP(I)=1.·ALP°·2-U°·2-V°·2-W°·2
CALL SIMP(Z,CP,CL1,CL2,CL3,CL4,CL5,CL6)
CLN=(-CL6+CL1)
CDP=CLN·ALP
CLF=(-CL5+CL2)·DCOS(DLT)·(1.·ALP·APX·DTAN(DLT))
CDF=(-CL5+CL2)·DCOS(DLT)·(ALP·APX·DTAN(DLT))
CLT=(-CL4+CL3)·DCOS(DLT)·(1.·ALP·APX·DTAN(DLT))
CDT=(-CL4+CL3)·DCOS(DLT)·(ALP·APX·DTAN(DLT))
TCL=CLW+CLF+CLT
TCD=CDW+CDF+CDT
RTOR=1./ALP
C  WRITE(34,100)
C DO 46 I=1,406.5
C46 WRITE(34,200) Z(I),CP(I)
C WRITE(34,300) CL1,CDM,CLF,CLG,CLT,CLT,KLC,TCD,RTOR
100 FORMAT(2X,'Z',21X,'CP',10X)
200 FORMAT(10X,'3D11.4',5X,'3D11.4')
300 FORMAT(//7X,'6. LIFT BY INTEGRATION OF PRESSURE')//
*  10X,'LIFT OF MAIN WING' = 'D11.4/
* 10X,'DRAG OF MAIN WING' = 'D11.4/
* 10X,'LIFT OF ELAP' = 'D11.4/
* 10X,'DRAG OF ELAP' = 'D11.4/
* 10X,'LIFT OF TAP' = 'D11.4/
* 10X,'DRAG OF TAP' = 'D11.4/
* 10X,'CHECK FOR LIFT' = 'D11.4/
* 10X,'TOTAL LIFT' = 'D11.4/
* 10X,'TOTAL DRAG' = 'D11.4/
* 10X,'LIFT DRAG RATIO' = 'D11.4/
* 10X,'IDEAL RATIO' = 'D11.4)
RETURN
END

SUBROUTINE SIMP(Z,CP,CL1,CL2,CL3,CL4,CL5,CL6)
IMPLICIT INTEGER*4(I-N)
COMPLEX*16 Z
REAL*8 CL1,CL2,CL3,CL4,CP,CL5,CL6
DIMENSION Z(700),CP(700)
CL1=0.
CL2=0.
CL3=0.
CL4=0.
CL5=0.
CL6=0.
DO 10 I=1,100
10 CL1=CL1+CDABS(Z(I+1)-Z(I))* (CP(I+1) + CP(I))/2.
DO 20 I=102,201
20 CL2=CL2+CDABS(Z(I+1)-Z(I))* (CP(I+1) + CP(I))/2.
DO 30 I=203,302
30 CL3=CL3+CDABS(Z(I+1)-Z(I))* (CP(I+1) + CP(I))/2.
DO 40 I=304,403
40 CL4=CL4+CDABS(Z(I+1)-Z(I))* (CP(I+1) + CP(I))/2.
DO 50 I=405,504
50 CL5=CL5+CDABS(Z(I+1)-Z(I))* (CP(I+1) + CP(I))/2.
DO 60 I=506,605
60 CL6=CL6+CDABS(Z(I+1)-Z(I))* (CP(I+1) + CP(I))/2.
RETURN
END

SUBROUTINE PRESSURE(VW,GAM)
IMPLICIT INTEGER*4(I-N)
COMPLEX*16 VW,X,ES,A2P,A3M,A3P,A6M,C1,C2,C3,C4,CS,C6,
*                              C7,C8,C9,X1,SING1,SING2,SING3,SING4,Z1,Z2,Z
REAL*8 GAM,A2,A3,A5,A6,U,V,W,CP,ERR,PI,XK1,XK2,DLT
*                              ,ALP,APX
DIMENSION VW(10),GAM(10)
COMMON /DATA2/ XI,ERR,PI
COMMON /DATA3/ ALP,APX
COMMON /DATA5/ XK1,XK2,DLT
COMMON /DATA6/ A2,A3,A5,A6
WRITE(34,100)
DO 200 I=1,41
X=A2*(A6-A2)*FLOAT(I-1)/40.
       IF (DREAL(X).EQ.A2.OR.DREAL(X).EQ.A3.OR.
*                              DREAL(X).EQ.0.OR.DREAL(X).EQ.A5.OR.
*                              DREAL(X).EQ.A6) GO TO 200
ES=(A6-A2)/200.
A2P=A2+ES
A3M=A3-ES
A3P=A3+ES
A6M=A6-ES
C1=A2-A5
C2=A2-A6
C3=A2-A3
C4=A3-A5

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C5=A3-A6
C6=A3-A2
C7=A6-A5
C8=A6-A2
C9=A6-A3
SING1=2.*A2*CDSQRT(ES) *C1***(DLT/PI)/
  (CDSQRT(C2) *C3)***(DLT/PI))
SING2=PI*A3*C4***(DLT/PI) *(-ES)***(PI-DLT)/PI)/
  ((PI-DLT) *CDSQRT(C5) *CDSQRT(C6))
SING3=PI*A3*C4***(DLT/PI) *ES***(PI-DLT)/PI)/
  ((PI-DLT) *CDSQRT(C5) *CDSQRT(C6))
SING4=-2.*A6*C7***(DLT/PI) *CDSQRT(-ES)/
  (CDSQRT(C8) *C9***(DLT/PI))
IF (DREAL (X) .LE. DREAL (A2P)) GO TO 200
IF (DREAL (X) .LT. DREAL (A3P)) GO TO 1
IF (DREAL (X) .LE. DREAL (A3P)) GO TO 200
IF (DREAL (X) .LT. DREAL (A6P)) GO TO 2
GO TO 200
1 CALL ROMB:S,G(1,A2P,X,21)
Z=Z1+SING1
GO TO 300
2 CALL ROMB:S,G(1,A3P,X,22)
Z=X1*XK1+SING3-Z2
300 CALL VEL(VW,CAM,X,Z,U,V,W)
CP=1.+ALP**2-U**2-V**2-W**2
C WRITE(6,*) Z,CP.U,V,W
CONTINUE
200 FORMAT(*X,'.4. PRESSURE DISTRIBUTION '/
  1X,'Z',1X,'CP',1X,'U',1X,'V',1X,'W',1X)
600 FORMAT(*X,2D11.4,4(2X,2D11.4))
RETURN
END
SUBCUTINE VEL(VW,CAM,X,Z,U,V,W)
IMPLICIT INTEGER*4(I-N)
COMPLEX*16 VW,A,FUN4,X,Z,XI
REAL*8 CAM,U,V,W,ERR,PI,APX,XK1,XK2,DLT
DIMENSION VW(10),CAM(10)
COMMON /DATA2/ XI,ERR,PI
COMMON /DATA3/ ALP,APX
COMMON /DATA5/ XK1,XK2,DLT
A=FUN4(VW(1),CAM(1),X)
V=DREAL(A)
W=DREAL(XI*A)
IF (DREAL (X) .GT. O.) GO TO 1
U=1.+DREAL(ALP*X*XI*CAM(1)) * (CDLOG(X-VW(1)) -
  CDLOG(X-DCONJG(VW(1))) +2.*PI*XI)/(2.*PI)
   -DREAL(X-V-DIMAG(2)*W)*APX/XK1
GO TO 2
1 U=1.+DREAL(ALP*X*XI*CAM(1)) * (CDLOG(X-VW(1)) -
FUNCTION FUN4(V, G, X)
IMPLICIT INTEGER*4(I-N)
COMPLEX*16 V, X, FUN4, XI
REAL*8 G, ERR, PI, ALP, APX, XK1, XK2, DLT, A2, A3, A5, A6
COMMON /DATA2/ XI, ERR, PI
COMMON /DATA3/ ALP, APX
COMMON /DATA5/ XK1, XK2, DLT
COMMON /DATA6/ A2, A3, A5, A6
FUN4=(ALP*XI*G*(1./X-V)-1./CDCONJG(V))/(2.*PI))
* CDSQRT(X-A6)*CDSQRT(X-A2)*(X-A3)**(DLT/PI)/
* (X*(X-A5)**(DLT/PI))
RETURN
END

SUBROUTINE LIFT(GAM, U, CL)
IMPLICIT INTEGER*4(I-N)
COMPLEX*16 W, XI, A2P, A3M, A6H, ES, C1, C2, C3, C4, CS, C5, C7,
* C8, C9, C10, XINT1, XINT2, XINT3, SING1, SING2,
* SING3, SING4, CLL
REAL*8 GAM, CL1, CL2, CL, ALP, APX, XK1, XK2, DLT, A2, A3, A5, A6,
* ERR, PI
DIMENSION GAM(10), W(10)
COMMON /DATA2/ XI, ERR, PI
COMMON /DATA3/ ALP, APX
COMMON /DATA5/ XK1, XK2, DLT
COMMON /DATA6/ A2, A3, A5, A6
ES=(A6-A2)/1000.
C1=A2-A5
C2=A2-A6
C3=A2-A3
C4=A3-A5
C5=A3-A6
C6=A3-A2
C7=A6-A5
C8=A6-A2
C9=A6-A3
C10=A5
A2P=A2+ES
A3M=A3-ES
A5M=A6-ES
CALL ROMERG(2, A2P, A3M, XINT1)
CALL ROMERG(2, A3P, C10, XINT2)
CALL ROMERG(2, C10, A5M, XINT3)
SING1=2.0*A2*2*C1***(DLT/PI)**CDSQRT(ES)/
* (CDSQRT(C2)*C3***(DLT/PI))
SING2=-PI*A3**2*C4**(DLT/PI)**(ES)**((PI-DLT)/PI)/
* ((PI-DLT)*CDSQRT(C5)*CDSQRT(C6))
SING3=-2.0*A6**2*C7**(DLT/PI)*CDSQRT((-ES))/
* (CDSQRT(C8)*C9***(DLT/PI))
SING4=PI*A3**2*C4**(DLT/PI)*ES**((PI-DLT)/PI)/
* ((PI-DLT)*CDSQRT(C5)*CDSQRT(C6))
CLL=XINT1+SING1+SING2*XINT3+SING3
* (XINT2*SING4)*DCOS(2.0*DLT)
CL2=-4.0*APX*(DIMAG(CLL)*AX**/(XX1*(1.-XX1)*DCOS(DLT))
CL=CL1+CL2
WRITE(6, *) CL1, CL2, CL
WRITE(100,100) CL1, CL2, CL
100 FORMAT(/SX,'5. Lift coefficients: '/
* 6X,' Lift from vortex = ',DL1.6/
* 6X,' Lift from attached flow = ',DL1.4/
* 6X,' Total lift = ',DL1.6/
RETURN
END

FUNCTION FUNS(X)
IMPLICIT INTEGER*4(I-N)
COMPLEX*16 FUN5, X, XI
REAL*3 ERR, XII, XX1, XX2, DLT, A2, A3, A5, A6
COMMON /DATA2/ XI, ERR, PI
COMMON /DATA5/ XX1, XX2, DLT
COMMON /DATA6/ A2, A3, A5, A6
IF(DREAL(X) .EQ.0 .OR. DREAL(X) .EQ.A5) GO TO 1
FUNS=X**2*(X-A5)**(DLT/PI)/
* (CDSQRT(X-A6)**CDSQRT(X-A2)**(X-A3)**(DLT/PI))
GO TO 2
1 FUNS=0.
2 RETURN
END
SUBROUTINE DECOMP (A, IPVT, OA)
IMPLICIT INTEGER*4 (I-N)
IMPLICIT REAL*8 (A-H, O-Z)
DIMENSION A(10, 10), NORM(10), OA(10, 10), Z(10), IPVT(10)
COMMON /DATA1/ IMAX, JMAX
N=2*IMAX
DO 1 I=1,N
    DO 1 J=1,N
        OA(I,J)=A(I,J)
        CONTINUE
    IPVT(N)=1
    IF (N.NE.1) GO TO 10
    NNL=N-1
    ANORM=1
    DO 10 J=1,N
        T=0.
        DO 5 I=1,N
            T=T+DABS(A(I,J))
            CONTINUE
        IF (T.GT.ANORM) ANORM=T
        CONTINUE
    DO 35 K=1,NNL
        KP1=K+1
        N=K
        DO 15 I=KP1,N
            IF (DABS(A(I,K)).GT.DABS(A(M,K))) M=I
            CONTINUE
        IF (M.NE.K) IPVT(N)=-IPVT(N)
        T=A(M,K)
        A(M,K)=A(K,K)
        A(K,K)=T
        IF (T.EQ.0.) GO TO 35
        DO 20 I=KP1,N
            A(I,K)=-A(I,K)/T
            CONTINUE
        DO 30 J=KP1,N
            T=A(M,J)
            A(M,J)=A(K,J)
            A(K,J)=T
            IF (T.EQ.0.) GO TO 30
            DO 25 I=KP1,N
                A(I,J)=A(I,J)+A(I,K)*T
                CONTINUE
            CONTINUE
        DO 35 K=1,N
            T=0.
        IF (K.EQ.1) GO TO 45
        KNL=K-1
        CONTINUE
    1 CONTINUE
    5 CONTINUE
    10 CONTINUE
    15 CONTINUE
    20 CONTINUE
    25 CONTINUE
    30 CONTINUE
    35 CONTINUE
    45 CONTINUE
```
DO 40 I=1,KM1
   T=T+A(I,K)*WORK(I)
40   CONTINUE

45   EK=1.
   IF (T.LT.0.) EK=-1.
   IF (A(K,K).EQ.0.) GO TO 90
   WORK(K)=-(EK*T)/A(K,K)
50   CONTINUE
DO 60 K3=1,NM1
   K=N-K3
   T=0.
   KP1=K+1
   DO 55 I=KP1,N
      T=T+A(I,K)*WORK(K)
55   CONTINUE
   WORK(K)=T
   M=IPVT(K)
   IF (M.EQ.K) GO TO 60
   T=WORK(M)
   WORK(K)=WORK(K)
   WORK(K)=T
60   CONTINUE
   YNORM=0.
   DO 65 I=1,N
      YNORM=YNORM+DABS(WORK(I))
65   CONTINUE
   CALL SOLVE(A,WORK,IPVT,Z)
   ZNORM=0.
   DO 70 I=1,N
      ZNORM=ZNORM+DABS(Z(I))
70   CONTINUE
   COND=ANORM*ZNORM/YNORM
   IF (COND.LT.1.) COND=1.
   RETURN
80   COND=1.
   IF (A(1,1).NE.0.) RETURN
90   COND=1.E+32
   RETURN
END

SUBROUTINE SOLVE(A,B,IPVT,X)
IMPLICIT INTEGER*4(I-N)
IMPLICIT REAL*8(A-H,O-Z)
DIMENSION IPVT(10),A(10,10),B(10),X(10)
COMMON /DATA1/ IMAX,JMAX
N=2*IMAX
DO 1 I=1,N
   X(I)=B(I)
1 CONTINUE
```
IEN N.EQ. 1  GO TO 50
N0=10-1 DO 20 K=1, 10
M=1+1(K)
T=X(T(K)) X(K)=T
DO 10 I=2, P1, N
X(I)=X(I)+A(1,1).T
10 CONTINUE
DO 20 I=2, P1, N
X(1)=X(1)+A(1,1).T
20 CONTINUE
DO 30 J=1, 10
X(I)=X(I)/A(I,1)
30 CONTINUE
END
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