EVALUATION OF A QUARTZ BOURDON PRESSURE GAGE FOR WIND TUNNEL MACH NUMBER CONTROL SYSTEM APPLICATION

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EVALUATION OF A QUARTZ BOURDON PRESSURE GAGE FOR WIND TUNNEL
MACH NUMBER CONTROL SYSTEM APPLICATION

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SUMMARY

A theoretical and experimental study was undertaken to determine the feasibility of using the National Transonic Facility's high accuracy Mach number measurement system as part of a closed-loop Mach number control system. The theoretical and experimental procedures described are applicable to the engineering design of pressure control systems. The results show that the dynamic response characteristics of the NTF Mach number gage (a Ruska DDR-6000 quartz absolute pressure gage) coupled to a typical length of pressure tubing were only marginally acceptable within a limited range of the facility's total pressure envelope and could not be used in the Mach number control system.

INTRODUCTION

The automatic control of the operating Mach number in a wind tunnel is normally achieved using moderately accurate, high response pressure instrumentation and then measuring that Mach number using high accuracy, steady-state pressure instrumentation. For the National Transonic Facility (NTF), there is a requirement to incorporate the high accuracy pressure measuring system (ref. 1) into the Mach number control system to achieve Mach number control to within ±0.002 over the 0.2 to 1.2 M range. The NTF Mach number system is a complex system utilizing nine Ruska DDR-6000 quartz gages having ranges from 0.1 to 1.0 MPa (15 to 150 psia). The transducing mechanism in this pressure gage consists of a hollow heli-quartz Bourdon tube, two attached coils positioned in a magnetic field, and a curved mirror mounted to the tube. As an input pressure is introduced into the Bourdon tube, the mirror reflects a light beam to a pair of balanced photocells thus generating an off-null signal. The resulting electrical current from the photocells is amplified and fed through the force balancing coils thus creating an electromagnetic torque equal and opposite to that caused by the input pressure which forces the Bourdon tube back to the null position. The current also flows through a precision resistor to produce an analog voltage directly proportional to the input pressure. This pressure gage has an internal volume of 18 cm³ and has a sintered filter in its inlet line to eliminate contamination in the gage. This pressure gage has a nonrepeatability of 0.002% FS, a nonlinearity of 0.001% FS, and nondetectable hysteresis. The dynamic characteristics of this gage along with a suitable length of pressure tubing were experimentally measured to determine its suitability for use in a closed-loop control system.
SYMBOLS

Values are given in SI units and also in U.S. units. Measurements and calculations were made in U.S. units.

\( f \) \hspace{1cm} \text{frequency of sinusoidal fluctuations, Hz}
\( f_b \) \hspace{1cm} \text{3 db frequency in a first order term, Hz}
\( f_c \) \hspace{1cm} \text{gain cross-over frequency, Hz}
\( f_g \) \hspace{1cm} \text{frequency at which the phase angle is -180°, Hz}
\( f_n \) \hspace{1cm} \text{undamped natural frequency in a quadratic term, Hz}
\( G(jf) \) \hspace{1cm} \text{approximate transfer function in the frequency domain}
\( G(S) \) \hspace{1cm} \text{approximate LaPlace transfer function}
\( j \) \hspace{1cm} \sqrt{-1}
\( K_g \) \hspace{1cm} \text{gain margin, db}
\( S \) \hspace{1cm} \text{LaPlace transform variable, sec}^{-1}
\( T \) \hspace{1cm} \text{time constant in a first order term, sec}
\( \zeta \) \hspace{1cm} \text{dimensionless damping coefficient in a quadratic term}
\( \theta_c \) \hspace{1cm} \text{phase lag at the gain crossover frequency, deg}
\( \theta_p \) \hspace{1cm} \text{phase margin, deg}
\( \tau \) \hspace{1cm} \text{transportation lag time, sec}
\( \omega_n \) \hspace{1cm} \text{undamped radian frequency} = 2 \pi f_n \text{ rad/sec}

Abbreviations:

i.d. \hspace{1cm} \text{inside diameter, cm}

d.a. \hspace{1cm} \text{double amplitude}
PROCEDURE

Determination of Sinusoidal Pressure Response

The sinusoidal response was measured for the system consisting of a 30 meter (approximately 100 feet) length of 0.48 cm (0.19 in.) i.d. steel tubing (approximating that used in NTF) and the 0.34 MPa absolute Ruska DDR-6000 pressure gage. This combination is alternatively referred to in this paper as the "tubulation-pressure gage system." Furthermore, the Ruska DDR-6000 pressure gage itself is referred to as the "pressure gage." Measurements were made at static pressures of 0.0517 MPa absolute (7.5 psia), 0.101 MPa absolute (14.7 psia), 0.172 MPa absolute (25 psia), and 0.207 MPa absolute (30 psia). The range of frequencies was from 0.1 to 6 Hz.

The block diagram of the apparatus used is shown in figure 1. The desired static pressure with a small superimposed sinusoidal pressure was applied to the inlet of the tubing. The electronic manometer system converted this pressure into an analog electrical signal, nulled out the dc voltage produced by the static pressure component, and amplified the sinusoidal component of voltage. This voltage was applied to one channel of the strip chart recorder through one channel of the filter, the need of which is discussed later. The output of the pressure gage was an analog electrical signal containing both a static and a dynamic component. The static component was nulled out with specially designed dc voltage offset circuitry to be discussed later. The sinusoidal voltage output was amplified by the variable gain amplifier and then applied through the second channel of the filter to the second channel of the strip chart recorder. The ratio of the magnitude of the sinusoidal component of pressure at the pressure gage to that at the tubing inlet was determined from known calibration factors. The phase shift was determined from the relative position of the peaks of the recorded sine waves. The sinusoidal frequency was measured with the frequency counter at the output of the sine wave voltage generator. The dc voltage component from the pressure gage was measured with voltmeter no. 1 to determine the static pressure.

The system for generating the static and sinusoidal pressure components consisted of the pneumatic pressure generator, the sine wave voltage generator, the nitrogen gas supply tank and pressure regulator, and vacuum pump no. 1. As shown in figure 1, a vacuum, required for proper operation, was produced by vacuum pump no. 1. The nitrogen gas supply tank and pressure regulator provided gas at a pressure slightly higher than that desired. The exact static pressure was obtained by adjusting a precision pressure regulator in the pneumatic generator itself. The sinusoidal pressure component was obtained by applying a voltage from the sinusoidal voltage generator to an electrically driven modulating valve in the pneumatic generator. The amplitude generating capability of the system decreased with increasing frequency. For example, at 0.0517 MPa static pressure, a 1380 Pa (0.2 psi) d.a. signal could be generated at 0.1 Hz and only a 34 Pa (0.005 Pa) d.a. signal could be generated at 6 Hz. At 0.207 MPa static pressure, the pressure generated at 0.1 Hz was 2670 Pa (0.4 psi) d.a. and was 96 Pa (0.014 psi) d.a. at 6 Hz. These pressures were measured with a variable capacitance electronic manometer consisting of a differential pressure head, signal conditioning, and dc voltage null unit. The pressure head had a diaphragm resonant frequency of 3 kHz and an internal volume of 5 cm³. It was coupled to the inlet of the tubulation-pressure gage system by a tube with a diameter of 0.3 cm and a length of 10 cm. A sinusoidal response calculation showed the response of this tubulation pressure head
combination to be flat to within 1% up to 20 Hz, which is well above the frequency range of interest. The reference port of the pressure head was evacuated with vacuum pump no. 2. It was necessary to null out the dc component of voltage from the pressure gage to enable the sinusoidal component to be amplified without saturating the variable gain amplifier. For this purpose, the circuit shown in figure 2 was designed. In this circuit, the purpose of amplifier no. 1 and associated circuitry was to provide essentially an infinite impedance to the output \( V_g \) of the pressure gage. The gain of this stage was equal to unity. The purpose of amplifier no. 2, the associated circuitry, and the battery was to provide a continuously variable source of nulling dc voltage \( V_b \). Amplifier no. 3 and associated circuitry was a difference amplifier whose output was the difference between the voltages \( V_g \) and \( V_b \). The voltage \( V_b \) essentially cancelled out the dc component of \( V_g \) so that only the sinusoidal component of the pressure gage output was applied to the input of the variable gain amplifier.

At first, the two-channel filter was not used. Without the filter, however, a 60 Hz signal appeared on the pressure gage trace on the strip chart recorder. An investigation revealed that this signal was an electrical signal generated by the pressure gage itself and was not affected by the presence of other units in the test setup; therefore, further efforts were not made to eliminate the source of the signal. The effect of this 60 Hz signal was reduced to an insignificant level by using the low-pass filter with its cutoff frequency set at 30 Hz. However, this filter produced a significant phase shift in the frequency range of interest, 0.1 to 6 Hz. By applying the inlet pressure analog signal to the second channel of the filter, also set to 30 Hz cutoff frequency, this phase shift was equalized on each channel.

Determination of the Transient Response

The purpose of this test was to determine the transportation lag time (dead time) of the tubulation-pressure gage system. The block diagram for performing this test is shown in figure 3. A comparison of figures 1 and 3 shows that the same apparatus was used as for the sinusoidal response tests except that the components for producing a sinusoidal pressure were replaced with those needed for producing a pressure transient, ideally a step function. The test was performed in the following manner: Initially the needle valve and the solenoid valve were open so that the total system was at atmospheric pressure. The solenoid valve was closed. Then, the system pressure was raised 140 Pa (0.2 psi) above atmospheric pressure by using the bellows. Next, the needle valve was closed. With the recorder running, the solenoid valve was opened. The time between the initial and final values of inlet pressure (rise time), as recorded on the strip chart recorder, was 0.016 seconds. The transportation lag time of the tubulation-pressure gage system in air was determined to be 0.106 seconds. The purpose of the needle valve was to eliminate the extra unwanted system volume contributed by the bellows.

RESULTS

The experimental results are shown as the discrete points in parts a and b of figures 4, 5, 6, and 7. In part a, the points show the ratios in db of the magnitude of the sinusoidal component of pressure at the pressure gage to that at the tubing inlet. In part b, the points show the phase shifts of the sinusoidal component at the pressure gage relative to the sinusoidal component at
the tubing inlet. The solid lines in part a and b show the magnitudes in db and phase angles, respectively, of calculated frequency domain transfer functions G(jf) which approximate the experimental data. These functions were synthesized by means of a specially developed computer program from a transportation lag term, and either first order terms, quadratic terms, or a combination of these. These terms and the synthesis procedure are discussed later in more detail. The transfer functions are also presented in table 1. In part c of figures 4 through 7, polar (Nyquist) plots of the calculated functions G(jf) are shown.

Approximate LaPlace transfer functions G(S) were obtained from the frequency domain transfer functions by letting jf = S/2π. These functions are tabulated in table 2.

Next, the individual functions used to synthesize the overall transfer functions are discussed. A first order term in the frequency domain has the form:

\[ \left[ 1 + j \frac{f}{f_b} \right]^{-1} \]

where \( f \) is the sinusoidal frequency, and \( f_b \) is the 3 db frequency of a log magnitude plot. The corresponding term in the S (LaPlace transform) domain, obtained by letting jf = S/2π has the form:

\[ (TS + 1)^{-1} \]

where \( T = 1/2\pi f_b \).

In the S domain, a term with an exponent of -1 contributes a pole on the negative real axis, while a term with an exponent of +1 contributes a zero on the negative real axis. The pole or zero is located at \( S = -1/T \). A quadratic term in the frequency domain has the form:

\[ \left[ 1 - \left( \frac{f}{f_n} \right)^2 + j2\zeta \frac{f}{f_n} \right]^{-1} \]

where \( f_n \) is the undamped natural frequency, and \( \zeta \) is the dimensionless damping coefficient. The corresponding term in the S domain has the form:

\[ \left[ \frac{S^2}{\omega_n^2} + \frac{2\zeta}{\omega_n} S + 1 \right]^{-1} \]

where \( \omega_n = 2\pi f_n \)

In the S domain, for \( \zeta < 1 \), a term with an exponent of -1 contributes a pair of complex conjugate poles, while a term with an exponent of +1 contributes a pair of complex conjugate zeros. The poles or zeros are located in the left plane at:
The damping coefficient \( \zeta \) was always chosen to be \(< 1 \) because if \( \zeta > 1 \), the resulting expression can be factored into two first order terms.

The transportation lag time term in the S domain is \( \exp(-\tau S) \) where \( \tau \) is the transportation lag time. In the frequency domain, this term is \( \exp(-j2\pi f\tau) \). In phasor notation, if the phase angle is expressed in degrees, this term is \( 1/\sin(360f\tau) \).

Next, the computer program and the procedure for synthesizing the transfer functions are discussed. The experimental data were stored on a floppy disk for storage and entry into the computer. The following parameters were varied until satisfactory fit to the magnitude and phase angle data were obtained: number of first order numerator terms; number of first order denominator terms; number of quadratic numerator terms; number of quadratic denominator terms; for first order terms, the values of the 3 dB frequencies; for second order terms, the undamped natural frequency \( f_n \), and dimensionless damping coefficient \( \zeta \). Although it was also possible to vary the transportation lag term, it was allowed to remain constant. It was not necessary that any one transfer function contain all of the five possible types of components. The synthesized functions could be computed and superimposed on the experimental data. Once a satisfactory fit was obtained, the corresponding terms in the S domain were computed and printed out. For small pressure disturbances, the transportation lag time contributed by the tubing is equal to the tubing length divided by the speed of sound. For each test, the ambient temperature was close to 24°C. Therefore, the speed of sound and, consequently, the transportation lag time were assumed to be the same at each static pressure. For air, the transportation lag time at 24°C and a tubing length of 30 meters was calculated to be 0.088 seconds, while the measured value was 0.106 seconds. The difference was probably due to a small transportation lag in the pressure gage itself. Because the speed of sound for nitrogen is 1.6% greater than for air, a transportation lag time of 0.104 sec was used in obtaining the transfer functions for nitrogen.

The concepts of gain margin and phase margin are important in stability studies of closed-loop control systems. In the following discussion, a unity feedback system is assumed. The gain margin \( K_g \) (ref. 2) is defined as the reciprocal of the magnitude at the frequency \( f_g \) at which the phase angle is \(-180^\circ\). Expressed in db

\[
K_g = 20 \log_{10} \frac{1}{|G(jf_g)|}
\]

On a polar plot, \( |G(jf_g)| \) is the magnitude of \( G(jf) \) on the negative real axis. The phase margin \( \phi \) (ref. 2) is defined as the amount of additional phase lag at the gain cross-over frequency \( f_c \) required to bring the system to the verge of instability. The gain cross-over frequency is the frequency at which the gain magnitude is unity. In db, \( 20 \log_{10} |G(jf_c)| = 0 \). This quantity can be determined by finding \( f_c \) on the magnitude plot and then finding the corresponding phase angle \( \phi_c = -\theta_c \) on the phase-angle plot. Then

\[
\phi = 180^\circ - \phi_c
\]
For a system to be stable, the gain and phase margins must be positive. The larger these numbers, the less likely it is that the closed-loop system will be unstable. The gain and phase margins determined for the tubulation-pressure gage system from figures 4 through 7 are as follows:

<table>
<thead>
<tr>
<th>STATIC PRESSURE (MPa absolute)</th>
<th>GAIN MARGIN (db)</th>
<th>PHASE MARGIN (deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0517</td>
<td>10.8</td>
<td>180</td>
</tr>
<tr>
<td>0.101</td>
<td>6.4</td>
<td>180</td>
</tr>
<tr>
<td>0.172</td>
<td>2.6</td>
<td>50</td>
</tr>
<tr>
<td>0.207</td>
<td>1.6</td>
<td>20</td>
</tr>
</tbody>
</table>

It is important, of course, to realize that the preceding numbers apply to the tubulation-pressure gage system alone. A complete closed-loop system would contain additional components such as amplifiers, motors, and servo valves which would contribute to the overall response. Motors would decrease the phase margin especially. Amplifiers with gain greater than unity would generally decrease both gain and phase margins.

CONCLUDING REMARKS

The NTF Mach number system was designed to provide very high accuracy, steady-state pressure measurements over a range of 0.1 to 0.9 MPa (approximately 15 to 130 psia) to determine Mach number from 0.2 to 1.2. Experiments have been performed to measure the dynamic response characteristics of an NTF Mach number gage, a Ruska DDR-6000 quartz absolute pressure gage, coupled to a typical length of pressure tubing to determine if the system could be used in the NTF's closed-loop Mach number control system. In the frequency domain, analytical transfer functions approximating the experimental data have been synthesized from a transportation lag term and either first order or quadratic terms or a combination of these. Approximate LaPlace transfer functions have been derived from these frequency domain transfer functions. From the log-magnitude, phase angle, and polar plots in the frequency domain, gain margin and phase margin were determined.

As a rough design criterion, the phase margin of the total system should be between 30° and 60°, and the gain margin should be greater than 6 db. The tubulation-pressure gage system tested, even without the additional components needed for a complete system, does not meet the gain margin criterion at static pressures of 0.172 MPa absolute (25 psia) and 0.207 MPa absolute (30 psia). It just marginally does so at 0.101 MPa absolute (14.7 psia). Furthermore, this tubulation-pressure gage system by itself does not meet the phase margin criteria at 0.207 MPa absolute pressure. With the needed additional components, the phase margin would probably be unsatisfactory at 0.172 MPa absolute pressure. At static pressures higher than the maximum experimental static pressure of 0.207 MPa, the gain margins and phase margins will probably be even less. Compensation techniques could conceivably be used to bring about some improvement in response. However, because the tubulation-pressure gage system transfer functions depend on the absolute pressure, one compensation network would probably not be suitable for the entire range of absolute pressures.
Because of the preceding considerations, it is concluded that the Ruska DDR-6000 absolute pressure gage should not be used as part of the NTF Mach number control system.

REFERENCE


TABLE 1. - APPROXIMATE FREQUENCY DOMAIN TRANSFER FUNCTIONS

0.0517 MPa absolute static pressure

\[
G(jf) = \frac{\exp(-0.653f) [1 + j0.714f]}{[1 + j2.00f] [1 + j0.333f] [1 + j0.182f]}
\]

0.101 MPa absolute static pressure

\[
G(jf) = \frac{\exp (-0.653f) [1 - 0.063f^2 + j0.400f]}{[1 - 0.207f^2 + j0.882f] [1 - 0.030f^2 + j0.243f]}
\]

0.172 MPa absolute static pressure

\[
G(jf) = \frac{\exp (-0.653f) [1 - 0.082f^2 + j0.269f]}{[1 - 0.207f^2 + j0.455f] [1 - 0.033f^2 + j0.156f]}
\]

0.207 MPa absolute static pressure

\[
G(jf) = \frac{\exp(-0.653f) [1 - 0.098f^2 + j0.293f]}{[1 - 0.227f^2 + j0.381f] [1 - 0.033f^2 + j0.104f]}
\]
<table>
<thead>
<tr>
<th>Pressure Level (MPa)</th>
<th>Transfer Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0517</td>
<td>[ G(S) = \frac{\exp(-0.104S)(0.114S + 1)}{(0.32S + 1)(0.053S + 1)(0.029S + 1)} ]</td>
</tr>
<tr>
<td>0.101</td>
<td>[ G(S) = \frac{\exp(-0.104S)(0.0016S^2 + 0.064S + 1)}{(0.0052S^2 + 0.140S + 1)(0.0008S^2 + 0.039S + 1)} ]</td>
</tr>
<tr>
<td>0.172</td>
<td>[ G(S) = \frac{\exp(-0.104S)(0.0021S^2 + 0.043S + 1)}{(0.0052S^2 + 0.073S + 1)(0.0008S^2 + 0.025S + 1)} ]</td>
</tr>
<tr>
<td>0.207</td>
<td>[ G(S) = \frac{\exp(-0.104S)(0.0025S^2 + 0.047S + 1)}{(0.0057S^2 + 0.061S + 1)(0.0008S^2 + 0.026S + 1)} ]</td>
</tr>
</tbody>
</table>
Figure 1.— Block diagram of apparatus for determining sinusoidal response of tubulation-DDR-6000 pressure gage system
Figure 2.-Circuit for nulling the dc component of voltage output for Ruska DDR-6000 pressure gage
Figure 3.- Block diagram of apparatus for determining transient response of tubulation-DDR-6000 pressure gage system.
Figure 4. - Frequency response of tubulation-pressure gage system at a static pressure of 0.0517 MPa absolute.
Figure 5. - Frequency response of tubulation-pressure gage system at a static pressure of 0.101 MPa absolute.
Figure 6. - Frequency response of tubulation-pressure gage system at a static pressure of 0.172 MPa absolute.
Figure 7. Frequency response of tubulation-pressure gage system at a static pressure of 0.207 MPa absolute.
### Abstract

A theoretical and experimental study was undertaken to determine the feasibility of using the National Transonic Facility's high accuracy Mach number measurement system as part of a closed loop Mach number control system. The theoretical and experimental procedures described are applicable to the engineering design of pressure control systems. The results show that the dynamic response characteristics of the NTF Mach number gage (a Ruska DDR-6000 quartz absolute pressure gage) coupled to a typical length of pressure tubing were only marginally acceptable within a limited range of the facility's total pressure envelope and could not be used in the Mach number control system.

### Key Words (Suggested by Authors(s))

Mach number, Pressure Measurement, Wind Tunnel Control