TRANSFER FUNCTION ANALYSIS OF THERMOSPHERIC PERTURBATIONS

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Applying perturbation theory, a spectral model (Mayr et al., JGR, 1984) in terms of vector spherical harmonics (Legendre polynomials, specifically) is used to describe the short term thermospheric perturbations originating in the auroral regions. The source may be Joule heating, particle precipitation or ExB ion drift-momentum coupling. A multiconstituent atmosphere is considered, allowing for the collisional momentum exchange between species including Ar, O₂, N₂, O, He and H. The coupled equations of energy, mass and momentum conservation are solved simultaneously for the major species N₂ and O. Applying homogeneous boundary conditions, the integration is carried out from the Earth’s surface up to 700 km. In our analysis, the spherical harmonics are treated as eigenfunctions, assuming that the Earth’s rotation (and prevailing circulation) do not significantly affect perturbations with periods which are typically much less than one day. Under these simplifying assumptions, and given a particular source distribution in the vertical, a two-dimensional transfer function is constructed to describe the three dimensional response of the atmosphere. In the order of increasing horizontal wave numbers (order of polynomials), this transfer function reveals five components: (A) The trapped component which is confined to the source region and decays slowly, with the time constant depending on the height of energy deposition and the magnitude of eddy diffusion for example. (B) The quasi-horizontally propagating gravity wave which is represented by the lower cut-off and the first resonance maximum in the transfer function. In the thermosphere, it is also the dominant maximum. The horizontal wave length (order of 1000 km) and propagation velocity (700 m/s) are large. (C) The obliquely propagating wave generated through partial reflection from the base of the thermosphere. This wave appears as a broad secondary maximum in the transfer function; its horizontal wave length and propagation velocity (about 350 m/s) is smaller; this wave is important near the source but cannot propagate very far horizontally. (D) The ducted wave which is produced in the non-dissipative lower atmosphere by total reflection from the Earth’s surface and partial reflection from the mesopause temperature minimum. Leaking back into the thermosphere where it originates, this wave has a relatively short wavelength but can travel large distances away from the source region (pole to equator); the horizontal propagation velocity is about 250 m/s. (E) The waves reflected from the surface which appear in the transfer function as broad secondary maxima. In general, these waves have relatively short horizontal wavelengths and dissipate rapidly above the mesopause. Thus, they cannot propagate far away from the source. To compile the transfer function, the numerical computations are very time consuming (about 100 hours on a VAX for one particular vertical source distribution). However, given the transfer function, the atmospheric response in space and time (using Fourier integral representation) can be constructed with a few

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seconds of CPU. This model is applied in a case study of wind and temperature measurements on the Dynamics Explorer B, which show features characteristic of a ringlike excitation source in the auroral oval. The data can be interpreted as gravity waves which are focused (and amplified) in the polar region and then are reflected to propagate toward lower latitudes.
Figure 1. Block diagram illustrating the analysis. With a fixed height distribution of the excitation source \( h(z) \), and for frequencies \( \omega \), and horizontal wave numbers \( k \), the transfer function is constructed from a numerical integration of the perturbation equations (model). The model describes the perturbation amplitudes in a static background atmosphere with a height dependent temperature distribution, \( T(z) \), and corresponding density variations. The temporal and horizontal variations of the source are spectrally analyzed (source spectrum) and folded into the transfer function to construct (or synthesize) the wave response.
Figure 2. Assuming that the Earth’s rotation and prevailing circulation do not significantly affect thermospheric perturbations with periods much less than one day, one should expect that gravity waves tend to propagate radially away from a “source pixel” as illustrated in the shading. To first order, the response is “source symmetric”. Under this condition, spherical harmonics with zonal wave number $m = 0$ (Legendre polynomials) which normally describe axi-symmetric configurations, can be used to describe an arbitrary three dimensional configuration representing a conglomerate of individual source pixels. The two dimensional transfer function, $F_h (z, \theta, \omega)$, can then be used to describe the three dimensional wave response.
Figure 3. For a wave period of half an hour, we show the computed transfer function of the relative temperature amplitude. The perturbations are excited by a height distributed heat source due to Joule heating. Adhering to the order in the abstract, the four propagating wave modes are identified: (B) the dominant, quasi-horizontally propagating gravity wave near the lower cutoff at the horizontal wave number $\lambda = 32$, consistent with the theory of Hines (1960). (C) The wave reflected from the temperature gradient at the base of the thermosphere (Richmond, 1978). (D) The ducted wave returning from the lower atmosphere, which is formed through total reflection at the surface and partial reflection from the mesopause temperature minimum at 80 km (Hines, 1960; Friedman, 1966); this wave appears in the transfer function as a narrow resonance maximum near $\lambda = 90$ and persists throughout the atmosphere. (E) The broad secondary maxima at higher wave numbers which are due to reflection from the surface (Francis, 1974).
Figure 4. Computed transfer function for the temperature amplitude at 300 km, shown versus the frequency and the ratio between speed of sound and horizontal propagation velocity (proportional to \( \omega / \omega_c \), for \( \omega > 2 \)). The frequency range extends from 0 to 140 cycles per day, i.e., down to periods of 10 minutes. Note that the individual features of the transfer function line up along constant values of \( \omega / \omega_c \), consistent with classical gravity wave theory (Hines, 1960). The labels B through E identify the wave modes discussed in Figure 3. At the lower end of the frequency spectrum, labelled A, the transfer function shows the property of the thermospheric low pass filter (Volland and Mayr, 1970); the global scale and long term variations in the temperature (and density) are preferentially excited. This property is determined by the long time constant for dissipative processes such as heat conduction and accounts for the slow decay of the "trapped" density and temperature perturbations in the source region.
Figure 5. Analogous to Figure 4, we show the computed transfer functions for the vertical and horizontal velocities at 300 km. Note that the amplitudes are relatively small at lower frequencies; in contrast to the temperature (Figure 4), the low pass filter (labelled A) is not important. With increasing frequency, the amplitude of the vertical velocity grows and eventually becomes comparable to that of the horizontal velocity.
Figure 6. Temperature and wind measurements from the DE-2 WATS experiment (Spencer et al., 1982) during a magnetic substorm on December 25, 1981. (For quiet conditions, the temperatures are about 1100 K). Near 65° latitude, a region of energy deposition is indicated as inferred from the magnetic field and particle flux measurements on DE-2 (Farthing, Sugiura et al., 1981; Winningham et al., 1981). Airglow measurements of the 1500 A emission from DE-1 (Frank et al., 1981) made four hours earlier showed the auroral oval centered near the magnetic pole.
Figure 7. Based on the measurements from the Dynamics Explorers, we adopt the source geometry shown in this figure. The source, due to Joule heating, is turned on abruptly, then is kept constant for two hours when it is turned off again. The meridian is indicated along the satellite orbit for which the wave response is simulated.
Figure 8. During the transient process of energization, a broad spectrum of frequencies is prominent in the source. The frequency spectrum of the resultant thermospheric disturbance is determined by the source spectrum and the transfer function. Thus, the frequencies preferentially excited are those which match, through the resonances in the transfer function, the characteristic horizontal dimension of the source. This in turn produces the local ringing and the wave disturbances which propagate away from the source region. Shown here is a three dimensional display of the computed temperature perturbation plotted versus colatitude and time (at $t = 0$ the source is turned on). One perturbation propagates toward the equator, while a second one propagates toward the center, i.e. the magnetic pole, where convergence leads to amplification. The latter wave is "reflected" from the center and then also propagates equatorwards. Both wave trains are launched when the source is turned on and off, thus producing a complicated disturbance.
Figure 9. Along a satellite pass one would observe only a snapshot of this disturbance. Shown here are latitudinal cross sections of the computed vertical velocity and relative temperature perturbation about half an hour (24 minutes (1) and 36 minutes (2)) after the source is turned on. In this time interval, the numerical results can simulate the observations from DE-2 that are shown in Figure 6. The height integrated (localized) energy source required to produce this perturbation is 160 erg/cm²/sec and is very large when compared with the EUV input. During magnetically disturbed conditions, however, such heating rates can be supplied through joule heating (M. Sugiura, this workshop).