\[
\dot{\varepsilon}_{ij} = \frac{(1+\nu)}{E} \sigma_{ij} - \frac{\nu}{E} \sigma_{kk} \delta_{ij} + \dot{\varepsilon}^\text{PL}_{ij}
\]

where the inelastic strain rate tensor \( \dot{\varepsilon}^\text{PL}_{ij} \) is the plastic strain rate and is

\[
\dot{\varepsilon}^\text{PL}_{ij} = f \ S_{ij}
\]

where \( S_{ij} \) are the deviatoric stress components and where

\[
f = \frac{b B}{M \tau_0} N_m (\sqrt{J_2} - D / N_m) \frac{e^{-Q / kT}}{\sqrt{J_2}}
\]
ADVANCED SILICON SHEET

Note 1

PRE BUCKLING - IN PLANE STRESSES
ASSUMES $k > k_0$
(NO BUCKLING)
\[ \nabla^2 (\sigma_{xx} + \sigma_{yy}) = -\alpha E \nabla^2 \mathbf{T} + \frac{1}{\nu} \int_0^x \left( \frac{3\varepsilon^2_{xx} PL}{ay^2} + \frac{3\varepsilon^2_{yy} PL}{ax^2} - 2 \frac{3\varepsilon^2_{xy} PL}{3ax3y} \right) \mathbf{E} \, du \quad (8) \]

\[ \sigma_{xx} = \frac{z}{h} \left[ \varepsilon_{xx} \frac{1}{2} \varepsilon_{yy} + \frac{1}{2} \varepsilon_{yy} - z \frac{3\varepsilon^2_{xx}}{2} - z \frac{3\varepsilon^2_{yy}}{2} \right] \]

where \( w^c \) is the creep (viscoplastic) portion of the transverse displacement.

The moment intensity is related to the stress by the basic definition.

\[ M_{xx} = - \int_{-h/2}^{h/2} \sigma_{xx} \, dz \]

The "elastic moment" \( M_{xx} \) is then

\[ M_{xx} = \frac{Eh^3}{12(1-\nu^2)} \left[ \frac{3\varepsilon^2_{xx}}{ax^2} + \frac{3\varepsilon^2_{yy}}{ay^2} \right] \]

while the corresponding "inelastic" moment component is

\[ M_{xx} = \frac{1}{f} \left[ \frac{3\varepsilon^2_{xx}}{ax^2} + \frac{1}{2} \frac{3\varepsilon^2_{yy}}{ay^2} \right] \frac{h^3}{12} \]

Since the moments are the same, the displacements are clearly related by

\[ \frac{3\varepsilon^2_{w}}{ax^2} = \frac{FE}{(1-\nu^2)} \frac{3\varepsilon^2_{w}}{ax^2} \quad (7) \]

\[ \sigma_{xx} \frac{\partial^2 w}{\partial x^2} = \frac{fE}{12(1-\nu^2)} \left( \frac{N}{ax^2} \frac{\partial^2 w}{\partial x^2} + 2N \frac{\partial^2 w}{\partial xy3x3y} + N \frac{\partial^2 w}{\partial y^2} \right) \]

\[ + \frac{fE}{12(1-\nu^2)} \left[ N \frac{\partial^2 w}{\partial x^2} + 2N \frac{\partial^2 w}{\partial xy3x3y} + N \frac{\partial^2 w}{\partial y^2} \right] \quad (9) \]
ADVANCED SILICON SHEET

\[ w^e(x,y,t) = g(t)w(x,y) \quad (10) \]

and obtain

\[ g - \lambda^2 g = 0 \quad (11) \]

for the time part and

\[ D \nabla^4 W = N_{ab} \left( 1 + \frac{fF}{12(1-v^2)\lambda^2} \right) \frac{\partial^2 W}{\partial x_\alpha \partial x_\beta} \quad (12) \]

Hence one can see that the inelastic material behavior results in buckling very much like the elastic case but with the pseudo in-plane forces given by

\[ N_{ab} \left( 1 + \frac{fF}{12(1+v^2)} \frac{1}{\lambda^2} \right) \quad (13) \]

The separation parameter \( \lambda^2 \) in Eq (13) reflects how "fast" the lateral deflections grow from some initial value. Clearly the presence of \( f(x,y) \) in the numerator of Eq (13) makes similar interpretation impossible for \( \lambda^2 \) except as given in Eq (11).

To obtain values for \( \lambda^2 \), we use a Galerkin method on Eq (12) and find that

\[ \lambda^2 = \frac{2 h^3 \int \int e^2 \nabla^4 W \, da}{3(h^3 - h_{cr}^3) \int \int \nabla^4 W \, da} \quad (14) \]
Normal Strain Rate $XX$ Along $Y = 0$ (Centerline) for $T = 1440 \times \text{Exp} (-0.08X)$ Width = 6.0 CM

LINE 1 IS TOTAL, LINE 2 IS ELASTIC, LINE 3 IS PLASTIC AND LINE 4 IS THERMAL STRAIN RATE.
The Dislocation Density Contour Plot
for $T = 1440 \cdot \exp (-0.08X)$
Unit of $X$ and $Y = \text{CM}$, Unit of $Z = 10^3 \text{ Per CM}^2$
ADVANCED SILICON SHEET

\[ 6 \text{ cm} \times 6 \text{ cm} \]
\[ T = T_{\text{cr}} \]

The results are

<table>
<thead>
<tr>
<th>( N_i / \text{cm}^2 )</th>
<th>( N_f \text{ max} / \text{cm}^2 )</th>
<th>( \sigma_{yy} \text{ max} \text{ MPa} )</th>
<th>( \sigma_{xx} \text{ max} \text{ MPa} )</th>
<th>( t_{1\text{cr}} \text{ mm} )</th>
<th>( t_{2\text{cr}} \text{ mm} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>2497</td>
<td>-22.64</td>
<td>15.23</td>
<td>0.1936(c)</td>
<td>0.1375(t)</td>
</tr>
<tr>
<td>0.15</td>
<td>1092</td>
<td>-22.65</td>
<td>15.99</td>
<td>0.1965(c)</td>
<td>0.1388(t)</td>
</tr>
<tr>
<td>0.01</td>
<td>266</td>
<td>-23.69</td>
<td>17.1</td>
<td>0.1982(c)</td>
<td>0.1394(t)</td>
</tr>
</tbody>
</table>

---

D+10D  
Larger backstress

| 0.3  | 1811  | -22.64 | 15.41  | 0.1948(c) | 0.1381(t) |

Table I
ADVANCED SILICON SHEET

The 20th Elastic Buckling Mode for Westinghouse Temperature Profile
Critical Thickness = 0.19817 MM
Unit of X and Y = CM
### Advanced Silicon Sheet

<table>
<thead>
<tr>
<th>Mode</th>
<th>hcr (mm)</th>
<th>$\lambda^2$ (sec$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.1936</td>
<td>+0.06668</td>
</tr>
<tr>
<td>2</td>
<td>0.1375</td>
<td>+0.01124</td>
</tr>
<tr>
<td>3</td>
<td>0.1132</td>
<td>+0.00742</td>
</tr>
<tr>
<td>4</td>
<td>0.1031</td>
<td>+0.00452</td>
</tr>
<tr>
<td>5</td>
<td>0.0893</td>
<td>+0.00365</td>
</tr>
<tr>
<td>6</td>
<td>0.08503</td>
<td>+0.00153</td>
</tr>
<tr>
<td>7</td>
<td>0.07565</td>
<td>+0.00698</td>
</tr>
<tr>
<td>8</td>
<td>0.06755</td>
<td>+0.00117</td>
</tr>
<tr>
<td>9</td>
<td>0.06644</td>
<td>+0.006555</td>
</tr>
<tr>
<td>10</td>
<td>0.04987</td>
<td>-0.0000416</td>
</tr>
<tr>
<td>11</td>
<td>0.04086</td>
<td>+0.0000832</td>
</tr>
</tbody>
</table>

$T = T_W$

$No = 3/\text{cm}^2$

6cm x 6cm
## ADVANCED SILICON SHEET

<table>
<thead>
<tr>
<th>Mode</th>
<th>( hcr ) (mm)</th>
<th>( \lambda^2 ) (sec(^{-1}))</th>
<th>critical case</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.1965</td>
<td>0.02937</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0.1388</td>
<td>0.04919</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0.1099</td>
<td>0.003094</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>0.1074</td>
<td>0.0018414</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>0.09146</td>
<td>0.0016733</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>0.08562</td>
<td>0.000734</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>0.076924</td>
<td>0.001504</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>0.069803</td>
<td>0.000597</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>0.06800</td>
<td>0.000367</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>0.062011</td>
<td>0.0006800</td>
<td></td>
</tr>
</tbody>
</table>

\[
T = T_w
\]
\[
N_0 = 0.15/cm^2
\]

6cm x 6cm
ADVANCED SILICON SHEET

The First Positive Buckling Mode
for Westinghouse Temperature Profile
Critical Thickness = 0.76154 MM
Unit of X and Y = CM
ADVANCED SILICON SHEET

<table>
<thead>
<tr>
<th>Mode</th>
<th>hcr (mm)</th>
<th>$\lambda^2\text{sec}^{-1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>.4204</td>
<td>$-1.706 \times 10^{-3}$</td>
</tr>
<tr>
<td>2</td>
<td>.3872</td>
<td>$-7.164 \times 10^{-3}$</td>
</tr>
<tr>
<td>3</td>
<td>.3346</td>
<td>$-5.623 \times 10^{-4}$</td>
</tr>
<tr>
<td>4</td>
<td>.3080</td>
<td>$+2.8909 \times 10^{-3}$</td>
</tr>
<tr>
<td>5</td>
<td>.2789</td>
<td>$+4.9641 \times 10^{-3}$</td>
</tr>
<tr>
<td>6</td>
<td>.2663</td>
<td>$-7.8054 \times 10^{-4}$</td>
</tr>
<tr>
<td>7</td>
<td>.2337</td>
<td>$+2.3418 \times 10^{-4}$</td>
</tr>
<tr>
<td>8</td>
<td>.2281</td>
<td>$-1.1527 \times 10^{-3}$</td>
</tr>
<tr>
<td>9</td>
<td>.2044</td>
<td>$-6.0790 \times 10^{-4}$</td>
</tr>
<tr>
<td>10</td>
<td>.1974</td>
<td>$+1.2900 \times 10^{-4}$</td>
</tr>
<tr>
<td>11</td>
<td>.1801</td>
<td>$-2.0870 \times 10^{-4}$</td>
</tr>
<tr>
<td>12</td>
<td>.1724</td>
<td>$+1.0930 \times 10^{-4}$</td>
</tr>
</tbody>
</table>

$T =$ Modified EFG  
$N_o = 2 \times 10^{-7} / \text{cm}^2$

6cm x 6 cm

Run V-0545  
8 June
The Effectiveness Stress Contour Plot for Modified EFG Profile
Unit of X and Y = CM, Z = MPA

\[ N_0 = 2 \times 10^{-7}/\text{cm}^2 \]
ADVANCED SILICON SHEET

The Normal Stress YY Contour Plot
for Modified EFG Profile
Unit of X and Y = CM, Z = MPA

\[ N_0 = 2 \times 10^{-7}/\text{cm}^2 \]

LEGEND: Z
-215.4, -191.8
-168.2, -144.6
-121.0, -97.4
-73.8, -50.2
-26.6, -3.0
20.6, 44.2
67.8, 91.4
The Normal Stress XX Contour Plot
for Modified EFG Profile
Unit of X and Y = CM, Z = MPA

\[ N_0 = 2 \times 10^{-7}/\text{cm}^2 \]
ADVANCED SILICON SHEET

The 13th Buckling Mode
for Modified EFG Profile
Critical Thickness = 0.027877 MM
Unit of X and Y = CM
Fig. I. Surface profile traces illustrating typical edge buckling for ribbon no. 18-102-2 grown at a speed of 3.0 cm/min. Traces are taken along the growth direction, with respect to the width dimension as marked.

MAR 1979
for the case of a 6 cm x 6 cm ribbon pulled at \( v_0 = 0.0005 \text{ m/sec} \). The results are shown in Table III.

<table>
<thead>
<tr>
<th>( M ) /cm</th>
<th>( K_0 ) cm/2</th>
<th>( N_F ) /cm (^2)</th>
<th>( N_{\text{tel}} ) /cm (^2)</th>
<th>( \sigma_{YY_{\text{max}}} ) MPa</th>
<th>( T_F ) mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.75</td>
<td>.5</td>
<td>Diverge</td>
<td>4.65 \times 10^8</td>
<td>-151.1*</td>
<td>.4698</td>
</tr>
<tr>
<td>1.0</td>
<td>.3</td>
<td>Diverge</td>
<td>3.175 \times 10^8</td>
<td>-101.7*</td>
<td></td>
</tr>
<tr>
<td>.25</td>
<td>.3</td>
<td>Diverge</td>
<td>.5941 \times 10^4</td>
<td>-17.4*</td>
<td></td>
</tr>
<tr>
<td>.240625</td>
<td>.3</td>
<td>1.06 \times 10^4</td>
<td>.3137 \times 10^4</td>
<td>-17.8</td>
<td>.31364</td>
</tr>
<tr>
<td>.2375</td>
<td>.3</td>
<td>1984</td>
<td>.2527 \times 10^4</td>
<td>-16.8</td>
<td></td>
</tr>
<tr>
<td>.225</td>
<td>.3</td>
<td>963</td>
<td>1049</td>
<td>-15.0</td>
<td></td>
</tr>
<tr>
<td>.200</td>
<td>.3</td>
<td>173</td>
<td>174</td>
<td>-12.1</td>
<td></td>
</tr>
</tbody>
</table>

Table III

The * in the last column indicates these are the elastic stresses, because plastic ones are not obtained.

\[
T_{\text{NEG}} = \frac{1200 - 125x + 485e^{-1.75x}}{3}
\]

This led to divergent solutions under conditions when the Westinghouse profile did not. With this situation in mind we considered the family of thermal profiles, defined by

\[
T(x) = \frac{1200 - 125x + 485e^{-1.75x}}{3}
\]
ADVANCED SILICON SHEET

OF POOR QUALITY

\[ N_i^0 = 5.914 \, \text{cm}^{-2} \]

\[ N(1) = 9 \, \text{cm}^{-2} \]

\[ N(1.5) = 11.52 \, \text{cm}^{-2} \]

\[ N(1.8) = 11.71 \, \text{cm}^{-2} \]

\[ T(x) = 1000 - \frac{125}{3} x + 452 e^{-2.35x} \]

\[ N(2) = 2527 \, \text{cm}^{-2} \]

FIRST ITERATION

WE BASED ON ELASTIC STRESSES

AFTER 41 ITERATIONS, \( N(5) = 1984 \, \text{cm}^{-2} \)

\[ s = 3 \quad 4 \quad 5 \quad 6 \quad x(\text{cm}) \]
ADVANCED SILICON SHEET
ADVANCED SILICON SHEET

Really New Science
Dislocations as part of the stress analysis.
That is $N \neq$ constant!

New
Creep buckling (lowest mode does not dominate!)

Practical
Elastic very useful
Plastic - residual stress
$\tau_{cr} = \tau_{cr}$ (elastic)
Keep $N$ small
Very sensitive
(ala melting)