WHY HAVE HYDROSTATIC BEARINGS BEEN AVOIDED AS A STABILIZING ELEMENT FOR Rotating Machines?

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The paper discusses the advantages of hydrostatic, high pressure bearings as providing higher margin of stability to the rotor/bearing systems.

INTRODUCTION

Despite the fact that the hydrostatic bearing was invented in 1862 by L. Girard, it appears to have been carefully avoided as a means of stabilizing rotor systems.

This is very interesting because it is perfectly obvious that if a cylindrical bearing could provide a higher Direct Dynamic Stiffness term (see Appendix) in the low eccentricity region, then the stability of a rotor system is, in normal situations, greatly enhanced. The hydrostatic principle not only provides this stiffness term easily, it may also supply its controllability.

It is an interesting exercise to examine the history of rotor dynamics and lubrication theory to guess what went right and what went wrong, and for what reasons. In this instance, as is typical in other such histories, there appears to be other engineering considerations which caused the path to go in the wrong direction.

Very early in lubrication theory (in 1919), Harrison correctly pointed out that a fully lubricated cylindrical bearing is inherently unstable. As a result of this natural behavior, Newkirk and Kimball introduced the pressure dam modification in order to provide a static load to the bearing to alleviate the instability when they showed the basic rules of oil whirl and oil whip in 1924. Numerous research since that time deals with stabilization by means of static loading, and this method of static load to a seal or bearing to hold it at high eccentricity position. At high eccentricity position, of course, the Direct Dynamic Stiffness term is always very high, thus providing stability to the system.

In addition to internal static loading (the pressure dam) and external static loading (by whatever means, such as gravity or deliberate misalignment), there has been a great deal of work on various modifications of the cylindrical form of bearings. It is the author's observation that all of these modifications are helpful for the simple reason that they provide a modification of the flow pattern, as a pure cylinder inside a cylinder is the worst possible situation, if stability is desired. There are lobe bearings, offset halves, tapered land, and hundreds of other helpful geometric modifications of bearings. These methods, however, cannot be applied to seals.

With all these studies and designs, surely someone must have tried hydrostatic principles on the basic bearing. Even so, the only known high pressure lubrication
systems for rotating machinery are very successful designs by a few manufacturers using a combination of tapered land bearings plus high pressure supply. Beyond these, the field appears barren.

As nearly as can be reconstructed, the blockade against hydrostatics occurred in the 1950s. Papers published in this period noted that the sleeve bearing went unstable if the bearing was 360° (fully) lubricated, but was stable with partial lubrication. (This is more or less correct; there usually is wider stability margin with partial lubrication.) The obvious and unfortunate conclusion, however, appears to have been that high pressure oil supply can cause 360° lubrication, therefore, use only low pressure oil supply!

Thus, while correct in its own context, this conclusion apparently blocked the use of Girard's great invention. It is interesting to note that Rayleigh followed upon the original hydrostatic bearing to make the first of the more widely used hydrostatic thrust bearing, but no one applied this idea to oil and gas-lubricated radial bearings.\[1\].

**HIGH PRESSURE BEARING RESULTS**

The plots of the Direct Dynamic Stiffness as a function of static load-related eccentricity ratio (a) for a bearing with "normal" oil supply, i.e., with hydrodynamic pressure or about 20 psi, and (b) for the same bearing with four oil supply ports and four hydrostatic segments generating pressure about ten times high is shown in Figure 1. The increase of the Direct Dynamic Stiffness, especially for low eccentricity ratios, is very significant, as this increase powerfully increases the stability margin of the rotor system if the values are significant, especially for low eccentricity ratio.

The Quadrature Dynamic Stiffness (see Appendix) is virtually unaffected by the addition of the hydrostatic bearing, exhibits very regular relationship with the eccentricity ratio for various values of rotative speed and circular preloads, as shown in Figure 2.

For a more dramatic presentation of the extent of control over instability at the bearing, the Direct Dynamic Stiffness as a function of perturbation frequency yielded from perturbation testing is shown for the same bearing at low, medium, and high oil supply pressures in Figure 3a.

Figure 3b shows the Quadrature Dynamic Stiffness. It is, as in the steady-state tests, essentially independent of oil supply pressure.

Very similar to the high pressure bearing effect was described in the paper reviewing the Lomakin effect. All that is required to match up to Lomakin effect is that the fluid pressure to the seals (or other rotor/stator periphery) goes up with the square of rotative speed, because the hydrostatic stiffness increases linearly with oil pressure. Obviously this creates a Direct Dynamic Stiffness which increases with square of speed, therefore looks like the Lomakin effect of the "negative mass."

**CONCLUSIONS**

The conclusions are as follows:
1. It is readily apparent that deliberate use of hydrostatic bearing high pressure lubricated (any gas or liquid) can easily be used to build higher stability margin into rotating machinery, in spite of the thirty years bias against high pressure lubrication.

2. Since this supply pressure in controllable, the Direct Dynamic Stiffness at lower eccentricity is also controllable, so that within some rotor system limits, the stability margin and dynamic response of the rotor system is more readily controllable.

3. It may be possible to take advantage of this effect in the various seals, as well as the bearings, to assist with stability margin and dynamic response of rotating machinery.

4. The stability of the bearing can be additionally improved by taking advantage of the anti-swirling concept. The high pressure fluid supply inlets should be located tangentially at the bearing circumference and directed against rotation. The incoming fluid flow creates stability by reducing the swirling rate.

REFERENCES


APPENDIX

It is the easiest to explain Dynamic Stiffness terms on the example of a one-degree-of freedom mechanical oscillator, modelled by the equation:

\[ M\ddot{x} + D\dot{x} + Kx = Fe^{j\omega t}, \quad j = \sqrt{-1} \tag{1} \]

where \( M \) is mass, \( D \) is damping coefficient, \( K \) is stiffness coefficient, \( x \) is vibrational displacement and \( F \) represents the amplitude of the periodic exciting force with frequency \( \omega \). Dots indicate derivation with respect to time, \( t \).

The forced response of the oscillator has the following form:

\[ x = Ae^{j(\omega t + \alpha)} \tag{2} \]

where

\[ Ae^{j\alpha} = \frac{F}{K - M\omega^2 + Dj\omega} \tag{3} \]

In the expression (2) \( A \) is the amplitude of the vibration response, \( \alpha \) is the phase of response with respect to the forcing function \( F \). The product \( Ae^{j\alpha} \) has a name of the Complex Response Vector. It is proportional to the excitation amplitude \( F \) and inverse proportional to the Complex Dynamic Stiffness:
Complex Dynamic Stiffness $= K - M\omega^2 + D\omega$ (4)

Complex Dynamic Stiffness has the components:

Direct Dynamic Stiffness $= K - M\omega^2$ (5)

and

Quadrature Dynamic Stiffness $= D\omega$ (6)

The equation (3) yields

Direct Dynamic Stiffness $= \frac{F}{A} \cos \alpha$ (7)

Quadrature Dynamic Stiffness $= -\frac{F}{A} \sin \alpha$ (8)

The expressions (5) through (8) are used for the identification of the system parameters.

Plotted versus excitation frequency $\omega$ the Direct Dynamic Stiffness is a symmetric parabola; the Quadrature Dynamic Stiffness is a straight line crossing the origin of the coordinates.

Similar formulation of the Dynamic Stiffness terms can be applied to the systems modelled by more sophisticated equations.

In particular, for a symmetric rotor supported in one rigid and one fluid lubricated bearing the model is as follows:

$$M\ddot{z} + M_f(2\lambda \omega_R \dot{z}^2 - \lambda^2 \omega_R^2 z) + D(\dot{z} - j\lambda \omega_R z) + Kz + K_b z = Fe^{jwt}$$ (9)

where $M$, $K$ are rotor mass and stiffness respectively, $M_f$, $D$, $K_b$ are bearing fluid inertia, radial damping, and radial stiffness correspondingly, $\omega_R$ is rotative speed, $\lambda$ is the average oil swirling ratio, $\omega$ is perturbation (excitation) frequency. The variable $z = x + jy$ represents the rotor radial displacement composed with the horizontal ($x$) and vertical ($y$) displacements.

For the steady-state periodic response

$$z = Ae^{j(\omega t + \alpha)}$$ (10)

the Direct and Quadrature Dynamic Stiffnesses for the rotor model (9) are as follows:

Direct Dynamic Stiffness $= \frac{F}{A} \cos \alpha = K - M\omega^2 - M_f(\omega - \lambda \omega_R)^2 + K_b$ (11)

Quadrature Dynamic Stiffness $= -\frac{F}{A} \sin \alpha = D(\omega - \lambda \omega_R)$ (12)

The Direct Dynamic Stiffness versus frequency $\omega$ is a parabola shifted from the symmetric origin due to fluidic inertia. The effect of higher pressure, which causes an increase of $K_b$ is shown in Figure 3. For small values of $K + K_b$ the Direct Dynamic Stiffness at zero frequency $\omega$ can be negative (Fig. 3).
The Quadrature Dynamic Stiffness versus frequency $\omega$ is a straight line crossing the vertical axis at its negative side. This value is equal to the bearing "cross" stiffness coefficient, $-D\omega R$, Fig. 3b.

Figure 1. - Direct dynamic stiffness versus eccentricity ratio for a hydrodynamic bearing (a) and a hydrostatic bearing (b). (Eccentricity ratio = ratio of journal displacement to radial clearance).

Figure 2. - Experimentally obtained quadrature dynamic stiffness versus eccentricity ratio, indicating insensitivity to oil pressure.
Figure 3. - Dynamic stiffnesses versus frequency for several values of oil supply pressure, indicating stability margins.