MEASURING DYNAMIC OIL FILM COEFFICIENTS OF SLIDING BEARING

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This paper presents a method for determining the dynamic coefficients of bearing oil film. By varying the support stiffness and damping, eight dynamic coefficients of the bearing have been determined. Simple and easy to apply, the method can be used in solving practical machine problems.

INTRODUCTION

An understanding of the properties of sliding bearings is important for solving the vibration problems of rotating machines, such as steam turbine generators. The dynamic oil film coefficients of a sliding bearing, as well as pedestal dynamic coefficients, are necessary for calculating rotor critical speed, unbalance response, system stability, etc. Many experts have set up test rigs for performing experiments on sliding bearings to obtain the eight dynamic oil film coefficients and calculate the experiment results, but the test rigs are often very complex.

This paper presents a method for determining the dynamic coefficients of the bearing oil film, using a new continually-changing stiffness pedestal. By varying twice the support stiffness, eight dynamic coefficients of the bearing have been determined. The continuously-changing stiffness pedestal is described.

1. A MEASURING PROCEDURE

Figure 1 illustrates the measuring procedure for determining the dynamic coefficient of bearing oil film. The procedure requires two steps:

1.1 Determining the pedestal dynamic coefficients:

The electric or hydraulic actuator is first used for actuating the pedestal vibration. According to the actuating force amplitude and phase and the pedestal vibration amplitude and phase, the pedestal dynamic coefficients can be determined. Next, the pedestal stiffness is changed, and the experiment is repeated to obtain the second pedestal dynamic coefficients.

The two pedestal dynamic coefficients are important for determining the dynamic coefficients of the bearing oil film.

1.2 Determining the dynamic coefficients of bearing oil film:

The rotor is run at the different speeds to obtain the pedestal vibration and the relative rotor vibration measurements. Next the pedestal stiffness is changed, and the experiment is repeated. According to the amplitude and phase
measurements for the rotor and pedestal, the eight dynamic coefficients of sliding bearing will be determined.

2. DETERMINING THE BEARING DYNAMIC OIL FILM COEFFICIENTS

Figure 2 illustrates the model of sliding bearing and pedestal.

The following notation has been used:

- \( K_{xx}, K_{yy} \) = horizontal and vertical bearing oil film stiffness coefficients
- \( K_{xy}, K_{yx} \) = bearing oil film cross-stiffness coefficients
- \( C_{xx}, C_{yy} \) = horizontal and vertical bearing oil film damping coefficients
- \( C_{xy}, C_{yx} \) = bearing oil film cross-damping coefficients
- \( K'_{xx}, K'_{yy} \) = horizontal and vertical pedestal stiffness coefficients
- \( K'_{xy}, K'_{yx} \) = pedestal cross-damping coefficients
- \( C'_{xx}, C'_{yy} \) = horizontal and vertical pedestal damping coefficients
- \( C'_{xy}, C'_{yx} \) = pedestal cross-damping coefficients

The force between the rotor and the bearing oil film is

\[
\begin{bmatrix}
P_{rx} \\
P_{ry}
\end{bmatrix} = 
\begin{bmatrix}
K_{xx} & K_{xy} & C_{xx} & C_{xy} \\
K_{yx} & K_{yy} & C_{yx} & C_{yy}
\end{bmatrix}
\begin{bmatrix}
x_r \\
y_r \\
x_r' \\
y_r'
\end{bmatrix}
\]

where: \( P_{rx}, P_{ry} \) = the rotor unbalance force being transmitted to the oil film in the horizontal and vertical direction, \( x_r, y_r \) = the rotor center displacement in the horizontal and vertical direction relative to the pedestal.

The force being transmitted through the oil film to the pedestal can be represented by the following equation:

\[
\begin{bmatrix}
P_x \\
P_y
\end{bmatrix} = 
\begin{bmatrix}
K'_{xx} - w^2 m_{px} & K'_{xy} & C'_{xx} & C'_{xy} \\
K'_{yx} & K'_{yy} - w^2 m_{py} & C'_{yx} & C'_{yy}
\end{bmatrix}
\begin{bmatrix}
x_a \\
y_a \\
x_a' \\
y_a'
\end{bmatrix}
\]

where: \( x_a, y_a \) = pedestal center displacement in the horizontal and vertical direction, \( m_{px}, m_{py} \) = the mass of the pedestal in the horizontal and vertical direction.

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Omitting consideration of the oil inertia, the equations of motion can be represented by

\[ P_x = P_{rx} \quad P_y = P_{ry} \]  

(3)

When the rotor rotates at \( \omega \), synchronous vibrations of the rotor occur. Assuming that the bearing and pedestal are in contact, the vibrations are given by

\[
\begin{align*}
  x_{a_1} &= X_{a_1} \sin(\omega t - \alpha_1) \\
  y_{a_1} &= Y_{a_1} \sin(\omega t - \alpha_1) \\
  x_{r_1} &= X_{r_1} \sin(\omega t - \theta_1) \\
  y_{r_1} &= Y_{r_1} \sin(\omega t - \theta_1)
\end{align*}
\]  

(4)

In the same condition when the pedestal stiffness is changed, the new vibration of the rotor and the pedestal are given by

\[
\begin{align*}
  x_{a_2} &= X_{a_2} \sin(\omega t - \alpha_2) \\
  y_{a_2} &= Y_{a_2} \sin(\omega t - \alpha_2) \\
  x_{r_2} &= X_{r_2} \sin(\omega t - \theta_2) \\
  y_{r_2} &= Y_{r_2} \sin(\omega t - \theta_2)
\end{align*}
\]  

(5)

Substituting for equation (2) using equations (4) and (5), according to equations (1) and (3) the necessary equation is yielded:

\[
\begin{bmatrix}
  K_{xx} \\
  K_{xy} \\
  C_{xx} \\
  C_{xy}
\end{bmatrix}
= [A]^{-1}[G]
\]

(6)

where:

\[
[A] =
\begin{bmatrix}
  X_{r_1}\cos\theta_1 & Y_{r_1}\cos\beta_1 & wX_{r_1}\sin\theta_1 & Y_{r_1}\w sin\beta_1 \\
  -X_{r_1}\sin\theta_1 & -Y_{r_1}\sin\beta_1 & wX_{r_1}\cos\theta_1 & Y_{r_1}\w cos\beta_1 \\
  X_{r_2}\cos\theta_2 & Y_{r_2}\cos\beta_2 & wX_{r_2}\sin\theta_2 & Y_{r_2}\w sin\beta_2 \\
  -X_{r_2}\sin\theta_2 & -Y_{r_2}\sin\beta_2 & wX_{r_2}\cos\theta_2 & Y_{r_2}\w cos\beta_2
\end{bmatrix}
\]
Similarly, in the direction Y, the equation is given by

\[
\begin{bmatrix}
K_{yy} \\
K_{yx} \\
C_{yy} \\
C_{yx}
\end{bmatrix} = [B]^{-1}[W]
\]

where:

\[
[B] =
\begin{bmatrix}
y_1 \cos \beta_1 & x_1 \cos \theta_1 & y_1 \sin \beta_1 & x_1 \sin \theta_1 \\
-y_1 \sin \beta_1 & -x_1 \sin \theta_1 & y_1 \cos \beta_1 & x_1 \cos \theta_1 \\
y_2 \cos \beta_2 & x_2 \cos \theta_2 & y_2 \sin \beta_2 & x_2 \sin \theta_2 \\
-y_2 \sin \beta_2 & -x_2 \sin \theta_2 & y_2 \cos \beta_2 & x_2 \cos \theta_2
\end{bmatrix}
\]

\[
[W] =
\begin{bmatrix}
(K'_{yy} - u^2 m_y) y a_1 \cos \alpha_1 + C' y a_1 \sin \alpha_1 \\
-(K'_{yy} - u^2 m_y) y a_1 \sin \alpha_1 + C' y a_1 \cos \alpha_1 \\
(K'_{yy} - u^2 m_y) y a_2 \cos \alpha_2 + C' y a_2 \sin \alpha_2 \\
-(K'_{yy} - u^2 m_y) y a_2 \sin \alpha_2 + C' y a_2 \cos \alpha_2
\end{bmatrix}
\]

According to the experimental set-up, assuming that cross-stiffness and damping coefficients are equal to zero, the bearing oil film coefficients are calculated by using equations (6) and (7). If the cross-stiffness and damping are not equal to zero, the calculation is similar.

3. EXPERIMENTAL SET-UP AND MEASURING INSTRUMENTS

3.1 Experimental equipment

Figure 3 illustrates the construction of the pedestal, in which the stiffness can be changed by varying the oil pressure. When the oil pressure is changed, it affects the stiffness of the butterfly spring.

3.2 Measuring instruments

Figure 4 illustrates the measuring instruments. Two eddy current probes are
used to measure rotor vibration. Two velocity probes are used to measure pedestal vibration. The Digital Vector Filter (DVF 2) is used to measure the speed (rpm), phase, and amplitude.

CONCLUSIONS AND RECOMMENDATIONS

Table 1 shows the experimental results of Ø20 sliding bearing oil film stiffness and damping coefficients. Most of the data are reasonable. The individual data are scattered, such as individual cross-stiffness coefficients being larger than horizontal and vertical stiffness coefficients at a specific speed, due to measuring errors. For example, the measuring phase error of 2° causes a coefficient error of 5%. When applying this method to the measurement of oil film coefficients, it is important to use the same pedestal and foundation. Changing the pedestal and foundation causes measurement errors of the oil film coefficients.

By varying the support stiffness, eight dynamic coefficients of the bearing have been determined. The method is simple and easy to apply.

REFERENCES

### Table 1: The Experimental Result for #20 Sliding Bearing Oil Film Dynamic Coefficients

<table>
<thead>
<tr>
<th>Speed (rpm)</th>
<th>$k_{xx}$ (kg/μ)</th>
<th>$k_{yy}$ (kg/μ)</th>
<th>$k_{xy}$ (kg/μ)</th>
<th>$k_{yx}$ (kg/μ)</th>
<th>$c_{xx}$ (kg·s/cm)</th>
<th>$c_{yy}$ (kg·s/cm)</th>
<th>$c_{xy}$ (kg·s/cm)</th>
<th>$c_{yx}$ (kg·s/cm)</th>
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<tr>
<td>2000</td>
<td>0.19</td>
<td>0.120</td>
<td>-0.061</td>
<td>0.057</td>
<td>0.67</td>
<td>0.038</td>
<td>0.45</td>
<td>0.69</td>
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<td>3000</td>
<td>0.245</td>
<td>0.163</td>
<td>-0.11</td>
<td>0.043</td>
<td>0.84</td>
<td>0.61</td>
<td>-0.002</td>
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<td>0.287</td>
<td>-0.23</td>
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<td>-0.075</td>
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<td>-0.67</td>
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<tr>
<td>5000</td>
<td>0.04</td>
<td>0.064</td>
<td>-0.081</td>
<td>0.081</td>
<td>-0.042</td>
<td>-0.078</td>
<td>0.37</td>
<td>0.043</td>
</tr>
<tr>
<td>6000</td>
<td>0.134</td>
<td>0.07</td>
<td>0.096</td>
<td>0.037</td>
<td>0.66</td>
<td>0.45</td>
<td>-0.018</td>
<td>-0.051</td>
</tr>
<tr>
<td>7000</td>
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<td>0.021</td>
<td>-0.030</td>
<td>0.26</td>
<td>-0.24</td>
<td>-0.78</td>
<td>0.087</td>
</tr>
</tbody>
</table>

### Figure 1: Measuring Procedure.

### Figure 2: Model of sliding bearing and pedestal.
Figure 3. - Construction of pedestal with variable stiffness.

Figure 4. - Measuring instruments and their connection.