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INTRODUCTION

The Institute for Computer Applications in Science and Engineering (ICASE) is operated at the Langley Research Center (LaRC) of NASA by the Universities Space Research Association (USRA) under a contract with the Center. USRA is a nonprofit consortium of major U. S. colleges and universities.

The Institute conducts unclassified basic research in applied mathematics, numerical analysis, and computer science in order to extend and improve problem-solving capabilities in science and engineering, particularly in aeronautics and space.

ICASE has a small permanent staff. Research is conducted primarily by visiting scientists from universities and from industry, who have resident appointments for limited periods of time, and by consultants. Members of NASA's research staff also may be residents at ICASE for limited periods.

The major categories of the current ICASE research program are:

a. Numerical methods, with particular emphasis on the development and analysis of basic numerical algorithms;

b. Control and parameter identification problems, with emphasis on effective numerical methods;

c. Computational problems in engineering and the physical sciences, particularly fluid dynamics, acoustics, and structural analysis;

d. Computer systems and software, especially vector and parallel computers.

ICASE reports are considered to be primarily preprints of manuscripts that have been submitted to appropriate research journals or that are to appear in conference proceedings. A list of these reports for the period October 3, 1985, through March 31, 1986, is given in the Reports and Abstracts section which follows a brief description of the research in progress.

Presently, ICASE is operated at NASA Langley Research Center, Hampton, VA under the National Aeronautics and Space Administration, NASA Contract No. NAS1-18107. In the past, support has been provided by NASA Contracts No. NAS1-17070, NAS1-17130, NAS1-15810, NAS1-16394, NAS1-14101, and NAS1-14472.
RESEARCH IN PROGRESS

Saul Abarbanel

Work of S. Abarbanel, A. Bayliss, and L. Lustman continues on the construction of non-reflecting boundary conditions for the compressible Navier-Stokes equations. The boundary conditions are obtained by linearizing the Navier-Stokes equations around a model mean profile. We then compute an approximate dispersion relation between the frequency and the waves that can be supported by the resulting linear system. The approximate dispersion relation is valid in the limit of long wavelengths. This analysis has been completed for flows over a flat geometry. We have verified that the new boundary condition can significantly accelerate the convergence to a steady state. A report describing this work was completed. We have completed the analysis for flows in a channel and are presently testing the resulting boundary conditions numerically.

H. T. Banks

The standard proofs of convergence in identification techniques for distributed parameter systems involves a "uniform compactness" type assumption on the parameter sets and their family of approximates. This compactness also allows one to prove "method-stability" results (e.g., continuity of parameter estimates with respect to observations) and has an analogue in the Tikhonov regularization formulation for these problems. We are currently pursuing investigations into the computational necessity of imposing this compactness through some type of functionally-constrained optimization algorithms. A numerical study of the algorithms including a comparison with regularization techniques is underway.

H. T. Banks and K. Ito

We have investigated the use of a hybrid method for the computation of feedback gains in a general class of linear quadratic Gaussian (LQG)
problems. The methods are easily applied to large but finite dimensional interconnected systems as well as approximate systems for infinite dimensional distributed parameter control problems. The methods entail using a Chandrasekhar algorithm to obtain start ups for a Newton-Kleinman algorithm with a variable step Smith method to solve the resulting Lyapunov equations in the iterative method. Numerical experiments suggest that the method is superior to the standard eigenvector type techniques (Potter, Laub-Schur) in problems where the number of states is large compared to the number of control inputs.

H. T. Banks and Robert K. Powers

Large space structures currently proposed often contain long flexible members. The size of such structures makes it intractable to test full scale models in slewing and control maneuvers, making it necessary to have reliable mathematical models for computer simulation. Current research is underway in conjunction with J. Juang (Structures and Dynamics Division, LaRC) to model damping due to air during slewing maneuvers of flexible beams. Finite element and parameter identification techniques are being applied to models of flexible beams to determine the coefficients of nonlinear damping terms.

H. T. Banks and Georg Propst

Together with R. Silcox (Acoustics Division, LaRC) a noise suppression problem has been formulated as a control problem governed by the wave equation, and the numerical performance of approximations to this model is being evaluated. Several foundations of a one-dimensional version of the problem are being studied for computational feasibility, in addition, extension of the ideas to multi-dimensional domain problems are being considered.
H. T. Banks and I. G. Rosen

We have continued our efforts on the difficult problems associated with the estimation of spatially varying material parameters such as stiffness and damping in cantilevered beams with tip mass or tip body. Results of our comparisons of several types of finite element state approximation schemes for various state formulations (e.g., weak variational, conservative) are now being prepared for publication. The enhancement of methods and the effects on "identifiability" via applied shear forces and moments at the tip have been investigated.

In a related study we have developed and used similar ideas for a model of the "RPL experiment" (a beam with tip thrusters and a flexible hose). We have successfully identified physically meaningful parameters in a proof-mass (i.e., spring-mass-dashpot) - beam hybrid model using acceleration data from a series of experiments.

Alvin Bayliss

Work of A. Bayliss, L. Maestrello (Transonic Aerodynamics Division, LaRC), P. Parikh (Vigyan Research Associates, Inc.), and E. Turkel continues on the numerical simulation of the active control of compressible flows by localized periodic surface heating. We have completed our study of the control of two-dimensional flows over a curved surface. The results show that the pressure gradient induced by the surface curvature significantly enhances the receptivity of the flow to surface heating. Numerical simulations demonstrate that unstable disturbances can be stabilized by a proper choice of the phase of the surface heating. A report describing this work is being prepared. We have begun simulating three-dimensional disturbances in a two-dimensional mean flow.

Shahid H. Bokhari

The classical (circa 200 BC) Sieve of Eratosthenes for finding prime numbers is one of the oldest parallel algorithms known to man. In recent
years it has seen much use as a benchmark algorithm for serial computers while its intrinsically parallel nature has gone largely unnoticed.

We have implemented a parallel version of the Sieve on the Flex/32 multiprocessor and are evaluating its performance. The Sieve is a good test of the capabilities of a multiprocessor since it requires rapid access to shared memory by a number of processes that are created and destroyed on the fly. It is also a challenge for the algorithm designer since, by the very nature of the algorithm, it is impossible to get good speedups without some form of dynamic load-balancing. We expect that this parallel version of the Sieve will be useful in comparing the performance of shared memory multiprocessors.

Shahid H. Bokhari and Marsha Berger

Research on a binary partitioning technique for non-uniform problems has been completed and a technical report prepared. We have compared the performance of nearest neighbor arrays and hypercubes when using our partitioning strategy. Some of our results are unexpected—they indicate that hypercubes perform better than meshes only if the problem domain has moderate non-uniformity. For the case of mildly distorted or heavily distorted domains, the performance of hypercubes is essentially equal to that of meshes.

Shahid H. Bokhari and M. Y. Hussaini

In collaboration with Gordon Erlebacher (High-Speed Aerodynamics Division, LaRC), we have embarked on a major effort to transport an existing Navier-Stokes code from the Cray-2 to the Flex/32. The objective is to parallelize the code and demonstrate speedup on the Flex/32 in anticipation of the availability of parallel processing on the Cray-2 and its successors. By carefully developing the synchronization and spawning routines needed for parallelism, we have succeeded in moving the code to the Flex/32 with very little modification. In fact, other than calls to the synchronization
routines and changes in the do loop ranges, there is almost no change in the main body of the code. This work is thus likely to have a major impact on the utilization of parallel machines for large fluid mechanics problems over the next five years.

Shahid H. Bokhari and M. Ashraf Iqbal

We are studying the problem of optimally assigning the modules of a parallel/pipelined program over the processors of a multiple computer system under certain restrictions on the interconnection structure of the program as well as the multiple computer system. We have established that for a variety of such programs it is possible to determine in linear time if a partition of the program exists in which the load on any processor is within a certain bound. This method, when combined with a binary search over a finite range, provides an approximate solution to the partitioning problem.

Another research effort addresses the problem of finding a minimal super binary tree which contains a given set of given binary trees. Such a problem arises in the design of the ALU pipeline of the Navier-Stokes computer under development at Princeton University. The ALU pipeline is composed of individual units connected in the form of a binary tree. As each different application may require a distinct binary tree interconnection of the ALU chips, it is useful to hard wire the ALU pipeline in the form of a minimal super binary tree which contains all required trees. We have constructed the minimal supertree for a specific set of trees and are now developing an efficient algorithm for the general problem.

Shahid H. Bokhari, M. Ashraf Iqbal, and Joel H. Saltz

We have studied the problem of uniformly distributing the load of a parallel program over a multiprocessor system. We have compared the performance of four strategies for load balancing. These are (1) the optimal static assignment which is guaranteed to yield the best static solution, (2)
the static binary dissection method which is very fast but sub-optimal, (3) the greedy algorithm, a static fully polynomial time approximation scheme, which estimates the optimal solution to arbitrary accuracy, and (4) the predictive dynamic load balancing heuristic which uses information on the precedence relationships within the program and outperforms any of the static methods. A technical report has been prepared.

**Dennis W. Brewer**

Research is continuing on parameter estimation problems associated with linear evolution equations in infinite-dimensional spaces. A general algorithm based on quasi-linearization has been established along with its local convergence properties. The algorithm has been numerically tested on linear delay-differential equations. Numerical experiments indicate that the method converges rapidly when used to identify two unknown delays together with two or three unknown coefficients using simulated data. Future research will involve continuing numerical experimentation and improvements in the theory to accommodate a wider class of problems.

**Dennis W. Brewer and J. Steven Gibson**

We are continuing our efforts to develop robust software routines for identifying parameters in robotic manipulators. The programs use numerical integration of nonlinear differential equation models and nonlinear optimization algorithms. This work is in conjunction with J. Pennington and F. Harrison (Information Systems Division, LaRC) who have provided experimental data. Our previous research indicates a need to model the integrated electro-mechanical system to reduce parameter sensitivity. We have obtained data and plan to test models which include coupled physical and actuator dynamics. In addition, input optimization should be a fruitful area for future research.
Jan Cuny, Piyush Mehrotra, and John Van Rosendale

BLAZE is a scientific programming language developed at ICASE over the past two years. BLAZE programs are mapped onto parallel architectures by a program transformation system. This system translates BLAZE source programs through a sequence of intermediate forms to Extended-BLAZE. E-BLAZE is an explicit-tasking language consisting of BLAZE together with a number of lower-level constructs for expressing parallelism. It contains constructs for allocating and de-allocating processes, specifying array storage patterns, scheduling loops, performing interprocess communication and synchronization, and so forth. E-BLAZE is, in effect, a high level virtual architecture for BLAZE.

Though E-BLAZE is intended primarily as a compiler intermediate form, it may also prove useful for programmers wishing to fine-tune their algorithms for a particular architecture. This is especially true for non-shared memory architectures, such as the NCUBE or FPS hypercubes, or the Navier Stokes Computer being developed at Princeton University, where well developed software environments are currently unavailable. We are studying both the problem of mapping E-BLAZE to such architectures and the issue of its utility as an application programming language.

Pravir Dutt and Shlomo Ta'asan

Results have been obtained for parameter estimation problems involving elliptic partial differential equations using the equation error approach. Work is continuing on extending the method to nonelliptic problems such as the equations for oil recovery and the Euler-Berroulli beam equation.

Stefan Feyock

The previous report described the development of a Prolog escape predicate, which allowed convenient communication between Prolog and external software systems, in particular the FORTRAN-based database management system.
RIM. The resulting integration of Prolog with a large-scale commercial database was used to add expert-system capabilities to the database system. A RIM-based expert system that diagnosed jet engine malfunctions was developed and run successfully. Current research is focusing on the observation that addition of an Artificial Intelligence (AI) capability to pre-existing software systems is a common requirement and that an AI capability that can be called in slave mode from existing software would be a valuable programming tool. Means of providing such interfaces between AI tools and conventional software are being studied.

Dennis Gannon and Piyush Mehrotra

The principal open questions with BLAZE revolve around the family of program transformations needed to map BLAZE programs to multiprocessor architectures. We plan to complete preliminary implementations of BLAZE on several types of multiprocessors during the next six months. Our first target architectures are the BBN Butterfly, Flex/32, and Alliant multiprocessors. We are also planning to implement versions of BLAZE on the IBM RP3 prototype and the University of Illinois Cedar computer as soon as they become available.

Coupled with this effort to achieve preliminary implementations is a longer term effort to study the subtle issues involved in parallel run-time environments. While it is relatively easy to create naive implementations of BLAZE on shared memory multiprocessors, constructing implementations which fully exploit these architectures is difficult. For example, on the RP3 one could potentially exploit memory locality, caching, the fetch-add network, and could also allocate sub-groups of processors to cooperate on parallel tasks. With such architectural flexibility, deciding the best way to implement any given language construct can be quite complex.

To study such issues, a graphics based program transformation system has been written and is currently running on Apollos and Macintoshes. This system will be ported to the ICASE Suns over the summer and should prove useful in developing program transformations suited to the NASA Flex/32 multiprocessor.
To some extent, this work parallels work done by D. Kuck, K. Kennedy, F. Allen, and others on mapping Fortran programs to parallel architectures. The difference is that BLAZE provides a much better starting point, since the semantics of BLAZE were chosen with exactly this problem in mind. A Prolog based expert system has been developed to select the sequence of transformations to be performed, and preliminary results on the efficacy of this approach should be available soon.

James Geer

During the past six months, I have been working on the following projects in connection with NASA Langley:

Comparison of numerical and perturbation techniques to determine the aerodynamic forces on slender bodies. This study has involved the comparison of pressure coefficients for some two- and three-dimensional slender bodies of simple geometrical shape using standard numerical (panel) methods and uniform asymptotic methods. Some preliminary results have been reported in a recent ICASE report. Work is now continuing to try to determine if some of these results can be "pieced together" to describe the forces acting on more complicated geometrical shapes, such as a typical subsonic aircraft. This work is being done with Dr. Eddie Liu (Low Speed Aerodynamics Division, LaRC) and Prof. Lu Ting of New York University.

Reduced basis method extensions and applications. This work involves a study of the reduced basis method (a semi-numerical, semi-analytical method which has been used to solve a variety of nonlinear boundary value problems in elasticity) to determine how the method might be applied to some fluid dynamic and scattering problems. Several model problems are being studied both to understand the method better and to see what type of problems the method might be applied to. At present, these model problems include some two point boundary value problems with boundary layers and some simple exterior scattering problems. This work is being done with Dr. Carl Andersen of the College of William and Mary.
Nonlinear oscillations using symbolic computation. This study involves the use of the symbolic computation system MACSYMA and perturbation methods to investigate some free and forced nonlinear oscillations. The method of multiple time scales has been implemented using MACSYMA and applied to the problem of determining the transient responses of the van der Pol and Duffing oscillators when the applied force consists of a sum of periodic terms with different frequencies. The conditions under which the system will experience frequency entrainment are being investigated. This work is being done with Dr. Carl Andersen of the College of William and Mary.

Integral equations of the first kind. Investigations are underway concerning some fundamental properties (both from theoretical and computational points of view) of a class of "almost" singular integral equations of the first kind which are useful in representing the solution to certain elliptic exterior boundary value problems. These equations typically have the property that the domain of integration \( R \) is a proper subset of the domain of validity \( D \) of the equation. Special consideration is being given to the idea of analytically continuing the solution into the domain \( D \). Applications of the results will be useful in several two- and three-dimensional problems involving slender or thin bodies.

J. S. Gibson and I. G. Rosen

We are developing numerical approximation methods for the computation of optimal feedback control and observer gains for infinite dimensional, discrete-time, linear-quadratic regulator problems. In particular, we are considering systems in which the state transition operators are given in terms of linear semigroups of operators on infinite dimensional Hilbert spaces and the input and output operators are possibly unbounded. Our investigation is to include both theoretical (convergence arguments, etc.) and numerical components. We intend to apply our theory to a variety of test examples including systems describing transport phenomena (e.g., diffusion, heat conduction, etc.) and the vibration of flexible structures.
We are also studying numerical approximation methods for the estimation or identification of spatially varying coefficients in wave and beam equations based upon model data (e.g., natural frequencies and mode shapes). Specifically, we are looking at schemes which use spline-based finite element methods to discretize the infinite dimensional inverse problem. A scheme which uses modal data rather than time histories of displacement, velocity, strain, etc. to estimate unknown parameters has the potential to be much faster since the repeated integration of the initial-boundary value problem which governs the underlying dynamics (or more precisely, a high order finite dimensional approximation to it) is not required when the optimization is carried out.

David Gottlieb

Research continued on the problem of the behavior of high order schemes applied to discontinuities. It is clear that the notion of formal accuracy is not valid and, in fact, loss of accuracy is observed. During the last few years, we have realized that high order schemes do contain more information about the solution if they can be made stable. We are still working on the general theory, and we are making progress in applying the notion of "information content" to high order schemes. Joint work with D. Fishelov (University of California, Berkeley) demonstrates that accurate results can be recovered from spectral simulations of the small disturbance equation.

Together with M. Salas (Theoretical Aerodynamics Division, LaRC) research is in progress concerning the vortex breakdown problem. We study properties of solutions of the ODE model developed by Salas.

Together with T. Zang and G. Erlebacher (Computational Methods Division, LaRC) we study splitting methods for time dependent flows with small variations in the sound speed. The idea is to remove the stiffness of the system by splitting out the average (constant) sound speed.
Chester E. Grosch

A three-dimensional, time dependent Navier-Stokes code in vorticity and velocity variables has been developed in collaboration with T. Gatski (Viscous Flow Division, LaRC) and M. E. Rose. This code is being modified in order to increase its speed and efficiency and is being tested against several exact analytic solutions. The initial calculations for the flat plate receptivity problem have been completed. The results have been analyzed, written up, and submitted for publication.

The study, in collaboration with R. Fatoohi, of parallel algorithms for elliptic problems is continuing. The four color relaxation scheme is now working on the MPP at NASA Goddard. This has been quite successful for problems which fit on the array; the average processing rate for these problems is 175 MFLOPS. A modification of the algorithm for problems which are much larger than the array has been developed. This will be tested in the near future. Similarly, a modified algorithm has been developed for the MIMD FLEX/32 and is being tested.

Subramaniya I. Hariharan

Currently, we are making numerical simulation of atmospheric acoustic problems that were theoretically investigated during the last reporting period. The computer code for this problem is stable and consistent with the theoretical boundary conditions that we proposed. Results will be tested against an experiment that is planned in the fall of 1986 at Langley.

The major project that we are undertaking here is to study the effect of acoustics on flows past blunt bodies. Even the steady compressible viscous flow problem in the situation of "leading edge" is quite complicated. Acoustics is an unsteady phenomenon. Thus, as a first attempt we are investigating efficient methods, such as optimal time splitting, to obtain simulations of flow past a flat plate including the leading edge. Then the acoustic disturbances will be introduced in the flow. In this effort, well-posedness of the problem and appropriate outflow boundary conditions with and without acoustics will be examined.
Yousuff Hussaini

In a continuing program of research in hydrodynamic stability and transition, refined simulations of certain basic problems were carried out with a view to unravelling the underlying instability mechanisms. In particular, the simulation of the turbulent Rayleigh-Benard problem on a 124 x 64 x 64 grid yielded results in good agreement with experiments; these are reported in ICASE Report No. 86-6. Statistical analysis of the simulation data is continuing to assess the effect of buoyancy on subgrid scale modelling, etc. Another fundamental stability problem under study is the Taylor-Couette flow in a very short annulus allowing for an exchange between two-cell and single-cell modes. Of immediate interest is the temporal evolution of the order parameter which quantifies the symmetry-breaking bifurcation of a symmetric two-cell flow into an asymmetric single-cell flow. The long term objective is the simulation of the three-dimensional processes leading to transition.

Our investigation of the receptivity problem is continuing. We have constructed an accurate and efficient spectral method of solving the three-dimensional compressible Navier-Stokes equations relevant to certain physical experiments. The purpose is to assess the effect of Mach number on the instability mechanisms identified in low-speed flows.

D. Bushnell (Viscous Flow Division, LaRC) proposes a method of suppressing the lift-induced drag. It consists of essentially pushing down the front stagnation point and pushing the rear stagnation point up on the upper surface. This concept is being tested in the framework of the two-dimensional compressible Navier-Stokes simulation.

This program of research is being carried out in collaboration with G. Erlebacher, T. A. Zang (High-Speed Aerodynamics Division, LaRC), and C. L. Streett (Transonic Aerodynamics Division, LaRC).

Kazufumi Ito and Robert K. Powers

Efforts are continuing in the application of Chandrasekhar-type algorithms to infinite dimensional linear quadratic regulator (LQR) problems that have
unbounded inputs and outputs (i.e., boundary control and observation). Approximation of the associated gain operators via the Chandrasekhar equations has been shown to be more computationally efficient than approximation via the Riccati equation for many problems with bounded inputs and outputs. The features of the Chandrasekhar equations that make them computationally efficient are present in many problems involving boundary controls and observations. Numerical techniques for solving such problems are being sought. Research on application of these ideas to LQR problems with unbounded input and output operators is also being pursued.

Tom L. Jackson

An earlier paper titled "Shock-Induced Thermal Runaway" (SIAM J. Appl. Math., 1985) describes the initial stage in the birth of a detonation wave. The ignition stimulus was modelled by impulsively moving a piston into a combustible gas confined within a tube at constant speed. This creates a shock wave which runs ahead of the piston and ignites the gas at the piston face. The creation of the hot spot and its growth into a full-fledged explosion in the vicinity of the piston face was described. We are currently investigating techniques to develop the solution further in time, describing the complete history of transition to ignition. This work is being done in collaboration with A. K. Kapila.

Tom L. Jackson and Pravir Dutt

A mathematical model is being constructed to examine the combustion process that occurs when two gases, initially separated by a finite length splitter plate, come into contact and mix at the trailing edge. For large Reynolds numbers, the trailing edge region can be represented analytically by the so-called "triple deck" model of Stewartson and Messiter. It is found that mixing and ignition takes place only in the lower deck and can be described by the standard boundary layer equations subject to unusual boundary conditions. A code is currently being constructed to solve this set of equations. This work is being done in collaboration with M. Y. Hussaini.
Harry Jordan

The parallel programming environment known as the Force and described in ICASE Report 85-45 has been implemented under the ConCurrent Fortran system on the FLEX/32 computer. Programs previously run on the HEP multiprocessor have been used to test the implementation of the system. Work is currently underway to parallelize some of the Fortran routines in the NICE/SPAR system using the parallel extensions embodied in the Force. Of particular interest is whether extensions to the Force system are indicated as a result either of the FLEX/32 hardware and performance parameters or of the type of algorithms appearing in NICE/SPAR.

William D. Lakin

Instability of a two-dimensional boundary layer flow through the interaction of inviscid- and viscous-type disturbances. The basic flow in this investigation is a two-dimensional, parallel boundary layer profile with non-zero mean velocity components in the streamwise and spanwise directions. When a small amplitude, three-dimensional, inviscid-type disturbance is imposed on this mean flow, the equations governing the amplitudes of the velocity and pressure perturbations can be reduced to a Rayleigh equation involving two wavenumbers. The present work considers possible instabilities due to the interaction of these inviscid-type disturbances with viscous-type disturbances.

The initial focus of this work has been on understanding the nature of the inviscid disturbances. Frobenius-type solutions of the Rayleigh equation have been derived. The asymptotic behavior of the inviscid disturbances has also been obtained making possible determination of the appropriate combination of inviscid perturbations which remains bounded in the free stream. The possibility of resonant inviscid triads is currently being investigated. This work is being done in collaboration with P. Hall and M. Y. Hussaini.

Determination of realistic modal damping coefficients for a rotor-airframe helicopter model. Theoretical investigations of the forced response of a helicopter, modeled as a rotor-airframe structure, typically are based on a
finite-element model in which the modal damping coefficients are taken to be uniform for all modes. Recent comparison of theoretical predictions with experimental data shows that uniform modal damping produces poor correlation between theory and experiment. The present work investigates the feasibility of determining realistic, non-uniform damping coefficients which will bring about close correlation with experimental response curves in frequency bands away from resonant peaks. This work is being done jointly with Ray Kvaternik (Loads and Aeroelasticity Division, LaRC).

Liviu Lustman

The steady state elasticity problem for composite materials has been solved for the simple brick geometry. The compact scheme equations are solved by conjugate gradient. The numerical results compare favorably with calculations performed on the same problem by other algorithms.

Work is in progress on the more realistic problem of a plate with stiffeners, which can be reduced to the previous one—the domain considered may be decomposed into two bricks with a common interface. The boundary data for this problem are to be supplied from a two-dimensional finite element code, and the results—a three-dimensional analysis—may validate the two-dimensional model and add details to it. This work has been performed in collaboration with M. E. Rose and S. Ta'asan.

Joint research with D. Gottlieb and E. Tadmor has continued on spectral collocation methods applied to initial-boundary value problems for hyperbolic systems. We can prove convergence of the spectral approximations to the solutions of the differential equations.

The study of optimal numerical boundary conditions for steady state Navier-Stokes is continuing, in collaboration with S. Abarbanel and A. Bayliss. We have identified the parameters for channel flow, using a model O.D.E. eigenvalue problem. Test calculations are in progress for the full Navier-Stokes code with these novel boundary conditions.
Abrupt and drastic changes of structure can often occur in vortex flows. Theoretical investigations of such phenomena have focused on axisymmetric processes. Nevertheless, it has been experimentally verified that all transitions occurring in concentrated vortex flows are three-dimensional in nature. We will try and simulate numerically the transition sequence that has been observed experimentally in the swirling flow of an incompressible fluid in a pipe. Two fundamental parameters govern such a flow, namely, the amount of swirl imparted to the flow and its Reynolds number. As these parameters are altered from low values, flow visualization studies indicate that the axisymmetry of the initial laminar flow is broken and, while the ultimate form of transition to the "bubble" type of "vortex breakdown" (encountered when the governing parameters reach their largest values) appears to the eye to be axisymmetric, it is not, nor are any of the other structures that appear at lower values of the parameters.

Our investigation will focus on the three-dimensional aspects of these flows. The numerical method employed in the study will be a spectral one that is ideally suited for the simulation of incompressible flows. The velocity of the fluid is expanded in a series of divergence free vectors which ensures that the continuity equation for an incompressible fluid is satisfied exactly and the pressure term is eliminated from the Navier-Stokes equation by projecting it onto the space of inviscid vector fields. By choosing appropriate polynomials for the components of the divergence free vectors, we can achieve "exponential accuracy" for the components of the velocity. Such expansions have been used successfully in the past to solve certain stability problems and also for the simulation of fluid flows such as the well known Taylor-Couette flow.

Piyush Mehrotra and John Van Rosendale

BLAZE is a scientific programming language designed at ICASE during the last two years. It has a Pascal-style syntax, but employs functional procedure calls, making it relatively easy to restructure BLAZE programs for
execution on multiprocessor architectures (The BLAZE Language, ICASE Report No. 85-29). There are several advantages to this approach. It frees programmers from the necessity of dealing with the complex parallel run-time environment implicit in the use of explicit-tasking languages, such as Ada or Occam, and also allows the expression of parallel scientific programs at a higher and more natural level.

A BLAZE compiler which produces sequential code for Vaxes is currently available, and a number of parallel implementations of BLAZE are being developed. We are currently in the process of defining Version 2 of BLAZE. This will differ from the current language primarily through the introduction of abstractions and compilation modules. Using abstractions, one will be able to construct large objects, such as files, which have their state retained between procedure calls making it easier to write large scientific programs in BLAZE.

Vijay K. Naik

In the past two years, a variety of general purposes MIMD architectures has appeared as commercial products. Many algorithms suitable for parallel processing have been proposed. The development, on parallel machines, of application codes for solving problems of practical importance are also reported. In spite of these, several difficulties exist in implementing nontrivial problems efficiently on the multiprocessor systems. These difficulties are primarily because of the lack of methodologies and tools that can automate the process of matching the rates and the amounts of data that must be transferred among the various computational tasks with the structure and the capacity of the underlying interconnection network.

The study undertaken here is aimed at developing a methodology that will allow one to characterize MIMD architectures and will quantify the architecture and algorithm dependent parameters that contribute to the communication delays. Various existing, as well as proposed, architectures and important algorithms are currently under investigation. The outcome of this research will help towards the automation process of efficient parallel
implementations so that the user need not know the detailed anatomy of the underlying architecture. The research currently in progress with Shlomo Ta'asan (described below) addresses some of the issues involved in solving scientific problems on various currently existing architectures.

**Vijay K. Naik and Shlomo Ta'asan**

Algorithms based on multigrid techniques are known to be among the most efficient methods for solving partial differential equations. On sequential machines, a wide class of problems discretized on a grid with $n$ points requires only $O(n)$ arithmetic operations to obtain a solution to within the truncation error of the discretization. This makes the multigrid methods optimal on the sequential machines. This optimal property and the fact that many aspects of the multigrid algorithms are highly parallelizable, make these methods attractive for implementation on multi-processor systems. Several researchers have studied the effects of parallelizing the intra-grid and inter-grid operations. Some researchers have proposed special purpose architectures that allow one to map some important problems directly onto the hardware and thus reduce the communication delay effects. But none of these studies have conclusively shown that the optimality observed on the sequential machines is conserved in time and the number of processors needed when these methods are implemented on multiprocessors, nor is there any clear understanding about the communication and the synchronization costs involved.

The work undertaken here is to explore the above mentioned questions further for shared and non-shared memory architectures. The effort is aimed at studying the effects of various communication strategies and also at understanding the effects of various implementations on the convergence rates. We are also studying the advantages and disadvantages of using a large number of simple processors and those of using a small number of powerful processors. The model problem considered here is that of solving 2-D incompressible Navier-Stokes equations in vorticity-stream function formulation. Experiments are being carried out on commercially available MIMD machines such as the Intel Hyper-cube, Sequent, Encore, and Flex/32. In
addition to these studies, investigation is underway towards the development of new algorithms that possess some of the numerical properties of multigrid methods but do not require tight synchronizations.

David M. Nicol

A large class of parallel computations have run-time behavior which may vary in time. Examples of such computations include many numerical methods for solution of scientific problems and parallel simulations. Because of this stochastic variance, a good partitioning of the workload between processors can drift out of balance, leading to degraded performance. This performance degradation gives rise to two fundamental problems: (1) how to model this phenomenon, and (2) how to detect and respond to performance degradation due to stochastic variation. Future efforts will be focused on both of these issues.

Merrell L. Patrick

Computing systems of the future for large scale scientific and engineering computations will consist of many processors and memories working in parallel to solve a single problem. Efficient use and ease of use of these future systems are two basic problems facing potential users. The development of new parallel algorithms, parallel execution time models which predict the performance of the algorithms on different parallel architectures, and parallel programming environments for expressing the algorithms are essential for progress toward the solution of these problems.

We are developing parallel algorithms and corresponding parallel execution time models for solving the generalized eigenvalue problem. Results from the models will be used to predict the performance of the algorithms on message passing and shared memory MIMD machines. The parallel algorithms will be integrated into a structural analysis system, NICE/SPAR, running on the FLEX/32 as a program module used in the dynamic analysis of a structure in motion.
The algorithms developed will be used as a basis to continue work with T. Pratt and H. Jordan on defining the essential ingredients of parallel programming languages for scientific computations. Different programming paradigms will be used to program these algorithms on the FLEX/32 system.

Recent work with L. Adams and D. Reed determined optimal stencil/partition pairs for solving elliptic partial differential equations on MIMD machines. The optimality condition was to maximize the computation to communication ratio. Parallel execution time models were developed for message passing and shared memory architectures to predict the impact of the architecture on the choice of stencil/partition pairs. The results of the models will be validated using various available machines such as the FLEX/32, Sequent Balance 8000, and the Connection Machine.

A new study was begun with Saltz and Voigt which attempts to answer the question as to whether SIMD architectures are sufficient for carrying out most computations arising in the solution of partial differential equations. A thorough review of fundamental algorithms used in solving partial differential equations will be conducted in an attempt to identify those which require MIMD architectures.

Doug Peterson

The VAX and Ridge computer systems, as well as their installation configurations, both hardware and software have been learned. The Talaris laser printer has been brought on-line, and has been used with ditroff, pic, and System V graphics. The VAX has provided user support to the NAS project by sharing the tape drive with the Iris workstation as a network resource and as an interactive gateway to the computing resources at Ames Research Center through the micom switching system.

To better allocate ICASE computing resources, a patch panel was designed and installed in the VAX room to provide quick and reliable interconnections between user terminals, the VAX, and micom. The system has been documented as both a set of floor plan drawings and as a table of connections. To provide for future updates, the drawings are done with the Macintosh using MacDraw
software, and the tables are kept on the VAX. A similar set of floor plans has been developed for the ICASE office staff to aid in developing seating plans.

I have continued to provide consultation and support for other UNIX system administrators, both at Langley and other locations. User support for Langley is being absorbed by the Analysis and Computation Division, and I expect to reduce that activity at ICASE. However, the locally prepared manual "Introduction to Computing at ICASE" is being updated. I have offered to participate in the addition of UNIX systems to the LarcNet.

Plans have been made to replace the VAX with a system of Sun 3 diskless workstations and a file server, connected via ethernet. Delivery is expected by May 9, 1986, with installation on or about May 15, 1986.

Terrence W. Pratt

The PISCES parallel programming environment allows the scientific programmer to write parallel programs that are intended to run on various parallel architectures. Programming is in PISCES Fortran (an extension of Fortran 77) under the Unix operating system. Several "granularities" of parallel operations are provided.

The PISCES 1 environment has been implemented on the ICASE VAX 750 and on a network of Apollo Domain workstations at the University of Virginia. In PISCES 1, a program is organized in clusters of tasks that communicate by passing messages.

The PISCES 2 environment is being designed and implemented for the FLEX/32 at NASA Langley. PISCES 2 extends PISCES 1 to include the "force" constructs developed by H. Jordan which allow the programmer to make effective use of the shared memory of the FLEX/32. PISCES 2 also allows the programmer to control the mapping from PISCES clusters to the hardware processors.
Georg Propst

Approximation schemes for hereditary differential equations are studied. In order to prove applicability of the approximations to control problems, strong convergence of the adjoint systems is required. One subject of the current work is the investigation of the adjoints of a class of spline approximations. Another subject is a scheme that uses discontinuous piecewise linear functions and has favorable stability properties. If the cost functional ranges over an infinite time horizon, the proof of convergence of the approximating controls relies on the nonsingularity of the weight on the state trajectory. The necessity of this assumption is investigated.

Joel H. Saltz, Vijay Naik, and David M. Nicol

The efficient implementation of algorithms on multiprocessor machines requires that the effects of communication delays be minimized. The effects of these delays on the performance of a model problem on a hypercube multiprocessor architecture is investigated, and methods are developed for increasing algorithm efficiency. The model problem under investigation is the solution by red-black Successive Over Relaxation of the heat equation; most of the techniques to be described here also apply equally well to the solution of elliptic partial differential equations by red-black or multicolor SOR methods.

A complete report identifies methods for reducing communication traffic and overhead on a multiprocessor and presents the results of testing these methods on the Intel iPSC Hypercube. We examine methods for partitioning a problem's domain across processors, for reducing communication traffic during a global convergence check, for reducing the number of global convergence checks employed during an iteration, and for concurrently iterating on multiple time-steps in a time dependent problem. Our empirical results show that use of these methods can markedly reduce a numerical problem's execution time.
Joel H. Saltz and David M. Nicol

We consider the tradeoffs between the benefits and costs of balancing load in multiprocessor solutions of computational problems that describe the evolution of a physical system over the course of time. The type of architecture being assumed is one in which each processor has a local memory and that the processors pass messages through either a communications network or a shared memory. The responsibility for computations pertaining to a given portion of problem domain is assigned to each processor. The data dependencies inherent in the solution of a time dependent problem cause the most heavily loaded processor to limit the rate at which the problem is solved. Frequently, the load in the portion of the problem assigned to each processor will vary, and without redistribution of load, system performance declines. While performance can be improved by a redistribution of load, this redistribution itself exacts a sometimes large delay cost.

Two probabilistic load models are proposed and used to evaluate policies for deciding when load balancing should be performed. Through use of these models, it is possible to characterize the relative performance of a number of different policies designed to determine when load should be balanced. This performance characterization is carried out through the derivation of analytic expressions and through simulation. Estimates of how the cost of balancing load interact with model parameters describing the number and activity of processors are obtained. A description of this work entitled: "A Statistical Methodology for the Control of Dynamic Load Balancing" is to be published in the SIAM publication *Parallel Processing and Medium Scale Multiprocessors*.

Joel H. Saltz and Hillel Tal-Ezer

The use of spectral methods in space to solve time dependent problems makes the investigation of efficient high accuracy schemes to march the solution forward in time quite attractive. Richardson extrapolation of leapfrog and Crank Nicholson time integration methods for the solution of these partial differential equations can produce efficient high order schemes and can be used in a number of different ways to facilitate efficient
solutions of spectral problems on a variety of multiprocessor architectures. Our investigation takes into account both numerical convergence as well as algorithm performance on message passing and shared memory multiprocessors.

A detailed analysis is carried out of a model hyperbolic problem solved by the pseudospectral Fourier method in space and extrapolated Leap-Frog scheme in time. Estimates of parallel efficiency are calculated from the analysis of several ways of mapping the model problem onto a range of message passing and shared memory architectures. Given a desired error tolerance for time integration, taking into account the multiprocessor machine architecture, and the number of mesh points used to solve the model problem, we have developed a model that predicts the optimal number of extrapolations that should be carried out. Experimental results on the convergence and stability of the extrapolated algorithms have been encouraging, and an experimental analysis of the performance of these algorithms on the Flex/32 multiprocessor is being carried out.

Ke-Gang Shih

Study of a third order nonlinear ordinary differential equation describing laminar flow through porous channels with injection is reported in ICASE Report 86-17.

Problems under consideration are nonlinear singular perturbation problems of chemical kinetics, enzyme kinetics, etc. We hope to get good estimates to help the numerical treatment of such equations.

Shlomo Ta'asan

Multigrid treatment of highly oscillatory problems, such as exterior problems in acoustics, is the subject of another study. The main idea here is to use a ray representation of the solution (similar to geometrical optics) on coarse grids, which allows a natural treatment of radiation boundary conditions as well as other advantages. This work is in collaboration with A. Brandt (Weizmann Institute).
Another direction of multigrid development is for system identification. Here the problem is to estimate certain coefficients in an elliptic equation for which a right-hand side and a solution are given. Estimation of the coefficient is needed under some minimization constraint.

**Eitan Tadmor**

Work on the approximate solution of the gas dynamics and other conservative systems is underway. We have implemented our proposed second order accurate nonoscillatory central differencing code obtaining very reliable results. Research is continuing on the application of entropy stable approximations to those systems based on the entropy conservative finite element method devised earlier. In particular, ideas of compensated compactness are used in order to control oscillations. A minimum (therodynamic) entropy principle was derived for approximate Euler solvers.

We have developed a stability theory for (pseudo-) spectral approximations for the approximate solution to hyperbolic mixed initial-boundary value systems with D. Gottlieb and L. Lustman. Our stability normal-mode analysis is demonstrated for Chebyshev and Legendre collocation methods, and consequently convergence follows. Together with M. Goldberg (Israel Institute of Technology), we have devised new, easily checkable sufficient stability criteria for difference approximations to such mixed systems, extending previous work.

**Hillel Tal-Ezer**

In previous ICASE reports (84-8, 85-9), we have described a spectral algorithm (both in time and space) for solving partial differential equations. It has been shown to be an efficient tool for problems when the eigenvalues of the spatial operator have a specific orientation: either close to the imaginary axis or close to the negative real axis. Present research is focused on implementing the approach discussed in 84-8 for these kind of problems.
Work is continuing with C. Swanson (Transonic Aerodynamics Division, LaRC) on the properties of residual smoothing. The effects of the Navier-Stokes terms were analyzed. In addition, we studied the application of a Crocco type scheme to accelerate the convergence of the Runge-Kutta algorithm for viscous flow. Computations demonstrated that increased time steps could be realized for both laminar and turbulent problems. An ICASE report was completed on this. Further work is continuing on the analysis of residual smoothing for nonuniform meshes. Work is also continuing with V. Vatsa (Transonic Aerodynamics Division, LaRC) on the three dimensional viscous code. An ICASE report was completed with A. Kumar (High-Speed Aerodynamics Division, LaRC) and A. Moitra (Systems and Applied Sciences) on the extension of Runge-Kutta schemes to nozzle geometries for both subsonic and supersonic flows.

A joint project with S. Yaniv and U. Landau (Tel Aviv University) was begun to study the effect of nonuniform meshes on the accuracy and convergence rates of schemes. It was found that for some finite volume schemes that the method could even be inconsistent for arbitrary meshes. In general, when the meshes are not sufficiently smooth, there is a loss of accuracy in physical space even though the accuracy in computational space is not affected. Cell centered schemes are more sensitive to nonsmooth grids than nodal schemes. A way of increasing the accuracy by accounting for the nonuniform mesh while keeping conservation was proposed. Two ICASE reports were completed.

The study of preconditionings for slow flow was continued. Comparisons with incompressible preconditionings were also studied. Several different preconditionings for both compressible and incompressible steady flow were compared. An ICASE report was completed.

Joint work is continuing with A. Bayliss, L. Maestrello (Transonic Aerodynamics Division, LaRC) and P. Parikh (Vigyan Research Associates). This is a project to analyze time dependent compressible laminar viscous boundary layers by numerically solving the complete Navier-Stokes equations. A Fourth order, in space, code is used to resolve the unstable modes. Extensions were completed to treat flows over several ramp-like geometries. Unstable modes were found for several of these cases when the inflow was perturbed by a time periodic disturbance. Both passive and active controls could be found to
stabilize the flow over the variable geometry domains. Several ICASE reports appeared describing the method and the computational results of the control. The three dimensional code has been debugged and preliminary results have been obtained.

A methodology, called the force, supports the construction of programs to be executed in parallel by a force of processes. The number of processes in the force is unspecified but potentially very large. The force idea is embodied in a set of macros which produce multiprocessor Fortran code and has been studied on two shared memory multiprocessors of fairly different character. The method has simplified the writing of highly parallel programs within a limited class of parallel algorithms and is being extended to cover a broader class. This paper deals with the individual parallel constructs which comprise the force methodology. Of central concern are their semantics, implementation on different architectures, and performance implications.


The instabilities of some spatially and/or time-periodic flows are discussed; in particular, flows with curved streamlines which can support Taylor-Gortler vortices are described in detail. The simplest flow where this type of instability can occur is that due to the torsional oscillations of an infinitely long circular cylinder. For more complicated spatially varying time-periodic flows, a similar type of instability can occur and is spatially localized near the most unstable positions. When nonlinear effects are considered, it is found that the instability modifies the steady streaming boundary layer induced by the oscillatory motion. It is shown that a rapidly rotating cylinder in a uniform flow is susceptible to a related type of instability; the appropriate stability equations are shown to be identical to those which govern the instability of a Boussinesq fluid of Prandtl number unity heated time periodically from below.


The Taylor-Gortler vortex instability equations are formulated for steady and unsteady interacting boundary layer flows of the type which
arise in triple-deck theory. The effective Gortler number is shown to be a function of the wall shape in the boundary layer, and the possibility of both steady and unsteady Taylor-Gortler modes exists. As an example the steady flow in a symmetrically constricted channel is considered, and it is shown that unstable Gortler vortices exist before the boundary layers at the wall develop the Goldstein singularity discussed by Smith and Daniels (1981). As an example of an unsteady spatially varying basic state, we also consider the instability of high frequency large amplitude Tollmien-Schlichting waves in a curved channel. It is shown that they are unstable in the first "Stokes layer stage" of the hierarchy of nonlinear states discussed by Smith and Burggraf (1985). The Tollmien-Schlichting waves are shown to be unstable in the presence of both convex and concave curvature.


During L/U decomposition of a sparse matrix, it is possible to perform computation on many diagonal elements simultaneously. Pivots that can be processed in parallel are related by a compatibility relation and are grouped in a compatible set. The collection of all maximal compatibles yields different maximum sized sets of pivots that can be processed in parallel. Generation of the maximal compatibles is based on the information obtained from an incompatible table. This table provides information about pairs of incompatible pivots. In this paper, generation of the maximal compatibles of pivot elements for a class of small sparse matrices is studied first. The algorithm involves a binary tree search and has a complexity exponential in the order of the matrix. Different strategies for selection of a set of compatible pivots based on the Markowitz criterion are investigated. The competing issues of parallelism and fill-in generation are studied and results are provided. A technique for obtaining an ordered compatible set directly from the ordered incompatible table is given. This technique generates a set of compatible pivots with the property of generating few fills. A new heuristic algorithm is then proposed that combines the idea of an ordered compatible set with a limited binary tree search to generate several sets of compatible pivots in linear time. Finally, an elimination set to reduce the matrix is selected. Parameters are suggested to obtain a balance between parallelism and fill-ins. Results of applying the proposed algorithms on several large application matrices are presented and analyzed.

The physical properties of the commonly used second-order closure models are examined theoretically for rotating turbulent flows. Comparisons are made with results which are a rigorous consequence of the Navier-Stokes equations for the problem of fully-developed turbulent channel flow in a rapidly rotating framework. It is demonstrated that all existing second-order closures yield spurious physical results for this test problem of rotating channel flow. In fact, the results obtained are shown to be substantially more unphysical than those obtained from the simpler K-ε and K-λ models. Modifications in the basic structure of these second-order closure models are proposed which can alleviate this problem.


Consider the initial-boundary value problem for Burgers' equation. It is shown that its solution converge for t, to a unique steady state. The speed of the convergence depends on the boundary conditions and can be exponentially slow. Methods to speed up the rate of convergence are also discussed.


Discrete approximations to hyperbolic systems of conservation laws are studied. We quantify the amount of numerical viscosity present in such schemes, and relate it to their entropy stability by means of comparison. To this end, conservative schemes which are also entropy conservative are constructed. These entropy conservative schemes enjoy second-order accuracy; moreover, they admit a particular interpretation within the finite-element framework, and hence can be formulated on various mesh configurations. We then show that conservative schemes are entropy stable, if and--for three-point schemes--only if, they contain more viscosity than that present in the mentioned above entropy conservative ones.


We consider the effect of localized, time-periodic surface heating and cooling over a curved surface. This is a mechanism for the active control of unstable disturbances by phase cancellation and reinforcement. It is
shown that the pressure gradient induced by the curvature significantly enhances the effectiveness of this form of active control. In particular, by appropriate choice of phase, active surface heating can completely stabilize an unstable wave.


We match formal asymptotic expansions with differently scaled variables to obtain a uniform approximation to the similarity solution of the shock-wedge diffraction problem.


The problem of optimally assigning the modules of a parallel program over the processors of a multiple computer system is addressed. A Sum-Bottleneck path algorithm is developed that permits the efficient solution of many variants of this problem under some constraints on the structure of the partitions.

In particular, the following problems are solved optimally for a single-host, multiple satellite system: Partitioning multiple chain-structured parallel programs, multiple arbitrarily structured serial programs and single tree structured parallel programs. In addition, the problems of partitioning chain structured parallel programs across chain connected systems and across shared memory (or shared bus) systems are also solved under certain constraints. All solutions for parallel programs are equally applicable to pipelined programs.

These results extend prior research in this area by explicitly taking concurrency into account and permit the efficient utilization of multiple computer architectures for a wide range of problems of practical interest.


We consider the partitioning of a problem on a domain with unequal work estimates in different subdomains in a way that balances the work load
across multiple processors. Such a problem arises for example in solving partial differential equations using an adaptive method that places extra grid points in certain subregions of the domain. We use a binary decomposition of the domain to partition it into rectangles requiring equal computational effort. We then study the communication costs of mapping this partitioning onto different multiprocessors: a mesh-connected array, a tree machine and a hypercube. The communication cost expressions can be used to determine the optimal depth of the above partitioning.


Growth of unstable disturbances in a high Reynolds number compressible boundary layer is numerically simulated. Localized periodic surface heating and cooling as a means of active control of these disturbances is studied. It is shown that compressibility in itself stabilizes the flow but at a lower Mach number, significant nonlinear distortions are produced. Phase cancellation is shown to be an effective mechanism for active boundary layer control.


This paper deals with nearly singular, possibly indefinite problems for which the usual multigrid solvers converge very slowly or even diverge. The main difficulty is related to some badly approximated smooth functions which correspond to eigenfunctions with nearly zero eigenvalues. A correction to the usual coarse-grid equations is derived, both in Correction Scheme and in Full Approximation Scheme. The performance of the new algorithm using this correction is essentially as that of usual multigrid for definite problems.

A Chandrasekhar-type factorization method is applied to the linear-quadratic optimal control problem for distributed parameter systems. An aeroelastic control problem is used as a model example to demonstrate that if computationally efficient algorithms, such as those of Chandrasekhar-type, are combined with the special structure often available to a particular problem, then an abstract approximation theory developed for distributed parameter control theory becomes a viable method of solution. A numerical scheme based on averaging approximations is applied to hereditary control problems. Numerical examples are given.


The effect of non-uniform grids on the solution of the Euler equations is analyzed. We consider a Runge-Kutta type scheme based on a finite volume formulation. We show that for arbitrary grids the scheme can be inconsistent even though it is second-order accurate for uniform grids. An improvement is suggested which leads to at least first-order accuracy for general grids. Test cases are presented in both two- and three-space dimensions. Applications to finite difference and implicit algorithms are also given.


Downstream marching iterative schemes for the solution of the Parabolized or Thin Layer (PNS or TL) Navier-Stokes equations are described. Modifications of the primitive equation global relaxation sweep procedure result in efficient second-order marching schemes. These schemes take full account of the reduced order of the approximate equations as they behave like the SLOR for a single elliptic equation. The improved smoothing properties permit the introduction of Multi-Grid acceleration. The proposed algorithm is essentially Reynolds number independent and therefore can be applied to the solution of the subsonic Euler equations. The convergence rates are similar to those obtained by the Multi-grid solution of a single elliptic equation; the storage is also comparable as only the pressure has to be stored on all levels. Extensions to three-dimensional and compressible subsonic flows are discussed. Numerical results are presented.

This is a report of the results of an experiment: to adapt a Navier-Stokes code, originally developed on a serial computer, to concurrent processing on the ICL Distributed Array Processor (DAP). In this paper the algorithm used in solving the Navier-Stokes equations is briefly described. The architecture of the DAP and DAP Fortran is also described. The modifications of the algorithm so as to fit the DAP are given and discussed. Finally, performance results are given and conclusions are drawn.


Spectral methods have been successfully applied to the simulation of slow transients in gas transportation networks. Implicit time advancing techniques are naturally suggested by the nature of the problem.

The aim of this paper is to clarify the correct treatment of the boundary conditions in order to avoid any stability restriction originated by the boundaries. The Beam and Warming and the Lerat schemes are unconditionally linearly stable when used with a Chebyshev pseudospectral method. Engineering accuracy for a gas transportation problem is achieved at Courant numbers up to 100.


We treat a constant coefficient hyperbolic system in one space variable, with zero initial data. Dissipative boundary conditions are imposed at the two points \( x = \pm 1 \). This problem is discretized by a spectral approximation in space. We demonstrate sufficient conditions under which the spectral numerical solution is stable—moreover, these conditions have to be checked only for scalar equations. The stability theorems take the form of explicit bounds for the norm of the solution in terms of the boundary data. The dependence of these bounds on \( N \), the number of points in the domain (or equivalently the degree of the polynomials involved), is investigated for a class of standard spectral methods, including Chebyshev and Legendre collocations.

Godunov's method and several other methods for computing solutions to the equations of gas dynamics use Riemann solvers to resolve discontinuities at the interface between cells. A new method is proposed here for solving the Riemann problem based on a global existence proof for the solution to the Riemann problem. The method is found to be very reliable and computationally efficient.


The efficient implementation of algorithms on multiprocessor machines requires that the effects of communication delays be minimized. The effects of these delays on the performance of a model problem on a hypercube multiprocessor architecture is investigated and methods are developed for increasing algorithm efficiency. The model problem under investigation is the solution by red-black Successive Over Relaxation [YOUN71] of the heat equation; most of the techniques to be described here also apply equally well to the solution of elliptic partial differential equations by red-black or multicolor SOR methods.

This paper identifies methods for reducing communication traffic and overhead on a multiprocessor, and reports the results of testing these methods on the Intel iPSC Hypercube. We examine methods for partitioning a problem's domain across processors, for reducing communication traffic during a global convergence check, for reducing the number of global convergence checks employed during an iteration, and for concurrently iterating on multiple time-steps in a time-dependent problem. Our empirical results show that use of these models can markedly reduce a numerical problem's execution time.


A multi-stage Runge-Kutta method is analyzed for solving the two-dimensional Euler equations for external and internal flow problems. Subsonic, supersonic, and highly supersonic flows are studied. Various techniques for accelerating the convergence to a steady state are described
and analyzed. Effects of the grid aspect ratio on the rate of convergence are evaluated. An enthalpy damping technique applicable to supersonic flows is described in detail. Numerical results for supersonic flows containing both oblique and normal shocks are presented confirming the efficiency of the method.


The three-dimensional, incompressible Navier-Stokes and energy equations with the Boussinesq assumption have been directly simulated at a Rayleigh number of $3.8 \times 10^5$ and a Prandtl number of 0.76. In the vertical direction, wall boundaries were used and in the horizontal, periodic boundary conditions were used. A spectral/finite difference numerical method was used to simulate the flow. The flow at these conditions is turbulent and a sufficiently fine mesh was used to capture all relevant flow scales. The results of the simulation are compared to experimental data to justify the conclusion that the small scale motion is adequately resolved.


A central problem to parallel processing is the determination of an effective partitioning of workload to processors. The effectiveness of any given partition is dependent on the stochastic nature of the workload. We treat the problem of determining when and if the stochastic behavior of the workload has changed enough to warrant the calculation of a new partition. We model the problem as a Markov decision process, and derive an optimal decision policy. Quantification of this policy is usually intractable; we empirically study a heuristic policy which performs nearly optimally. Our results suggest that the detection of change is the predominant issue in this problem.

We present here a convergence proof for spectral approximations for hyperbolic systems with initial and boundary conditions. We treat in detail Chebyshev collocation, but the final result is readily applicable to other spectral methods, such as Legendre collocation or tau-methods.


A small perturbation analysis, in the long wavelength regime, is used to obtain the downstream boundary condition for the pressure for the flow over a flat plate. The methodology is extendable to other geometries. Numerical results for high Reynolds number laminar flows show great improvement in convergence rate to steady state as well as in the quality of the results.


In this paper we consider a model problem that simulates an atmospheric acoustic wave propagation situation that is nonlinear. The model is derived from the basic Euler equations for the atmospheric flow and from the regular perturbations for the acoustic part. The nonlinear effects are studied by obtaining two successive linear problems in which the second one involves the solution of the first problem. Well-posedness of these problems is discussed and approximations of the radiation boundary conditions that can be used in numerical simulations are presented.


We discuss techniques for the estimation of nonlinearities and state-dependent coefficients in parabolic differential equations. Applications to density-dependent population dispersal and nonlinear growth/predation models are presented. Computational results using parallel and vector architectures are discussed.
Integrating and differentiating matrices allow the numerical integration and differentiation of functions whose values are known at points of a discrete grid. Previous derivations of these matrices have been restricted to one-dimensional grids or to rectangular grids with uniform spacing in at least one direction. The present work develops integrating and differentiating matrices for grids with non-uniform spacing in both directions. The use of these matrices as operators to reformulate boundary value problems on rectangular domains as matrix problems for a finite-dimensional solution vector is considered. The method requires non-uniform grids which include "near-boundary" points. An eigenvalue problem for the transverse vibrations of a simply supported rectangular plate is solved to illustrate the method.

We consider the problem of uniformly distributing the load of a parallel program over a multiprocessor system. We analyze a program whose structure permits the computation of the optimal static solution. We then describe four strategies for load balancing and compare their performance.

The four strategies are: (1) the optimal static assignment algorithm which is guaranteed to yield the best static solution, (2) the static binary dissection method which is very fast but sub-optimal, (3) the greedy algorithm, a static fully polynomial time approximation scheme, which estimates the optimal solution to arbitrary accuracy, and (4) the predictive dynamic load balancing heuristic which uses information on the precedence relationships within the program and outperforms any of the static methods.

It is also shown that the overhead incurred by the dynamic heuristic (4) is reduced considerably if it is started off with a static assignment provided by either (1), (2), or (3).

Acceleration methods are presented for solving the steady state incompressible equations. These systems are preconditioned by introducing artificial time derivatives which allow for a faster convergence to the
steady state. We also consider the compressible equations in conservation form with slow flow. Two arbitrary functions $\alpha$ and $\beta$ are introduced in the general preconditioning. An analysis of this system is presented and an optimal value for $\beta$ is determined given a constant $\alpha$. It is further shown that the resultant incompressible equations form a symmetric hyperbolic system and so are well-posed. Several generalizations to the compressible equations are presented which generalize previous results.


An abstract approximation framework is developed for the finite and infinite time horizon discrete-time linear-quadratic regulator problem for systems whose state dynamics are described by a linear semigroup of operators of an infinite dimensional Hilbert space. The schemes included in the framework yield finite dimensional approximations to the linear state feedback gains which determine the optimal control law. Convergence arguments are given. Examples involving hereditary and parabolic systems and the vibration of a flexible beam are considered. Spline-based finite element schemes for these classes of problems, together with numerical results, are presented and discussed.


In the optimal linear quadratic regulator problem for finite dimensional systems, the method known as an $\alpha$-shift can be used to produce a closed-loop system whose spectrum lies to the left of some specified vertical line; that is, a closed-loop system with a prescribed degree of stability. This paper treats the extension of the $\alpha$-shift to hereditary systems. As in finite dimensions, the shift can be accomplished by adding $\alpha$ times the identity to the open-loop semigroup generator and then solving an optimal regulator problem. However, this approach does not work with a new approximation scheme for hereditary control problems recently developed by Kappel and Salamon. Since this scheme is among the best to date for the numerical solution of the linear regulator problem for hereditary systems, an alternative method for shifting the closed-loop spectrum is needed. An $\alpha$-shift technique that can be used with the Kappel-Salamon approximation scheme is developed. Both the continuous-time and discrete-time problems are considered. A numerical example which demonstrates the feasibility of the method is included.

We establish the existence of concave solutions of Berman's equation which describes the laminar flow in channels with injection through porous walls. It is found that the (unique) concave solutions exist for all injection Reynolds number $R < 0$.


This collection of papers covers some recent developments in numerical analysis and computational fluid dynamics. Some of these studies are of a fundamental nature. They address basic issues such as intermediate boundary conditions for approximate factorization schemes, existence and uniqueness of steady states for time-dependent problems, pitfalls of implicit time stepping, etc. The other studies deal with modern numerical methods such as total variation-diminishing schemes, higher-order variants of vortex and particle methods, spectral multidomain techniques, and front-tracking techniques. There is also a paper on adaptive grids. The fluid dynamics papers treat the classical problems of incompressible flows in helically-coiled pipes, vortex breakdown, and transonic flows.


An automatic history matching algorithm is developed based on bi-cubic spline approximations of permeability and porosity distributions and on the theory of regularization to estimate permeability or porosity in a single-phase, two-dimensional areal reservoir from well pressure data. The regularization feature of the algorithm, the theoretical details of which are described by Kravaris and Seinfeld is used to convert the ill-posed history matching problem into a well-posed problem. The algorithm employs the conjugate gradient method of Nazareth as its core minimization method. A number of numerical experiments are carried out to evaluate the performance of the algorithm. Comparisons with conventional (non-regularized) automatic history matching algorithms indicate the superiority of the new algorithm with respect to the parameter estimates obtained. A quasioptimal regularization parameter is determined without requiring a priori information on the statistical properties of the observations.
ICASE COLLOQUIA

October 1  Mr. Gary Schmidt, Mr. Henno Allik and Mr. Bob Thomas, Bolt, Beranek and Newman, Inc.: Overview and Use of the Butterfly System

October 7  Professor James Gimma, Courant Institute of Mathematical Sciences: An Overview of the Front Tracking Method

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