

OPTIMAL COOPERATIVE CONTROL SYNTHESIS OF ACTIVE DISPLAYS

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ABSTRACT

The utility of augmenting displays to aid the human operator in controlling high order complex systems is well known. Analytical evaluations of various display designs for a simple k/s^2 plant in a compensatory tracking task using an Optimal Control Model (OCM) of human behavior is carried out. This analysis reveals that significant improvement in performance should be obtained by skillful integration of key information into the display dynamics. The cooperative control synthesis technique previously developed to design pilot-optimal control augmentation is extended to incorporate the simultaneous design of performance enhancing augmented displays. The application of the cooperative control synthesis technique to the design of augmented displays is discussed for the simple k/s^2 plant. This technique is intended to provide a systematic approach to design optimally augmented displays tailored for specific tasks.

I. INTRODUCTION

With the advent of high performance aircraft, the amount of information to be processed by the pilot to successfully accomplish the assigned task has increased tremendously. It has, therefore, become critical to determine and limit information to the best informational set needed by the pilot so as to reduce his workload and improve his performance by reducing complex, unusual tasks to simpler, familiar ones. The need for providing augmented displays to the pilot to achieve this objective is very well understood. In the present paper, analytical evaluation of various display "quickenings" control laws for a simple k/s^2 plant is carried out. The evaluation is done for a tracking task using an Optimal Control Model (OCM) [1] of human behavior.

A methodology to design pilot-optimal display/control augmentation systems which analytically takes into account the control and information processing limitations of the human controller is proposed. This methodology is an extension of the cooperative control synthesis technique previously developed to design pilot optimal control augmentation [2,3,4]. Though the proposed methodology has been developed so as to be applicable to simultaneous synthesis of pilot optimal control augmentation and display augmentation, the present discussion focuses on the application of the technique to display design only.

The cooperative display design technique is applied to synthesize performance enhancing augmented compensatory displays for the k/s^2 plant in the tracking task. The displays thus obtained show improved tracking performance for much reduced mean square pilot input when evaluated using the OCM. Moreover, the methodology offers considerable potential as a tool for providing a systematic approach to task tailoring of augmented displays.

II. DISPLAY DESIGN FOR k/s^2 PLANT

Consider the k/s^2 plant dynamics as discussed by Klienman et al. in [1]. The system state equations are

$$\begin{Bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{Bmatrix} = \begin{bmatrix} -2 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u(t) + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} w(t)$$

or in concise form

$$\dot{\bar{x}} = \begin{matrix} \cdot \\ \bar{A} \\ \bar{B} \\ \bar{D} \end{matrix} \bar{x} + \begin{matrix} \cdot \\ \bar{B} \\ \bar{D} \end{matrix} u + \begin{matrix} \cdot \\ \bar{D} \\ \bar{C} \end{matrix} w \quad (2.1)$$

Here $k=1$ in./in. and the state $x_1(t)$, a first order Markov process having a break frequency of 2 rads/sec, is the velocity of the command. $w(t)$ has intensity $W = 0.217$ to give $E\{x_1^2\} = 0.054$ in.²

$$\begin{aligned}
y_1 &= x_2 = e: \text{ error} \\
y_2 &= x_1 + \dot{x}_3 = \dot{e}: \text{ error rate}
\end{aligned}
\tag{2.2}$$

where the pilot is assumed to be able to reconstruct the error rate by observing the error itself. For the OCM model, the pilot's cost function is taken to be

$$J(u) = E\{e^2\} + rE\{\dot{u}^2\} \tag{2.3}$$

where "r" is chosen so as to give a neuromuscular lag time constant, $\tau_N = 0.1$ secs.

For all the analysis carried out in this section, the following parameters were set for the OCM pilot model

- a. Pilot's observation time delay set to 0.2 seconds
- b. Observation noise ratio was set at -20 dB
- c. Motor noise ratio was set at -25 dB
- d. The weighting on the control rate in the pilot's cost function was always adjusted to yield $\tau_N = 0.1$ secs.
- e. Very low values of thresholds were used for the observations made available to the pilot.

With the above parameter settings, the OCM analysis of system (2.1)-(2.3) gave results that are compatible with those given in [1]. These results are as shown in the last row of Table 1.

Next consider the display dynamics having the form

$$\dot{x}_d = a_d x_d + u_d \tag{2.4}$$

with the display quickening control u_d given by

$$u_d = G_d \bar{y}_d \tag{2.5}$$

where \bar{y}_d is the vector of plant outputs which are available for driving the display and G_d is the set of display control gains being determined, or

$$\bar{y}_d = C_d \bar{x} \tag{2.6}$$

The dynamics of the display augmented system can then be written as

$$\begin{bmatrix} \dot{\bar{x}} \\ \dot{x}_d \end{bmatrix} = \begin{bmatrix} A_o & 1 & 0 \\ G_d C_d & 1 & a_d \end{bmatrix} \begin{bmatrix} \bar{x} \\ x_d \end{bmatrix} + \begin{bmatrix} B_o \\ 0 \end{bmatrix} u + \begin{bmatrix} D_o \\ 0 \end{bmatrix} w \tag{2.7}$$

The pilot's observations for the display augmented system are

$$\begin{aligned} y_1 &= x_d \\ y_2 &= \dot{x}_d \end{aligned} \quad (2.8)$$

where it is again assumed that the pilot is able to reconstruct the rate of display by observing the displayed variable itself. The pilot's performance objective for the display augmented system is to minimize the cost

$$J_d(u) = E\{x_d^2\} + rE\{u^2\} \quad (2.9)$$

With the above formulation in mind, the performance of the display augmented system is evaluated using the OCM model for various values of a_d and various combinations of the display control gains G_d . Two cases of \bar{y}_d are considered. The first is when the display state is driven only by the error, i.e. state x_2 , and the second is when \bar{y}_d consists of both the error as well as the plant velocity state x_3 .

Case(a) $y_d = x_2$:

This is the simplest possible case for display of the form (2.4). For this case the displayed variable is just lagged error. The display dynamics are given by

$$\dot{x}_d = a_d x_d + g_{d2} x_2 \quad (2.10)$$

where g_{d2} is the display control gain on state x_2 . For $g_{d2} = -a_d$ (2.10) can be written in transfer function form as

$$x_d(s) = \frac{-a_d}{s-a_d} x_2(s) \quad (2.11)$$

Since $x_2 = e$, it is clear from (2.11) that in the steady state the displayed variable will closely approximate the error.

The OCM results for various values of a_d are presented in Table 1. The results of Table 1 are also plotted in Fig. 1 and Fig. 2, and correspond to the curve marked ①. From these plots it is clear that with only error driving the display, the pilot's performance is worse than the idealized no-display case. As $a_d \rightarrow -\infty$, the pilot's performance approaches that of the case with no display augmentation. The no-display case which then corresponds to an infinitely fast display is not desirable because of the inherent limitations on the pilot's ability to perceive fast changing signals, and the need to provide filtering of the noisy outputs. It might be reasonable to select a display which has a slightly higher bandwidth than the pilot, so a_d in the range -10 to -20 sec⁻¹ is desirable since the pilot's minimum neuro-muscular lag time constant is approximately 0.1 secs.

Case(b) $\bar{y}_d = [x_2, x_3]^T$:

For this case, the display dynamics have the form

$$\dot{x}_d = a_d x_d + g_{d2} x_2 + g_{d3} x_3 \quad (2.12)$$

where g_{di} , $i = 2, 3$ is the display gain on the state x_i .

Since $x_3(t)$ is the plant velocity state, the above form of display will provide lead information to the pilot. The pilot's performance can then be expected to improve as the gain g_{d3} is increased.

OCM analysis is carried out for two values of a_d : -10 and -20 sec^{-1} . For each of these values of a_d , $g_{d2} = -a_d$ and g_{d3} is varied from 1 to 6 in steps of 1. The results of this analysis are presented in Table 2 and are also plotted in Fig. 1 and Fig. 2 so as to compare them with the case of $g_{d3} = 0$. In the two figures, the curve marked ② corresponds to $a_d = -10 \text{ sec}^{-1}$ and that marked ③ to $a_d = -20 \text{ sec}^{-1}$.

Fig. 1 is a plot of mean square error vs. mean square control rate (\dot{u}) for the various display cases discussed above and Fig. 2 is a plot of mean square error vs. the mean square control input (u). The point marked A corresponds to the no display case in the two figures. From these two figures it is clear that the mean square input and the mean square control rate both decrease as the display control gain g_{d3} is increased. What is most interesting is that the mean square error initially decreases as g_{d3} is increased and then starts increasing beyond a certain value of g_{d3} that depends on the choice of the display bandwidth and the display gain g_{d2} . Noting that earlier work [5,6] has shown that the pilot's workload is directly related to the mean square control rate, this means that it is possible to improve performance (of which mean square error is a measure) while at the same time decreasing pilot's workload and the control energy required by a skillful integration of key information into the display dynamics. Moreover, the results indicate that for a given display bandwidth there is an optimal choice of display control gains which leads to the best possible performance. For instance, in Figures 1 and 2, point C is such an optimal display design for $a_d = -20 \text{ sec}^{-1}$, and for this case the performance is slightly better than the no-display case. Meanwhile the pilot's workload and the control effort required are both significantly reduced.

It then appears desirable to develop a systematic approach to display augmentation which will make it possible to directly synthesize the optimal display design without having to resort to trial and error. In the following sections an extension of the optimal cooperative control synthesis technique is proposed as a methodology to synthesize pilot-optimal display/control augmentation systems.

TABLE 1: OCM RESULTS FOR VARYING DISPLAY BANDWIDTH ($g_{d2} = -a_d$)

a_d (sec^{-1})	r ($\tau_N = 0.1$ secs)	M.S. Error (in.^2)	M.S. Input (in.^2)	M.S. Control rate ($\text{in.}^2/\text{sec}^2$)
-5	4.6×10^{-5}	0.0215	2.176	106.91
-10	5.8×10^{-5}	0.0177	1.543	76.47
-20	6.2×10^{-5}	0.0157	1.353	67.44
-50	6.25×10^{-5}	0.0142	1.261	62.9
-100	6.3×10^{-5}	0.0135	1.223	60.98
NO DISPLAY	7.0×10^{-5}	0.0131	1.141	54.73

TABLE 2: OCM RESULTS FOR VARYING DISPLAY CONTROL GAINS

g_{d3}	$a_d = -10, g_{d2} = 10$ (sec^{-1})			$a_d = -20, g_{d2} = 20$ (sec^{-1})		
	M.S. Error (in.^2)	M.S. Input (in.^2)	M.S. Control rate ($\text{in.}^2/\text{sec}^2$)	M.S. Error (in.^2)	M.S. Input (in.^2)	M.S. Control rate ($\text{in.}^2/\text{sec}^2$)
1	0.0144	1.113	54.75	0.014	1.175	58.06
2	0.0138	0.733	35.92	0.013	0.968	47.47
3	0.0143	0.486	23.71	0.0127	0.789	38.49
4	0.0157	0.339	16.52	0.0128	0.639	30.97
5	0.0175	0.248	12.05	0.0131	0.521	25.15
6	0.0195	0.187	9.01	0.0136	0.427	20.46

II. OPTIMAL COOPERATIVE CONTROL/DISPLAY DESIGN METHODOLOGY

PROBLEM FORMULATION:

In this section the mathematical formulation of the cooperative control synthesis technique is presented, and necessary conditions for the simultaneous optimality of the display and control augmentation systems are developed. The procedure followed here is very similar to that of [3, 4].

Consider the dual controller system described by the linear time invariant set of first order differential equations

$$\dot{\bar{x}} = A \bar{x} + B_{10} \bar{u}_1 + B_{20} \bar{u}_2 + D \bar{w} \quad (3.1)$$

with $\bar{x} \in \mathbb{R}^n$, $\bar{u}_1 \in \mathbb{R}^{m_1}$, $\bar{u}_2 \in \mathbb{R}^{m_2}$ and \bar{w} a zero-mean Gaussian white noise process with intensity W . The two controls represent two physically independent controllers.

The display dynamics are assumed to be of the form

$$\dot{\bar{x}}_d = A_d \bar{x}_d + B_{d0} \bar{u}_d \quad (3.2)$$

with $\bar{x}_d \in \mathbb{R}^d$, $u_d \in \mathbb{R}^{m_d}$, and \bar{u}_d is the display quickening controller. The objective is to find the optimal cooperative controllers 1 and 2 (\bar{u}_1 and \bar{u}_2) along with the optimal display control law \bar{u}_d .

Controller 1 (\bar{u}_1) has noisy observations available for feedback given by

$$\bar{y}_1 = C_{10}\bar{x} + C_{d1}\bar{x}_d + C_u\bar{u}_d + \bar{v}_y \quad (3.3)$$

where \bar{v}_y is also a zero-mean Gaussian white noise process with intensity V_y .

The augmentation controller \bar{u}_2 and the display control law \bar{u}_d are assumed to have noise-free system outputs \bar{y}_2 and \bar{y}_d , respectively, available for feedback, where

$$\bar{y}_2 = C_{20}\bar{x}; \quad \bar{y}_d = C_d \begin{Bmatrix} \bar{x} \\ \bar{x}_d \end{Bmatrix} \quad (3.4)$$

Note that the above formulation does not allow feedback of the display states into the augmentation controller \bar{u}_2 .

Finally, these two controllers are constrained to have the direct output feedback form

$$\begin{aligned} \bar{u}_2 &= G_2\bar{y}_2 = G_2C_{20}\bar{x} \\ \bar{u}_d &= G_d\bar{y}_d = G_dC_d \begin{Bmatrix} \bar{x} \\ \bar{x}_d \end{Bmatrix} \end{aligned} \quad (3.5)$$

which is consistent with the desire for simple, easy to implement control laws.

The interaction between the different controllers is shown in the block diagram of Figure 3.

DESIGN OBJECTIVES:

Controller 1 is to be optimal with respect to the cost

$$J_1 = E\left\{\lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T (\bar{x}^T \Omega_{10} \bar{x} + \bar{x}_d^T \Omega_{1d} \bar{x}_d + \bar{u}_1^T R_1 \bar{u}_1 + \bar{u}_2^T F_1 \bar{u}_2) dt\right\} \quad (3.6)$$

in the presence of the action of control inputs \bar{u}_2 and \bar{u}_d . Here $E\{\cdot\}$ indicates the expected value operator and the weighting matrices are $\Omega_{10} \succ 0$, $\Omega_{1d} \succ 0$, $R_1 \succ 0$, $F_1 \succ 0$.

Conversely, Controller 2 (\bar{u}_2) and the display control law \bar{u}_d are to be optimal with respect to the cost

$$J_2 = E\left\{\lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T (\bar{x}^T \Omega_{20} \bar{x} + \bar{x}_d^T \Omega_{2d} \bar{x}_d + \bar{u}_2^T R_2 \bar{u}_2 + \bar{u}_1^T F_2 \bar{u}_1 + \bar{u}_d^T F_d \bar{u}_d) dt\right\} \quad (3.7)$$

in the presence of the control action \bar{u}_1 . The weighting matrices are $\Omega_{2o} > 0$, $\Omega_{2d} > 0$, $R_2 > 0$, $F_2 > 0$, $F_{2d} > 0$. Augmenting the system dynamics (3.1) with the display dynamics (3.2), the state-space description of this augmented system is obtained to be

$$\begin{Bmatrix} \dot{\bar{x}} \\ \dot{\bar{x}}_d \end{Bmatrix} = \begin{bmatrix} A_{o1} & 0 \\ 0 & A_d \end{bmatrix} \begin{Bmatrix} \bar{x} \\ \bar{x}_d \end{Bmatrix} + \begin{bmatrix} B_{1o} \\ 0 \end{bmatrix} \bar{u}_1 + \begin{bmatrix} B_{2o} \\ 0 \end{bmatrix} \bar{u}_2 + \begin{bmatrix} 0 \\ B_{do} \end{bmatrix} \bar{u}_d + \begin{bmatrix} D_o \\ 0 \end{bmatrix} \bar{w} \quad (3.8)$$

Defining $\bar{\chi} = \text{COL}(\bar{x}, \bar{x}_d)$, (3.8) can be written in a compact form with appropriate definitions for the matrices as

$$\dot{\bar{\chi}} = A\bar{\chi} + B_1\bar{u}_1 + B_2\bar{u}_2 + B_d\bar{u}_d + D\bar{w} \quad (3.9)$$

The outputs can similarly be written as

$$\begin{aligned} \bar{y}_1 &= C_1\bar{\chi} + C_u\bar{u}_d + \bar{v}_y \\ \bar{y}_2 &= [C_{2o} \quad 0]\bar{\chi} = C_2\bar{\chi} \end{aligned} \quad (3.10)$$

The two cost functions can then be expressed in terms of the augmented state vector $\bar{\chi}$ as

$$\begin{aligned} J_1 &= E\left\{\lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T (\bar{\chi}^T \Omega_1 \bar{\chi} + \bar{u}_1^T R_1 \bar{u}_1 + \bar{u}_2^T F_1 \bar{u}_2) dt\right\} \\ J_2 &= F\left\{\lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T (\bar{\chi}^T \Omega_2 \bar{\chi} + \bar{u}_1^T R_2 \bar{u}_1 + \bar{u}_2^T F_2 \bar{u}_2 + \bar{u}_d^T F_{2d} \bar{u}_d) dt\right\} \end{aligned} \quad (3.11)$$

where the weighting matrices Ω_1 and Ω_2 are appropriately defined.

SOLUTION FOR \bar{u}_1 :

In the presence of the action of control inputs \bar{u}_2 and \bar{u}_d , as given by (3.5), the dynamics of the augmented system are obtained to be

$$\begin{aligned} \dot{\bar{\chi}} &= A_{\text{aug}} \bar{\chi} + B_1 \bar{u}_1 + D\bar{w} \\ \bar{y}_1 &= C_{\text{aug}} \bar{\chi} + \bar{v}_y \end{aligned} \quad (3.12)$$

where

$$\begin{aligned} A_{\text{aug}} &\triangleq (A + B_2 G_2 C_2 + B_d G_d C_d) \\ C_{\text{aug}} &\triangleq (C_1 + C_u G_d C_d) \end{aligned} \quad (3.13)$$

and the performance index J_1 becomes

$$J_1 = F\left\{\lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T (\bar{\chi}^T (\Omega_1 + C_2^T G_2^T F_1 G_2 C_2) \bar{\chi} + \bar{u}_1^T R_1 \bar{u}_1) dt\right\} \quad (3.14)$$

Equations (3.12) and (3.14), in the case of uncorrelated process and measurement noises and for $V_y > 0$, describe the standard non-singular linear quadratic Gaussian regulator problem. When stabilizability and detectability conditions for the system are satisfied, the optimal controller is known [7] to have the form

$$u_1 = k_1 \hat{\bar{X}} \quad (3.15)$$

where $\hat{\bar{X}}$ is the minimum mean-square estimate of the system state vector \bar{X} .

The gain matrix k_1 is given by

$$k_1 = -R_1^{-1} B_1^T P \quad (3.16)$$

with $P > 0$ the symmetric solution of the algebraic Riccati equation

$$A_{aug}^T P + P A_{aug} + (O_1 + C_2^T G_2^T F_1 G_2 C_2) - P B_1^T R_1^{-1} B_1 P = 0 \quad (3.17)$$

The dynamics of the state estimator are

$$\dot{\hat{\bar{X}}} = A_{aug} \hat{\bar{X}} + B_1 \bar{u}_1 + M_1 (\bar{y}_1 - C_{aug} \hat{\bar{X}}) \quad (3.18)$$

where the Kalman filter gain matrix M_1 is given by

$$M_1 = \Sigma C_{aug}^T V_y^{-1} \quad (3.19)$$

with $\Sigma > 0$ the symmetric solution of the algebraic equation

$$A_{aug} \Sigma + \Sigma A_{aug}^T + D D^T - \Sigma C_{aug}^T V_y^{-1} C_{aug} \Sigma = 0 \quad (3.20)$$

SOLUTION FOR \bar{u}_2 AND \bar{u}_d :

The optimal controller \bar{u}_1 as derived above has the form

$$\bar{u}_1 = k_1 \hat{\bar{X}}; \quad \dot{\hat{\bar{X}}} = A_1 \hat{\bar{X}} + M_1 \bar{y}_1 \quad (3.21)$$

where $A_1 \triangleq (A_{aug} + B_1 k_1 - M_1 C_{aug})$.

Then in the presence of the control action \bar{u}_1 , the system dynamics can be written in terms of the augmented state vector $\bar{q} \triangleq \text{COL}(\bar{X}, \hat{\bar{X}})$ as

$$\dot{\bar{q}} = \begin{bmatrix} A & | & B_1 k_1 \\ \hline M_1 C_1 & | & A_1 \end{bmatrix} \bar{q} + \begin{bmatrix} B_2 \\ 0 \end{bmatrix} \bar{u}_2 + \begin{bmatrix} B_d \\ M_1 C_u \end{bmatrix} \bar{u}_d + \begin{bmatrix} D & | & 0 \\ \hline 0 & | & M_1 \end{bmatrix} \begin{Bmatrix} \bar{w} \\ \bar{v}_y \end{Bmatrix} \quad (3.22)$$

which can further be written in a compact form with appropriate definitions of matrices as

$$\dot{\bar{q}} = A_1' \bar{q} + B_2' \bar{u}_2 + B_d' \bar{u}_d + D' \bar{w}' \quad (3.23)$$

The intensity of the process \bar{w}' is $W' = \begin{bmatrix} W & 0 \\ 0 & V \\ & & y \end{bmatrix}$.

The index of performance then becomes

$$J_2 = E\left\{\lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T (\bar{q}^T \bar{q}' + \bar{u}_2^T F_2 \bar{u}_2 + \bar{u}_d^T F_{2d} \bar{u}_d) dt\right\} \quad (3.24)$$

with

$$\bar{q}' \triangleq \begin{bmatrix} \bar{q}_2 & 0 \\ 0 & k_1^T R_2 k_1 \end{bmatrix}.$$

The design objective can then be stated as to find the optimal controller \bar{u}_2 and optimal display control \bar{u}_d which minimize the cost J_2 as given by (3.24).

Proceeding in a way as detailed in [4], it can be shown that the gains G_2 and G_d which correspond to the simultaneous optimality of the two controllers \bar{u}_2 and \bar{u}_d are given by

$$G_2 = -F_2^{-1} [B_2^T \ 0] HL \begin{bmatrix} C_2^T \\ 0 \end{bmatrix} \left([C_2^T \ 0] L \begin{bmatrix} C_2^T \\ 0 \end{bmatrix} \right)^{-1} \quad (3.25)$$

and

$$G_d = -F_{2d}^{-1} \begin{bmatrix} B_d \\ M_1 C_u \end{bmatrix}^T HL \begin{bmatrix} C_d^T \\ 0 \end{bmatrix} \left([C_d^T \ 0] L \begin{bmatrix} C_d^T \\ 0 \end{bmatrix} \right)^{-1} \quad (3.26)$$

with $L = E\{\bar{q} \bar{q}^T\}$ satisfying the relation

$$A_c L + LA_c^T + D'W'D'^T = 0 \quad (3.27)$$

and H satisfying

$$A_c^T H + HA_c + \bar{0} = 0 \quad (3.28)$$

where the following definitions have been used

$$A_c \triangleq \begin{bmatrix} A_{aug} & B_1 k_1 \\ M_1 C_{aug} & A_1 \end{bmatrix}; \quad \bar{0} \triangleq \bar{0}' + \begin{bmatrix} C_2^T G_2^T F_2 G_2 C_2 + C_d^T G_d^T F_{2d} G_d C_d & 0 \\ 0 & 0 \end{bmatrix}$$

Though the methodology developed above is applicable for simultaneous synthesis of optimal control and display augmentation, only the application to display design will be discussed in this paper. For the case of display design only, the controller \bar{u}_2 is inactive and the system dynamics and corresponding conditions for² optimality are accordingly simplified.

IV. APPLICATION OF DISPLAY DESIGN METHODOLOGY TO k/s^2 PLANT

A computer code was developed to determine the optimal display control gains using the above methodology. The details of a similar computer code are documented in [4]. The algorithm is iterative using a gradient search technique. Given a starting display gain matrix (including the null matrix) the display gains that satisfy the conditions of optimality as stated in Section III are determined.

The dynamics of the k/s^2 , plant augmented with the display, are as in Section II. The application of the methodology to optimal display design for the case of $a_d = -20\text{sec}^{-1}$ with both x_2 and x_3 driving the display will be discussed.

The controller \bar{u}_1 is analogous to the control rate \bar{u}_p of the OCM, so in order to be consistent with the pilot's stated objective of regulating the display, the cost J_1 is defined as

$$J_1 = E\{x_d^2\} + R_1 E\{u_1^2\} \quad (4.1)$$

where R_1 is chosen so as to satisfy the requirement of $\tau_N = 0.1$ seconds for the pilot's neuro-muscular lag. Also the process noise and the measurement noise in the problem formulation are chosen such that the controller \bar{u}_1 for the beginning display dynamics is compatible with the OCM model corresponding to the dynamics. (The reader is referred to [4] for details of how to achieve this).

The cost J_2 is defined as

$$J_2 = \alpha_e E\{e^2\} + R_2 E\{u_1^2\} + F_{2d} E\{u_d^2\} \quad (4.2)$$

which is reflective of the overall objective of reducing the tracking error through the means of an "intelligent" display. Note that in (4.2), F_{2d} needs to be positive definite in order to get a finite optimal solution to the problem. However, since the display control does not reflect any measure of energy, the weighting F_{2d} may be chosen small such that its contribution to the cost J_2 is not significant. For the results presented in this section $F_{2d} = 0.001$ was used.

The results obtained using the optimal cooperative design methodology for various values of α_e and R_2 are presented in Table 3. For all these cases the starting display gains were taken to be $G_d = [20, 0]$. In Table 3 the optimal display gains are listed as well as the results of evaluation of the corresponding augmented dynamics using the OCM. The parameters that define the OCM were set to the values stated in Section II. The OCM analysis results for the no-display case and the cases

of $G_d = [20, 0]$ and $G_d = [20, 3]$ for $a_d = -20\text{sec}^{-1}$, are also listed in Table 3 to provide a comparison. The results of Table 3 are also plotted in Figures 4 and 5.

Note that as the relative weighting on the error is increased in the cost J_2 , the optimal cooperative display design methodology does lead to display gains which give improved performance at the expense of increased control activity. Thus this methodology, through a proper choice of weightings in the cost function J_2 , provides a systematic approach to design of task-tailored display augmentation.

Also note that for all the 5 cases of display design using this methodology, the final optimal display gains were such that the performance is significantly improved as compared to the beginning display and at the same time the workload (\dot{u}) and control effort (u) are considerably reduced. If the weighting on the error is made high enough (cases 4 and 5), performance comparable to the no display case and the best case corresponding to $G_d = [20, 3]$ of Section II is obtained for significantly reduced workload and control effort. Moreover it is clear that for the display bandwidth such that $a_d = -20\text{sec}^{-1}$, performance better than that of case 5 cannot be obtained. Increasing the weight on error in the cost function J_2 any further would only have the effect of leading to a display design requiring higher control effort without any noticeable improvement in performance.

TABLE 3: OCM RESULTS FOR OPTIMAL DISPLAYS FOR k/s^2 PLANT
 $a_d = -20\text{sec}^{-1}$, $G_d = [g_{d2}, g_{d3}]$

S.N.	Q_e	R_2	Optimal G_d	M.S. (in^2) Error	M.S. (in^2) Input	M.S. (in^2/sec^2) Control Rate
1	1	20×10^{-5}	[36.6, 11.9]	0.014	0.389	18.64
2	2	20×10^{-5}	[49.2, 13]	0.0132	0.492	23.72
3	2	10×10^{-5}	[64.4, 16.3]	0.0131	0.514	24.78
4	4	10×10^{-5}	[88.1, 17.3]	0.01272	0.650	31.58
5	4	5×10^{-5}	[118.2, 22.6]	0.01269	0.665	32.40
A	NO DISPLAY			0.0131	1.141	54.73
B	BEG. DISPLAY FOR DESIGN			0.0157	1.353	67.44
C	BEST PER. DISPLAY (II)			0.0127	0.789	38.49

CONCLUSIONS

Through OCM analysis of a simple k/s^2 plant it was shown that the performance of a human controller can be improved and his workload significantly reduced by providing him an active display which integrates information for the system dynamics being controlled. A methodology based on the optimal cooperative control synthesis technique was suggested as a means to synthesize optimal display gains, tailored for specific tasks. The application of this methodology to the k/s^2 plant was discussed and the results presented show that the methodology has potential for providing a systematic approach to display design.

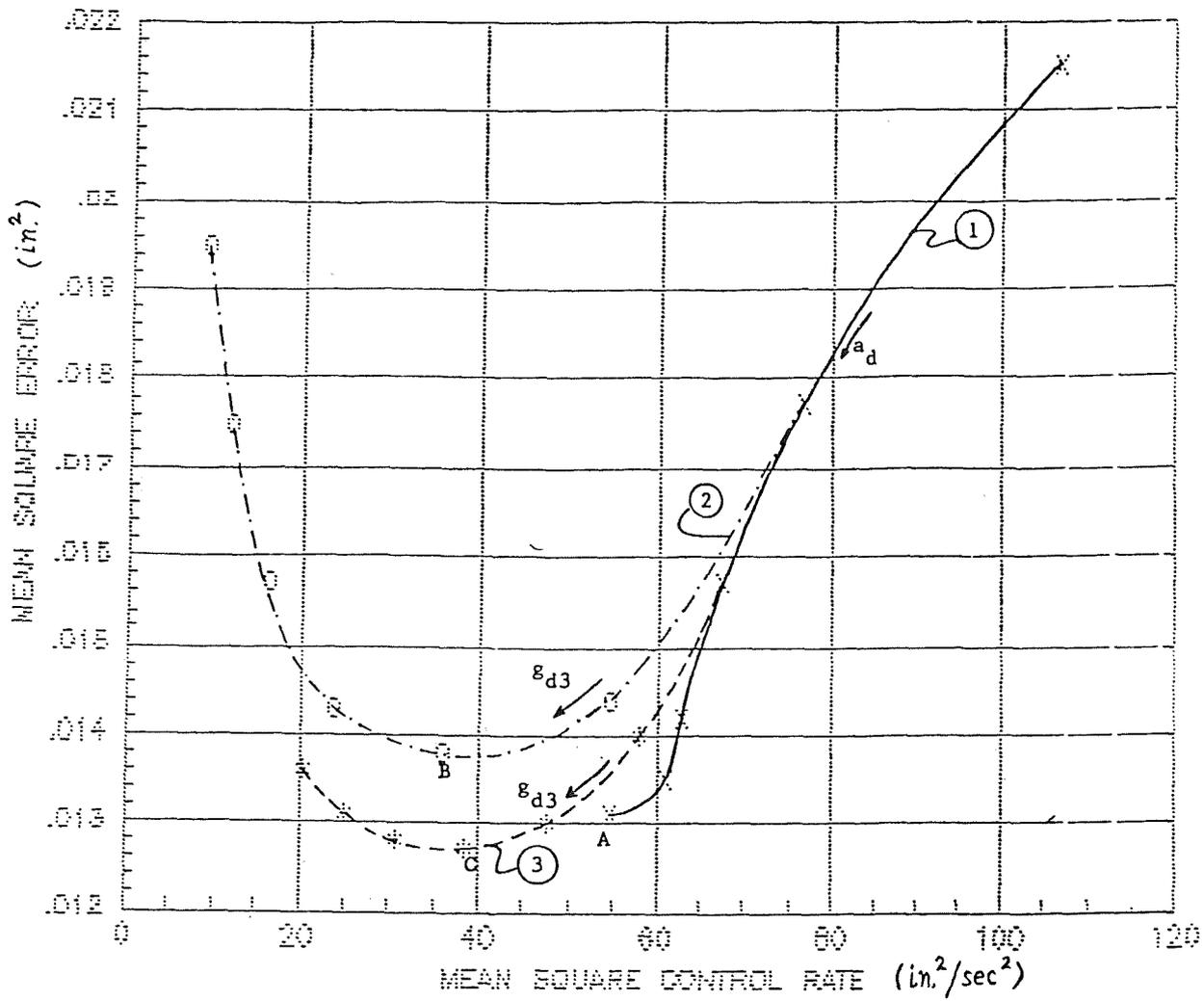
The results obtained for the k/s^2 plant need to be experimentally verified with man in the loop simulation in order to validate the display design methodology. Research in the area of applying the proposed design methodology to high order dynamical systems in a complex multi-control task scenario is presently ongoing and the preliminary results are quite encouraging.

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REFERENCES

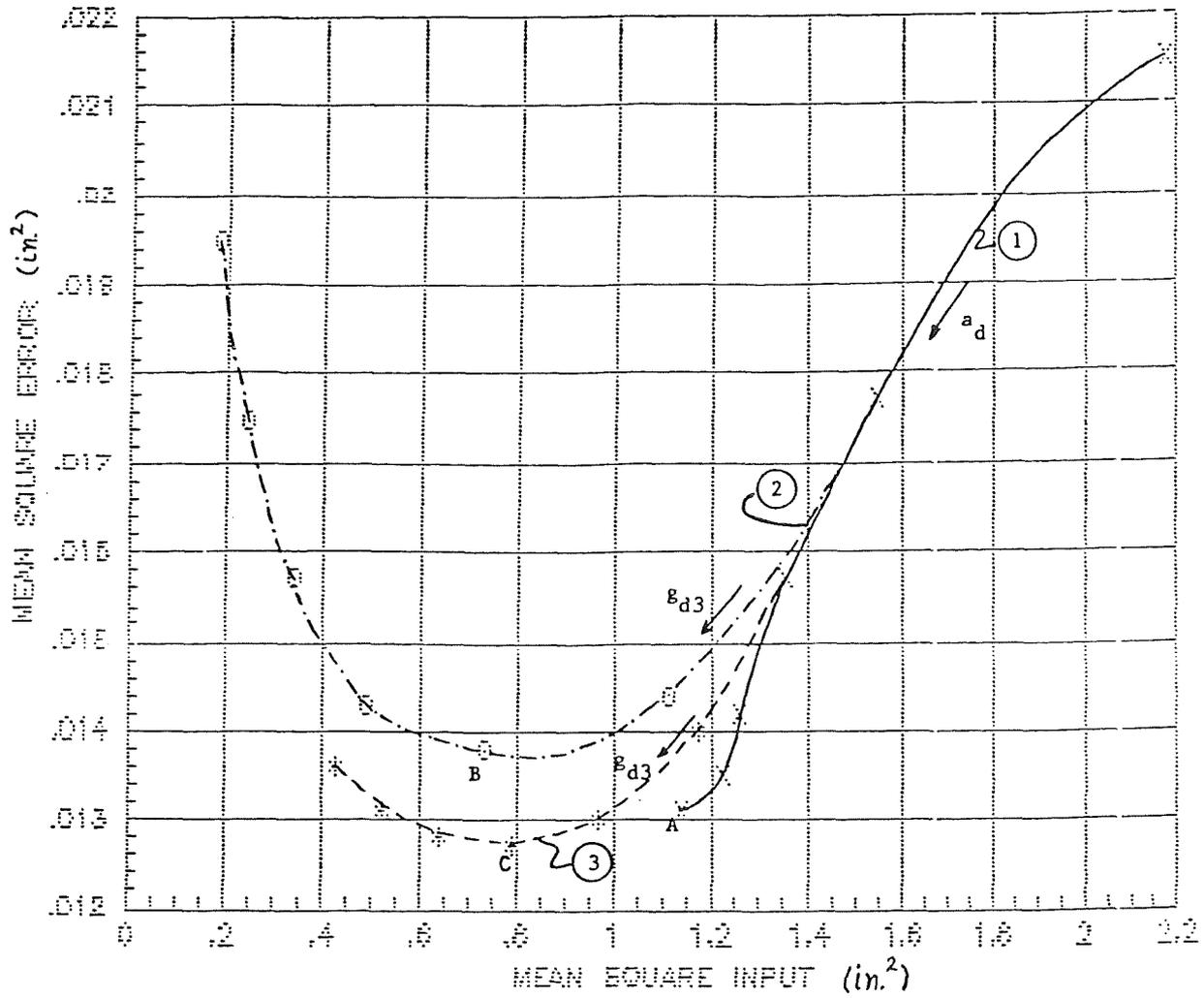
- [1] Kleinman, D.R., Baron, S., and Levison, W.H., "An Optimal Control Model of Human Response," Parts I and II, *Automatica*, Vol. 6, pp. 357-383, 1970.
- [2] Schmidt, D.K., "Optimal Flight Control Synthesis via Pilot Modeling," *AIAA Journal of Guidance and Control*, July-August 1979.
- [3] Schmidt, D.K., and Innocenti, M., "Pilot Optimal Multivariable Control Synthesis by Output Feedback," NASA CR-16312, July 1981.
- [4] Innocenti, N., "Cooperative Pilot-Optimal Augmentation System Synthesis for Complex Flight Vehicles," Ph.D. Thesis, Purdue University, West Lafayette, Indiana, May 1983.
- [5] Schmidt, D.K., "On the Use of the OCM's Quadratic Objective Function as a Pilot Rating Metric," 17th Annual Conference on Manual Control, Los Angeles, CA, June 1981.
- [6] Wierwille, Walter W., and Connor, Sydney A., "Evaluation of 20 Workload Measures Using a Psychomotor Task in a Moving-Base Aircraft Simulator", *Human Factors*, Vol. 25, pp. 1-16, 1983.
- [7] Kwakernaak, H., and Sivan, R., "Linear Optimal Control Systems," Wiley-Interscience, 1972.



NOMENCLATURE

- ① Only x_2 driving display, $a_d = -5, -10, -20, -50, -100$
 A - no display
- ② $a_d = -10, g_{d2} = 10, g_{d3} = 1, 2, 3, 4, 5, 6$
 B - $g_{d3} = 2$
- ③ $a_d = -20, g_d = 20, g_{d3} = 1, 2, 3, 4, 5, 6$
 C - $g_{d3} = 3$

FIG. 1 PERFORMANCE VS WORKLOAD



NOMENCLATURE

- ① Only x_2 driving display, $a_d = -5, -10, -20, -50, -100$
A - no display
- ② $a_d = -10, g_{d2} = 10, g_{d3} = 1, 2, 3, 4, 5, 6$
B - $g_{d3} = 2$
- ③ $a_d = -20, g_d = 20, g_{d3} = 1, 2, 3, 4, 5, 6$
C - $g_{d3} = 3$

FIG. 2 PERFORMANCE VS CONTROL EFFORT

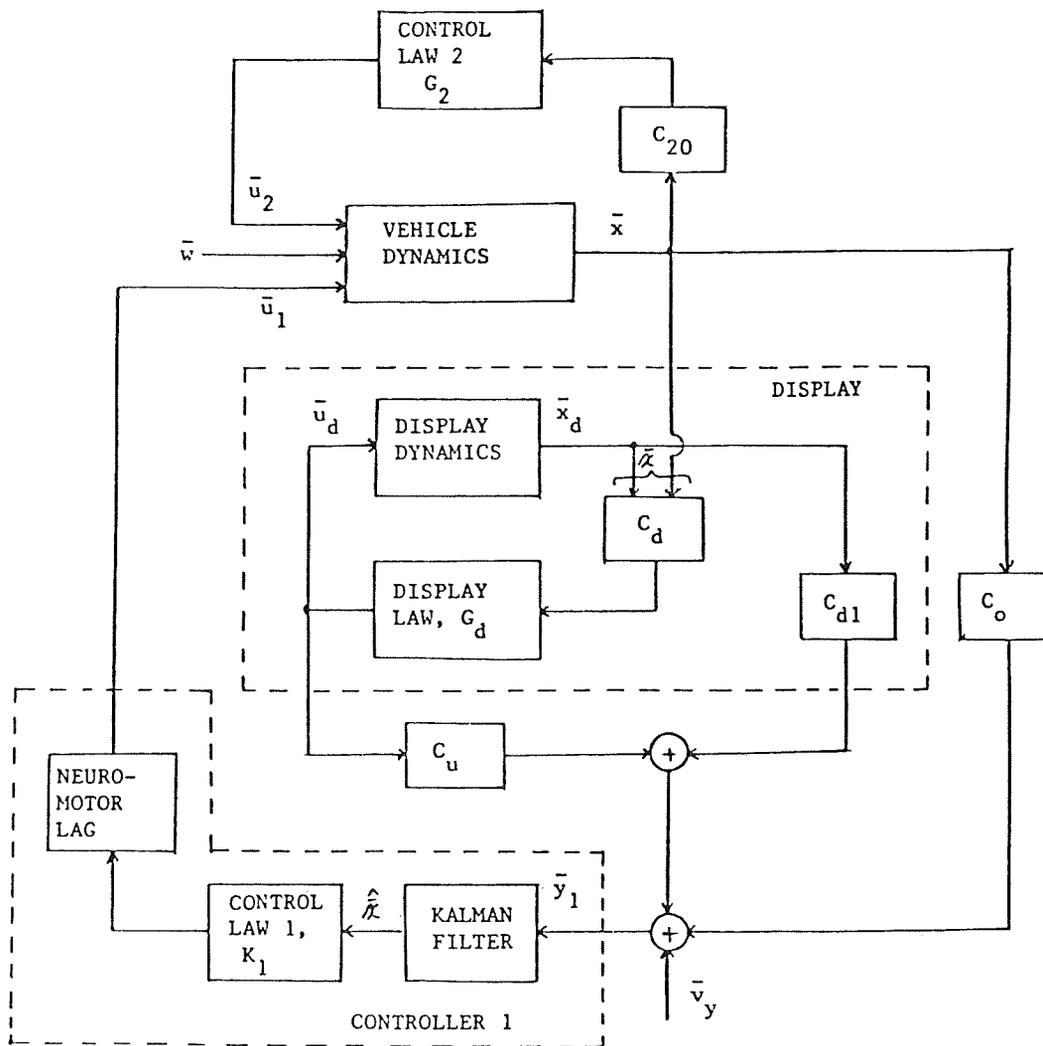
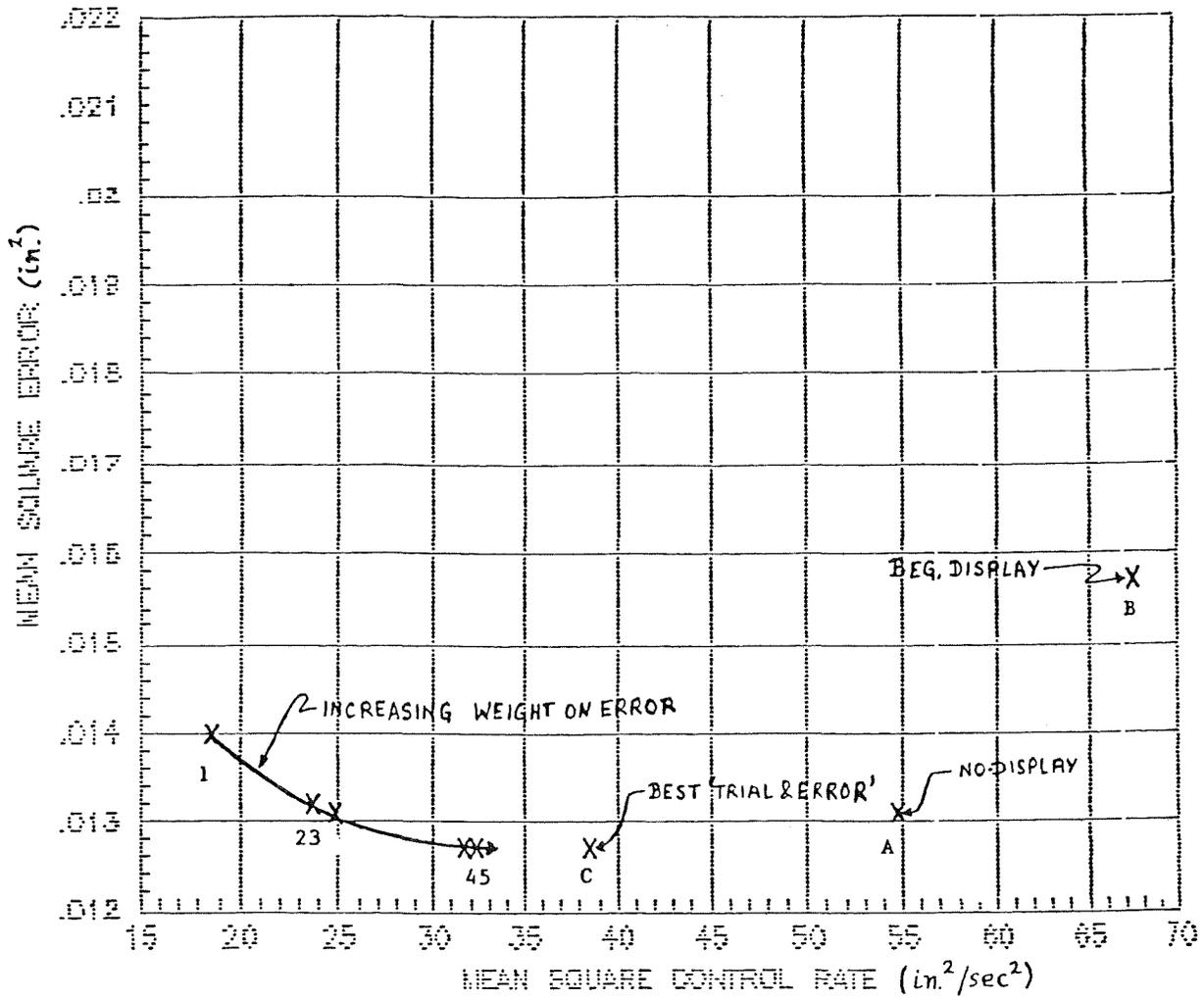


FIG. 3 BLOCK DIAGRAM FOR CONTROL AND DISPLAY AUGMENTATION



NOMENCLATURE

A: NO-DISPLAY

B: $G_d = [20, 0]$

C: $G_d = [20, 3]$

1: $Q_e = 1, R_2 = 20 \times 10^{-5}$

2: $Q_e = 2, R_2 = 20 \times 10^{-5}$

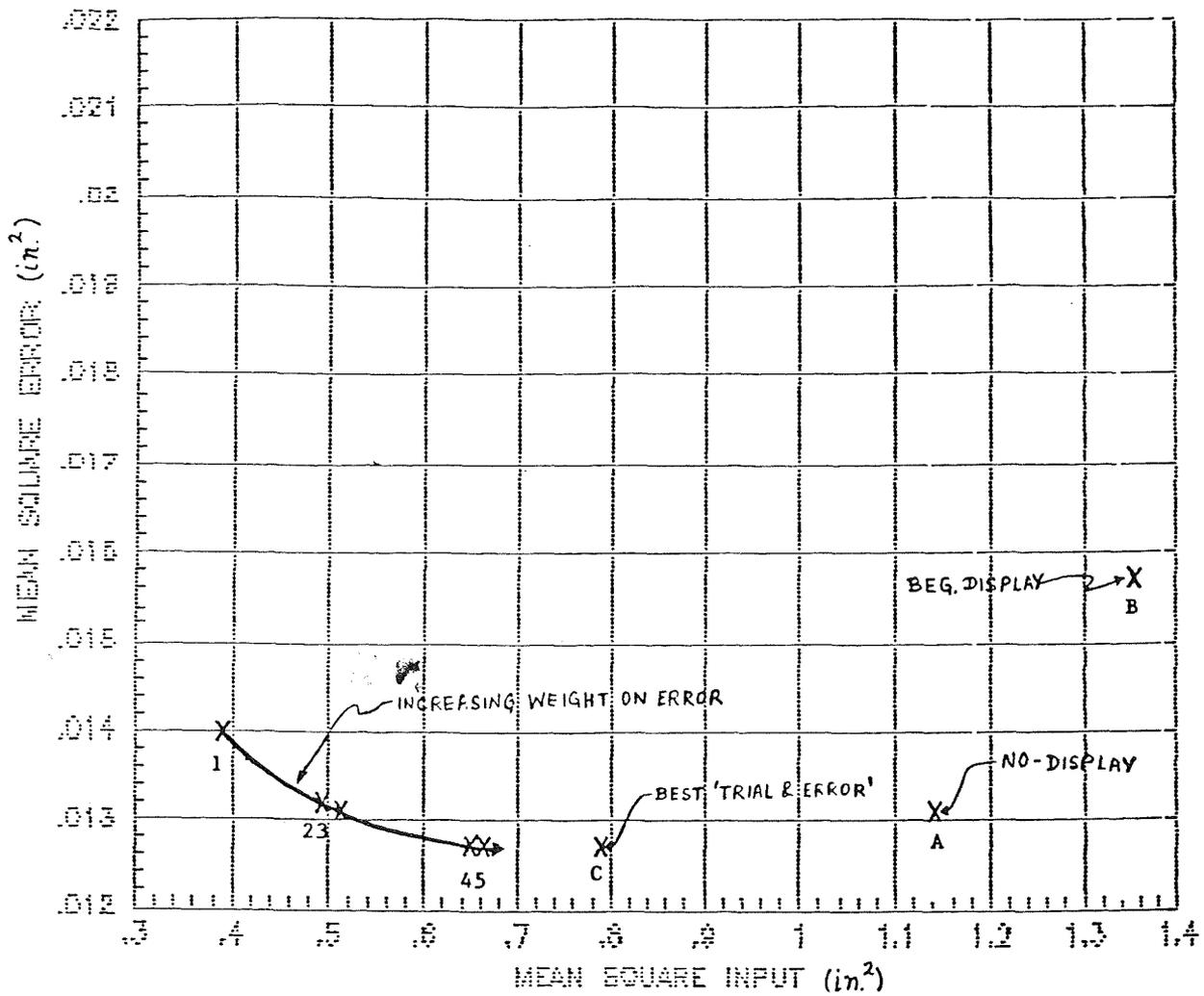
3: $Q_e = 2, R_2 = 10 \times 10^{-5}$

4: $Q_e = 4, R_2 = 10 \times 10^{-5}$

5: $Q_e = 4, R_2 = 5 \times 10^{-5}$

} OPTIMAL DISPLAYS

FIG. 4. PERFORMANCE VS WORKLOAD FOR OPTIMAL DISPLAYS



NOMENCLATURE

A: NO-DISPLAY

B: $G_d = [20, 0]$

C: $G_d = [20, 3]$

1: $Q_e = 1, R_2 = 20 \times 10^{-5}$

2: $Q_e = 2, R_2 = 20 \times 10^{-5}$

3: $Q_e = 2, R_2 = 10 \times 10^{-5}$

4: $Q_e = 4, R_2 = 10 \times 10^{-5}$

5: $Q_e = 4, R_2 = 5 \times 10^{-5}$

} OPTIMAL DISPLAYS

FIG. 5. PERFORMANCE VS CONTROL EFFORT FOR OPTIMAL DISPLAYS