MANUAL CONTROL OF UNSTABLE SYSTEMS

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ABSTRACT

Under certain operational regimes and failure modes, air and ground vehicles can present the human operator with a dynamically unstable or divergent control task. Research conducted over the last two decades has explored the ability of the human operator to control unstable systems under a variety of circumstances. This paper will review past research and summarize human operator control capabilities. A current example of automobile directional control under rear brake lockup conditions is also reviewed. A control system model analysis of the driver's steering control task is summarized, based on a generic driver/vehicle model presented at last year's Annual Manual. Results from closed course braking tests are presented that confirm the difficulty the average driver has in controlling the unstable directional dynamics arising from rear wheel lockup.

INTRODUCTION

Unstable vehicle dynamics present a rather specific task demand on the human operator. Vehicle system states tend to diverge exponentially, and the human controller must be alert and attentive enough to counteract this divergent system behavior. In many situations, due to a transition in vehicle behavior (e.g., component failures or a change in operating conditions), unstable dynamics may occur unexpectedly. In this case the human operator must detect the change and adapt to the vehicle's new response characteristics. Some attention has been devoted to control of unstable dynamic systems at past manual control conferences (e.g., Refs. 1-3).

In this paper we will start off with a simple analysis of the response of unstable vehicles. Next we will consider the ability of the human operator to control unstable dynamics. Then we will analyze an unstable vehicle control problem, i.e., a car with the rear wheels locked up during braking. Following this, the closed-loop stability properties of cars with and without rear wheel lockup are analyzed. Finally, field test data is presented which illustrates the ability of the average driver in controlling unstable automobile dynamics.

BACKGROUND

It is important to focus on the nature of unstable vehicle dynamics in order to appreciate the task difficulty imposed on the human operator.
Basically, simple unstable vehicle dynamics result in the exponential divergence of state variables and their derivatives:

\[
X = Ke^{t/T_\lambda}
\]

\[
\dot{X} = \frac{K}{T_\lambda} e^{t/T_\lambda}
\]

\[
\ddot{X} = \frac{K}{T_\lambda^2} e^{t/T_\lambda}
\]

where

\[t = \text{time}\]

\[K = \text{multiplying constant}\]

\[T_\lambda = \text{divergence time constant}\]

This effect occurs without any forcing function, and it should be noted that all variables have the same characteristic exponential time response, differing only by a multiplying constant as indicated above.

This exponential divergence characteristic is apparent in both field test and simulation data associated with simple unstable dynamics. To observe this, first note that the time required for an exponential curve to double in amplitude is related to the divergence time constant of the exponential as derived in Table 1:

\[
T_\lambda = 1.44 \Delta t_{2/1}
\]

Given vehicle response test data, this relationship can be used to identify divergence time constants as will be discussed subsequently.

**HUMAN OPERATOR CAPABILITY**

Given that unstable vehicle dynamics result in an exponential state variable divergence, can the human operator be expected to control such an occurrence? This question has been addressed extensively in the literature, involving a variety of situations including aircraft piloting, tracking task research, and a vehicle mounted task for screening drunk drivers. A summary of this research is given in Table 2 including the limiting divergence time
TABLE 1. RELATIONSHIP BETWEEN TIME TO DOUBLE AND DIVERGENCE TIME CONSTANT

A system with an unstable root \( s = \lambda \) will have an exponentially divergent response given by

\[
X = Ke^{t/T_\lambda}
\]

where \( T_\lambda = 1/\lambda \)

Now evaluate \( X \) at two time points

\[
X_1 = Ke^{t_1/T_\lambda} \quad ; \quad X_2 = 2X_1 = Ke^{t_2/T_\lambda}
\]

then

\[
\frac{X_2}{X_1} = 2 = e^{t_2/T_\lambda} = e^{(t_2 - t_1)/T_\lambda}
\]

\[
\therefore \quad \frac{t_2 - t_1}{T_\lambda} = \ln 2
\]

and \( T_\lambda = \frac{t_2 - t_1}{\ln 2} \)

Finally

\[
T_\lambda = 1.44 \ (t_2 - t_1) = 1.44 \ \Delta t_2/1
\]

32.3
### TABLE 2. SUMMARY OF RESEARCH ON HUMAN CONTROL OF TASKS WITH UNSTABLE DYNAMICS

<table>
<thead>
<tr>
<th>REF.</th>
<th>STUDY</th>
<th>UNSTABLE CONTROL LIMIT</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$\Delta t_2/1$ (sec)</td>
</tr>
<tr>
<td>4</td>
<td>Cheatham (1954): study of the characteristics of human pilot control response to simulated aircraft lateral motions using rudder pedals</td>
<td>0.3</td>
</tr>
<tr>
<td>5</td>
<td>Jex, et al. (1960): correlation of theoretical limits with past experimental results</td>
<td>0.23</td>
</tr>
<tr>
<td>6</td>
<td>Sadoff, et al. (1961): experimental study of aircraft longitudinal control problems limit unacceptable</td>
<td>0.58</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.4</td>
</tr>
<tr>
<td>7</td>
<td>Taylor &amp; Day (1961): controllability limits determined from simulator and flight tests Long. practiced Inexperienced</td>
<td>0.3</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.5</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Lat. practiced Inexperienced</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>Jex &amp; Cromwell (1962): theoretical and experimental study of aircraft longitudinal handling qualities parameters</td>
<td>0.23</td>
</tr>
<tr>
<td>9</td>
<td>Young &amp; Meiry (1965): manual control of unstable systems with visual and motion cues</td>
<td>0.3</td>
</tr>
<tr>
<td>10</td>
<td>Washizu &amp; Miyajima (1965): theoretical and experimental study of human pilot lateral controllability limits practiced Inexperienced</td>
<td>0.17</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.20</td>
</tr>
<tr>
<td>11</td>
<td>Jex, et al. (1966): studied well practiced limits of human controllability using a laboratory tracking task (Critical Tracking Task or CTT) and isometric control stick</td>
<td>0.11</td>
</tr>
<tr>
<td>12</td>
<td>Allen, et al. (1983): CTT mounted in a car, used as a drunk driver detection system practiced Inexperienced</td>
<td>0.14</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.28</td>
</tr>
</tbody>
</table>

constant that subjects were able to control. This summary suggests the following regarding human operator capability:

1) inexperienced operators can nominally handle divergence time constants greater than 0.5 sec.

2) well-practiced vehicle operators can handle divergence time constants on the order of 0.3 sec.

3) the well-practiced human operator's ultimate limit is on the order of 0.2 sec when a car steering wheel is used as a control device. When stiff "fly-by-wire" aircraft sticks are used as a control device, the controllable divergence limit can be reduced to 0.15 sec.
The above results are overly optimistic (i.e., time constants are too low) for cases where operators are surprised by a sudden change in vehicle response properties. There is a body of literature that relates to this situation. This literature is summarized in Ref. 13, along with the following summary statement:

"The process of adaptive control is thought to consist of four phases: retention of prefailure dynamics, detection of the failure, identification of the failure and adaptation of appropriate dynamic form for the postfailure situations, and, finally, optimization of postfailure control. ...

... Typical detection times for laboratory experiments with sudden changes in gain or velocity range from 0.5 to 3 sec. Times to detect failures involving higher order plants are increased to several seconds and may be considerably longer if emergency training is insufficient.

In the case where the human operator is controlling a vehicle that transitions to unstable operation, any delay in counteracting divergent state variables can be critical. As noted from Table 1, the state variable for a first-order unstable plant will double in less than one divergent time constant (i.e., $\Delta t_2/1 = 0.69 T_x$). Thus, state variables could easily diverge over several doubling times for a system with a divergence time constant of less than one second before the human operator detects and recognizes the problem and takes appropriate action. Whether or not the operator can then regain control depends on whether the system has diverged to an uncontrollable state before corrective action is taken.

A CAR DRIVING EXAMPLE

As a common example of a potentially unstable vehicle consider hard braking in an automobile. If the rear brakes should lock first (as can happen in cars with misbalanced brakes or pickup trucks with no cargo), then the vehicle will exhibit a directional instability. A simple approximation for this vehicle behavior can be derived as follows:

1) Assume a simple free body diagram as shown in Fig. 1. This is similar to several approaches that have been discussed in the literature (e.g., Refs. 14, 15).

2) Develop two degree of freedom force and moment equations from the free body diagram as shown in Table 3.

3) Derive the yaw rate transfer function from the Laplace transform of the Table 3 force and moment equations as given in Table 4. Now, for rear wheel lockup, since a locked and sliding wheel cannot develop any side force, set the rear side force coefficient ($Y_{2\alpha}$) to zero. Then the transfer function reduces to an unstable form as shown in Table 4.
\[ \delta_A = \text{Ackerman Steer Angle (deg)} \]
\[ \delta_W = \text{Front Steer Angle (deg)} = \delta_{SW}/N_G \]
\[ \alpha_1 = \text{Front Tire Slip Angle (deg)} \]
\[ \alpha_2 = \text{Rear Tire Slip Angle (deg)} \]
\[ R = \text{Path Radius (ft)} \]
\[ N_G = \text{Steering Ratio} \]
\[ F_S = \text{Tire Side Force} \]
\[ F_T = \text{Tire Traction Force} \]

Figure 1. Free Body Diagram and Constant Radius Turn Definitions
TABLE 3. TWO DEGREE OF FREEDOM VEHICLE DYNAMICS INCLUDING LOAD TRANSFER

**Force Equation:**

\[ m \left( \frac{\dot{v}}{} + U_0 r \right) = - \left( \frac{Y_{\alpha_1} + Y_{\alpha_2}}{U_0} \right) v - \left( \frac{a Y_{\alpha_1} - b Y_{\alpha_2}}{U_0} \right) \]

\[ + \left( Y_{\alpha_1} - F_{T_1} \right) \delta_w \]

**Moment Equation:**

\[ I \dot{r} = - \left( \frac{a Y_{\alpha_1} - b Y_{\alpha_2}}{U_0} \right) v - \left( \frac{a^2 Y_{\alpha_1} + b^2 Y_{\alpha_2}}{U_0} \right) r \]

\[ + a \left( Y_{\alpha_1} - F_{T_1} \right) \delta_w \]

<table>
<thead>
<tr>
<th>v</th>
<th>side slip velocity</th>
<th>r</th>
<th>yaw rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>m</td>
<td>mass</td>
<td>I</td>
<td>moment of inertia</td>
</tr>
<tr>
<td>U_0</td>
<td>longitudinal speed</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Y_{\alpha_1}</td>
<td>front axle side force coeff. (left + right)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Y_{\alpha_2}</td>
<td>rear axle side force coeff. (left + right)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>F_{T_1}</td>
<td>front axle traction force</td>
<td></td>
<td></td>
</tr>
<tr>
<td>a</td>
<td>distance from front axle to c.g.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>b</td>
<td>distance from real axle to c.g.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

32.7
TABLE 4. LAPLACE TRANSFORM TRANSFER FUNCTIONS FOR YAW RATE RESPONSE TO STEERING COMMANDS DEVELOPED FROM TABLE 3 EQUATIONS

Complete Transfer Function:

\[
\frac{r}{\delta_w} = \frac{\left(\frac{Y_{a1} - F_{T1}}{Y_{a1}}\right) \cdot \left(\frac{m a U_0}{l Y_{a2}} \cdot s + 1\right)}{\frac{m U_0 I}{l Y_{a1} Y_{a2}} s^2 + \left[\frac{m}{l} \left(\frac{a^2}{Y_{a2}} + \frac{b^2}{Y_{a1}}\right) + \frac{1}{l} \left(\frac{1}{Y_{a1}} + \frac{1}{Y_{a2}}\right)\right] s + \frac{l}{U_0} + \frac{m U_0}{l} \left(\frac{b}{Y_{a1}} - \frac{a}{Y_{a2}}\right)}
\]

where \(l = \text{wheelbase} = a + b\)

setting \(Y_{a2} = 0\):

\[
\frac{r}{\delta_w} = \frac{\left(\frac{Y_{a1} - F_{T1}}{Y_{a1}}\right) \cdot \frac{a}{I} \cdot s}{s^2 + \frac{Y_{a1}}{U_0} \left[\frac{a^2}{I} + \frac{1}{m}\right] s - \frac{a Y_{a1}}{I}}
\]

Negative constant term in denominator characteristic equation indicates basic dynamic instability
4) Find roots of the unstable transfer function using the quadratic formula as shown in Table 5. Using typical front wheel drive/passenger car parameters it is apparent from Table 5 that speed only has a minor effect on the divergent time constant, and that typical values for $T_\lambda$ are in the region of 0.3 seconds.

CLOSED-LOOP VEHICLE CONTROL

Now consider a closed-loop vehicle control model including visual and motion cue feedbacks shown in Fig. 2 that was presented at this conference last year (Ref. 16). Operating in this mode, the car driver ordinarily has a rather easy control task. Past analysis (Ref. 17) has shown that the driver's control parameters can be derived in a fairly straightforward manner. What we wish to consider here is what happens to closed-loop stability when the rear wheels lock up and how must the driver change his/her behavior to maintain stable closed-loop operation.

As has been derived in the past (Ref. 16) the closed-loop stability properties of the Fig. 2 model can be assessed by considering an opened-loop transfer function for the loop broken at the equivalent of the visual feedback point:

$$G_{OL}(s) = \frac{s + K'}{s} \cdot \frac{K_y e^{-\tau_y s}}{s} \cdot \frac{s + U_o/R}{s} \cdot \frac{G_{MOT}}{s}$$

(2)

where $G_{MOT}$ is the closed-loop transfer function for the motion feedback loop:

$$G_{MOT} = \frac{G_{NM} \cdot G_v}{1 + G_{NM} \cdot G_v \cdot e^{-\tau_m s}}$$

(3)

and

$G_{NM} = \text{neuromuscular dynamics}$

$G_v = \text{vehicle directional control dynamics}$

$\tau_m = \text{motion feedback delay}$
TABLE 5. ROOTS OF CHARACTERISTIC EQUATION

Equation:  \( s^2 + Bs + C \)

where

\[ B = \frac{Y_{\alpha 1}}{U_o} \left( \frac{a^2}{I} + \frac{1}{m} \right) \quad ; \quad C = -\frac{a Y_{\alpha 1}}{I} \]

Quadratic Roots:

\[ s = \frac{1}{2} (-B \pm \sqrt{B^2 - 4C}) \]

Typical Front Wheel Driver Passenger Car Parameters:

\[ m = 89 \text{ lb}-\text{sec}^2/\text{ft} \quad ; \quad I = 1475 \text{ lb}-\text{ft}-\text{sec}^2 \]

\[ a = 3 \text{ ft} \quad ; \quad b = 5.75 \text{ ft} \quad ; \quad \ell = a + b = 8.75 \text{ ft} \]

\[ B = 0.0173 \cdot \frac{Y_{\alpha 1}}{U_o} \quad ; \quad C = 0.00203 \cdot Y_{\alpha 1} \]

for \( Y_{\alpha 1} = 15,000 \)

\[ s = \frac{1}{2} \left( \frac{260}{U_o} \pm \sqrt{\frac{67340}{U_o^2} + 122} \right) \]

<table>
<thead>
<tr>
<th>SPEED, ( U_o )</th>
<th>ROOTS (rad/sec)</th>
<th>DIVERGENCE TIME CONST. ( T_\lambda ) (sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>mph</td>
<td>ft/sec</td>
<td>STABLE</td>
</tr>
<tr>
<td>30</td>
<td>44</td>
<td>-9.21</td>
</tr>
<tr>
<td>40</td>
<td>58.7</td>
<td>-8.16</td>
</tr>
<tr>
<td>50</td>
<td>73.4</td>
<td>-7.57</td>
</tr>
<tr>
<td>60</td>
<td>88</td>
<td>-7.19</td>
</tr>
</tbody>
</table>

32.10
The properties of the Eq. 2 transfer function have been discussed in the past (Refs. 16-18), and for nominal driver behavior with stable vehicle dynamics, a Bode plot of Eq. 2 appears as shown in Fig. 3. In order to maintain stable operation the driver must adjust his visual feedback gain $K_\psi$ to lie within the stable phase region as shown. Now consider unstable car dynamics due to rear wheel lock. The driver/vehicle transfer function in Fig. 4 assumes that the driver has maintained his pretransition behavior, and it is obvious that under these circumstances the closed-loop operation will be unstable for any level of visual feedback gain $K_\psi$ because the open-loop phase curve never has less than 180° phase lag!

It is clear from the above results that the driver must change behavior and adapt to rear wheel lockup conditions in order to maintain stable closed-loop vehicle control. Basically the driver must reduce system open-loop phase lag, and this can be accomplished in several phases as follows:

1) Change gain in the motion feedback loop ($K_r$) to reduce high frequency phase lag shown in Fig. 4.

2) Eliminate trimming behavior ($K' = 0$) to reduce low frequency phase lag as shown in Fig. 4.

3) Increase lookahead distance $R$ (equivalent of reducing outer loop gain) in order to further reduce low frequency phase lag as shown in Fig. 4.
Figure 3. Bode Plot for Normal Stable Driver/Vehicle Control

- \[ G_v(S) = \frac{2.07(S+5.61)}{(S^2+9.7S+39.2)} \]
- \[ K' = 0.4 \]
- \[ K_r = 550 \]
- \[ K_{\phi} = 12.8 \]
- \[ \tau_v = 0.05 \text{ sec} \]
- \[ \tau_m = 0.95 \text{ sec} \]
- \[ U_0 = 80.7 \text{ fps} \]
- \[ \zeta_{nm} = 0.774 \]
- \[ \omega_{nm} = 6.26 \]
- \[ U_0/R = 1.5 \text{ rad/s} \]

Frequency (rad/sec)

Figure 4. Bode Plot for Driver/Vehicle Control with Unstable Vehicle Dynamics Due to Rear Wheel Lockup. Figure 3 Driver Gains Give Unstable Closed-Loop Response
With this adapted driver behavior it can be seen in Fig. 5 that a small phase angle region of $K_\psi$ stability is allowed. At this stage, any further improvement in stability is limited by the driver's time delay and neuromuscular lag. The closed-loop control will not be very good under these circumstances because the closed-loop phase margin will be very low, but since the driver is slowing rapidly (for rear wheels locked, deceleration can be on the order of 0.3-0.4g's) he/she only has to maintain control until the vehicle comes to rest. Also, based on the Table 5 analysis, the vehicle becomes less unstable as speed decreases.

FIELD TEST EXPERIMENT

Methods and Procedures

A field test was conducted to determine driver behavior under actual wheel lockup conditions. The test course layout which defined the task to be performed by the drivers is illustrated in Fig. 5. The basic task was for the driver to stop safely and quickly within the 180 ft stopping zone as defined by the sets of orange cones indicated in Fig. 5. The approach speed to the test course was nominally 40 miles an hour, which would permit the driver to stop in 180 ft at a nominal deceleration of 0.3g. Drivers were told to imagine that the stopping barrier indicated by two orange cones was a car that had pulled out in front of them or possibly pedestrians that had moved into their path and that they were to do their best to stop within the lane before reaching this barrier. Subjects were not told anything about the objectives of the tests other than that we were testing stopping behavior and would be making some variations in the car characteristics.

![Test Course Layout](image)

Figure 5. Test Course Layout
The test car was outfitted with a special valve that permitted changing the proportioning of brake pressure going to the rear brakes. Valve settings were setup to achieve three experimental conditions:

A - Significant tendency for front brakes to lockup  
B - Moderate tendency for rear brakes to lockup  
C - Significant tendency for rear brakes to lockup

In braking, driver's do have the option to modulate their brakes and avoid or at least minimize wheel lockup, and the above experimental condition's allowed for observing this behavior over a range of possible brake balance conditions.

The above three brake bias conditions were tested for each subject in the design indicated in Table 6. The conditions were tested on consecutive runs for each subject. In order to avoid biasing the results, the ordering of the test conditions was changed between subjects as indicated in Table 6.

### TABLE 6. EXPERIMENTAL DESIGN

<table>
<thead>
<tr>
<th>Test Conditions:</th>
</tr>
</thead>
<tbody>
<tr>
<td>A - 1:1 valve setting (front bias)</td>
</tr>
<tr>
<td>B - 1:2 valve setting (smaller rear bias)</td>
</tr>
<tr>
<td>C - 1:3 valve setting (larger rear bias)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Condition Orders Assigned to Subjects in Sequential Order:</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) A, B, C 4) B, A, C</td>
</tr>
<tr>
<td>2) C, B, A 5) C, A, B</td>
</tr>
<tr>
<td>3) B, C, A 6) A, C, B</td>
</tr>
</tbody>
</table>

**Results**

In Fig. 6 distributions of directional control performance metrics are given. For final heading deviations it is noted that the worst performance was encountered under condition C. The best or smallest heading angle deviations were achieved under the front bias condition (A) as might be expected since front wheel lockup does not tend to excite the directional mode of the vehicle or result in unstable dynamics. Final heading angle deviation is an overall directional control metric and it should be noted that only the
a) Dist. of Abs. Final Heading

Bake Balance Conditions:
A - Significant front wheel lock tendency
B - Moderate rear wheel lock tendency
C - Significant rear wheel lock tendency

b) Dist. of Abs. Peak Yaw Rate (sc)

Figure 6. Distributions of Directional Control Performance Measures
poorest third of the subjects are having a significant control problem. Referring to peak yaw rate distributions in Part b of Fig. 6, note that this intermediate directional control metric gives the same ranking of the brake bias conditions as did heading angle, but tends to be a more sensitive measure in that now fully half of the subject population is having trouble with the rear bias brake conditions.

In general the field test results tend to confirm that rear brake lockup leads to directional control problems, which will cause problems for some portion of the driving public. Although the vehicle dynamics alone represent a dynamic instability which is characterized by an exponentially divergent heading mode, the driver can exert some influence over vehicle heading through steering actions. In many cases even though the rear brakes were locked up and the vehicle itself was unstable drivers were able to exert positive steering control on the vehicle and maintain adequate directional control. There were a few runs, however, where drivers exerted little or no steering action and the vehicle spinout was basically a classical exponential divergence. There were 12 such runs and from yaw rate gyro strip chart records of these few runs we were able to measure a divergent time constant. The distribution of these divergent time constants is illustrated in Fig. 7. Note that one half of these runs or 6 runs in total were near the theoretical vehicle only divergence time constant given in Table 5.

![Image: Driver/Vehicle System Divergence Time Constants Measured from Yaw Rate Recordings for Runs Exhibiting Little or No Driver Control](image-url)
CONCLUDING REMARKS

Human operator control of unstable vehicle dynamics is a fairly well understood problem based on over two decades of research. Limiting human operator capability is constrained to a large extent by internal perceptual and processing time delays. Training and other system characteristics have some influence on limit performance. Analysis of driver/vehicle behavior under rear wheel lockup conditions shows a classical unstable vehicle control problem which leads to loss of control for some portion of the driver population. Experimental results are consistent with a driver/vehicle system stability analysis and past research on limit control capabilities and unexpected transition of vehicle dynamics.

ACKNOWLEDGEMENTS

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REFERENCES


