# Guidebook for Analysis of Tether Applications

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Final Report on Contract RH4-394049 with the Martin Marietta Corporation

William Nobles, Technical Monitor

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Effects of Tether Deployment and Release

\[ M_1 r_1 + M_2 r_2 = M_{12} r_{12} \]

- \( 7L \) if hanging release
- \( <14L \) if swinging release
- \( >14L \) if spun or winched

[Diagram showing orbit and effects of tether deployment and release]
This Guidebook is intended as a tool to facilitate initial analyses of proposed tether applications in space. The guiding philosophy is that at the beginning of a study effort, a brief analysis of ALL the common problem areas is far more useful than a detailed study in any one area. Such analyses can minimize the waste of resources on elegant but fatally flawed concepts, and can identify the areas where more effort is needed on concepts which do survive the initial analyses.

In areas in which hard decisions have had to be made, the Guidebook is:

- Broad, rather than deep
- Simple, rather than precise
- Brief, rather than comprehensive
- Illustrative, rather than definitive

Hence the simplified formulas, approximations, and analytical tools included in the Guidebook should be used only for preliminary analyses. For detailed analyses, the references with each topic & in the bibliography may be useful. Note that topics which are important in general but not particularly relevant to tethered systems analysis (e.g., radiation dosages) are not covered.

CREDITS

This Guidebook was prepared by the author under subcontract RH4-394049 with the Martin Marietta Corporation, as part of its contract NAS8-35499 (Phase II of a Study of Selected Tether Applications in Space) with the NASA Marshall Space Flight Center. Some of the material was adapted from references listed with the various topics, and this assisted the preparation greatly. Much of the other material evolved or was clarified in discussions with one or more of the following: Dave Arnold, James Arnold, Ivan Bekey, Guiseppe Colombo, Milt Contella, Dave Criswell, Don Crouch, Andrew Cutler, Mark Henley, Don Kessler, Harris Mayer, Jim McCoy, Bill Nobles, Tom O'Neil, Paul Penzo, Jack Slowey, Georg von Tiesenhausen, and Bill Thompson. The author is of course responsible for all errors, and would appreciate being notified of any that are found.

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## Generic Issues in Various Tether Applications

### Major Constraints in Momentum-Transfer Applications

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### Major Constraints with Permanently-Deployed Tethers

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<td>Gravity Use: Hanging Spinning</td>
<td>Libr-sensitive</td>
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<td>&lt;.1 gee only. Docking awkward</td>
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Basic orbit nomenclature & equations are needed frequently in following pages. Comparison of tether & rocket operations requires orbit transfer equations.

The first equation in the box is known as the Vis Viva formulation, and to the right of it is the equation for the mean orbital angular rate, \( n \). Much of the analysis of orbit transfer \( \Delta V_s \) and tether behavior follows from those two simple equations. Some analyses require a close attention to specific angular momentum, \( h \), so an expression for \( h \) (for compact objects) is also given here.

In general, six parameters are needed to completely specify an orbit. Various parameter sets can be used (e.g., 3 position coordinates & 3 velocity vectors). The six parameters listed at right are commonly used in orbital mechanics. Note that when \( i = 0 \), \( \Omega \) becomes indeterminate (and unnecessary); similarly with \( \omega \) when \( e = 0 \). Also, \( i \) & \( \Omega \) are here referenced to the central body's equator, as is usually done for Low Earth Orbit (LEO). For high orbits, the ecliptic or other planes are often used. This simplifies calculation of 3rd body effects.

The effects of small \( \Delta V_s \) on near-circular orbits are shown at right. The relative effects are shown to scale: a \( \Delta V \) along the velocity vector has a maximum periodic effect 4 times larger than that of the same \( \Delta V \) perpendicular to it (plus a secular effect in \( \theta \) which the others don't have). Effects of oblique or consecutive \( \Delta V_s \) are simply the sum of the component effects. Note that out-of-plane \( \Delta V_s \) at a point other than a node also affect \( \Omega \).

For large \( \Delta V_s \), the calculations are more involved. The perigee and apogee velocities of the transfer orbit are first calculated from the Vis Viva formulation and the constancy of \( h \). Then the optimum distribution of plane change between the two \( \Delta V_s \) can be computed iteratively, and the required total \( \Delta V \) found. Typically about 90% of the plane change is done at GEO.

To find how much a given in-plane tether boost reduces the required rocket \( \Delta V \), the full calculation should be done for both the unassisted and the tether-assisted rocket. This is necessary because the tether affects not only the perigee velocity, but also the gravity losses and the LEO/GEO plane change split. Each m/s of tether boost typically reduces the required rocket boost by \( 0.89 \) m/s (for hanging release) to \( 0.93 \) m/s (for widely librating release).

Note that for large plane changes, and large radius-ratio changes even without plane changes, 3-impulse "bi-elliptic" maneuvers may have the lowest total \( \Delta V \). Such maneuvers involve a boost to near-escape, a small plane and/or perigee-adjusting \( \Delta V \) at apogee, and an apogee adjustment (by rocket or aerobrake) at the next perigee. In particular, this may be the best way to return aerobraking OTVs from GEO to LEO, if adequate time is available.

REFERENCES
Orbit & Orbit Transfer Equations

\[ r_{apo} = a(1+e) \quad r_{per} = a(1-e) \]

\[ r = \frac{p}{1+e \cos \theta} \]

\[ a = \text{semi-major axis} \]
\[ e = \text{eccentricity} \]
\[ i = \text{inclination} \]
\[ \Omega = \text{long. of asc. node} \]
\[ \omega = \text{argument of periapsis} \]
\[ \mu_0 = \text{position at epoch} \]

\[ V^2 = \mu(\frac{2}{r} - \frac{1}{a}) \]
\[ n = \frac{\mu}{a^2} \]
\[ h = \sqrt{\mu a} = r^2 \phi = r \cos \phi \]
\[ V_{circ} = \frac{\mu}{r} \]
\[ V_{esc} = 2\mu/r \]
\[ \mu_x = G \times \text{Mass of x} \]

\[ M = M_0 + nt \]

**Effects of Small \( \Delta V \)s on Near-Circular Orbits**

\[ \Delta r = \sin \theta \Delta V_n/\mu \]
\[ \Delta \theta \approx 2(\cos \phi - 1) \Delta V_n/\mu \]
\[ \Delta \Omega = \frac{2(1 - \cos \phi)}{\mu} \Delta V_n/\mu \]
\[ \Delta \omega \approx \frac{4 \sin \phi \Delta V_n}{\mu} \]

\[ \Delta \Omega \approx \frac{\Delta V}{V_{circ}} \] (in radians)

\[ \Delta \Omega = 0 \]

\[ \Delta V_2 = V_{\text{GEO}} - V_{\text{apo}} \]
\[ V_{\text{apo}} = V_{\text{per}} \frac{r_{\text{LEO}}}{r_{\text{GEO}}} \]

\[ \Delta V_1 = V_{\text{per}} - V_{\text{LEO}} \]
\[ v_{\text{per}} = \frac{\mu}{r_{\text{LEO}} + r_{\text{GEO}}} \]

**Total \( \Delta V \) is minimized when**
\[ \frac{\sin y_{\text{LEO}}}{\sin y_{\text{GEO}}} = \frac{r_{\text{LEO}}}{r_{\text{GEO}}} \]

Large Orbit Transfers (e.g., LEO—GEO)
KEY POINTS

Differential nodal regression severely limits coplanar rendezvous windows. Apsidal recession affects STS deboost requirements from elliptical orbits. Third bodies can change the orbit plane of high-orbit facilities.

*The geoid (earth's shape) is roughly that of a hydrostatic-equilibrium oblate ellipsoid, with a 296:297 polar:equatorial radius ratio. There are departures from this shape, but they are much smaller than the 1:297 oblateness effect and have noticeable effects only on geosynchronous and other resonant orbits.*

The focus here is on oblateness, because it is quite large and because it has large secular effects on Ω and ω for nearly all orbits. (Oblateness also affects n, but this can usually be ignored in preliminary analyses.) As shown at right, satellites orbiting an oblate body are attracted not only to its center but also towards its equator. This force component imposes a torque on all orbits that cross the equator at an angle, and causes the direction of the orbital angular momentum vector to regress as shown.

Ω is largest when i is small, but the plane change associated with a given ∆Ω varies with sin i. Hence the actual plane change rate varies with sin i cos i, or sin2i, and is highest near 45°. For near-coplanar rendezvous in LEO, the required out-of-plane ∆V changes by 78 sin 2i m/s for each phasing "lap". This is independent of the altitude difference (to first order), since phasing & differential nodal regression rates both scale with ∆a. Hence even at best a rendezvous may require an out-of-plane ∆V of 39 m/s. At other times, out-of-plane ∆Vs of 2 sin i sin(Ω/2) V_circ (=up to 2 V_circ') are needed.

The linkage between phasing and nodal regression rates is beneficial in some cases: if an object is boosted slightly and then allowed to decay until it passes below the boosting object, the total ∆Ω is nearly identical for both. Hence recapture need not involve any significant plane change.

Apsidal recession generally has a much less dominant effect on operations, since apsidal adjustments (particularly of low-e orbits) involve much lower ∆Vs than nodal adjustments. However, tether payload boosts may often be done from elliptical STS orbits, and perigee drift may be an issue. For example, OMS deboost requirements from an elliptical STS orbit are tonnes lower (and payload capability much higher) if perigee is near the landing site latitude at the end of the mission. Perigee motion relative to day/night variations is also important for detailed drag calculations, and for electrodynamic day-night energy storage (where it smears out and limits the eccentricity-pumping effect of a sustained day-night motor-generator cycle).

Just as torques occur when the central body is non-spherical, there are also torques when the satellite is non-spherical. These affect the satellite's spin axis and cause it to precess around the orbital plane at a rate that depends on the satellite's mass distribution and spin rate.

In high orbits, central-body perturbations become less important and 3rd-body effects more important. In GEO, the main perturbations (~47 m/s/yr) are caused by the moon and sun. The figure at right shows how to estimate these effects, using the 3rd body orbital plane as the reference plane.

NOTES

REFERENCES

Orbital Perturbations

Nodal Regression in LEO:

\[ \dot{\Omega} = \frac{-63.6 \cos i \text{ rad/yr}}{(a/re)^{3.5} (1-e^2)^2} \]

\( (r_e = 6378 \text{ km}) \)

For sun-synchronous orbits: \( (i=100^\circ \pm \delta) \)
\[ \cos i = -0.0988(a/re)^{3.5}(1-e^2)^2 \]

For coplanar \( \Delta V \) rendezvous between 2 objects \( (e_i = e_s = 0, I_i = I_s) \), nodal coincidence intervals are:
\[ \Delta t_{nc} = \frac{180 (r_e/\text{km})^{4.5}}{\Delta A |\cos i|} \text{ km yrs} \]

Apsidal recession in LEO:

\[ \dot{\omega} = \frac{63.6(2 - 2.5 \sin^2 i)}{(a/re)^{3.5} (1-e^2)^2} \text{ rad/yr} \]

\( i<63.4^\circ \quad i=63.4^\circ \quad i>63.4^\circ \)

Motion of the longitude of perigee with respect to the sun's direction ("noon") is:
\[ \bar{\omega}_s = \dot{\omega} + \dot{\Omega} - 2\pi/\text{yr} \]

Third-Body Perturbations (non-resonant orbits)

\[ \dot{\omega}_3 = -7.5 \cos i_{3}, \mu_3/r_3^3 \]

"Smeared out" 3rd body
Tether drag affects tether shape & orbital life; atomic oxygen degrades tethers. Out-of-plane drag component can induce out-of-plane tether libration. The main value of payload boosting by tether is the increased orbital life. Unboosted orbital life of space facilities is affected by tether operations.

The figure at right shows the orbiter trolling a satellite in the atmosphere, as is planned for the 2nd TSS mission in the late 1980s. The tether drag greatly exceeds that on the end-masses and should be estimated accurately. The drag includes a small out-of-plane component that can cause libration.

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Tether drag is experienced over a range of altitudes, over which most of the terms in the drag equation vary: the air density \( \rho \), the airspeed \( V_{rel} \), and the tether width & angle of attack. In free-molecular flow, \( C_L \) is small, and \( C_D \) (if based on \( A_L \)) is nearly constant at 2.2. (\( C_D \) rises near grazing incidence, but then \( A_L \) is low.)

Only \( \rho \) varies rapidly, but it varies in a way which lends itself to simple approximations. Empirical formulae have been developed by the author and are shown at right. They give values that are usually within 25% of ref. 1, which is still regarded as representative for air density as a function of altitude & exosphere temperature. These estimates hold only for \( \rho > 10^{-14} \), beyond which helium & hydrogen dominate & the density scale height \( H \) increases rapidly.

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Note that over much of LEO, atomic oxygen is the dominant species. Hyperthermal impact of atomic oxygen on exposed surfaces can cause rapid degradation, and is a problem in low-altitude applications of organic-polymer tethers.

The space age began in 1957 at a 200-yr high in sunspot count. A new estimate of mean solar cycle temperatures (at right, from ref. 2), is much lower than earlier estimates. Mission planning requires both high & mean estimates for proper analysis. Ref. 2 & papers in the same volume discuss models now in use.

If the tether length \( L \) is \(< H \), the total tethered system drag can be estimated from the total \( A_L \) & the midpoint \( V \) & \( \rho \). If \( L >> H \), the top end can be neglected, the bottom calculated normally, and the tether drag estimated from \( L \rho_{bottom} \)

As shown at right, the orbital life of more compact objects (such as might be boosted or deboosted by tether) can be estimated analytically if \( T_{eq} \) is known. For circular orbits with the same \( r \), \( V_{rel} \) & \( \rho \) both vary with \( i \), but these variations tend to compensate & can both be ignored in first-cut calculations.

The conversion of elliptical to "equal-life" circular orbits is an empirical fit to an unpublished parametric study done by the author. It applies when apsidal motions relative to the equator and relative to the diurnal bulge are large over the orbital life; this usually holds in both low & high-i orbits. For a detailed study of atmospheric drag effects, ref. 3 is still useful.

Aerodynamic Drag

\[ F_{\text{drag}} = 0.5 \rho C_D \frac{V_{\text{rel}}^2 \Delta r}{\text{Width} \delta r} \]

\[ V_{\text{rel}} = V_{\text{orb}} - V_{\text{air}} \]

\[ V_{\text{e}} = 0.465 \cos(\text{Lat}) \text{ km/sec} \]

Lift & Drag in Free-Molecular Flow

\[ (\lambda > D_{\text{tether}}; \lambda = 10^{-7} \text{kg/m}^3) \]

Air Density as Function of Altitude & Exosphere Temperature

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<th>Pressure Ratio</th>
<th>Height (km)</th>
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<tr>
<td>70 &lt; Alt &lt; 118</td>
<td>( \rho = 11 \exp(-\text{Alt}/6) )</td>
<td>(-\rho/\rho_0 = H = 6 \text{ (km)})</td>
<td></td>
</tr>
<tr>
<td>118 &lt; Alt &lt; 200</td>
<td>( \rho = (\text{Alt}-95)^2/2600 )</td>
<td>( H = (\text{Alt}-95)/3 )</td>
<td></td>
</tr>
<tr>
<td>200 &lt; Alt &lt; 400</td>
<td>( \rho = 1.47 \times 10^{-16} \text{Tex}(3000 - \text{Tex}) )</td>
<td>( H = 0.1(\text{Alt}-200) + \text{Tex}/29 )</td>
<td></td>
</tr>
<tr>
<td>Alt &gt; 400</td>
<td>( \rho &gt; 1 \times 10^{-14} )</td>
<td></td>
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Circular Orbit Life

\[ \text{Orbit Life} \approx \frac{0.15 m^2 \text{yr}}{\text{kg}} \frac{M}{C_D A} \left( \frac{1 + 2.9(r-6578)}{\text{Tex}} \right)^{11} \]

\((-14 < \log \rho < -10)\)

Equal-Life

\[ \frac{\text{Perigee}}{2} + \frac{\text{Apo} - \text{Per}}{0.154(\text{Apo} - \text{Per})/H_{\text{Per}}} \]

\(-7-\)
Aerothermal heating of tethers is severe at low altitudes (<120 km). Tether temperature affects strength, toughness, & electrical conductivity. Extreme thermal cycling may degrade pultruded composite tethers. "View factors" are also used in refined micrometeoroid risk calculations.

Key Points

Preliminary heat transfer calculations in space are often far simpler than typical heat transfer calculations on the ground, since the complications introduced by convection are absent. However the absence of the "clamping" effect of large convective couplings to air or liquids allows very high or low temperatures to be reached, and makes thermal design important.

At altitudes below about 140 km in LEO, aerodynamic heating is the dominant heat input on surfaces facing the ram direction. The heating scales with ρ as long as the mean free path λ is much larger than the object's radius. It is about equal to the energy dissipated in stopping incident air molecules. In denser air, shock & boundary layers develop. They shield the surface from the incident flow and make Q rise slower as ρ increases further. (See ref 1.)

Because tethers are narrow, they can be in free molecular flow even at 100 km, and may experience more severe heating than the (larger) lower end masses do. Under intense heating high temperature gradients may occur across non-metallic tethers. These gradients may cause either overstress or stress relief on the hot side, depending on the sign of the axial thermal expansion coefficient.

Notes

At higher altitudes the environment is much more benign, but bare metal (low-emittance) tethers can still reach high temperatures when resistively heated or in the sun, since they radiate heat poorly. Silica, alumina, or organic coatings >1 μm thick can increase emittance and hence reduce temperatures. The temperature of electrodynamic tethers is important since their resistance losses (which may be the major system losses) scale roughly with T_{abs}.

For a good discussion of solar, albedo, and longwave radiation, see ref. 2. The solid geometry which determines the gains from these sources is simple but subtle, and should be done carefully. Averaged around a tether, earth view-factors change only slowly with altitude & attitude, and are near .3 in LEO.

Surface property changes can be an issue in long-term applications, due to the effects of atomic oxygen, UV & high-energy radiation, vacuum, deposition of condensable volatiles from nearby surfaces, thermal cycling, etc. Hyper-thermal atomic oxygen has received attention only recently, and is now being studied in film, fiber, and coating degradation experiments on the STS & LDEF.

Continued thermal cycling over a wide range (such as shown at bottom right) may degrade composite tethers by introducing a maze of micro-cracks. Also, temperature can affect the strength, stiffness, shape memory, and toughness of tether materials, and hence may affect tether operations and reliability.

References

Thermal Balance

\[ \hat{Q}_{\text{emitted}} = A \varepsilon \varepsilon_0 T^4 \]  
\[ (\varepsilon = 5.68 \times 10^{-8} \text{ W/m}^2 \text{K}) \]

\[ \dot{Q}_\text{eq} = \frac{\sum \theta \sin \theta}{A \varepsilon \varepsilon_0} \]

\[ \dot{Q}_\text{emitted} = A \varepsilon \varepsilon_0 T^4 \]  
\[ (\varepsilon = 5.68 \times 10^{-8} \text{ W/m}^2 \text{K}) \]

\[ \dot{Q}_\text{internal} = \dot{Q}_\text{thermal} + \text{any others} \]

\[ \dot{Q}_\text{aerodyn.} = 0.5 \rho A V^3 \text{ (see "Aero. Drag")} \]

\[ \dot{Q}_\text{albedo} = 0.37 (\pm 0.3) \times 1368 (\pm 40) \text{ W/m}^2 \]

\[ \dot{Q}_\text{earth} = 215 (\pm 100) \text{ W/m}^2 \]

- Graph showing Earth Viewfactors in LEO

- Table showing Inclination and Beta Range:
  - 0°: 0-23.5°
  - 28.5°: 0-51°
  - >56.5°: 0-90°

- Graph showing Tether Temperature Over 1 Orbit

(G = Sun out-of-plane angle)
KEY POINTS

Micrometeoroids can sever thin tethers & damage tether protection/insulation.
Orbiting debris can sever tethers of any diameter.

At the start of the space age, estimates of meteoroid fluxes varied widely. Earth was thought to have a dust cloud around it, due to misinterpretation of data such as microphone noise caused by thermal cycling in spacecraft. By the late 1960s most meteoroids near earth were recognized to be in heliocentric rather than geocentric orbit. The time-averaged flux is mostly sporadic, but meteor showers can be dominant during their occurrence.

There is a small difference between LEO and deep-space fluxes, due to the focusing effect of the earth's gravity (which increases the velocity & flux), and the partial shielding provided by the earth & "sensible" atmosphere. For a typical meteoroid velocity of 20 km/sec, these effects combine to make the risk vary as shown at right in LEO, GEO, and beyond. The picture of a metal plate after hypervelocity impact is adapted from ref. 3.

The estimated frequency of sporadic meteoroids over the range of interest for most tether applications is shown by the straight line plot at right, which is adapted from ref. 4 & based on ref. 1. (Ref 1 is still recommended for design purposes.) For masses<1E-6 gm (<15 mm diam. at an assumed density of .5), the frequency is lower than an extension of that line, since several effects clear very small objects from heliocentric orbits in geologically short times.

Over an increasing range of altitudes and particle sizes in LEO, the main impact hazard is due not to natural meteoroids but rather to man-made objects. The plots at right, adapted from refs 4 & 5, show the risks presented by the 5,000 or so objects tracked by NORAD radars (see ref. 6). A steep "tail" in the 1995 distribution is predicted since it is likely that several debris-generating impacts will have occurred in LEO before 1995. Such impacts are expected to involve a 4-40 cm object striking one of the few hundred largest objects and generating millions of small debris fragments.

Recent optical detection studies which have a size threshold of about 1 cm indicate a population of about 40,000 objects in LEO. This makes it likely that debris-generating collisions have already occurred. Studies of residue in small surface pits on the shuttle and other objects recovered from LEO indicate that they appear to be due to titanium, aluminum, and paint fragments (perhaps flaked off satellites by micrometeoroid hits). Recovery of the Long Duration Exposure Facility (LDEF) later this year should improve this database greatly, and will provide data for LEO exposure area-time products comparable to those in potential long-duration tether applications.

NOTES

REFERENCES

Micrometeoroids & Debris

Relative μm Risks in LEO

\[ \text{RelRisk} = (1 - F_{\text{Earth}})(0.57 + 0.43 r_e / r) \]

Population Corrected to 4 cm Limiting Size

Observed Debris Flux (corrected to 4-cm limiting size)

Altitude, km

Observed Debris Flux (corrected to 4-cm limiting size)

Threshold Diameter, cm

Cumulative Flux in 1995 (600-1100 km)

Relative Collision Frequency

Debris Impact Velocity

Orbital Motion

Relative Frequency of Space Debris Flux as a Function of Direction of Approach (Alt = 500 km, i = 30°)
"Microgee" environments are possible only in small regions (~5 m) of a LEO facility. Milligee-level gravity is easy to get & adequate for propellant settling, etc.

The figure at right shows the reason for gravity-gradient effects. The long tank-like object is kept aligned with the local vertical, so that the same end always faces the earth as it orbits around it. If one climbs from the bottom to the top, the force of gravity gradually decreases and the centrifugal force due to orbital motion increases. Those forces cancel out only at one altitude, which is (nearly but not exactly) the altitude of the vehicle's center of mass.

At other locations an object will experience a net force vertically away from the center of mass (or a net acceleration, if the object is allowed to fall). This net force is referred to as the "gravity-gradient force." (But note that 1/3 of the net force is actually due to a centrifugal force gradient!) Exact and approximate formulas for finding the force on an object are given at right.

The force occurs whether or not a tether is present, and whether or not it is desirable. Very-low-acceleration environments, which are needed for some types of materials processing and perhaps for assembling massive structures, are only available over a very limited vertical extent, as shown at right. Putting a vehicle into a slow retrograde spin can increase the "height" of this low-gee region, but that then limits the low-gee region's other in-plane dimension.

Since gravity gradients in low orbits around various bodies vary with \( \mu/r^3 \), the gradients are independent of the size of the body, and linearly dependent on its density. Hence the gradients are highest (~3-4 milligee/km) around the inner planets and Earth's moon, and 60-80% lower around the outer planets. In higher orbits, the effect decreases rapidly (to 1.6 microgee/km in GEO).

The relative importance of surface tension and gravity determines how liquids behave in a tank, and is quantified with the Bond number, \( \text{Bo} = \sigma r_\ell a^2/\alpha \). If \( \text{Bo} > 10 \), liquids will settle, but higher values (\( \text{Bo} = 50 \)) are proposed as a conservative design criterion. On the other hand, combining a small gravity gradient effect (\( \text{Bo} < 10 \)) with minimal surface-tension fluid-management hardware may be more practical than either option by itself. Locating a propellant depot at the end of a power-tower structure might provide an adequate gravity-gradient contribution. If higher gravity is desired, but without deploying the depot, another option is to deploy an "anchor" mass on a tether, as shown at right.

Many nominally "zero-gee" operations such as electrophoresis may actually be compatible with useful levels of gravity (i.e., useful for propellant settling, simplifying hygiene activities, keeping objects in place at work stations, etc). This needs to be studied in detail to see what activities are truly compatible.

NOTES

REFERENCES
Gravity Gradient Effects

Origin of "Gravity-Gradient" Forces

Two Propellant-Settling Options

Potential Overlap of Regions for Low-Gee & Gee-Dependent Operations
Libration periods are independent of length, but increase at large amplitude. Out-of-plane libration can be driven by weak forces that have a 2n component. Tethers can go slack if $\theta_{\text{max}}>65^\circ$ or $\phi_{\text{max}}>60^\circ$.

The two figures at right show the forces on a dumbbell in circular orbit which has been displaced from the vertical, and show the net torque on the dumbbell, returning it towards the vertical. The main difference between the two cases is that the centrifugal force vectors are radial in the in-plane case, and parallel in the out-of-plane case. This causes the net force in the out-of-plane case to have a smaller axial component and a larger restoring component, and is why $\phi$-libration has a higher frequency than $\theta$-libration.

Four aspects of this libration behavior deserve notice. First, the restoring forces grow with the tether length, so libration frequencies are independent of the tether length. Thus tether systems tend to librate "solidly", like a dumbbell, rather than with the tether trying to swing faster than the end-masses as can be seen in the chain of a child's swing. (This does not hold for very long tethers, since the gravity gradient itself varies.) For low orbits around any of the inner planets or the moon, libration periods are roughly an hour.

Second, tethered masses would be in free-fall except for the tether, so the sensed acceleration is always along the tether (as shown by the stick-figures). Third, the axial force can become negative, for $\phi>60^\circ$ or near the ends of retrograde in-plane librations $>65.9^\circ$. This may cause problems unless the tether is released, or retrieved at an adequate rate to prevent slackness.

And fourth, although $\theta$-libration is not close to resonance with any significant driving force, $\phi$-libration is in resonance with several, such as out-of-plane components of aerodynamic forces (in non-equatorial orbits that see different air density in northward and southward passes) or electrodynamic forces (if tether currents varying at the orbital frequency are used). The frequency droop at large amplitudes (shown at right) sets a finite limit to the effects of weak but persistent forces, but this limit is quite high in most cases.

The equations given at right are for an essentially one-dimensional structure, with one principal moment of inertia far smaller than the other two: $A<<B<C$. If $A$ is comparable to $B$ & $C$, then the $\theta$-restoring force shrinks with $(B-A)/C$, and the $\theta$-libration frequency by $\sqrt{(B-A)/C}$. Another limitation is that a coupling between $\phi$ & $\theta$ behavior (see ref. 1) has been left out. This coupling is caused by the variation of end-mass altitudes twice in each $\phi$-libration. This induces Coriolis accelerations that affect $\phi$. This coupling is often unimportant, since $4n$ is far from resonance with $1.73n$.

Libration is referenced to the local vertical, and when a dumbbell is in an eccentric orbit, variations in the orbital rate cause librations which in turn exert periodic torques on an initially uniformly-rotating object. In highly eccentric orbits this can soon induce tumbling.

Dumbbell Libration in Circular Orbit

In-Plane Libration ($\theta$)

$$\dot{\theta} = -3n^2 \sin \theta \cos \theta = -1.5n^2 \sin(2\theta)$$

$$\dot{\theta} = \pm \sqrt{3} \ n \sin^2 \theta \max \sin^2 \theta$$

($\dot{\theta} = \pm \sqrt{3} \ n \sin \theta \max$ when $\theta=0$)

$$n_\theta \approx n \sqrt{3} \cos \theta \max$$

Out-Of-Plane Libration ($\phi$)

$$\dot{\phi} = -4n^2 \sin \phi \cos \phi = -2n^2 \sin(2\phi)$$

$$\dot{\phi} = \pm 2n \sin^2 \phi \max \sin^2 \phi$$

($\dot{\phi} = \pm 2n \sin \phi \max$ when $\phi=0$)

$$n_\phi \approx 2n \sqrt{3} \cos \phi \max$$

Tension Variations in Librating Dumbbells (compared to tension in hanging dumbbells)

Libration Freq. vs Amplitude
Open-loop control is adequate for deployment; full retrieval requires feedback. Tension laws can control θ & ϕ-libration plus tether oscillations. Many other options exist for libration, oscillation, & final retrieval control.

The table at right shows half a dozen distinct ways in which one or more aspects of tethered system behavior can be controlled. In general, anything which can affect system behavior (and possibly cause control problems) can be part of the solution, if it itself can be controlled without introducing other problems.

Thus, for example, stiff tethers have sometimes been considered undesirable, because the stiffness competes with the weak gravity-gradient forces near the end of retrieval. However, if the final section of tether is stiff AND nearly straight when stress-free (rather than pig-tail shaped), then "springy beam" control laws using a steerable boom tip might supplement or replace other laws near the end of retrieval. A movable boom has much the same effect as a stiff tether & steerable boom tip, since it allows the force vector to be adjusted.

The basic concepts behind tension-control laws are shown at right. Libration damping is done by paying out tether when the tension is greater than usual and retrieving it at other times. This absorbs energy from the libration. As shown on the previous page, in-plane libration causes large variations in tension (due to the Coriolis effect), so "yoyo" maneuvers can damp in-plane librations quickly. Such yoyo maneuvers can be superimposed on deployment and retrieval, to allow large length changes (>4:1) plus large in-plane libration damping (or initiation) in less than one orbit, as proposed by Swet.¹

Retrieval laws developed for the TSS require more time than Ref. 1, because they also include damping of out-of-plane libration built up during stationkeeping. Rupp developed the first TSS control law in 1975;² much of the work since then is reviewed in (3). Recent TSS control concepts combine tension and thrust control laws, with pure tension control serving as a backup in case of thruster failure.⁴ Axial thrusters raise tether tension when the tether is short, while others control yaw & damp out-of-plane libration to allow faster retrieval.

A novel concept which in essence eliminates the final low-tension phase of retrieval is to have the end mass climb up the tether.³ Since the tether itself remains deployed, its contribution to gravity-gradient forces and stabilization remains. The practicality of this will vary with the application.

---

# Tether Control Strategies

**EFFECTIVENESS OF VARIOUS CONTROL CONCEPTS**

<table>
<thead>
<tr>
<th>APPLICATION</th>
<th>Libration</th>
<th>Tether Oscillations</th>
<th>Endmass Attitude Osc.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>In-plane</td>
<td>Out-of-plane</td>
<td>Longitudinal Transverse</td>
</tr>
<tr>
<td>Tension</td>
<td>Strong</td>
<td>Weak</td>
<td>Strong</td>
</tr>
<tr>
<td>(Note: tension control is weak when tether is short)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>El. Thrust</td>
<td>Only if M1 ≠ M2</td>
<td>None</td>
<td>Only odd harmonics</td>
</tr>
<tr>
<td>Thruster</td>
<td>Strong, but costly if prolonged</td>
<td>None</td>
<td>Strong, but costly if prolonged</td>
</tr>
<tr>
<td>Movable mass</td>
<td>Good w/short tether</td>
<td>Possible but awkward</td>
<td>None</td>
</tr>
<tr>
<td>Stiff tether, Movable boom</td>
<td>Strong if tether is very short; weak otherwise</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Aerodynamic</td>
<td>High drag—use only if low altitude needed for other reasons.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

![Diagram of Tether Control Concepts](image)

**Equation:**

\[ \text{Tension} = k1(L - L_c) + k2\ell \]

(k1 & k2 are control gains; L & L_c are the actual and the commanded tether length.)

**Deployment & retrieval paths of tip:**

- Deployment & retrieval paths of tip
- Full retrieval takes ~6 hours with thrusters & ~24 without.
- 80 km deployed in 4.6 hours.
Tethers merely redistribute angular momentum; they do not create it. Changes in tether length, libration, and spin all redistribute momentum. Momentum transfer out-of-plane or in deep space is possible but awkward.

The two figures at right show two different tether deployment (and retrieval) techniques. In both cases, the initial deployment (which is not shown) is done with RCS burns or a long boom. In the case at left, the tether is paid out under tension slightly less than the equilibrium tension level for that tether length. The tether is slightly tilted away from the vertical during deployment, and librates slightly after deployment is complete.

In the other case, after the initial near-vertical separation (to about 2% of the full tether length), the two end masses are allowed to drift apart in near-free-fall, with very low but controlled tension on the tether. Just under one orbit later, the tether is almost all deployed and the range rate decreases to a minimum (due to orbital mechanics). RCS burns or tether braking are used to cushion the end of deployment and prevent end mass recoil. Then the tether system begins a large-amplitude prograde swing towards the vertical.

In both cases, the angular momentum transferred from one mass to the other is simply, as stated in the box, the integral over time of the radius times the horizontal component of tether tension. In one case, transfer occurs mainly during deployment; in the other, mainly during the libration after deployment. In each case, momentum transfer is greatest when the tether is vertical, since the horizontal component of tether tension changes sign then.

An intermediate strategy—deployment under moderate tension—has also been investigated. However, this technique results in very high deployment velocities and large rotating masses. It also requires powerful brakes and a more massive tether than required with the other two techniques.

As discussed under Tether Control Strategies, changing a tether's length in resonance with variations in tether tension allows pumping or damping of libration or even spin. Due to Coriolis forces, in-plane libration and spin cause far larger tension variations than out-of-plane libration or spin, so in-plane behavior is far easier to adjust than out-of-plane behavior. Neglecting any parasitic losses in tether hysteresis & the reel motor, the net energy needed to induce a given libration or spin is simply the system's spin kinetic energy relative to the local vertical, when the system passes through the vertical.

Two momentum transfer techniques which appear applicable for in-plane, out-of-plane, or deep-space use are shown at right. The winching operation can use lighter tethers than other tethered-momentum-transfer techniques, but requires a very powerful deployer motor. The tangential ΔV simply prevents a collision.

The spin-up operation (proposed by Harris Mayer) is similar to the winching operation. It uses a larger tangential ΔV, a tether with straight and tapered sections, and a small motor. Retrieval speeds up the spin by a factor of \( L^2 \). Surprisingly, the long tapered section of tether can be less than half as massive as the short straight section that remains deployed after spin-up.

Momentum Transfer During Deployment & Retrieval

Deployment Followed by Winching (in orbit or in deep space)

One Spin-Up Technique For Use in Deep Space

Momentum Transfer During Libration (after low-tension deployment)

\[ \Delta (Mvr) = rT \sin \theta t \]

High Tension During Swing

Low Tether Tension

Libration Pumping

Spin Pumping

Small \( \Delta V_s \)

Straight Tether Section

Tapered Tether Section
The achievable orbit change scales with the tether length (as long as $\Delta r \ll r$). Retrograde-libration releases are inefficient, but allow concentric orbits. Apogee & perigee boosts have different values in different applications. Tethered capture can be seen as a time-reversal of a tether release operation.

The figures to the right show the size of the orbit changes caused by various tether operations. When released from a vertical tether, the end masses are obviously one tether length apart in altitude. The altitude difference $1/2$ orbit later, $\Delta h$, varies with the operation but is usually far larger. The linear relationship shown becomes inaccurate when $\Delta r$ approaches $r$. Tethered plane changes are generally limited to a few degrees and are not covered here.

Tether release leaves the center-of-mass radius at each phase angle roughly unchanged; if the upper mass is heavier, then it will rise less than the lower mass falls, and vice-versa. Note that the libration amplitude, $\theta_{\text{max}}$, is taken as positive during prograde libration & negative during retrograde libration. Hence retrograde libration results in $\Delta r < 7L$. In particular, the pre-release & post-release orbits will all be concentric if $\theta_{\text{max}} = -60^\circ$. But since methods of causing $-60^\circ$ librations usually involve $+60^\circ$ librations (which allow much larger boosts by the same tether), prograde releases may usually be preferable unless concentric orbits are needed or other constraints enter in.

The relative tether length, mass, peak tension, and energy absorbed by the deployer brake during deployment as a function of (prograde) libration angle are all shown in the plot at right. Libration has a large effect on brake energy. This may be important when retrieval of a long tether is required, after release of a payload or after tethered-capture of a free-flying payload.

The double boost-to-escape operation at right was proposed by A. Cutler. It is shown simply as an example that even though momentum transfer is strictly a "zero sum game", a tethered release operation can be a "WIN-win game" (a large win & a small one). The small win on the deboost-end of the tether is due to the reduced gravity losses $1/2$ orbit after release, which more than compensate for the deboost itself. Another example is that deboosting the shuttle from a space station can reduce both STS-deboost & station-reboost requirements.

Rendezvous of a spacecraft with the end of a tether may appear ambitious, but with precise relative-navigation data from GPS (the Global Positioning System) it may not be difficult. The relative trajectories required are simply a time-reversal of relative trajectories that occur after tether release. Approach to a hanging-tether rendezvous is shown at right. Prompt capture is needed with this technique: if capture is not achieved within a few minutes, one should shift to normal free-fall techniques. Tethered capture has large benefits in safety (remoteness) and operations (no plume impingement; large fuel savings). The main hazard is collision, due to undetected navigation or tether failure.

**REFERENCES**

Orbit Transfer by Tethered Release or Capture

Effects of Tether Deployment and Release

\[ \Delta r_0 = L \]

\[ \Delta r_X = 1L (-60°) \]

\[ \Delta r_X = 7L (0°) \]

\[ \Delta r_X = 13L (+60°) \]

Effect of Libration on Boost (release at middle of swing)

\[ L_0 = 6 \]°

\[ \Delta r_X = L \ast (7 + \sqrt{46} \sin \theta_{\text{max}}) \]

Effects of Tether Mass, Tether Length, Brake Energy, and Amplitude

- Tether Mass \( \propto \text{Length} \times \text{MaxTension} \)
- Brake Energy \( \propto \frac{1}{(1 + 0.866 \sin \theta_{\text{max}})} \)
- Amplitude \( \propto \text{RelTension/Length} \)
- Tether Length \( \propto (\text{Length} \times \cos \theta_{\text{max}})^2 \)

Effects of Libration (for equal-\( \Delta v \) boosts)

If done right, a tether boost/deboost operation can reduce \( \Delta v \)-to-escape for both end masses!

Phasing orbit(s)

\[ \sim 12L \rightarrow 6L \rightarrow L \]}

Trajectory for Tethered Capture from Above
(in tether-centered LV-LH reference frame)

STS hovers till captured
OMV "chases" passive target
Tether operations cause higher-order repartitions of energy & angular momentum. First-order approximations that neglect these effects may cause large errors. Extremely long systems have strange properties such as positive orbital energy.

The question and answer at right are deceptively simple. The extent to which this is so, and the bizarre effects which occur in extreme cases, can be seen in the 3 graphs at right. At top, deploying & retrieving two masses on a very long massless tether changes not only the top & bottom orbital radii but also that of the CM. In addition, the free-fall location drops below the CM. Other key parameter changes under the same conditions are plotted underneath.

Note that when the tether length exceeds about 30% of the original orbital radius, the entire system lies below the original altitude. Also, at a radius ratio near L95:1, the maximum tether length compatible with a circular orbit is reached. At greater lengths (and the initial amount of angular momentum), no circular orbit is possible at any altitude.

Tether retrieval at the maximum-length point can cause the system to either rise or drop, depending on the system state at that time. If it continues to drop, there is a rapid rise in tether tension, and the total work done by the deployer quickly becomes positive. This energy input eventually becomes large enough (at 2.89:1) to even make the total system energy positive. The system is unstable beyond this point: any small disturbance will grow and can cause the tether system to escape from the body it was orbiting. (See ref. 2.)

The case shown is rather extreme: except for orbits around small bodies such as asteroids, tethers either will be far shorter than the orbital radius, or will greatly outweigh the end masses. Either change greatly reduces the size of the effects shown. The effects on arbitrary structures can be calculated using the equations listed at right, which are based on a generalization of the concept of "moments" of the vertical mass distribution. Changes in tether length or mass distribution leave h unchanged, so other parameters (including r_{CM}, n, and E) must change. (For short tethers, the changes scale roughly with the square of the system's radius of gyration.) In many cases different conditions are most easily compared by first finding the orbital radius that the system would have if its length were reduced to 0, r_{Lt0}.

The mechanism that repartsitions energy and angular momentum is that length changes cause temporary system displacements from the vertical. This causes both torques and net tangential forces on the system, which can be seen by calculating the exact net forces and couples for a non-vertical dumbbell. The same effect occurs on a periodic basis with librating dumbbells, causing the orbital trajectory to depart slightly from an elliptical shape.

Other topics which are beyond the scope of this guidebook but whose existence should be noted are: eccentricity changes due to deployment, orbit changes due to resonant spin/orbit coupling, and effects of 2- & 3-dimensional structures.


REFERENCES

-22-
Question: What are the sources of the dumbbell spin angular momentum and deployer brake energy?

Answer: Orbit changes which repartition $h$ & $E$.

For arbitrary nearly-one-dimensional vertical structures in circular orbit, analysis can be based on 5 "moments":

$$I_N = \sum M_i r_i^N \quad \text{(for } N: -2..2)$$

Each of these has physical meaning:

- $F_{grav} = -\mu \frac{I_{-2}}{r^2}$
- $E_{pot} = -\mu \frac{I_{-1}}{r}$
- Mass = $I_0$
- $F_{cen} = n^2 \frac{I_1}{r}$
- $h_{tot} = n \frac{I_2}{r}$
- $E_{kin} = \frac{1}{2}n^2 \frac{I_2}{r}$

Some other useful equations include:

$$r_{cm} = \frac{I_1}{I_0}$$

$$n^2 = \mu \frac{I_{-2}}{I_1}$$

$$E = \mu \left(\frac{1}{2}n^2 \frac{I_2}{r} - I_{-1}\right)$$

$$r_{(u^t=0)} = \frac{I_{-2}^2}{(I_1^*I_0^2)}$$

Very-long-tether effects:

Changes in energy, tension, & period

Angular momentum repartitioning, tether length, & deployer work

Equal-angular-momentum orbits

Max length for $h=1$

Orbital period

Unstable orbit ($E>0$)

Orbital tension

Exponential
KEY POINTS

Tether strength/weight ratio constrains performance in ambitious operations. Required tether mass is easily derivable from deltaV and payload mass.

Usable specific strength can be expressed in various ways. Three ways are shown at right. Vc, Lc, and Llg are here defined in terms of a typical design stress (new/m2) rather than the (higher) ultimate stress. Including the safety factor here streamlines the subsequent performance calculations. Higher safety factors are needed with non-metals than with metals since non-metals are often more variable in their properties, brittle, abrasion-sensitive, and/or creep-sensitive. A safety factor of 4 (based on short-term fiber strength) is typical for Kevlar, but the most appropriate safety factor will vary with the application.

The "characteristic velocity," Vc, is the most useful parameter in tether-boost calculations, because the tether mass can be calculated directly from \( \Delta V/Vc \), independently of the orbit, and nearly independently of the operation. The table at the bottom, which lists tether/rocket combinations that have the lowest life-cycle mass requirements, holds whenever kVc=1 km/sec & Isp=350 sec.

The characteristic length Lc is useful in hanging-tether calculations. It varies with the orbital rate n. (The simple calculation given assumes L<<r; if this is not true, 1/r effects enter in, and calculations such as those used in refs 3-5 must be used.) The safe 1-gee length Llg is mainly useful in terrestrial applications, but is included since specific strength is often quoted this way. (Note that Vc and Lc vary with Sqrt(strength), and Llg directly with strength.)

The specific modulus is of interest because it determines the speed of sound in the tether (C=the speed of longitudinal waves), the strain under design load (\( \Delta L/L=\left(Vc/C\right)^2 \)), & the recoil speed after failure under design load (\( = Vc^2/C \)).

Tether mass calculations are best done by considering each end of the tether separately. If Mp1>>Mp2, then Mt1 can be neglected in preliminary calculations.

Du Pont's Kevlar is the highest-specific-strength fiber commercially available. Current R&D efforts on high-performance polymers indicate that polyester can exhibit nearly twice the strength of Kevlar. Two fiber producers have already announced plans to produce polymers with twice the specific strength of Kevlar.

In the long run, the potential may be greater with inorganic fibers like SiC & graphite. Refs. 3-5 focus on the requirements of "space elevators." They discuss laboratory tests of single-crystal fibers and suggest that 10-fold improvements in specific strength (or 3-fold in Vc & Lc) are conceivable.

REFERENCES

Specific Strength and Required Tether Mass

C = 20 km/s
C = speed of sound
= \sqrt{\frac{\text{modulus}}{\text{density}}} 10 km/s

<table>
<thead>
<tr>
<th>TETHER STRENGTH PARAMETERS</th>
<th>5 km/s</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sqrt{\text{density}} ) = Char. vel = Vc:</td>
<td>0.25</td>
</tr>
<tr>
<td>Vc/\sqrt{3} n = Char. length = Lc:</td>
<td>125</td>
</tr>
<tr>
<td>Vc^2/g = Safe 1g length = Lg:</td>
<td>6</td>
</tr>
</tbody>
</table>

Specific Strength & Modulus of Several Tether Materials

* Design stress is assumed to be 1/2 the ultimate strength for metals and 1/4 the short-term individual fiber strength for other materials.

**Gaussian "normal" bell-shaped curve (if L < L << r)**

\[ L < L_c \quad L = L_c \quad L > L_c \]
\[ M_t < M_p \quad M_t = M_p \quad M_t > M_p \]

Tether Length & Required Mass

\[ X = \frac{\Delta V}{kV_c}, \text{ or } L/L_c \]

Required Tether Mass (Mt)

\[ X \leq 1 \]

\[ X > 1 \]

\[ \frac{M_t}{M_p} \rightarrow \frac{1}{\sqrt{2}} X e^{2X} \]

Expected # of uses

<table>
<thead>
<tr>
<th>1</th>
<th>10</th>
<th>100</th>
<th>1000</th>
</tr>
</thead>
<tbody>
<tr>
<td>Best tether ( \Delta V ), km/s</td>
<td>.14</td>
<td>.9</td>
<td>1.8</td>
</tr>
<tr>
<td>Required Mt/Mp</td>
<td>.02</td>
<td>1</td>
<td>11</td>
</tr>
</tbody>
</table>

(For \( kV_c = 1 \) km/s and rocket \( Isp = 350 \) seconds; marginal deployer & dry rocket masses neglected.)

Best Tether \( \Delta V \) for Combined Tether/Rocket Boosts
Micrometeoroids can sever thin tethers & damage tether protection/insulation. Orbiting debris (or other tethers) can sever tethers of any diameter. Debris could impact an Earth-based "Space Elevator" over once per year.

KEY POINTS

Sporadic micrometeoroids are usually assumed to have an typical density of about .5 and a typical impact velocity in LEO of approximately 20 km/sec. At impact speeds above the speed of sound, solids become compressible and the impact shock wave has effects like those of an explosion. For this reason, the risk curve assumes that if the EDGE of an adequately large meteoroid comes close enough to the center of the tether (within 45° or .35 Dt), failure will result.

Experiments done by Martin Marietta on TSS candidate materials have used glass projectiles fired at 6.5 km/sec, below the (axial) speed of sound in Kevlar. Two damaged tethers from those tests are shown at right. The scaling law used (ρ^0.5v^0.67) indicates that this is representative of orbital conditions, but that law (used for impacts on sheet metal) may not apply to braided fibers.

For tethers much thicker than 10 mm or so (depending on altitude), the risk does not go down much as Dt increases, because even though the micrometeoroid risk still decreases, the debris risk (which INCREASES slightly with Dt) begins to dominate. As with micrometeoroids, the tether is assumed to fail if any part of the debris passes within .35 Dt of the center of the tether.

The debris risk at a given altitude varies with the total debris width at that altitude. This was estimated from 1983 CLASSEY radar-cross-section (RCS) data, by simply assuming that W = Sqrt(RCS) and summing Sqrt(RCS) over all tracked objects in LEO. This underestimates W for objects with appendages, and overestimates it for non-librating elongated objects without appendages.

CLASSEY RCS data are expected to be accurate for RCS > 7 m2. The 700 objects with RCS > 7 m2 account for 3 km of the total 5 km width, so errors with smaller objects are not critical. Small untracked objects may not add greatly to the total risk: 40,000 objects averaging 2 cm wide would increase the risk to a 1-cm tether by only 20%. W was assumed independent of altitude, so the distribution of risk with altitude could be estimated by simply scaling Figure 1 from Ref. 4.

As shown at right, debris impact with a space elevator could be expected more than once per year at current debris populations. The relative density at 0° latitude was estimated from data on pp. 162-163 of ref. 6.

Similar calculations can be made for two tethers in different orbits at the same altitude. If at least one is spinning or widely librating, the mutual risks can exceed .1 cut/km.yr. This makes "tether traffic control" essential.

REFERENCES

Impact Hazards for Tethers

Braided Kevlar, grazed

Stainless steel wire, direct hit

Effective Width, $W$
(Any position between the 2 extremes shown cuts the tether.)

Debris Risk to the Lowest 4000 km of an Earth-based Space Elevator:

$$\text{Risk} = \sum \text{Width} \cdot \bar{V} \cdot \text{RelDensity at } \lambda=0 \over \text{Earth "Surface Area" at Alt}$$

$$= \frac{5 \text{ km} \cdot \sim 7.3 \text{ km/sec} \cdot \sim 0.72}{4 \cdot \pi \cdot \text{Sqr}(\sim 7378 \text{ km})}$$

$$= 3.9 \times 10^{-8} / \text{sec} \approx 1.2 \text{ cuts/year}$$
Tether (and other) resistance can limit the output of electrodynamic tethers. Electron collection methods & effectiveness are important—and uncertain.

Since the publication of ref. 1, 20 years ago, electrodynamic tether proposals and concepts have been a frequent source of controversy, mainly in these areas:

1. What plasma instabilities can be excited by the current?
2. What is the current capacity of the plasma return loop?
3. What is the best way to collect electrons from the plasma?

The first Tethered Satellite mission may do much to answer these questions. The discussion below and graphics at right merely seek to introduce them.

The current flowing through an electrodynamic tether is returned in the surrounding plasma. This involves electron emission, conduction along the geomagnetic field lines down to the lower ionosphere, cross-field conduction by collision with neutral atoms, and return along other field lines.

The tether current causes a force on the tether (and on the field) perpendicular to both the field and the tether (horizontal, if the tether is vertical). Motion of the tether through the geomagnetic field causes an EMF in the tether. This allows the tether to act as a generator, motor, or self-powered ultra-low-frequency broadcast antenna. The motion also causes each region of plasma to experience only a short pulse of current, much as in a commutated motor.

Based on experience with charge neutralization of spacecraft in high orbit, it has been proposed that electrons be collected by emitting a neutral plasma from the end of the tether, to allow local cross-field conduction. In GEO, the geomagnetic field traps a plasma in the vicinity of the spacecraft, and "escape" along field lines may not affect its utility. This may also hold in high-inclination orbits in LEO. But in low inclinations in LEO, any emitted plasma might be promptly wiped away by the rapid motion across field lines.

A passive collector such as a balloon has high aerodynamic drag, but an end-on sail can have an order of magnitude less drag. The electron-collection sketch at bottom right is based on a preliminary analysis by W. Thompson. This analysis suggests that a current moderately higher than the electron thermal current (≈Ne * ~200 km/sec) might be collected on a surface normal to the field. This is because collecting electrons requires that most ions be reflected away from the collection region as it moves forward. This pre-heats and densifies the plasma ahead of the collector. The voltage required for collection is just the voltage needed to repel most of the ions, about 12 V.

Electrodynamic Tether Principles

Electrons

DECELERATING FORCE

CURRENT

EARTH'S MAGNETIC FIELD

ORBITAL VELOCITY

PLASMA CONTACTOR

SPACE STATION

POWER (GENERATOR)

THRU5T (MOTOR)

Electron collector

Tether

Load

Electron emitter

180 V/km * cos i

Collisional cross-field conduction in lower ionosphere.

Max Efficiency

Useful output

Generator Performance

Sunspot maximum:

- daytime

Sunspot minimum:

- daytime

Low density plasma region

Geomagnetic field

Top View of Electron Collection

Log_{10} n_e/m^3

-29-
Electrodynamic tether use will affect the orbit—whether desired or not. Stationkeeping and/or large orbit changes without propellant use are possible.

The offset dipole approximation shown at right is only a first approximation to the geomagnetic field: harmonic analyses of the field give higher-order coefficients up to 20% as large as the fundamental term. Ref. 1 contains computerized models suitable for use in detailed electrodynamic studies.

The geomagnetic field weakens rapidly as one moves into higher orbits, and becomes seriously distorted by solar wind pressure beyond GEO. However, ohmic losses in a tether are already significant in LEO, so electrodynamic tethers are mainly useful in low orbits where such distortions are not significant.

As the earth rotates, the geomagnetic field generated within it rotates also, and the geomagnetic radius and latitude of a point in inertial space vary over the day. If a maneuvering strategy which repeats itself each orbit is used (necessary unless the spacecraft has large diurnal power storage capacity), then the average effect, as shown at right, will be a due east thrust vector.

Variations in geomagnetic latitude (and thus in Bh) cancel out variations in the component of flight motion perpendicular to the field, so these variations do not cause large voltage variations in high-inclination orbits. (Note that the relevant motion is motion relative to a rotating earth.) Out-of-plane libration, variations in geomagnetic radius, and diurnal variation of the "geomagnetic inclination" of an orbit can all cause voltage variations. Peak EMFs (which drive hardware design) may approach 400 V/km.

However these variations need not affect the thrust much if a spacecraft has a variable-voltage power supply: neglecting variations in parasitic power, constant power investment in a circular orbit has to give constant in-plane thrust. The out-of-plane thrust is provided "free" (whether desired or not). Average voltage & thrust equations for vertical tethers are shown at right.

The table shows how to change all six orbital elements separately or together. Other strategies are also possible. Their effects can be calculated from the integrals listed. For orbits within 11° of polar or equatorial, diurnally-varying strategies become more desirable. Computing their effects requires using the varying geomagnetic inclination instead of i (moving it inside the integral). Note that the "DC" orbit-boosting strategy also affects i. This can be cancelled out by superimposing a $-2 \cos(2\theta)$ current on the DC current.

As discussed under Electrodynamic Libration Control Issues, eccentricity and apside changes can strongly stimulate $\dot{\phi}$-libration unless the spacecraft center of mass is near the center of the tether. Other maneuvers should not do this, but this should be checked using high-fidelity geomagnetic field models.


REFERENCES
**Electrodynamic Orbit Changes**

**Offset Dipole Approximation to Geomagnetic Field**

\[
B_x = B_0 \cos \left( \frac{r}{a} \right)^3 \\
B_y = B_0 \tan \lambda \\
B_z = 0.35 \text{ Gauss} \\
\approx 35 \mu \text{Tesla}
\]

**Tilt \approx 11^\circ**  
**Offset \approx 436 \text{ km}**

**How to Change Orbits Using an Electrodynamic Tether**

<table>
<thead>
<tr>
<th>Element</th>
<th>Strategy</th>
<th>Thrust Vector</th>
<th>Effect</th>
</tr>
</thead>
<tbody>
<tr>
<td>Semimajor axis</td>
<td>DC</td>
<td>0° \rightarrow 360°</td>
<td>\Delta a = \cos(1) \frac{k_1}{m} \int i , dt</td>
</tr>
<tr>
<td>Phase</td>
<td>Sawtooth</td>
<td></td>
<td>\Delta \phi = \cos(1) \frac{k_1}{m} \int \phi \cos(\theta) , dt</td>
</tr>
<tr>
<td>Eccentricity</td>
<td>\cos(\theta)</td>
<td></td>
<td>\Delta e = \cos(1) \frac{k_1}{m} \int i \cos(\theta) , dt</td>
</tr>
<tr>
<td>Line of apsides</td>
<td>\sin(\theta)</td>
<td></td>
<td>\Delta \omega = \cos(1) \frac{k_1}{m} \int i \sin(\theta) , dt</td>
</tr>
<tr>
<td>Inclination</td>
<td>\cos(2\phi)</td>
<td></td>
<td>\Delta \iota = \frac{k_1}{2m} \int i \sin(\phi) \cos^2(\phi) , dt</td>
</tr>
<tr>
<td>Ascending node</td>
<td>\sin(2\phi)</td>
<td></td>
<td>\Delta \Omega = \frac{k_1}{2m} \int i \sin(\phi) \cos(\phi) , dt</td>
</tr>
</tbody>
</table>

\( \phi = \text{position of vehicle with reference to its perigee} \)  
\( \phi = \text{position with reference to ascending node} \)  
\( k = -4 \text{ tonnes per ampere} \)  
\( \gamma = (r_e/\gamma)^{1.5} \)  
\( \lambda = \text{tether length} \)  
\( \mu = \text{total vehicle mass} \)  
\( \omega = \text{orbital angular rate} \)
Properly controlled AC components can be used to control $\theta$ and $\phi$-libration. Solar-energy storage and $e$ or $\omega$ changes strongly stimulate $\phi$-libration. AC currents other than 1 & 3/orbit should not affect $\phi$-libration much.

The maneuvering strategies on the previous page have assumed that electrodynam-ic tethers will stay vertical. However, as shown at right, the distributed force on the tether causes bowing, and that bowing is what allows net momentum transfer to the attached masses. Note that net momentum can be transferred to the system even if the wire is bowed the wrong way (as when the current is suddenly reversed); momentum transferred to the wire gets to the masses later.

This figure also illustrates two other issues:
1. Bowing of the tether causes it to cross fewer field lines.
2. Unequal end masses and uniform forces cause overall torques & tilting.

The bowing causes the tether to provide less thrust while dissipating the same parasitic power. The net force on the system is the same as if the tether were straight but in a slightly weaker magnetic field.

The torque on the system causes it to tilt away from the vertical, until the torque is balanced by gravity-gradient restoring torques. For a given system mass and power input, disturbing torques vary with $L$ and restoring torques with $L^2$, so longer systems can tolerate higher power. The mass distribution also affects power-handling capability, as seen in the sequence at top right.

Modulating the tether current modulates any electrodynamic torques. Current modulation at $L73 \pi$ can be used to control in-plane libration. Out-of-plane torques can also be modulated, but another control logic is required. This is because the once-per-orbit variation in out-of-plane thrust direction makes a current with frequency $F$ (in cycles per orbit) cause out-of-plane forces and torques with frequencies of $F-1$ and $F+1$, as shown in the Fourier analysis at bottom right. Hence $\phi$-libration control ($F=2$) requires properly phased $F=1$ or $F=3$ currents. Higher frequencies can damp odd harmonics of any tether bowing oscillations. Control of both in- & out-of-plane oscillations may be possible since they have the same frequencies and thus require different currents.

Applications that require significant $F=1$ components for other reasons can cause problems. Four such strategies are shown at right. Sin & Cos controls allow adjustment of $e$ or $\omega$. The two "Sign of ..." laws allow constant power storage over 2/3 of each orbit and recovery the rest of the orbit. These laws would be useful for storing photovoltaic output for use during dark periods.

These strategies drive out-of-plane libration (unless the center of mass is at the center of the tether). The libration frequency decreases at large amplitudes, so if the system is not driven too strongly, it should settle into a finite-but-large-amplitude phase-locked loop. This may be unacceptable in some applications, due to resulting variations in gravity or tether EMF. In some cases, such as eccentricity changes, adding a $F=3$ component might cancel the undesired effect of an $F=1$ current while keeping the desired effect.

Electrodynamic Libration Control Issues

INCREASING STABILITY
(for fixed total length & mass & I)

FOR CONTROL OF: | MODULATE I AT:
--- | ---
Out-of-plane libration* | 1 n or 3 n
In-plane libration* | 1.73 n
Tether oscillations | >5 n

* I or mass distribution must be lopsided

Tether Current:
\[ I = 1.0 \]

\[ I = \sin \phi \]
\[ I = \cos \phi \]

\[ I = \text{Sign of} \ (0.5 + \sin \phi) \]
\[ I = \text{Sign of} \ (0.5 + \cos \phi) \]

\[ I = \sin 2\phi \]
\[ I = \cos 2\phi \]

Fourier Analysis of Out-Of-Plane Forces:

<table>
<thead>
<tr>
<th>( n )</th>
<th>( 2n )</th>
<th>( 3n )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1n</td>
<td>0.50</td>
<td>0.00</td>
</tr>
<tr>
<td>2n</td>
<td>0.00</td>
<td>0.50</td>
</tr>
<tr>
<td>3n</td>
<td>0.00</td>
<td>0.75</td>
</tr>
</tbody>
</table>

* drives \( \phi \) libration
BIBLIOGRAPHY OF SPACE-TETHER LITERATURE

The number of references listed with each topic in the body of the Guidebook was intentionally limited. A more comprehensive bibliography follows. It was compiled by merging bibliographies previously compiled by Mark Henley, Peter Swan, Georg von Tiesenhausen, and others. It will be expanded and updated in future revisions of the Guidebook.

Many references (particularly final reports) cover several topics. However they are listed here only under the most specific relevant heading. For example, transport concepts explicitly intended for use on a space station are listed under space stations. For thoroughness, check related headings.

BIBLIOGRAPHY HEADINGS:

General
The Tethered Satellite System
Space Station & Constellation Applications
Transportation
Electrodynamics
Dynamics, Controls, & Simulations (incl. TSS)
Beanstalks and Other Ambitious Concepts
Tether Materials

SOME FREQUENTLY OCCURRING ABBREVIATIONS:

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Full Form</th>
</tr>
</thead>
<tbody>
<tr>
<td>CSI</td>
<td>California Space Institute, SIO/UCSD, La Jolla CA</td>
</tr>
<tr>
<td>IAF</td>
<td>International Astronautical Federation</td>
</tr>
<tr>
<td>JSC</td>
<td>Johnson Space Center, Houston TX</td>
</tr>
<tr>
<td>MSFC</td>
<td>Marshall Space Flight Center, Huntsville AL</td>
</tr>
<tr>
<td>SAO</td>
<td>Smithsonian Astrophysical Observatory, Cambridge, MA</td>
</tr>
</tbody>
</table>

GENERAL


—, Movies for Gemini Missions XI and XII, available from Public Affairs Office, AP2, NASA JSC, Houston TX 77058. (See tether-experiment sequences.)


-34-


THE TETHERED SATELLITE SYSTEM


—, Shuttle Tethered Satellite System Definition - Final Study Report, NAS8-32853, Ball Aerospace Systems Division, April 1979.

—, Shuttle Tethered Satellite System, Final Report from the Facility Requirements Definition Team; sponsored by MSPC under NASA Contract NAS8-33383 to the Center for Atmospheric and Space Sciences, Utah State Univ., May 1980.


Mariani, F., "Science by Tethered Satellite", Dipartimento di Fisica, Universita di Roma "La Sapienza", Piazzale Aldo Moro, 2-00185, Roma, Italy.


SPACE STATION AND CONSTELLATION APPLICATIONS


Carroll, J.A., Tether Mediated Rendezvous, report to Martin Marietta on Task 3 of contract RH3-393855, March 1984, CSI.


-36-

Colombo, G., A Straightforward Use of the Shuttle ET, presented at the Workshop on the Utilization of the External Tanks of the STS, 1982, CSI.


Lorenzini, E., Analytical Investigation of Dynamics of Tethered Constellations in Earth Orbit, Aug. 17, 1984, NASA contract NAS 8-35497, SAO.


Nobles, W., Selected Tether Applications in Space, July 31, 1984, Martin Marietta final report/presentation on NASA Contract NAS 8-35499.


TRANSPORTATION


Bentz, D.J. Tethered Momentum Launching Systems, NASA Lewis Research Center.


Contella, M.C., Tethered Deorbit of the External Tank, April 24, 1984, NASA JSC.


Martinez-Sanchez, M., The Use of Large Tethers for Payload Orbital Transfer, MIT, 1983.


---

ELECTRODYNAMICS


Anderson, J., D. Arnold, G. Colombo, M. Grossi, & L. Kirshner, Orbiting Tether's Electrodynamic Interactions, final report on NAS5-25077, April 1979, SAO.


Dobrowolny, M., Wave and Particle Phenomena Induced by an Electrodynamic Tether, SAO Special Report #388, November 1979.


Finnegan, P.M. A Preliminary Look at Using a Tethered Wire to Produce Power on a Space Station, NASA LERC, May 10, 1983.

Giudici, R., Electrodynamic Tether for Power or Propulsion Design Considerations, March 20, 1984, NASA MSFC-PD.


Williamson, P.R., P.M. Banks, and K. Oyama, "The Electrodynamic Tether," report on NASA Contract NAS5-23837, 1978, Utah State University, Logan UT.

TETHER DYNAMICS, CONTROLS AND SIMULATIONS (incl. TSS)


Beletskii, V.V. and M. Guivertz, "The Motion of an Oscillating Rod Subjected to a Gravitational Field", Kosmitcheskie Issledovania 5, no. 6, 1967.
Beletskii, V.V., The Motion of Celestial Bodies, Nauka, Moscow, 1971.

Beletskii, V.V., "Resonance Phenomena at Rotations of Artificial and Natural Celestial Bodies," COSPAR-IAU-IUTAM Satellite Dynamics Symposium, June 1974, Sao Paulo, Brazil; Springer-Verlag, 1975.


Colombo, G., Tether Dynamics Software Review; High Resolution Tether Dynamics; and Advanced Tether Applications (Damping through Reel Motor Control; Payload Acquisition by Space Station), Jan. 1984, NASA Contract NAS 8-35036, SAO.


BEANSTALKS & OTHER AMBITIOUS TETHER CONCEPTS


Polyakov, A Space 'Necklace' about the Earth, (Kosmicheskoye ozherel'ye zemli), Teknika Molodezhi, No. 4, 41-43 (1977). (NASA TM-75174)


Tsiolkovsky, K.E., Grezy O. Zemie i nebe (i) Na Veste (Speculations between earth & sky, & on Vesta; science fiction works). Moscow, izd-vo AN SSSR, 1959.
TETHER MATERIALS


The following reports are available from Du Pont's Textile Fibers Department:

Kevlar 49 Aramid for Pultrusion. 1975.
The Effect of Ultraviolet Light on Products Based on Fibers of Kevlar 29 and Kevlar 49 Aramid. 1977. Information only on terrestrial applications.