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Modification of a Variational Objective Analysis Model for New Equations for Pressure Gradient and Vertical Velocity in the Lower Troposphere and for Spatial Resolution and Accuracy of Satellite Data

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1. Introduction

Beginning in late 1982, the NASA supported research to extend and improve upon the Achtemeier (1975) numerical variational objective analysis model to include observations from space based platforms for the diagnosis of cyclone scale weather systems. The goal of this research is a variational data assimilation method that incorporates as dynamical constraints the primitive equations for a moist, convectively unstable atmosphere and the radiative transfer equation. Variables to be adjusted include the three-dimensional vector wind, height, temperature, and moisture from rawinsonde data, and cloud-wind vectors, moisture, and radiance from satellite data. This presents a formidable mathematical problem. In order to facilitate thorough analysis of each of the model components, we defined four variational models that divide the problem naturally according to increasing complexity. The first of these variational models (MODEL I), which is the subject of this report, contains the two nonlinear horizontal momentum equations, the integrated continuity equation, and the hydrostatic equation. MODEL I is based upon Sasaki's (1958, 1970) method of variational objective analysis. Problems associated with an internally consistent finite difference method, a nonlinear hybrid terrain-following vertical coordinate, formulations for the pressure gradient terms, formulations for the velocity tendency terms and the development of a convergent solution sequence are addressed with MODEL I.

The theoretical development, coding for numerical computation, and preliminary testing of MODEL I was completed by August, 1985. The results of these tests indicated that before the theoretical complexity of the model was increased by the inclusion of the energy equation as a fifth dynamic constraint, it was advisable to modify MODEL I, if necessary, a) to improve the way the large nonmeteorological contributions to the pressure gradient force were reduced, b) to generalize the integrated continuity equation, and c) to introduce horizontal variation in the precision modulus weights for the observations. The results of our work on these three topics are the subject of this report.

Section 2 summarizes our work with the rederivation and implementation of an improved hydrostatic equation and pressure gradient force algorithm. This work was especially critical to the success of the overall MODEL I development after it was found that the original derivation was in error. All current and future publications including the final report (Achtemeier, etal., 1986) present results from the corrected version of MODEL I. Section 3 introduces the generalized integrated continuity equation and Section 4 presents the progress toward formulations for the horizontal variation in the precision modulus weights.

2. Reformulation of the Hydrostatic Equation and the Horizontal Pressure Gradient Force

The motivation for changing the method for removing the large nonmeteorological contributions by unlevel terrain in the hydrostatic equation and in the horizontal pressure gradient terms in the nonlinear horizontal momentum equations came about from a detailed evaluation of the MODEL I variational objective analysis model. We had found an anomously large filling of a deep synoptic scale trough that was located over the Great Basin and Rocky Mountain highlands areas. Our analysis of the contributions of the individual constraints to the final fields of geopotential height did not reveal that dynamic balancing or filtering inherent in the model was the cause for the filling. Preliminary calculations indicated that a modification in a "terrain correction temperature" could realize a correction in the pressure gradient force which, when translated into the spatial distribution of geopotential height, was of the magnitude of the filling of the synoptic scale trough. Further investigation revealed that the algorithm appeared to be correct in differential form for pressure surfaces but there was doubt about whether it was valid in difference form on sigma surfaces.

Given the uncertainities in the original formulations, it was decided to rederive the hydrostatic equation and the pressure gradient force terms of the horizontal momentum equations to remove the orographic effects in a more rigorous fashion. This posed a major reprogramming effort because 1) the terms changed are important terms in the variational adjustment and any major modifications in their form would result in equally major modifications in the Euler-Lagrange equations, 2) the terms are expressed explicitly in the derivations to form a diagnostic adjustment equation for the geopotential height through the reduction of the number of variables and their modification would require rederivation of the adjustment equation and introduce additional complicating terms, and 3) as described in the paragraph below, these terms determine the form in which the initial data must enter the variational model.

The procedure for acquiring the variationally adjustable part of the thermodynamic variables once they have been interpolated into the sigma coordinates first requires that all variables be nondimensionalized. Second, we remove a hydrostatic component that includes much of the vertical variations of the lower coordinate surface due to variable topography. Third, a hydrostatic reference atmosphere is removed and finally, the residual fields are multiplied by the ratio of the Rossby number to the Froud number to bring them into compatibility with the nondimensionalized dynamic equations. The variational adjustments are carried out on these residual variables.

Several formulations were made to achieve the requirements of the second step. In order to minimize the reprogramming tasks, we sought formulations that did not change the forms of the major terms nor introduce nonconstant coefficients. These formulations are summarized below.

a) No removal of topographic effects

The first approach at correcting the formulations for the removal of the nonmeteorological contribution to the geopotential heights by the underlying coordinate surface was to not remove them. The programs for processing the initial data were modified so that there was no removal of a terrain component from the initial data. Several additional modifications to MODEL I were necessary to maintain compatibility. Fig. 1a shows the heights of the lower coordinate surface at 1200 GMT 10 April 1979 with the large contribution by unlevel terrain included. The figure shows mostly the variation of height due to elevation. The Appalachian highlands and the Mississippi Valley are visible in the east and the western highlands are visible over Colorado and Utah. The smoothed elevations, which extend to about 1600 m, do not resolve the higher mountain ranges. Fig. 1b shows only the meteorological heights as they appear on the 1000 mb surface. A low about 100 m deep is located over Colorado. This meteorological system is an order of magnitude smaller than the terrain height. If the latter is not balanced by the pressure field which is part of the second term of the pressure gradient force as it appears in terrain following coordinates, considerable error can be introduced into the adjusted height gradients and, through the solution of the dynamic constraints, introduced into the wind field.

Our calculations show that the height field in Fig. 1a is mostly balanced by the pressure field (not shown) through second term of the pressure gradient force. However, the variational equations separate these terms and build the large nonmeteorological heights into the solution. Upon running MODEL I, we found that the cyclical adjustment sequence did not converge to a solution. It is imperative therefore that the terrain effects be separated from the height adjustments as was originally attempted.

b) Removal of the Nonmeteorological Hydrostatic Component: Method A

Assume that observations of temperature and height have been gridded onto the nonlinear sigma coordinate surfaces and nondimensionalized according to classical scale theory. Assume also that the pressure has been determined in a manner that is consistent with the heights and temperatures. The nondimensional hydrostatic equation is

$$\frac{\partial \phi_w}{\partial \sigma} + \overline{T}_w \frac{\partial \ln p_w}{\partial \sigma} = 0 \tag{1}$$

where the subscript, w, identifies the whole or total untransformed variable. We partition the nonmeteorological component through the definitions for height and pressure, viz., $\phi_w = \phi_a + \phi_T$ and $p_w = p_T + p$ where the subscript, T, identifies the terrain contribution. The hydrostatic equation is then

$$\frac{\partial \phi_a}{\partial \sigma} + \gamma \overline{T}_w + \beta' = 0, \tag{2}$$

where

$$\gamma_w = \frac{p}{p_w} \frac{\partial \ln p}{\partial \sigma}, \tag{3}$$

$$\beta' = \frac{\partial \phi_T}{\partial \sigma} + \frac{\overline{T}_w}{p_w} \frac{\partial p_T}{\partial \sigma}. \tag{4}$$

Equation (2) is identical in form to the original version in MODEL I. The coefficient γ is horizontally variant however, and this increases the complexity of the MODEL I equations over the original derivation.

If we assign the pressure for the lowest three layers to be 800, 900, and 1000 mb, respectively, then the remaining partition variables may be determined by setting $\beta' = 0$ and solving for the terrain height with the initial, unadjusted temperature for $\overline{T}_w$. Then we remove the reference atmosphere for each sigma level by defining $\phi_a = \phi_R + \phi$, $\gamma_w = \gamma_R + \gamma$, and $\overline{T}_w = \overline{T}_R + \overline{T}$ where the subscript, R, refers to the reference atmosphere. Substitution into (2) leaves

$$\frac{\partial \phi_R}{\partial \sigma} + \frac{\partial \phi}{\partial \sigma} + (\gamma_R + \gamma)(\overline{T}_R + \overline{T}) + \beta' = 0, \tag{5}$$

We also require that $\overline{T}_R$ be found through

$$\frac{\partial \phi_R}{\partial \sigma} + \gamma_R \overline{T}_R = 0. \tag{6}$$

Then the hydrostatic equation for the adjustable meteorological residual is

$$\frac{\partial \phi}{\partial \sigma} + \gamma_w \bar{T} + \beta = 0 \tag{7}$$

where $\bar{\beta} = \beta' + \gamma T_R$. Note that though β' was set to zero in order to obtain the terrain height, it will not be zero once the intial temperature has been replaced by the variational correction.

Fig. 1c shows the heights on the 1000 mb equivalent pressure surface after the removal of the terrain contribution by Method A. Although the large height anomaly present in Fig. 1a has been removed, the use of Method A is undesirable for the following reasons. First, Method A removed too much height from the areas of high elevation, ie., it overestimated the depth of the low center over Colorado in comparison with the actual heights of the 1000 mb surface (Fig. 1b). The mean layer temperature used for the calculation of the thickness between 700-1000 mb was estimated by the mean layer temperature between 700 mb and the pressure of the lowest sigma surface (810 mb at the highest elevation). Use of this relatively cold temperature in the hypsometric equation underestimates the thickness of the layer which, when subtracted from the heights at 700 mb, should underestimate the depth of the low at 1000 mb. Thus Method A may have overestimated the depth of the low by as much as 120 meters.

Second, scale analysis of the terms of (7) revealed that the variational adjustment of the temperature in the second term of (4) was equal in magnitude to the correction of the second term of (7). Thus (7), written in the form that can be most easily adapted into MODEL I, does not guarantee convergence of the solution sequence. Indeed we found convergence only when the temperature in the second term of (4) was held left unadjusted.

c) Removal of the Nonmeteorological Hydrostatic Component: Method B

As in Method A, Method B begins with the nondimensional hydrostatic equation valid for the whole or total thermodynamic variables,

$$\frac{\partial \phi_w}{\partial \sigma} + \overline{T}_w \frac{\partial \ln p_w}{\partial \sigma} = 0 , \tag{8}$$

We define the meteorological and terrain partitions of the height and pressure as before, $\phi_W = \phi_a + \phi_T$, and $p_w = p_T + p$, and substitute into (8) this time expanding the logarithmic term also. The resulting hydrostatic equation is

$$\frac{\partial \phi_a}{\partial \sigma} + \gamma \overline{T}_w + \beta' = 0 , \tag{9}$$

where

$$\gamma = \frac{\partial \ln p}{\partial \sigma} , \tag{10}$$

$$\beta' = \frac{\partial \phi_T}{\partial \sigma} + (\frac{p_w}{p} - 1) \frac{\partial \phi_w}{\partial \sigma} + \frac{\overline{T}_w}{p} \frac{\partial p_T}{\partial \sigma} . \tag{11}$$

In this formulation, γ is horizontally invariant and thus retains the simplified form of the hydrostatic equation as it appeared in the original derivation of MODEL I. We again define $p = 800,\ 900,\ 1000$ mb as equivalent pressure surfaces as a distinction between the heights obtained by Method B and the heights of the original pressure surfaces as obtained directly from the observations. If ϕ is averaged over each layer to get ϕ_R, the reference atmosphere may be removed subject to the condition that

$$\frac{\partial \phi_R}{\partial \sigma} + \gamma \overline{T}_R = 0, \tag{12}$$

if $\phi_a = \phi_R + \phi$ and $\overline{T}_w = \overline{T}_R + \overline{T}$. Then (9) reduces to

$$\frac{\partial \phi}{\partial \sigma} + \gamma \overline{T} + \beta = 0, \tag{13}$$

where, upon variational adjustments for height and temperature, β takes the form

$$\beta = \left(\frac{p_w}{p} - 1\right) \frac{\partial}{\partial \sigma} (\phi - \phi^o) + \frac{1}{p} \frac{\partial p_T}{\partial \sigma} (\overline{T} - \overline{T}^o). \tag{14}$$

The advantages of Method B are that γ is horizontally invariant and that scale analysis reveals that the adjustment terms of (14) are at least an order of magnitude smaller than the second term of (13). Thus (13) will not impede the convergence of the solution sequence of MODEL I. The difficulty with Method B is that the total and equivalent pressures appear as undifferentiated coefficients. Thus the accuracy of (14) and hence (13) is critically dependent upon a method to determine a representative pressure for each sigma layer. After some experimentation, it was found that an accurate representative pressure was given by the average of the arithmetic mean plus twice the geometric mean,

$$p = \frac{1}{3} [\frac{1}{2} (p_2 + p_1) + 2 \sqrt{p_2 p_1}] \, . \tag{15}$$

Fig. 1d shows the height of the 1000 mb equivalent pressure surface after the application of Method B. The resemblance of all features to the heights of the actual 1000 mb surface (Fig. 1b) is evident except for the smaller central height of the low center over Colorado. The underestimation of this feature was expected because of the cold layer thicknesses there as described in reference to Method A. Since we have merely partitioned the heights, not neglected height terms, the remaining heights that make up the difference in the heights between Fig. 1d and Fig. 1b must be contained in the residual term. Method B was adopted for the hydrostatic constraint in MODEL I.

Once the terrain and meteorological variables have been partitioned through the hydrostatic equation, it is a simple matter to calculate the pressure gradient force terms of the horizontal momentum equations. The nondimensional pressure gradient term in the x-direction is given by

$$PGX = \frac{\partial \phi_w}{\partial x} + \overline{T}^x_w \frac{\partial \ln p_w}{\partial x} , \tag{16}$$

where the superscript, x, implies that the temperature has been averaged over the interval of differentiation. Partitioning of the heights and pressures leaves

$$PGX = \frac{\partial \phi_a}{\partial x} + \frac{\partial \phi_T}{\partial x} + \overline{T}^x_w \frac{\partial \ln p_T}{\partial x} \, . \tag{17}$$

We note that before the partitioning, the height of the lower coordinate surface was entirely nonmeteorological by the definition of the terrain-following vertical coordinate. The pressure term contained both the terrain and meteorological contributions to the pressure gradient force. The partitioning has forced part of the meteorological pressure into the heights as $\partial p/\partial x = 0$ and also into ϕ_T through p_T in (11). Therefore, the pressure gradient is not given totally by the first term of (17) as is evident by the differences between the height analyses in Figs. 1b and 1d, but by the first term plus elements of the second and third terms. The partitioning is not a coordinate transformation, hence the term equivalent pressure surface, nor is it a perfect separation of the terrain from the meteorological pressure gradient. It is only a method to reduce the large nonmeteorological component between the original pressure gradient terms.

Introduction of the reference atmosphere and application of scale analysis modifies (17) as follows,

$$PGX = \frac{\partial \phi}{\partial x} + \eta_x , \tag{18}$$

where

$$\eta_x = \overline{T}^x \frac{\partial \ln p_w}{\partial x} + {}^{R_o}/_F \frac{\partial}{\partial x} (\phi_T + T_R \ln p_w) , \tag{19}$$

The temperature in the first term of (19) is subject to the variational adjustment. Since the remaining terms are determined from initial variables and held fixed, we may rewrite η_x as

$$\eta_x = \eta_x^o + (\overline{T}^x - \overline{T}^{ox}) \frac{\partial \ln p_w}{\partial x} , \tag{20}$$

to give the remaining meteorological part of the horizontal pressure gradient force.

3. Generalization of the Integrated Continuity Equation

The transformation of the continuity equation from pressure coordinates into the nonlinear sigma coordinates involved [redacted] a correction term that is the vertical velocity multiplied by a coefficient that is a nonlinear function of sigma. This coefficient appears as an integral upon the solution of the integrated continuity equation for the vertical velocity. An exact solution of the integral was not obtained at the time the variational model was being coded. Our attempts to approximate a solution by averaging failed because of the highly nonlinear relationship between pressure and sigma. Rather than hold up the model development with attempts to approximate this term, it was decided to set the term equal to one and proceed. The result was that packing of the sigma coordinate surfaces over elevated terrain was not taken into account in the calculation of vertical velocities. Low level divergences were accorded too much weight in the integration and subsequent adjustment of the continuity equation.

We now know that an exact solution for the integral can be obtained if the integration is taken over the pressure interval between sigma layers. We thus proceed to generalize the integrated continuity equation. The equation of continuity in nonlinear sigma coordinates is

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial \dot{\sigma}}{\partial \sigma} + q_1 \dot{\sigma} + F = 0 , \tag{21}$$

where

$$F = q_2 w_s + R_1 K^2 \left(\frac{\partial u/K}{\partial x} + \frac{\partial v/K}{\partial y}\right) . \tag{22}$$

In addition,

$$q_1 = \frac{-2[J - \alpha\,(p-p^*)]}{J^2} , \tag{23}$$

and

$$q_2 = \frac{(p-p^*)^3}{(p_s-p^*)^4} \; \frac{J_s[2\alpha J(p-p^*)]}{J^2} , \tag{24}$$

where

$$\alpha = \sigma^*/(p^*-p_u) , \tag{25}$$

and

$$J = 3\beta(p-p^*)^3 + \alpha(p-p^*) , \tag{26}$$

if

$$\beta = [1-\sigma^*(\frac{p_s-p_u}{p^*-p_u})] \; (p_s-p^*)^{-3} . \tag{27}$$

The definitions of the various symbols are as follows. $W_s = dp_s/dt$ is the vertical velocity in pressure coordinates along the lower sigma coordinate surface, $R_1 = 0.1$, K is the variable part of the nondimensionalized Lambert conformal map scale factor, p_s is the surface pressure, p_u is the pressure at the top of the analysis domain, and $p^* = 700$ mb is the pressure at a reference sigma level, σ^*, defined for the nonlinear sigma coordinate system. Refer to Achtemeier, etal.,

1986) for a more complete description of the dynamical constraints and the sigma coordinate system.

When solved for the vertical velocity, (21) takes the following form,

$$\dot{\sigma} = \dot{\sigma}_o Q_o - \int_{\sigma_o}^{\sigma_1} H\, Q\, d\sigma , \tag{28}$$

where

$$H = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + F , \tag{29}$$

and

$$Q = e^{-\int_{\sigma}^{\sigma_1} q_1\, d\sigma} . \tag{30}$$

We seek the solution of the integral term Q. Upon conversion to an integral over pressure, the exponent of (30) becomes

$$\int_{\sigma}^{\sigma_1} q_1\, d\sigma = \int_{p}^{p_1} q_1 \frac{\partial\sigma}{\partial p}\, dp . \tag{31}$$

From the definition of the sigma coordinate,

$$\sigma = \beta(p-p^*)^3 + \alpha(p-p_u) , \tag{32}$$

the derivative, $d\sigma/dp$, is found to be equal to

$$\frac{d\sigma}{dp} = 3\beta(p-p^*)^2 + \alpha , \tag{33}$$

Combining (33) with q_1 in (23) and integrating over pressure gives

$$\int_{p}^{p_1} q_1 \frac{\partial\sigma}{\partial p}\, dp = -\ln\, [3\beta(p-p^*)^2 + \alpha]_{p}^{p_1} , \tag{34}$$

and therefore, (30) becomes

$$Q = \frac{3\beta(p_1-p^*)^2+\alpha}{3\beta(p-p^*)^2+\alpha} . \tag{35}$$

Because the term, Q, is a variable coefficient that multiplies the horizontal divergence, it causes the introduction of additional terms in the Euler-Lagrange equations for the variational adjustment model. These terms increase the complexity of the diagnostic equation for the adjustment velocity potential. Therefore, the Euler-Lagrange equations which are affected by the generalization of (28) are rederived. All new terms are introduced into the equations in a manner consistent with the vertically staggard grid of the variational model. Then the variational model is rerun and a series of tests conducted to verify the coding of the formulation and determine how the correction term impacts upon the vertical velocity.

The third constraint, M_3, expressed in complete form is

$$
\begin{aligned}
M_3 = & \int_{\sigma_o}^{\sigma_1} (u_x+v_y)\, e^{-\int_{\sigma}^{\sigma_1} q_1\, d\sigma}\, d\sigma + (\dot{\sigma}-\dot{\sigma}_o\, e^{-\int_{\sigma_o}^{\sigma_1} q_1\, d\sigma}) \\
& + \int_{\sigma_o}^{\sigma_1} [\frac{Lh}{H\ell}\, q_2\, w_s + R_1\, \overline{K}^{xy}\, (u_x+v_y) \\
& - R_1\, (u^{-x} K_x^{-y} + v^{-y} K_x^{-x})]\, e^{-\int_{\sigma}^{\sigma_1} q_1\, d\sigma} .
\end{aligned}
\tag{36}
$$

Performing the variation only on the product of the Lagrangian multiplier with (36) gives the following terms in the adjustment equations for u, v, and $\dot{\sigma}$.

$$\delta u\ [\lambda_3 M_3] = -(\lambda_3 \int_{\sigma_o}^{\sigma_1} Q d\sigma)_x - R_1\ (\lambda_3 \int_{\sigma_o}^{\sigma_1} Q\ \overline{K}^{xy}\ d\sigma)_x$$

$$- R_1\ \overline{(\lambda_3 \int_{\sigma_o}^{\sigma_1} Q\ \overline{K}_x^y\ d\sigma)}^x) \tag{37}$$

$$\delta v\ [\lambda_3 M_3] = -\ (\lambda_3 \int_{\sigma_o}^{\sigma_1} Q\ d\sigma)_y - R_1\ (\lambda_3 \int_{\sigma_o}^{\sigma_1} Q\ \overline{K}^{xy}\ d\sigma)_y$$

$$- R_1\ \overline{(\lambda_3 \int_{\sigma_o}^{\sigma_1} Q\ \overline{K}_y^x\ d\sigma)}^y) \tag{38}$$

$$\delta\dot{\sigma}\ [\lambda_3 M_3] = \lambda_3\ . \tag{39}$$

Upon inspection of the terms of these equations, it becomes apparent that neither λ_3 nor the integral of Q is a function of sigma. Therefore we can combine both into a new Lagrangian multiplier,

$$\lambda_3^* = \lambda_3 \int_{\sigma_o}^{\sigma_1} Q\ d\sigma\ . \tag{40}$$

The variations on u, v, and $\dot{\sigma}$ (eqs. (37), (38), and (39)) simplify to

$$\delta u\ [\lambda_3 M_3] = -\lambda^*_{3x} - R_1\ (\lambda^*_3 \overline{K}^{xy})_x - R_1\ \overline{(\lambda^*_3 \overline{K}^y_x)}^x , \tag{41}$$

$$\delta v\ [\lambda_3 M_3] = -\lambda^*_{3y} - R_1\ (\lambda^*_3 \overline{K}^{xy})_y - R_1\ \overline{(\lambda^*_3 \overline{K}^x_y)}^y , \tag{42}$$

$$\delta \dot{\sigma}\ [\lambda_3 M_3] = \frac{\lambda^*_3}{\int_{\sigma_o}^{\sigma_1} Q\ d\sigma} , \tag{43}$$

These modifications are inserted into the full Euler-Lagrange equations that contain reference to λ_3. The Euler-Lagrange equations for u and v are

$$\Pi_1 u - \lambda^*_{3x} + \lambda_2 + F_1 = 0 , \tag{44}$$

$$\begin{aligned} F_1 = & -\Pi_1 u^o + R_1 \overline{C}^y \lambda_2 + R_o\ \{\overline{m}^y \overline{\lambda}^{xy} \overline{u}^x_x + \overline{m}^y \lambda_2 \overline{v}^y_x \\ & - [\overline{m}^{xy} \overline{\lambda}^y_1\ \overline{(u - c_x)}^x]_x - [m\ \overline{\lambda_1\ (v - c_y)}^x]_y \\ & - R_o\ (\dot{\sigma}\ \overline{\overline{\lambda}^x}^{xy\sigma})_\sigma\} - R_1\ [(\lambda^*_3 \overline{K}^{xy})_x + \overline{\lambda^*_3 \overline{K}^y_x}^x] , \end{aligned} \tag{45}$$

$$\Pi_1 v - \lambda^*_{3y} - \lambda_1 + F_2 = 0 , \tag{46}$$

$$F_2 = -\Pi_1 v^o - R_1\ \overline{c}^x\ \lambda_1 + R_o\ \{\overline{m}^x \lambda_1 \overline{u}^x_y + \overline{m}^x \overline{\lambda}^{xy}_2 \overline{v}^y_y$$

$$- [m\,\overline{\lambda_2\ \overline{(u-c_x)}^y}]_x - [\overline{m}^{xy}\overline{\lambda}_2^x\ \overline{(v-c_y)}^y]_y - R_o\ (\dot{\sigma}\ \overline{\overline{\lambda}^{y}}^{xy\sigma})_\sigma\}$$

$$- R_1\ [(\lambda_3^*\overline{K}^{xy})_y + \overline{\lambda_3^*\overline{K}_y^x}^y]\quad . \tag{47}$$

The Euler-Lagrange equation for $\dot{\sigma}$ is

$$\Pi_2\dot{\sigma} + \frac{\lambda_3^*}{\int_{\sigma_o}^{\sigma_1} Q\ d\sigma} + F_3 = 0\ , \tag{48}$$

$$F_3 = -\ \Pi_2\dot{\sigma}^o + R_o^2\ [\overline{\lambda}_1^{y\sigma}\ \overline{u}_\sigma^x + \overline{\lambda}_2^{x\sigma}\ \overline{v}_\sigma^y]\quad . \tag{49}$$

In addition, the integrated continuity equation is

$$\int_{\sigma_o}^{\sigma_1} Q(u_x+v_y)\ d\sigma + \dot{\sigma} + F_7 = 0\ , \tag{50}$$

$$F_7 = -\ Q_o\dot{\sigma}^o + \int_{\sigma_o}^{\sigma_1} [\frac{Lh}{H\ell}\ q_2 w_s + R_1\overline{K}^{xy}(u_x+v_y)$$

$$- R_1\ (\overline{u}^x\overline{K}_x^y + \overline{v}^y\overline{K}_y^x)]\ Q\ d\sigma\ , \tag{51}$$

Variables are eliminated to produce a diagnostic equation in λ_3. Dividing (44) and (46) by Π_1 and reformulating as components of the divergence gives

$$u_x - (\frac{1}{\Pi_1} \lambda_{3x}^*)_x + [\frac{1}{\Pi_1} (\lambda_2 + F_1)]_x = 0 , \tag{52}$$

$$v_x - (\frac{1}{\Pi_1} \lambda_{3y}^*)_y + [\frac{1}{\Pi_1} (-\lambda_1 + F_2)]_y = 0 . \tag{53}$$

Then (52) and (53) combine into the divergence and after integration over some interval $d\sigma$, become

$$- \int_{\sigma_o}^{\sigma_1} Q(u_x + v_y)\, d\sigma + (\lambda_{3x}^* \int_{\sigma_o}^{\sigma_1} \frac{Q}{\Pi_1}\, d\sigma)_x + (\lambda_{3y}^* \int_{\sigma_o}^{\sigma_1} \frac{Q}{\Pi_1}\, d\sigma)_y$$

$$- \int_{\sigma_o}^{\sigma_1} \{[\frac{1}{\Pi_1} (\lambda_2 + F_1)]_x + [\frac{1}{\Pi_1} (-\lambda_1 + F_2)]_y\} Q\, d\sigma = 0 . \tag{54}$$

Now $\dot{\sigma}$ is eliminated from (49) and (50) so that

$$\int_{\sigma_o}^{\sigma_1} Q(u_x + v_y)\, d\sigma - \frac{\lambda_3^*}{\Pi_2 \int_{\sigma_o}^{\sigma_1} Q\, d\sigma} - \frac{F_3}{\Pi_2} + F_7 = 0 . \tag{55}$$

A diagnostic equation in λ_3 is obtained upon elimination of the integrated divergence through combining (54) and (55):

$$(\lambda_{3x}^* \int_{\sigma_o}^{\sigma_1} \frac{Q}{\Pi_1}\, d\sigma)_x + (\lambda_{3y}^* \int_{\sigma_o}^{\sigma_1} \frac{Q}{\Pi_1}\, d\sigma)_y - \frac{\lambda_3^*}{\Pi_2 \int_{\sigma_o}^{\sigma_1} Q\, d\sigma} + F_{10} = 0 , \tag{56}$$

where

$$F_{10} = - \int_{\sigma_o}^{\sigma_1} \{[\frac{1}{\Pi_1} (\lambda_2 + F_1)]_x + [\frac{1}{\Pi_1} (-\lambda_1 + F_2)]_y\} Q\, d\sigma - \frac{F_3}{\Pi_2} + F_7 . \tag{57}$$

We also note that several terms of (56) obey an identity (Eq.(48) in Achtemeier et al., 1986). Therefore (56) transforms into the two-dimensional second-order elliptic partial differential equation with non-constant coefficients given by

$$\int_{\sigma_o}^{\sigma_1} \left(\frac{Q}{\Pi_1}\right) d\sigma\, \lambda^*_{3xx} + \int_{\sigma_o}^{\sigma_1} \left(\frac{\bar{Q}^y}{\Pi_1}\right) d\sigma\, \lambda^*_{3yy} + \int_{\sigma_o}^{\sigma_1} \left(\frac{Q}{\Pi_1}\right)_x d\sigma\, \lambda^*_{3x}$$

$$+ \int_{\sigma_o}^{\sigma_1} \left(\frac{Q}{\Pi_1}\right)_y d\sigma\, \lambda^*_{3y} - \frac{\lambda_3}{\Pi_2 \int_{\sigma_o}^{\sigma_1} Q\, d\sigma} + F_{10} = 0 \,. \tag{58}$$

In the event that Π_1 is horizontally invariant, (58) further simplifies to

$$\int_{\sigma_o}^{\sigma_1} \left(\frac{Q}{\Pi_1}\right) d\sigma\, \nabla^2 \lambda^*_3 - \frac{\lambda_3}{\Pi_2 \int_{\sigma_o}^{\sigma_1} Q\, d\sigma} + F_{10} = 0 \tag{59}$$

Model I was rerun with Q calculated from the above development and compared with Q=1.0 for all levels. Fig. 2 shows the vertical velocities at level 3 (700 mb). There is found an approximate 50 percent reduction in the vertical velocity for the analysis where variable layer thickness is a factor, the reduction having been made over the higher elevation areas west of the Great Plains. For example, a greater than 4 cm sec^{-1} center of rising motion for the Q=1.0 analysis (Fig.

2a) has been reduced to slightly greater than 2 cm sec^{-1} for the analysis with variable Q (Fig 2b). Elsewhere over lower elevations the impact of the layer thickness is much smaller.

4. Impact of Horizontal Variation of Precision Moduli

As a simplifying factor in the derivation of the MODEL I variational analysis, we made the precision moduli that weight the observations functions of height only. This is a reasonable assumption as long as the data are relatively evenly distributed over the analysis domain and the data quality are horizontally invariant. The TIROS-N data violates the assumption on both counts. The TIROS-N data we are using for the 10 April 1979 model verification case studies contains large gaps which are of regional scale. The method for objective gridding of the data fills the gaps by interpolation from surrounding data. The resulting temperature fields are only approximate in these areas and can be significantly in error if the data gaps coincide with temperature maxima or minima. Our analysis of 12 GMT 10 April 1979 was such a case.

Model I was designed for horizontally variant weights. No rederivations were necessary for this part of the study. We began sensitivity studies with the velocity adjustment potential equation number (58). The precision modulus weights were allowed to vary over five orders of magnitude giving adjustments that ranged from 100 percent restoration of the horizontal velocity divergence to 100 percent restoration of the initial vertical velocity. The weights were distributed over a number of geometrical patterns in order to determine the conditions which violate the stability criteria for (58). The results to date are too preliminary to report upon at this time however, early indications are that, in some of the cases we tested, the use of the five order of magnitude variation in the weights, even though satisfying stability criteria, can produce greatly exaggerated horizontal velocity fields.

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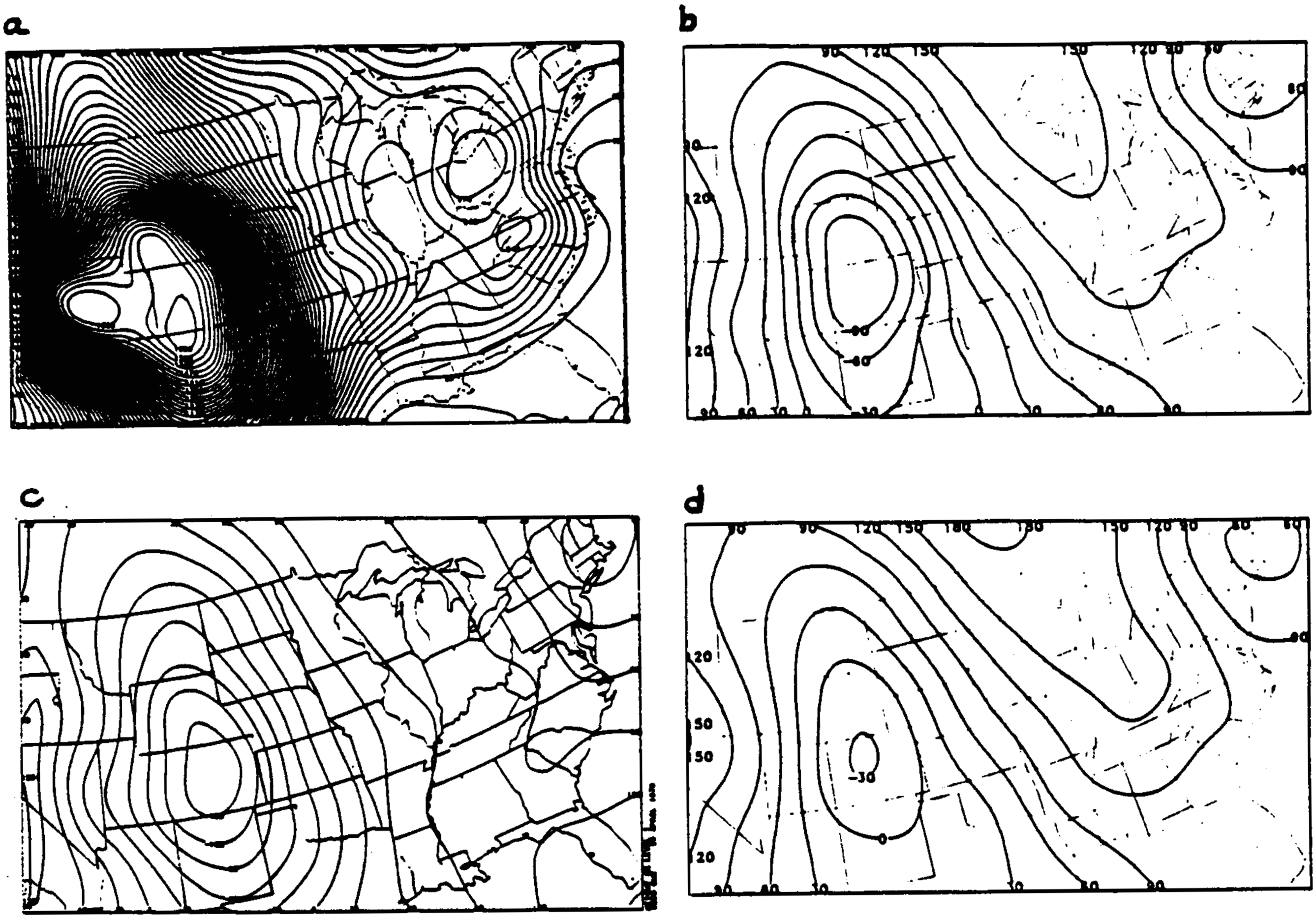

Figure 1. Heights of the lower coordinate surface for MODEL I at 1200 GMT 10 April 1979, a)the unlevel terrain included, b) the meteorological heights as they appear on the 1000 mb surface, c) the heights on the 1000 mb equivalent pressure surface after the removal of the terrain contribution by Method A, and d) the height of the 1000 mb equivalent pressure surface after the application of Method B.

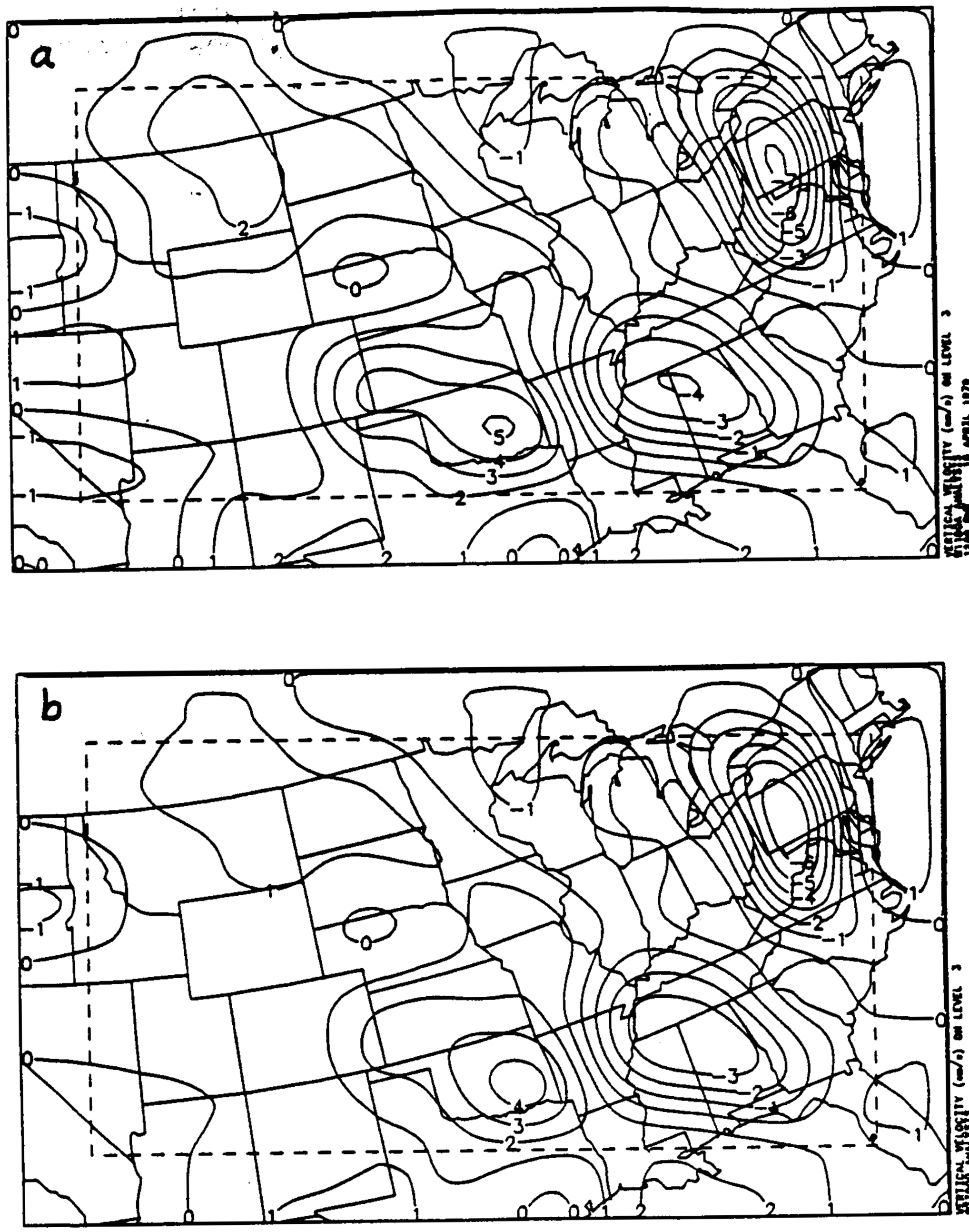

Figure 2. The vertical velocities at level 3 (700 mb), 1200 GMT 10 April 1979 for a) for the Q=1.0 analysis and b) for the analysis with variable Q.

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16. ABSTRACT

Beginning in late 1982, the NASA supported research to develop a numerical variational model for the diagnostic assimilation of conventional and space-based meteorological data. In order to analyze the model components, we defined four variational models that divide the problem naturally according to increasing complexity. The first of these variational models (MODEL I), which is the subject of this report, contains the two nonlinear horizontal momentum equations, the integrated continuity equation, and the hydrostatic equation.

This report summarizes the results of research a) to improve the way the large nonmeteorological parts of the pressure gradient force are partitioned between the two terms of the pressure gradient force terms of the horizontal momentum equations, b) to generalize the integrated continuity equation to account for variable pressure thickness over elevated terrain, and c) to introduce horizontal variation in the precision modulus weights for the observations.

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