Detonation Wave Compression in Gas Turbines

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INTRODUCTION

Aircraft propulsion currently is almost completely dominated by gas turbines with essentially isobaric combustion so that the thermodynamic cycle approximates the classical Brayton cycle with isentropic compression and expansion, and isobaric heat exchange.

It has been known virtually since the beginning of gas turbine development that significant gains in efficiency could be achieved if energy release could be accomplished at constant volume, rather than at constant pressure. The constant volume or Humphrey cycle is compared with the constant pressure Brayton cycle in Figure 1 in pressure-volume and temperature-entropy thermodynamic diagrams.

![Figure 1. Comparison of the Brayton and Humphrey Cycles](image)

The ideal gas thermodynamic efficiency of the closed Humphrey cycle is given by:

$$\eta_H = 1 - \gamma/R \frac{(T_3/T_2)^{1/\gamma} - 1}{T_3/T_2 - 1}$$

(1)
and that of the Brayton cycle by

$$\eta_B = 1 - \frac{1}{R^{\Gamma}}$$

Here $p$, $v$, and $T$, following convention, denote the pressure, volume, and temperature respectively and

- $R$ - pressure ratio $= \frac{p_2}{p_1}$
- $\gamma$ - isentropic exponent
- $\Gamma = 1 - \frac{1}{\gamma}$

Numerical subscripts denote the thermodynamic states in the cycle. Comparisons of the performance of the two cycles are shown in Figure 2. The displayed results indicate the significant superiority of the Humphrey cycle to the Brayton cycle even at modest temperature ratios.

Numerous attempts have been made to achieve constant volume combustion, but the complications of valves, such as those used by Holzwarth, did not result in a practical engine. Recent improvements in gas turbine performance have diminished somewhat the interest in the development of alternative engine cycles. However, improvements in performance have been gained at the expense of the cost and complications of multistage axial compressors. Such compressors are economically unacceptable for smaller gas turbine engines which might find applications in light aircraft or cruise missile.

Presented here is the summary of the second phase of a study aimed at demonstrating the feasibility of employing transient transverse detonation waves to augment the performance of gas turbines using low pressure ratio compressors. The ultimate objective is to replace multistage axial compressors in smaller gas turbines with single stage centrifugal units augmented by an array of detonation ducts. Alternately, specific fuel consumption and thrust coefficients of low-to-medium pressure ratio gas turbines could be enhanced through the use of detonation ducts. In the initial study (Reference 1) performance of pure jet engines with low pressure ratio compressors augmented by detonation ducts was studied parametrically, and it was shown that significant reductions in specific fuel consumption could be achieved concurrently with greatly
enhanced thrust coefficients. The main objective of the present study is to analyze the gas dynamics of the detonation duct-combustor interactions with the aim of determining the realistic bounds of performance in actual engines.

In the interest of clarity of presentation of the main objective of the program, details of the detonation duct gas dynamics have been placed in Appendix A, the supporting experimental work is in Appendix B, and the listings of computer codes developed specifically for this study are in Appendix C.

Figure 2. Comparison of Humphrey and Brayton Gas Turbine Cycles
THEORETICAL CONSIDERATIONS

The theoretical upper limit on performance could be achieved if the whole compressor output was processed by detonation waves. Lower limit corresponds to 100% of the compressor flow going through the combustor, which is the case with conventional gas turbines. It is of some interest to establish the upper limit as a measure of the potential performance improvements.

Detonation Wave Engines

In this case, the heat addition is by means of detonation waves. It is known (e.g., Stanyukovich2) that a Chapman-Jouguet detonation is essentially equivalent to a constant volume heat release. In the strong wave limit, the speed of the detonation wave, D, is given by:

\[ D^2 = 2(\gamma^2 - 1)Q \]  

with \( Q \) being the energy release per unit mass of the detonated mixture. The pressure, temperature, and density relations are:

\[ \rho_D = \rho D^2/(\gamma + 1) \]  
\[ T_D = 2\gamma (Q/c\gamma)/(\gamma + 1) \]  
\[ \rho_D = \rho(\gamma + 1)/\gamma \]

with the subscript D denoting conditions downstream of the detonation wave. Because of the assumption of the Chapman-Jouguet state, the gas behind the detonation wave is at Mach 1. Assumption of isentropic compression to stagnation conditions and ideal compressor and turbine performance results in the relation for engine efficiency which is given by:

\[ e = \gamma D^2(1 - c_T^{-\Gamma D}) - \gamma/H(\Gamma - 1) \]  

(5)
with
\[
H = \frac{Q}{c_v T_1} \quad R_D = 2 \left( \frac{D}{\gamma - 1} \right) \left( \frac{D}{2} \right)^{1/\gamma_D} \quad HR^{1/\gamma}
\]

Maximum efficiency is then achieved at:

\[
R_{opt} = \left[ \frac{\gamma_D}{\gamma - 1} \left( 2H D \frac{\gamma - 1}{\gamma - 1} \right)^{1/\gamma_D} \right]^{\frac{\gamma}{\gamma f + \gamma_D}}
\]  \hspace{1cm} (6)

with \( \gamma_D \) being the isentropic process exponent of the detonation products. Variations of efficiency with pressure ratios for a range of values of \( H \) is presented in Figure 3. For hydrocarbon fuels, \( H \) ranges from 5 to 10. It should be noted that the efficiency of about 53%, which is reached by the proposed cycle at a compressor pressure ratio of 5, is matched by a standard Brayton cycle at a pressure ratio of about 14. Alternatively, augmentation of a \( R = 5 \) Brayton cycle with detonation wave heat release increases the efficiency from 37% to 53%.
Figure 3. Performance of Augmented and Brayton Gas Turbines
Augmented Gas Turbine

A theoretical detonation wave engine considered above cannot be realized because of practical considerations such as complete detonation of the whole flow, starting problems, unacceptably high temperatures and maintenance of flow through the engine. An engine which could be made to work is shown in Figure 4 in pure jet engine form.

![Diagram of a Detonation Wave Augmented Jet Engine]

The station numbers are given for easy reference. The detonation duct could be a rectangular channel such as is shown in Figure 5. An array of injectors sends pulses of gaseous, or liquid, fuel into the duct where the mixture of air and fuel is detonated by an array of igniters fired in sequence with the injectors, or operated continuously. Maximum rate of energy release is obtained when the flow in the channel is at Mach 1. The rapid area increase at the upstream channel is necessary for the attenuation of shock waves. Area increase at the downstream end may not be necessary with ejector mixing-wave compression mode of operation. Preliminary experimental studies using a detonation duct with spark plug igniters are described in Appendix B.
Figure 5. Schematic Representation of the Detonation Duct

The range of operating conditions for the engine is bounded by no detonation duct flow and the maximum frequency cycling of the ducts. In the previous study, it was assumed that the combustor and detonation duct flows were mixed in an ejector. Since the cycling frequency of the duct was not determined at that time, the fraction of flow through the combustor $X_b$ was used as a free parameter. The studies were confined to a pure jet engine and representative performance parameters for a pressure ratio of 4 are shown in Figure 6. The limit of $X_b = 1$ represents no augmentation by detonation ducts. Even though the concept is aimed primarily at the use of very low pressure ratio compressors, it is interesting to note in Figure 7 that significant gains are possible with moderate pressure ratios.

In the present study, the pure ejector mixing model was rejected in favor of a mixed ejector-wave compression mixing of combustor and detonation duct flows. The more efficient mixing offset the lower than expected detonation wave compression which was measured in a preliminary experimental program.
Figure 6. Engine Performance Variation with Combustor Flow Fraction
Figure 7. Influence of Gas Dynamic Models on Engine Performance
BASIC CALCULATIONS

All of the physical quantities in the detonation wave augmented gas turbines are scaled on the detonation duct parameters evaluated prior to the detonations and denoted by the subscript 0. The natural scaling quantities are the detonation volume acoustic velocity, $a_0$, detonation volume cross-sectional area, and length, $A_0$ and $L_0$ detonation volume mass $\rho_0 A_0 L_0$, and internal energy $c_v T_0$. In the following analyses, these quantities appear:

Time

$$\theta = \frac{t_0}{L_0}$$

Area and length

$$A = \frac{A}{A_0} \text{ and } x/L_0$$

Mass

$$m = \frac{m}{\rho_0 L_0 A_0}$$

Energy

$$E = \frac{E}{\rho_0 L_0 A_0 c_v T_0}$$

All thermodynamic quantities and velocities scale on corresponding detonation volume parameters.

Baseline Gas Turbine

Performance of detonation wave augmented gas turbines is compared with a corresponding conventional turbine with pressure ratio $R$, compressor and turbine efficiencies $e_c$ and $e_T$ respectively, and a maximum to ambient temperature ratio $\phi = T_{max}/T_a$. The efficiency of such a turbine is given by:

$$\eta = \frac{e_c e_T \phi R^{-\Gamma} - 1}{e_c (\phi - 1)/(R^\Gamma - 1) - 1}$$

(7)
Compressor

The total temperature of a compressor with a pressure ratio $R$ and an isentropic efficiency $e_c$ is:

$$
T_T/T_a = 1 + (R^\Gamma - 1)/e_c 
$$

(8)

The ambient temperature $T_a$ is related to $T_0$ through the total temperature and isentropic relations. Compressor work, nondimensionalized by the detonated volume energy, is given by:

$$
W_c = \gamma(m_d + m_b)[1 + (\gamma - 1)M_o^2/2]/[1 + e_c/R^\Gamma - 1] 
$$

(9)

Detonation Duct

The dimensionless heat released in the detonation volume is:

$$
Q = (\rho_d/\rho_o)[T_D/T_d - 1] = (\rho_d/\rho_o)[(C_d/C_d)^2 - 1] 
$$

(10)

with the subscripts $d$, $D$ denoting the conditions prior to and subsequent to detonation. In the calculations, the acoustic velocity ratio is taken as 2.5 and therefore the energy release in the three sequenced firings is:

$$
Q_1 = 5.25 \quad Q_2 = 6.40 \quad Q_3 = 8.24
$$

Detailed calculations of the gas dynamics of the duct and its output for various modes of operation are given in Appendix A.

Combustor

Heat is assumed to be added isobarically during the period $\Delta \theta$. The dimensionless mass is:

$$
m = M_b(\rho_b/\rho_o)(C_b/C_o)(A_b/A_o)\Delta \theta 
$$

(11)
Initial combustor and detonation duct quantities are related through isentropic flow assumptions and Mach numbers $M_b$ and $M_o$. The temperature at the completion of combustion is:

$$T_{bf}/T_b = 1 + Q_c/Q_p \quad T_b = 1 + Q^*$$

(12)

with $Q^*$ being the heat release parameter. The total dimensionless heat added is:

$$Q_b = \gamma Q^* (T_b/T_0)^\Lambda (A_b/A_0) M_b \Delta \theta$$

(13)

with

$$\Lambda = (3\gamma - 1)/2(\gamma - 1)$$

Taking

$$e_c = 0.85 \quad M_b = 0.25 \quad Q^* = 2 \quad R = 5 \quad \gamma = 1.32$$

the maximum combustor temperature is calculated to be $T_{bf}/T_a = 4.62$ which corresponds to a turbine inlet temperature of about $2000^\circ F$. In order to maintain a constant Mach number in the combustor under the condition of the above parameters, the combustor flow area must increase by a factor of $3^{1/2}$ between the entry and exit.

**Mixing and Wave Compression**

The output of the combustor is steady while that of the detonation duct alternates volumes of fairly steady flow with sharp pulses of high pressure and temperature gas. When the detonation duct output is almost steady, the streams are mixed as if in an ejector with moderate pressure changes. At exit from the detonation duct, the pressure pulses expand suddenly and because of their much higher velocity and pressure, immediately compress the preceding volumes of mixed gases. In the initial phase of this process, the overall volume remains virtually unchanged while the flow equilibrates to an essentially uniform pressure and velocity. In the proposed system, the flows of 2 detonation ducts are mixed with the outflow from 1 combustor. The arrangement is something like that shown in Figure 8. Further equilibration of the flow could be achieved in the optional spiral collector. Without the spiral collector, the turbine would be operated in purely blowdown mode.
Figure 8. Collector and Gas Dynamic Augmentor
The mixing-wave compression section mixes the combustor and detonation duct flows and compresses them by the detonation wave moving downstream. The mixing is accomplished by momentum transfer as in ejectors. The high pressure detonation wave expands into the collector section and transfers energy to the mixed gases by doing expansion work on them. Curvature of the collector is considered necessary because in a real engine, this would add some length to ensure good mixing and would increase pressure by reflections of the detonation waves. A schematic representation of the collector section is shown in Figure 9.

Figure 9. Mixing-Wave Compression Section
The conditions at the end of mixing of the detonation duct and combustor streams are determined from the solution of the continuity, momentum, and energy equations. With all quantities normalized by the initial detonation duct values, the governing equations are:

Continuity

\[ \frac{P_m}{T_m} A_m U_m \Delta \theta = \sum m_i = M_m \]  

(14)

Momentum

\[ \sum (p_i A_i \Delta \theta + U_i m_i) = P_m A_m \Delta \theta + U_m M_m \]  

(15)

Energy

\[ \sum m_i [T_i + (\gamma - 1)U_i^2/2] = [T_m + (\gamma - 1)U_m^2/2]M_m \]  

(16)

with the definitions

\[ S_M = (T_m/U_m + U_m)M_m \]

and

\[ S_E = [T_m + (\gamma - 1)U_m^2/2]M_m \]

Subscripts \( i \) denote the individual volumes of gases entering the mixing section. The solution for the mixed velocity is:

\[ U_m = \frac{S_M}{(3 - \gamma)M_m} - \left[ \frac{S_M}{(3 - \gamma)M_m} \right]^2 - \frac{2S_E}{(3 - \gamma)M_m} \]  

(17)

The remaining quantities are then evaluated by simple substitution.

When the high pressure volume of the shock wave compressed flow exits the detonation duct, it undergoes a sudden expansion into an area \( A_m \). Accompanying this sudden expansion is an entropy increase or the so called "shock loss" which is accounted for here using the relations given by Rudinger. Immediately after expansion, the gas exchanges energy with the mixed gases by decelerating and expanding.
to some common pressure $P_e$ in a flow area $A_e$. In the first approximation, the equilibration process may be considered to be isentropic.

The continuity equation is written as:

$$\sum \Delta \theta_i = \sum \frac{U_i}{U_e} \frac{A_i}{A_e} \left( \frac{P_i}{P_e} \right)^{1/\gamma} \Delta \theta_i$$  \hspace{1cm} (18)$$

The statement of constant total energy is:

$$\frac{2}{\gamma (\gamma - 1)} \sum m_i \left\{ T_i \left[ 1 - \left( \frac{P_e}{P_i} \right)^{\gamma - 1/\gamma} \right] U_i^2 \right\} = M_m U_e^2$$  \hspace{1cm} (19)$$

When the wave compression exit area $A_e$ is specified, the 2 equations are solved by iteration for $P_e$ and $U_e$. This determines the thermodynamic state points of the wave compressed gas. The parameters of the mixing wave compression process are:

- $A_b/Ad$ - combustor flow area
- $A_e/Ad$ - wave compression exit area
- $A_m/Ad$ - mixing region area

Other parameters which bear directly on the problem are the combustor conditions and single or multiple firings of the detonation duct. A representative example of the output of the mixing-compression section is shown in Figure 10.
Figure 10. Gas States at Collector Exit
Parametric Studies

Extensive parametric studies were performed to determine the efficiency of detonation wave enhancement of the performance of low pressure ratio gas turbines. The basic engine parameters were:

\[ R = \text{pressure ratio} = 5 \]
\[ \epsilon_c = \text{compressor efficiency} = 0.85 \]
\[ \epsilon_T = \text{turbine efficiency} = 0.9 \]
\[ Q^* = \text{energy parameter} = 2 \]

With these parameters, the basic isobaric combustion conventional gas turbine has an efficiency of 0.256. The parameters of the augmented gas turbines are:

\[ A_b = \text{combustor area} \]
\[ A_m = \text{mixing area} \]
\[ A_e = \text{wave compression area} \]

There are two modes of firing of the detonation ducts which are discussed in Appendix A. In the single firing mode, the duct is operated at its nominal conditions, fuel is injected and the mixture is detonated. The flow in the duct is then allowed to return to its nominal conditions before it is detonated again. With 2 detonation ducts associated with each combustor, the maximum frequency is known to be controlled by the time for the detonation gases and associated wave systems to exit the duct \((\theta = 3)\) and for flow to be re-established, which is also \(\theta = 3\). Thus in \(\theta = 6\), which is the shortest theoretical period of the system, the detonation ducts will fire twice. The other limit is obtained when the detonation ducts have no flow going through them. Calculated efficiencies for the single firing mode of operation are shown in Table 1.

The detonation ducts can also be fired after the upstream contact surface exits the duct and the detonation duct volume is occupied by compressed gas. In principle, the duct could be fired the third time, but the pressure waves retard the flow significantly and the firing frequency diminishes with each firing quite rapidly. With double firing and two detonation ducts for each combustor, the minimum period is \(\theta = 10.2\). Calculated values of efficiencies are shown in Table 2.
It is clear that augmented gas turbines offer the potential for significant improvements over the conventional gas turbine efficiency of 0.256. Efficiency is higher for single firing mode of operation because greater fractions of the duct flow are processed by the detonation and shock waves. Efficiency also increases with decreasing mixer area, but the results for the lowest values of $A_m$ are probably not realizable because stable ejector operation may not be achievable. For $A_b = 1$ the combined detonation ducts and combustor exit area is 3.73 and a mixing area of 4 is probably realistic. The effect of the wave compression area on efficiency is minor so that it does not merit much consideration at this stage of the analysis. For $A_b = 2$ the combined areas are 5.4 and a mixing area of 6 is probably quite reasonable. Representative results for a single firing mode are therefore:

$$A_b = 1 \quad \eta = 0.45$$
$$= 2 \quad = 0.37$$

Similarly, for a double firing mode

$$A_b = 1 \quad \eta = 0.31$$
$$= 2 \quad = 0.29$$

Clearly the single firing mode is superior and significant improvements over the conventional gas turbine efficiency of 0.256 are indicated. Even if the losses in the mixing and compression are greater than are currently estimated, the potential for enhancing the performance of low pressure ratio gas turbines has been demonstrated.
Table 1. Efficiencies of Augmented Gas Turbines
Single Firing

<table>
<thead>
<tr>
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<th>$A_e$</th>
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Note - All areas are normalized by $A_d$
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Note - All areas normalized by $A_d$
The basic calculations presented in the previous section indicated the potential of the detonation wave augmentation in terms of thermodynamic cycle efficiency. Here, the discussion will be in conventional terms of performance of various types of gas turbines. In the following, the representative optimum augmented gas turbine will be a single firing mode, $A_B = 1$, $X_B = 0.35$, $\eta = 0.45$ system. The off-design condition will be a conventional gas turbine with $R = 5$ and $\eta = 0.26$. With the conditions stated previously and propane as fuel, the combustor air fuel ratio is about 30. At design conditions, the detonation duct air/fuel ratio is about 45. The overall fuel/air ratio is therefore:

$$\frac{m_f}{m_{air}} = \frac{X_B}{(m_{air}/m_f)B} + \frac{1 - X_B}{(m_{air}/m_f)B} - 0.0261$$

(19)

**Performance of a Pure Jet Engine**

The case considered is shown schematically in Figure 11.

![Schematic Representation of a Detonation Wave Augmented Jet Engine](image)

1 - INLET  
2 - COMPRESSOR  
3 - COMBUSTOR  
4 - DETONATION CHAMBERS  
5 - MIXING CHAMBER  
6 - TURBINE  
7 - NOZZLE

Figure 11. Schematic Representation of a Detonation Wave Augmented Jet Engine
In this case, the net work is simply the increase in the kinetic energy of the gases, or:

$$\eta = (1 + A_f) \frac{V_E^2}{2q_f}$$  \hspace{1cm} (20)

With thrust $F$ equal to the time rate of change of momentum, the fuel flow rate per unit of force is:

$$\frac{w_f}{F} = \frac{q}{2\eta q_f (1 + A_f)}$$  \hspace{1cm} (21)

In conventional terms, the performance of the optimum (maximum augmentation) and off-design engines is:

<table>
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<th>Condition</th>
<th>$w_f$</th>
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<tr>
<td>Optimum</td>
<td>0.83 lb/lb hr</td>
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<tr>
<td>Off-Design</td>
<td>1.23 lb/lb hr</td>
</tr>
</tbody>
</table>

Here, $q_f$ for propane was taken as 22,000 Btu/lb.

**Performance of a Turbofan**

In this case, the net work is used to power the fan, as shown in Figure 12. When the net work is equated to the change in kinetic energy of the total flow through the engine and the thrust is taken as the rate of change of momentum then:

$$\frac{w_f}{F} = \frac{V_E}{2\eta q_f} \frac{1 + \mu x^2/e_B}{1 + \mu x}$$  \hspace{1cm} (22)

with

$\mu$ - bypass ratio

$x$ - velocity ratio = $V_{EB}/V_E$

With $e_B = .85$, $x = 1/2$, $V_E = 2,200$ ft/s, $q_f = 22,000$ Btu/lb, and $\mu = 8$, the performance parameters are:

<table>
<thead>
<tr>
<th>Condition</th>
<th>$\frac{w_f}{F}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Optimum Operation</td>
<td>0.345 lb/lb hr</td>
</tr>
<tr>
<td>Off-Design</td>
<td>0.596</td>
</tr>
</tbody>
</table>
Performance of a Bypass Jet Engine

In this configuration, the flow from the detonation ducts is used directly for propulsion as shown in Figure 13. In this case, the turbine must deliver sufficient power to compress all the flow while only a fraction of it is going through the turbine itself.

Compressor work

\[ M_c \, c_p \, T_a (R^\gamma - 1)/e_c \]

Turbine work

\[ M_t \, c_p \, T_{bf} (1 - R^{-\gamma}) e_t \]

Equality of the above results in

\[ M_c/M_t = e_c \, e_t (T_{bf}/T_a) R^{-\gamma} \]
The fuel input is

\[ w_f = \left[ \frac{M_t}{A_f t} + \frac{(M_c - M_t)}{A_{fd}} \right] g \]

With the assumption that the detonation duct flow is expanded to ambient pressure, the specific fuel consumption is:

\[ \frac{w_f}{F} = \frac{1/A_f t (\mu - 1 + 1/A_{fd})}{\sum x_i c_p \left[ \frac{T_T}{T_a} \left( 1 - R^{-\Gamma} \right) \right]^{1/2}} \]  \hspace{1cm} (23)

Here

\[ \mu = \frac{M_c}{M_t} \]

\[ x_i = \text{mass function of total flow} \]

In conventional units, the performance is calculated to be

- **Optimum design** \( \frac{w_f}{F} = 2.4 \text{ lb/lb hr} \)
- **Off-Design** \( = 3.36 \)

The reason for the poor performance of this idea is that over 60% of the flow from the detonation ducts is at a low temperature and pressure. The high pressure pulses, which in this case do not compress the rest of the flow, give sharp thrust pulses but of very short duration.

![Figure 13. Bypass Jet Engine](image-url)
Performance of a Shaft Output Engine

In this case, the whole output of the gas turbine is taken as shaft power. The arrangement is shown in Figure 14. The specific fuel consumption is given in terms of the efficiency by the relation:

$$\frac{w_f}{W} = \frac{1}{\eta_f}$$  \hspace{1cm} (24)

In conventional units of specific fuel consumption, the performance of the shaft engine is calculated to be:

- Optimum engine $\frac{w_f}{W} = 0.257$ lb/hp hr
- Off-Design $\frac{w_f}{W} = 0.445$

Figure 14. Schematic Representation of a Shaft Output Engine

With the exception of the bypass jet engine, all the configurations indicate significant improvements in performance. Even if the full potential cannot be realized in a working engine, it appears very likely that detonation wave augmented engines could represent a major advance in technology.
CONCLUSIONS

The study employed highly simplified, but realistic models of the gas dynamics of transient transverse detonation waves in transonic duct flows to estimate the maximum frequency at which such ducts could be operated. Concurrently, a preliminary experimental effort not only proved the correctness of the hypothesis of the possibility of generating such waves, but also demonstrated that the induction distance for the formation of detonation waves was only about 1-2" and indicated that the pressure ratios across the wave were approximately 10. The combined theoretical and experimental results suggest that the salient features of the complex gas dynamic phenomena involved are modeled correctly. Therefore, the performance of detonation ducts operating as gas generators is predicted well enough for estimates of system performance. Losses due to mixing and sudden expansion into the mixing duct are accounted for using standard techniques, but it is possible that significant errors could result from the usual assumptions of quasi-one dimensional flows. Since the mixing process involves turbulent diffusion among streams at different pressures, calculated results of temperatures and velocities must be viewed with some caution. The pulse compression model for the transient flows of dissimilar volumes of gas contains the necessary conditions on energy and continuity equations but some uncertainty remains with regard to the ultimate state of the flow. The above considerations lead to the conclusion that even with highly reasonable models for the flow phenomena involved, considerable uncertainty remains as to the actual turbine entry conditions.

Calculations of performance of the gas turbines operations in various configurations demonstrate significant enhancement of performance. At pressure ratios of 5 the augmented gas turbines showed efficiencies of about 45% versus 26% for conventional engines. Even if the losses in the mixing and wave compression processes were doubled, a 50% increase in efficiency still appears to be realizable. In view of the high cost of additional compressor stages which would be required to achieve such an improvement in performance, the probability of using very inexpensive duct remains should merit further attention.
The theoretical work in this field has gone as far as is practical. Far more elaborate flow models could be invented but the same uncertainties would still remain. It is concluded that the concept has been shown theoretically to have a high probability of success and must now be validated in a simple engineering laboratory experiment.
## LIST OF SYMBOLS

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>area</td>
</tr>
<tr>
<td>Af</td>
<td>air/fuel ratio</td>
</tr>
<tr>
<td>cp</td>
<td>specific heat at constant pressure</td>
</tr>
<tr>
<td>cv</td>
<td>specific heat at constant volume</td>
</tr>
<tr>
<td>c</td>
<td>acoustic speed</td>
</tr>
<tr>
<td>D</td>
<td>detonation wave speed</td>
</tr>
<tr>
<td>e</td>
<td>efficiency</td>
</tr>
<tr>
<td>E</td>
<td>energy</td>
</tr>
<tr>
<td>F</td>
<td>thrust</td>
</tr>
<tr>
<td>g</td>
<td>gravitational constant</td>
</tr>
<tr>
<td>H</td>
<td>heat release parameter</td>
</tr>
<tr>
<td>Lo</td>
<td>length of the detonated volume</td>
</tr>
<tr>
<td>m</td>
<td>mass</td>
</tr>
<tr>
<td>M</td>
<td>Mach number</td>
</tr>
<tr>
<td>p</td>
<td>pressure</td>
</tr>
<tr>
<td>qf</td>
<td>heating value of fuel</td>
</tr>
<tr>
<td>Q</td>
<td>heat</td>
</tr>
<tr>
<td>Q*</td>
<td>heat release parameter</td>
</tr>
<tr>
<td>R</td>
<td>compressor pressure ratio</td>
</tr>
<tr>
<td>t</td>
<td>time</td>
</tr>
<tr>
<td>T</td>
<td>temperature</td>
</tr>
<tr>
<td>Tmax</td>
<td>maximum temperature in the engine</td>
</tr>
<tr>
<td>v</td>
<td>specific volume</td>
</tr>
</tbody>
</table>
\( V_E \)  exit velocity \\
\( w_f \)  fuel flow rate \\
\( W \)  thermodynamic work \\
\( x \)  axial distance \\
\( X_b \)  fraction of flow through the combustor \\
\( \gamma \)  isentropic exponent \\
\( \tau \)  \( 1 - 1/\gamma \) \\
\( \eta \)  engine efficiency \\
\( \Lambda \)  \( (3 - 1)/2(\gamma - 1) \) \\
\( \phi \)  \( T_{max}/T_a \) \\
\( \rho \)  density \\
\( \theta \)  dimensionless time \\

Subscripts 
- a: ambient 
- air: air 
- b: burner or combustor 
- B: bypass 
- bf: combustor exit conditions 
- c: compressor 
- d: detonated gas 
- D: detonation wave 
- e: wave compressor exit 
- f: fuel 
- m: mixing region 
- o: detonated volume initial conditions
REFERENCES


APPENDIX A

GAS DYNAMICS OF THE DETONATION DUCT
GAS DYNAMICS OF DETONATION DUCTS

Presented here is an outline of the modelling and gas dynamics calculations which were performed as part of the feasibility study of detonation wave compression augmentation of gas turbines. Surveys of open literature showed no comparable theoretical or experimental work so that this represents a new approach in the field of detonation waves. An independent laboratory experiment was performed to prove the existence of transverse detonation waves and to estimate some of the critical parameters. This work is described in Appendix B. Existing published work on detonation waves may be divided into three major categories:

a. Studies of propagating steady waves

b. Studies of stabilized waves in very high speed flows

c. Studies of transitions from deflagrations to detonations

The basic theory of steady detonation waves of the Chapman-Jouguet type has been known for about a century (e.g., Stanyukovich\(^1\)) but numerous questions still remain. A detonation wave represents an essentially constant volume heat release with an associated increase in temperature and pressure. Extensive experimental studies of various explosive mixtures indicate that the fundamental Chapman-Jouguet hypothesis of sonic outflow from the detonation wave front is essentially correct. Calculations (e.g., Eisen, et al\(^2\)) are generally in good agreement with experimental data. However, the nature of the detonation wave is itself in some question. While the Chapman-Jouguet detonation with sonic relative velocity of the exploded gas occurs most frequently, non-Chapman-Jouguet waves have also been observed (Gross and Oppenheim\(^3\), Schott\(^4\)). The latter cites instances of detonation products Mach numbers of 1.1. Pressure ratios across detonation waves are also somewhat uncertain, with Eisen, et al\(^2\), calculating stoichiometric H\(_2\)-Air pressure ratios to be 15.6, while Jost\(^5\) shows values of 15.2 and Sokoli\(_k\)\(^6\) shows a value of 13. Similar uncertainties exist in the values for pressures and temperatures after the detonation wave is reflected from the wall as a shock wave.
When a detonation wave is stabilized in high speed flow, total pressure is diminished, but this is deemed acceptable in ramjets because intake losses are reduced. Such a loss could not be tolerated in gas turbines. Because of the continuing interest in high speed ramjets, the problem of stabilizing detonation waves in undiffused hypersonic flow has received considerable attention. Among the published studies are those of Waltrup, et al\textsuperscript{7-9}. The latter is an extensive survey of the field which updates that of Dugger\textsuperscript{10}. Numerous other experimental studies of standing detonation waves demonstrated the feasibility of stabilization of oblique detonation waves (e.g., Nicholls and Dabora\textsuperscript{11} and Dunlap, et al\textsuperscript{12}) and advantages of supersonic combustion for certain ramjet applications were demonstrated by Dugger\textsuperscript{13}. While the field has been thoroughly explored, it is not clear that engineering applications for standing detonation waves do, in fact, exist.

The problem of transition from deflagration to detonation in ducts filled with quiescent explosive mixtures has also received considerable attention. These studies have concentrated on the determination of the effects of size, surface roughness, residual turbulence, ignition mechanism, and the thermodynamic state of the explosive mixture. It is well known that wall roughness or obstacles which generate turbulence in the flow can reduce the transition distance by an order of magnitude (e.g., Sokolik\textsuperscript{6} and Brinkley and Lewis\textsuperscript{14}). The configuration and the energy output of the igniter are also known to play an important role in the initial acceleration of the flame (Laderman, et al\textsuperscript{15}). Reference 14 describes experiments in which placement of obstructions near the igniter reduced the transition or induction distance from 60 to 5 tube diameters, for a wide range of tube diameters. Reference 2 gives a transition distance for a quiescent ethylene-oxygen mixture of only 6 cm. Jost\textsuperscript{5} shows that detonations tend to form in about 3-4 tube diameters for tube diameters greater than about 15 mm. Jost also shows that the transition distance decreases by 15-40% when the initial temperature is increased from 150°C to 180°C. The speed of the detonation wave appears to be very weakly dependent on the mixture pressure and temperature. When the detonation is formed in short tubes or vessels then the precursor pressure waves can compress the unexploded mixture to a pressure ratio as high as 5 (Brinkley and Lewis\textsuperscript{14}). A comprehensive parametric experimental study of Lee, et al\textsuperscript{16} showed the effects of mixture composition and obstacles in the tube on induction distances in hydrogen-air mixtures.
All of the above studies are for detonation waves in stationary fluids and thus the results have very little direct bearing on the transverse waves in high speed flows which are of interest here. A comprehensive literature search using the extensive facilities of the UCLA Engineering Library revealed no published data on transient transverse detonations in ducts. The only studies of the applications of travelling detonation waves in gas turbines appear to be those of Edwards\(^\text{17}\) who demonstrated that stable Chapman-Jouguet waves could be produced in an annulus. These results are interesting but they do not contribute to the gas dynamics problem considered here.

The large number of uncertainties in the fundamental processes and details of the gas dynamics of the flow situation renders exact calculations essentially meaningless and it is necessary to model the physics of the problem using reasonable approximations to the actual phenomena. Physical reasoning based on the experimental work performed in support of this program has led to the following mathematical models of the gas dynamics of transient, transverse detonation wave ducts.

**Initial Detonation**

Standing detonation waves were rejected for gas turbine applications because of the very high total pressure losses associated with the Mach 4-6 shock waves. Initially, transient waves propagating longitudinally in ducts were given some consideration, but the idea was rejected because of the difficulties of uniform ignition across the duct which would be necessary for the establishment of a planar wave. Evaluation of numerous other ideas led to the conclusion that transient transverse waves were most likely to be successful. This was confirmed by the experiments described in Appendix B. The basic idea is to ignite a volume of detonable mixture from one or more sides of a rectangular duct or from the centerline of a circular duct. A line of symmetry or a solid surface are necessary to support the pressure which accelerates the combustion front and leads to the transition from deflagration to detonation. Schematic representations of the ignition models are shown in Figure A.1 The model assumes that at the entrance to the detonation duct is a grid of fuel injectors which deliver fuel intermittently. Synchronized with the fuel injectors is the firing of the igniters at the sides or on the centerline of the duct. The latter is
not considered in depth at this time because of possible problems with unacceptably large induction distances for the formation of the detonation wave. After ignition the flame accelerates away from the wall while it is being swept downstream by the approximately Mach 1 channel flow. At some point a detonation wave is formed and is reflected from the far wall as a shock wave. The gas ahead of the detonation wave will be compressed by the precursor pressure waves generated by the accelerating flame. Some spreading of the flame in the longitudinal direction is inevitable as is turbulent diffusion so that the flame line will be quite irregular. It is also known (e.g., Jost\(^5\)) that detonation waves tend to exhibit transverse instabilities which in tubes manifest themselves as spin. It almost certain that in the highly turbulent transonic duct flow some transverse instabilities will appear and a simple trajectory of the detonation wave is unlikely. The sudden increase of pressure in a volume of gas will generate shock waves in the longitudinal direction and the gas dynamic situation will be somewhat like that shown in Figure A.2.

It is known from the limited test data obtained from a simple detonation duct that the detonation wave forms within a short distance from the igniters (approximately 1-2\(^\prime\)). With duct width/length ratio in the ignition region of about 3-4 and a detonation wave Mach number of 4-6, the traverse time in the transverse direction is an order of magnitude lower than the longitudinal traverse time for an acoustic pulse. A reasonable engineering approximation to this very complex situation is therefore a concept of a sudden appearance of a volume of high pressure and temperature gas. This leads to a sudden expansion which generates shock waves which propagate away from the detonated gas. In the limit this model is that of a shock tube in which diaphragms are burst on both sides of the chamber of high pressure gas.
Figure A.1. Formation and Propagation of Detonation Waves Moving Transversely to the Flow

Figure A.2. Expansion of the Detonation Products
The Shock Tube Problem

When regions 1 and 2, initially at different temperatures and pressures, begin to interact with each other, a shock wave is propagated into the low pressure region and a train of rarefaction waves moves into the high pressure region. The two gases are separated by an interface at which pressure and velocities are equal. The situation is depicted in Figure A.3 which also shows a shock wave entering an expanding duct.

Figure A.3. The Shock Tube Problem and Shock Wave in a Variable Area Duct
Analyses of the formation of the initial shock wave and the strength of the rarefaction waves follows the very clear exposition of Rudinger\textsuperscript{18}. The velocity of the gas in region 3 is given by the usual normal shock wave relation:

$$ u_e = \frac{2a_1 M_s (P_3/P_1 - 1)}{\left( \gamma_1 - 1 \right) \left( 1 + \frac{1}{\gamma_1} P_3/P_1 \right)} $$  \hspace{1cm} (A.1)

with $a$ being the acoustic velocity, $M_s$ the shock wave Mach number, $P$ pressure, $U$ velocity, $\gamma$ the isentropic exponent and subscripts denoting the corresponding regions. The Riemann invariant for region 4 is expressed as:

$$ u_4 = \frac{2a_2}{\gamma_2 - 1} \left[ 1 - \left( \frac{P_4}{P_2} \right)^{\frac{\gamma_2 - 1}{2 \gamma_2}} \right] $$  \hspace{1cm} (A.2)

with $P_3 = P_4$ and $U_3 = U_4$

$$ \frac{P_3}{P_1} = \frac{P_2}{P_1} \left[ 1 - \frac{\gamma_2 - 1}{\gamma_1 - 1} \frac{a_1}{a_2} \frac{(P_3/P_1 - 1) M_s}{1 + \frac{\gamma_1 + 1}{\gamma_1 + 1} P_3} \right]^{\frac{2}{\gamma_2 - 1}} $$  \hspace{1cm} (A.3)

The relation between pressure ratio and shock Mach number is shown in Reference 19 to be:

$$ M_s^2 = \frac{\gamma - 1}{2 \gamma} + \frac{\gamma + 1}{2 \gamma} \frac{P_3}{P_1} $$  \hspace{1cm} (A.4)

Solution of Equations A.3 and A.4 leads to the calculations of conditions across the shock and in the rarefaction fan.
Thus:

$$T_3/T_1 = (2/\gamma_2 + 1)^2 (\gamma_2 M_s^2 - (\gamma - 1)/2) (1 + (\gamma_2 - 1)/2 M_s^2)/M_s^2$$

and

$$\rho_3/\rho_1 = \frac{\gamma_1 + 1}{2} M_s^2 / \left(1 + \frac{\gamma - 1}{2} M_s^2\right)$$

Conditions in region 4 are evaluated by means of isentropic relations. These solutions constitute the initial conditions for the interaction phenomena which are studied using the method of characteristics.

The representative calculations exhibited in the following are based on the following initial conditions:

$$\gamma_1 = 1.4 \quad \gamma_2 = 1.25$$
$$a_2/a_1 = 2.5 \quad p_2/p_1 = 8.5$$

These values are based on theoretical considerations and preliminary test data. The initial values corresponding to these parameters are:

$$M_s = 1.99 \quad a_3/a_0 = 1.244$$
$$p_3/p_0 = 4.47 \quad \rho_3/\rho_0 = 2.66 \quad T_3/T_0 = 1.68$$

In all of the following calculations, it was assumed that in every flow region the gas was perfect and that the isentropic exponents were different but constant.
Initial Interactions

The initial expansion of the detonated mixture creates rarefaction wave fans, which intersect within the detonated volume, as shown in Figure A.4. The numbers on the characteristics are the Riemann invariants which in this case are defined as:

\[ I = 500 [1 + \frac{a}{a_0} + \frac{\gamma - 1}{2} \frac{U}{a_0}] \]  
(A.7a)

\[ II = 500 [1 + \frac{a}{a_0} - \frac{\gamma - 1}{2} \frac{U}{a_0}] \]  
(A.7b)

This identification of the invariants is purely arbitrary and is used here simply for convenience. The fundamental statement is that the quantities \([a + (\gamma - 1)U/2]\) remain constant on trajectories defined by \(U \pm a\).

The changes in state variables indicated in the above Figures are due to the interactions of the initial and reflected rarefaction wave trains and the passage of interfaces and shock waves.

The dimensionless time for the problem is:

\[ \theta = \frac{ta_0}{L_0} \]

and the dimensionless coordinate is \(X/L_0\), with \(X\) measured from the downstream boundary of the detonated volume. Variation of pressure with time throughout the detonated volume is shown in Figure A.5. The same variations at later times are shown in Figure A.6. All of the above quantities are shown in a coordinate system which moves at Mach 1 with the flow in the duct.
Figure A.4. Characteristics Net for the Initial Interaction in a Moving Coordinate System
Figure A.5. Distribution of Pressure in the Detonation Volume in a Moving Coordinate System
Figure A.6. Distributions of Temperature and Pressure within the Detonation Gas
Motion of Discontinuities

After the rarefaction wave trains intersect in the detonated volume, the waves intercept the contact surfaces. Interception of a contact surface by a rarefaction wave results in the deceleration of the contact surface, reflection of a rarefaction wave and a transmission of a weak rarefaction wave. The situation is shown in Figure A.7.

Figure A.7. Interaction of a Rarefaction Wave with a Contact Surface

Conditions for the reflected and transmitted waves are obtained from considerations of the Riemann invariants and the continuity of pressure and velocity at the contact surface. The calculated results for the initial interaction of the rarefaction waves with the contact surface are shown in Figure A.8. The transmitted waves propagate at sonic velocity and therefore overtake the shock wave. The resulting flow situation is somewhat similar to that shown in Figure A.7 except that there is no transmitted wave. Calculations of interactions of waves and contact surfaces in the inertial coordinate system of the duct are shown in Figure A.9. During the interactions, the flow is being swept downstream (to the left) at Mach 1 so that the upstream extent of the gas dynamic phenomena is rather limited. It should be noted that the contact surface which
initially moved upstream is swept back to its initial location at $$\theta = 1.1$$ and clears the detonated volume at $$\theta = 2.1$$. Therefore, in principle, the compressed gas in the detonation volume could be mixed with a new charge of fuel and detonated at some time $$\theta > 2.1$$. This is one of the alternative modes of operation which is considered later.

The shock wave which has an initial Mach number of about 2, in fact initially propagates upstream in the inertial coordinate system at a speed corresponding to Mach 1 since the duct flow is taken to be to the left at Mach 1. The first rarefaction waves overtake the shock wave at the point $$X/L_0 = 2.1$$ or about 1 detonated volume length upstream of the igniters. Motions of the shock wave and the upstream contact surface are shown in Figure A.10. It should be noted that at a distance of 2.4 $$L_0$$ from the igniters, the shock wave Mach number is reduced to about 1.4. Correspondingly, the pressure ratio has diminished from 4.5 to 2.1. This indicates that the rarefaction waves from the opposite side of the detonated volume alternate the upstream propagating shock wave quite rapidly. The phenomena of shock wave and contact surface interactions downstream of the detonated volume will be considered in the discussion of the duct output.
Figure A.8. Interactions with the Detonated Gas and the Shock Wave in a Moving Coordinate System
Figure A.9. Salient Features of the Primary Detonation in Inertial Coordinates
Figure A.10. Motion of Discontinuities in Inertial Coordinates
Attenuation of Upstream Shock Waves

The shock waves propagating upstream from the detonated volume must be attenuated to minor acoustic disturbances in order to minimize the pressure fluctuations which might affect compressor performance. It is known that rapid area increases are very effective in reducing the strength of transmitted shock waves. In general, the gas dynamic situation is as shown in Figure A.11.

![Figure A.11. Interactions of Shock Waves with Area Changes](image)

Here, following Rudinger\textsuperscript{18}, the area change is shown to be instantaneous. This approximation is discussed by Reference 18 and is shown to give good agreement with experimental results. The more elaborate analysis of Whitham\textsuperscript{20} which considers gradual area changes could be adapted to the solution of shock waves moving through variable area ducts with gas flows, but since only an estimate is needed here, the considerable analytical effort involved could not be justified at this point.

The actual flow situation will vary significantly with the strength of the incident shock wave and the Mach numbers of the flow in sections 1 and 2. Solutions of the problem are obtained by matching the conditions in region 5 which results from an isentropic expansion through the.
reflected waves with isentropic flow between area changes from region 5 to region 6 and in turn matching the velocity and pressure at the contact surface between regions 6 and 4. The process is quite straightforward and the required large number of interactions is easily performed on a computer. The procedure is as follows:

a. The velocity of the gas in region 3 is computed using:

\[ \frac{U_3}{a_1} = M_1 - M_5 (1 - R_{13}) \]  
\[ R_{13} = \frac{\rho_1}{\rho_3} \]  \hspace{1cm} (A.8)

b. The temperature ratio \( T_3/T_1 \) is calculated using shock wave relations

c. Mach number in region 3 is calculated using

\[ M_3 = \frac{U_3}{a_1} \left( \frac{T_1}{T_3} \right)^{1/2} \]  \hspace{1cm} (A.9)

d. With a chosen value of \( P_5 \) the Mach numbers in region 3 and 5 are connected using isentropic flow relations

e. Velocity in region 5 is calculated using isentropic flow relations

f. Transmitted shock wave Mach number is guessed and conditions in region 4 are calculated

g. Using isentropic relations and conditions in region 5, the conditions in region 6 are calculated

h. Calculated conditions in regions 6 and 4 are compared and the guesses for \( M_T \) and \( P_5 \) are adjusted.

The process is repeated until the contact surface conditions are satisfied to an arbitrary degree of precision. Solutions for strong incident shock waves in air flowing at \( M_1 = 1 \) are shown in Figure A.12.

A-20
In the particular situation considered here, the incident shock wave Mach number is 1.4 so that the problem is simplified greatly. Since the shock wave is moving into a counterflow at $M = 1$, its net speed in inertial coordinate system is only $M = 0.4$ and the gases in region 3 are swept downstream. The contact surface also moves downstream and the transmitted shock wave is extremely weak. In fact, at an area ratio of 1.34 which corresponds to a flow Mach number of 0.5, $M_T = 1.02$ and the pressure ratio is only 1.047. At an area ratio of 2, which corresponds to a flow Mach number of 0.3, the transmitted shock wave decays to an acoustic pulse.

Detailed calculations employing inviscid flow assumptions are not very meaningful when the shock waves are weak because of strong dissipation effects. However, the results indicate that even if different fuels are used, or the distance between the duct inlet and the detonated volume is reduced, shock waves with Mach numbers of 1.5 - 2.0 can be attenuated by area ratios of less than 5.
Output of the Detonation Duct

The detonation duct is a gas generator whose output varies with time. Here, and in all results exhibited in this Appendix, it is assumed that the air in the duct flows at sonic velocity, the compressor pressure ratio is 5:1, and that a stoichiometric mixture of propane and air is detonated. It is shown in Figure A.9 that the upstream contact surface clears the detonated volume after $\theta = 2.1$ and the flow is fairly uniform at $\theta = 3$. Similar situation exists at $\theta = 8$. In all of the exhibited results, it will be assumed that the duct is fired at $\theta = 0, 3, 8$. After each firing, the flow velocity in the duct is diminished and the cyclic frequency is reduced.

Summary of the thermodynamic states and flow conditions at the duct exit, which is defined as $X/L_0 = -1$ is presented in Table A.1 for 3 consecutive firings. Obviously, in actual operation the cut could be fired 1, 2, or 3 times. Variations of the static pressures, temperatures, and Mach numbers with dimensionless time are shown in Figure A.13. All quantities are normalized by the initial duct flow parameters which are denoted by the subscript o. Corresponding total pressures and temperatures are shown in Figure A.14. Flow velocities at the duct exit are shown in Figure A.15.

The most important parameter for the gas generator aspect of the detonation duct is the power output. Since only comparable quantities are of interest at this stage, the instantaneous power output is defined as isentropic expansion through a turbine to the initial pressure at the compressor inlet. This output is normalized by the energy flow rate through the detonation duct

$$W_0 = \rho_0 U_0 c_p T_0$$  \hspace{1cm} (A.10)

The output from such isentropic blowdown turbines supplied by the detonation ducts is compared with ideal expansion from a combustor with flow at $M_c = 0.25$ in Figure A.16. Comparisons of the total energy generated in a complete cycle consisting of 3 firings and recovery of the flow to initial conditions is 0.4 shown in Figure A.17. Obviously, the highly simplified calculations shown here must be viewed as indications of the potential of the detonation duct as a gas generator rather than from engineering quantities.
## Table A.1. Summary of Duct Output

<table>
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<th>EVENT</th>
<th>P/P₀</th>
<th>T/T₀</th>
<th>PT/P₀</th>
<th>TT/T₀</th>
<th>U/a₀</th>
<th>M</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>FIRST FIRING</strong></td>
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</tr>
<tr>
<td>0</td>
<td>0.00</td>
<td>1.20</td>
<td>1.00</td>
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More realistic power extraction models are used in the main report to evaluate potential gas turbine performance.
Figure A.13. Variations with Time of Mach Number, Static Temperature and Static Pressure at the Duct Exit
Figure A.14. Gas Dynamic Conditions at Duct Exit ($X/L_0 = -1$)

A-26
Figure A.15. Variation with Time of Gas Velocity at Duct Exit
Figure A.17. Energy Generated by the Detonation Duct
List of Symbols

- $a$: Acoustic velocity
- $c_p$: Specific heat at constant pressure
- $I$: Characteristic value
- $II$: Characteristic value
- $L_0$: Length of the detonated volume
- $M$: Mach number
- $P$: Pressure
- $R_{ij}$: Density ratio $= \rho_i/\rho_j$
- $t$: Time
- $T$: Temperature
- $U$: Particle velocity
- $W$: Thermodynamic work or energy
- $X$: Axial distance
- $\gamma$: Isentropic exponent of the gas
- $\rho$: Gas density
- $\theta$: Dimensionless time

Subscripts

- $0$: Initial conditions before detonation
- $1, 2, \ldots$: Regions
- $C$: Combustor
- $S$: Shock
- $T$: Transmitted shock
References


APPENDIX B

EXPERIMENTAL STUDIES OF DETONATION DUCTS
A simple experimental program was carried out in support of, but not as part of, the contract. Due to limitations of time and finances, the experimental program was aimed at demonstrating that transverse detonation waves could be produced, and to indicate the pressure levels which could be achieved.

From the beginning of the program of studies of detonation wave augmented gas turbines, serious questions were raised regarding the possibility of forming transverse detonation waves in transonic flows. With the exception of the rotating detonation waves of Edwards, which was noted in Appendix A, all previous work in detonation waves had been done in quiescent gases or stabilized waves in hypersonic flow. Because of the range of occasionally conflicting experimental data on the initiation and propagation of detonation waves, it became obvious that some experimental data were needed to support the fundamental assumptions underlying the theoretical work.

A rectangular detonation duct was fabricated out of heavy steel plates. Details of the duct and the assembled equipment are shown in Figures B.1-B.4. Pressures were measured using Kistler 602A high frequency pressure transducers mounted in instrumentation plugs such as may be seen in Figure B.5. Air was supplied continuously from a high pressure wind tunnel reservoir. Propane was injected intermittently from various tanks. Ignition was found to be the easiest and most effective when a row of spark plugs was fired by a series of high frequency interrupters or an automotive distributor. There was no direct control over the air/fuel ratio so that various air supply and propane reservoir pressures had to be tried until detonations could be obtained on every attempt. Occurrence of detonations was determined from transducer pressure outputs, and also audibly from the very loud sharp cracks, rather than dull thuds which followed a misfire or delayed ignition.

Representative results are shown in Table B.1. The relatively reliable velocities which are derived from the time increments between the pressure recording at two transducers correspond to shock waves whose pressure ratios are much higher than those exhibited in the Table. There was some uncertainty regarding the pressure level in the duct and it is possible that the detected pressures at
different transducers may have been caused by different waves. The data does indicate that detonation waves are formed in very short distances from the igniters (about 1-2"").

In the interest of using conservative basic data in the theoretical work, the bulk of the pressure data, which indicated pressure ratios of 9.6, was taken as the basis and reduced to 8.5 in the calculation.
Figure B.1. Details of the Side Plates

4. MATERIAL: MILD STEEL
3. DIMENSIONAL TOLERANCES: ±.XXX±.010, ±.XX±.03
2. DIMENSIONS ARE IN INCHES.
1. REMOVE ALL BURRS AND SHARP CORNERS.

NOTES: UNLESS OTHERWISE SPECIFIED.
Figure B.2. Details of the Bottom Plate

1. REMOVE ALL BURRS AND SHARP CORNERS.
2. DIMENSIONS ARE IN INCHES.
3. DIMENSIONAL TOLERANCES: ± .XXX ± .03
4. MATERIAL: MILD STEEL

NOTES: UNLESS OTHERWISE SPECIFIED
TOP PLATE

4. MATERIAL: MILD STEEL.
3. DIMENSIONAL TOLERANCES: \( \pm .010, \pm .03 \).
2. DIMENSIONS ARE IN INCHES.
1. REMOVE ALL BURRS AND SHARP CORNERS.

NOTES: UNLESS OTHERWISE SPECIFIED

Figure B.3. Details of the Top Plate
Figure 4.B. Detonation Duct Assembly
Figure 5.B. Details of an Instrumentation Plug
Table B.1. Experimental Results

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A detailed theoretical study was made of the concept of augmenting the performance of low pressure ratio gas turbines by detonation wave compression of part of the flow. The concept exploits the constant volume heat release of detonation waves to increase the efficiency of the Brayton cycle. In the models studied, a fraction of the compressor output was channeled into detonation ducts where it was processed by transient transverse detonation waves. Gas dynamic studies determined the maximum cycling frequency of detonation ducts, proved that upstream propagation of pressure pulses represented no problems and determined the variations of detonation duct output with time. Mixing and wave compression were used to recombine the combustor and detonation duct flows and a concept for a spiral collector to further smooth the pressure and temperature pulses was presented as an optional component. It was found that the best performance was obtained with a single firing of the ducts so that the flow could be re-established before the next detonation was initiated. At the optimum conditions of maximum frequency of the detonation ducts, the gas turbine efficiency was found to be 45 percent while that of a corresponding pressure ratio 5 conventional gas turbine was only 26 percent. Comparable improvements in specific fuel consumption data were found for gas turbines operating as jet engines, turbofans, and shaft output machines. Direct use of the detonation duct output for jet propulsion proved to be unsatisfactory. Careful analysis of the models of the fluid flow phenomena led to the conclusion that even more elaborate calculations would not diminish the uncertainties in the analysis of the system. Feasibility of the concept to work as an engine now requires validation in an engineering laboratory experiment.