Calculations of Electric Currents in Europa

D.S. Colburn and R.T. Reynolds

December 1986
Calculations of Electric Currents in Europa

D. S. Colburn,
R. T. Reynolds, Ames Research Center, Moffett Field, California

December 1986
SUMMARY

Model calculations of electrical currents expected to flow in Europa, which were summarized in a previous publication (Colburn and Reynolds, 1985) are presented in detail. The rationale that determined the choice of parameters is given, as well as the derivation of the equations defining the model, for clarification of the proposed current systems.

Electrical currents should flow in the Galilean satellite, Europa, because it is located in Jupiter's corotating magnetosphere. The possible magnitudes of these currents are calculated by assuming that Europa is a differentiated body consisting of an outer H2O layer and a silicate core. Two types of models are considered here: one in which the water is completely frozen and a second in which there is an intermediate liquid layer. For the transverse electric mode (eddy currents), the calculated current density in a liquid layer is approximately $10^{-5}$ Am$^{-2}$. For the transverse magnetic mode (unipolar generator), the calculated current density in the liquid is severely constrained by the ice layer to a range of only $10^{-10}$ to $10^{-11}$ Am$^{-2}$, for a total H2O thickness of 100 km, provided that neither layer is less than 4 km thick. The current density is less for a completely frozen H2O layer. If transient cracks were to appear in the ice layer, thereby exposing liquid, the calculated current density could rise to a range of $10^{-6}$ to $10^{-5}$ Am$^{-2}$ depending on layer thicknesses, which would require an exposed area of $10^{-9}$ to $10^{-8}$ of the Europa surface. The corresponding total current of $2.3 \times 10^9$ A could in 1 yr electrolyze $7 \times 10^5$ kg of water (and more if the cells were in series), and thereby store up to $10^{18}$ J of energy, but it is not clear how electrolysis can take place in the absence of suitable electrodes. Electrical heating would be significant only if the ice-layer thickness were on the order of 1 m, such as might occur if an exposed liquid surface were to freeze over; the heating under this condition could hinder the thickening of the ice layer.

INTRODUCTION

Jupiter's powerful magnetic field can induce sizable electric potentials on the large and relatively close Galilean satellites, particularly the two innermost of these satellites, Io and Europa. The presence of a conducting satellite in the magnetosphere of its primary can give rise to two possible electric current systems. In one of these, the transverse electric (TE) mode, eddy currents are set up in the satellite by the time-varying component of the magnetic field, which at Jupiter is caused principally by the $10^6$ tilt of the planetary dipole field with respect to the rotation axis. The currents remain within the satellite body and are not diminished by low conductivity at the surface or outside the body. In the other current system, the transverse magnetic (TM) mode, a unipolar generator is set up by
the relative velocity of the satellite through the corotating magnetosphere (Sonett and Colburn, 1967). Unipolar generator currents must close within the planet's magnetosphere or ionosphere and consequently might be limited by the electrical conductivity far from the satellite. The magnetic and electrical fields appropriate for both the TE and TM modes at Io and Europa are summarized in Table 1, adapted from Colburn (1980).

### Table 1. Magnetic and Electric Fields at Io and Europa

<table>
<thead>
<tr>
<th>Variable</th>
<th>Characteristic</th>
<th>Io</th>
<th>Europa</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B_0$</td>
<td>Magnetic field, steady component, nT</td>
<td>2000</td>
<td>500</td>
</tr>
<tr>
<td>$r_o$</td>
<td>Orbital distance, $R_j$</td>
<td>5.91</td>
<td>9.40</td>
</tr>
<tr>
<td>$v_o$</td>
<td>Orbital velocity with respect to corotating magnetosphere, km sec$^{-1}$</td>
<td>56.8</td>
<td>104.3</td>
</tr>
<tr>
<td>$E_{TM}$</td>
<td>TM mode field ($E_{TM} = v_o B_0$), V m$^{-1}$</td>
<td>.114</td>
<td>.0521</td>
</tr>
<tr>
<td>$V_0$</td>
<td>TM mode potential, per hemisphere ($V_0 = r E_{TM}$), kV</td>
<td>207</td>
<td>82</td>
</tr>
<tr>
<td>$B_1$</td>
<td>Magnetic field, alternating component, nT</td>
<td>800</td>
<td>200</td>
</tr>
<tr>
<td>$\omega$</td>
<td>Angular frequency of $B_1$, rad sec$^{-1}$</td>
<td>$1.347 \times 10^{-4}$</td>
<td>$1.554 \times 10^{-4}$</td>
</tr>
<tr>
<td>$r$</td>
<td>Satellite radius, km</td>
<td>1820</td>
<td>1569</td>
</tr>
<tr>
<td>$E_{TE}$</td>
<td>TE mode field ($E_{TE} = 0.5 w r B_1$), V m$^{-1}$</td>
<td>$9.81 \times 10^{-5}$</td>
<td>$2.44 \times 10^{-5}$</td>
</tr>
</tbody>
</table>

$^a R_j =$ radius of Jupiter = $7.1 \times 10^4$ km

Detection of TE-mode currents requires magnetic measurements near the surface and measurements of or assumptions regarding the driving field. TE-mode currents have been measured at the Moon (Sonett et al., 1971) and used to deduce a conductivity profile (see review by Sonett, 1982), but there has been no opportunity to observe these currents at Europa or Io. The TM mode is believed to have been observed at Io; it has been postulated as the explanation for the strong modulation of Jupiter's decametric radiation by Io's orbital position (Goldreich and Lynden-Bell, 1969), and a current was detected by the Voyager magnetometer while passing by Io's flux tube (Ness et al., 1979). From that experiment the magnitude of the current in the flux tube was estimated to be $10^6$ A, resulting in a current of $2 \times 10^6$ A at Io if northern and southern flux tubes contribute equally. Further analysis of the data yielded a value of $2.8 \times 10^6$ A (Neubauer, 1980; Acuña et al., 1981). For Europa, evidence for a modulation of the decametric radiation, summarized by Car and Desch (1976), is primarily negative. Among many unfruitful searches for the modulation effect, one ground-based study (Taininen, 1967) showed increased emission when Europa was $190^\circ$ from superior geocentric conjunction. During 1969, using RAE-1 spacecraft data, Carr and Desch found a strong Europa effect, also at approximately $190^\circ$, but in the following year the effect was indiscernible. Later Desch and Carr (1978) stated that their RAE-1 data do not show
a positive result concerning Europa. They cite the difficulty of sorting out Io and Europa effects. The approximate two-to-one ratio of the orbital periods of Europa and Io makes it difficult to isolate possible Europa effects in the presence of noise. Continuing the work of Desch, St. Cyr (1982, 1985) analyzed several ground-based decametric data bases, covering more than 20 yr, and found no clear evidence of the interaction of Europa or Ganymede with the Jovian magnetosphere. No Voyager measurement of flux-tube current for Europa was possible, since neither of the spacecraft trajectories came sufficiently close to possible flux-tube positions.

In a detailed study, Colburn (1980) showed that internal currents from either mode would be very small for Io, where a conducting ionosphere envelopes a poorly conducting cold crust composed of such materials as sulfur, silicates, and sulfur compounds. Thus, nearly all of the observed current must be confined to the ionosphere.

Although the induced fields are somewhat weaker at Europa (table 1), the lack of an observable ionosphere and the possibility of a better conducting surface material, H$_2$O ice, overlying the liquid water, suggest that currents in the solid body could be larger for Europa than for Io and consequently should be investigated.

The surface of Europa was characterized by the Voyager mission as having very low relief, an almost complete lack of visible craters, many linear albedo features, and a surface evidently consisting almost entirely of H$_2$O ice (Smith et al., 1979). The density suggests the presence of -6% ice by mass which, if the body is assumed to have differentiated, would form an outer layer of ~100 km of ice over an assumed iron-silicate core (Cassen et al., 1979). In this reference as well as in Cassen, Peale, and Reynolds (1980) the authors have further suggested the possibility of a liquid-water layer beneath an ice crust, which could be kept from freezing primarily by tidally generated heat. Squyres et al. (1983) have presented a case for a currently active surface for Europa involving an ongoing process of cracking and refreezing the surface, with only a very small fraction of the satellite surface being fractured at any one time. The mean subsurface temperature at Europa is estimated to be approximately 90 K.

In this paper we discuss electrical conductivities and calculate the TE and TM currents for Europa models that include a silicate interior, a surface ice layer, and in some cases an intermediate liquid layer. The TM-mode current densities are also estimated for an extreme Europa model in which surface cracks in the ice might allow increased current to flow. Some of the implications of such currents are considered.

ELECTRICAL CONDUCTIVITY

The electrical conductivity of rock materials has long been known to vary exponentially with the reciprocal of the temperature, as expressed in the relation

$$\sigma = \sigma_0 \exp[-E/(kT)]$$

(1)
where $\sigma_0$ and $E$ are constant over a broad temperature range, $\eta$ is Boltzmann's constant, and $T$ is the absolute temperature. Measurements of rock conductivity at a given temperature have varied by several orders of magnitude and differ appreciably, even with different samples of the same rock type. Parkhomenko (1967) has compiled many of these determinations. More recently, the conductivity has been found to be influenced strongly by the oxygen fugacity and by the concentration of trivalent elements in the sample (Duba et al., 1974; Huebner et al., 1979). Although conductivity measurements can be done fairly accurately, the various assumptions about the material composition of a planetary body can easily produce uncertainties of many orders of magnitude in electrical conductivity. Colburn (1980) listed several rock-conductivity functions with conductivities $(T = 273)$ ranging from $10^{-8}$ to $10^{-15}$ Sm$^{-1}$ (the siemens, S, is the reciprocal ohm). The highest conductivity function was ascribed to a lunar sample (Schwerer et al., 1971) and the lowest function was ascribed to an olivine sample (Bradley et al., 1964). Huebner et al. (1979) computed a conductivity function for a dunite representative of the outer lunar mantle, which gives a 273 K conductivity of $3 \times 10^{-20}$ Sm$^{-1}$. Parameters for the three samples are $\sigma_0 = 7.9$, 200, and 3.7 Sm$^{-1}$ and $E = 0.51$, 0.92, and 1.09 eV, respectively. Because the computed conductivity at 273 K is essentially an extrapolation of experimental measurements, its accuracy is questionable. However, even over this broad range of conductivity values, it will be shown that currents in ice and water, when present, are always large enough that currents in silicates can be considered negligible in comparison.

Measurements of the electrical conductivity of ice have varied widely, partly because of experimental errors. Hobbs (1974) has summarized these determinations, nearly all of which are taken at a temperature close to the melting point of ice rather than at the colder temperatures expected at the European surface. Measurements at nonzero frequencies (typically 1000 Hz) are not applicable to this problem, and dc measurements are often contaminated by potentials that develop at the electrodes. A measurement of low-temperature conductivity which appears reliable was made by Auvert (1973) and summarized in Auvert and Kahane (1973). Hydrogen plasma electrodes were used, which appears to be appropriate because laboratory experiments show that the method of conduction is principally proton migration (Hobbs, 1974). Auvert found a linear relation between the log of conductivity and the reciprocal temperature over the temperature range of Auvert and Kahane's experiment, 238 K to 158 K. For the present calculations we have used a linear fit to Auvert's measurements and extrapolated to lower temperatures. The activation energy is 0.224 V and the multiplying factor is 0.0391 Sm$^{-1}$. The conductivity ranges from $3 \times 10^{-6}$ Sm$^{-1}$ at 273 K to $1 \times 10^{-14}$ Sm$^{-1}$ at 90 K. Whereas the addition of impurities to distilled liquid water raises the conductivity dramatically because of the addition of charge carriers, impurities have less of an effect on the conductivity of ice. Laboratory measurements of this effect are subject to error unless performed at a temperature cold enough to assure that no liquid phase is present. Experiments using NH$_3$, HF, NH$_2$OH, and NH$_4$F have been reported; use of the first three can increase ice conductivity by one or two orders of magnitude, whereas use of NH$_4$F decreases the conductivity (Hobbs, 1974). It is clear that the conductivity of Europa's ice layer has not been satisfactorily determined.
If there is liquid water on Europa, it presumably contains dissolved salts from the silicates, as does the Earth's ocean. Salt water is much more conductive than distilled water. For a model of the liquid water on Europa, we use sea water of salinity 0.035 (standard for Earth's ocean); at 271 K (freezing point) the conductivity is 2.75 Sm\(^{-1}\) and is approximately proportional to the salinity. The conductivity increases with temperature (Montgomery, 1963); however, the temperature gradient within a liquid water layer should be nearly adiabatic and hence the temperature change across the layer should be quite small.

CURRENT DENSITIES IN THE TE MODE

In the TE mode, the eddy currents circle the lines of the alternating magnetic flux (Sonett and Colburn, 1968). The geometry of the TE mode for Europa is shown in figure 1. The dipole comprising the principal portion of Jupiter's magnetic field is tilted 10° from the rotation axis; consequently a dipole component, \(D_t\), rotates in the equatorial plane. The time-varying field of \(D_t\) at Europa can be resolved into two components, \(B_1\) in the \(x\)-direction, pointing toward Jupiter and \(B_2\) in the \(y\)-direction, that of Europa's orbital motion. The \(B_2\) component has half the magnitude of \(B_1\) and lags in its time phase by one-fourth cycle, so that the resultant field vector traces an ellipse in Europa's equatorial plane. An ensemble of eddy currents shown as \(I_1\) encircles \(B_1\), reaching the maximum at the time \(B_1\) passes through zero, and correspondingly an ensemble \(I_2\) encircles \(B_2\), reaching the maximum at the time \(B_2\) passes through zero. Currents \(I_1\) and \(I_2\) are thus 90° out of phase. The current density is fairly evenly distributed in any shell; the maximum occurs at the limb as viewed from Jupiter, whereas the minimum is less by a factor of only two, occurring along the sub-Jupiter meridian. For the work presented here, the elliptical vector was approximated by a single time-varying vector of magnitude \((B_1^2 + B_2^2)^{1/2}\).

Figure 1.- TE-mode geometry.
For a silicate body or one with an H$_2$O outer layer that is completely frozen, the conductivities are low enough that the eddy currents near the surface are not modified by currents in the interior; in this case, the maximum current-density is given by

$$i = \omega E$$ \hspace{1cm} (2)

where $E = 2.44 \times 10^{-5}$ Vm$^{-1}$ and the units of $i$ are Am$^{-2}$. These current densities are plotted as a function of temperature in figure 2. The calculation is for the equatorial surface. For the case of salt water, if the layer were thick, the eddy current system would reduce the current density shown here by a modest amount.

Calculations are shown for the three silicates previously described. For similar temperatures, the current density induced in ice is much larger than that induced in the silicates.

If there is an intermediate liquid layer of a thickness tens of kilometers, the currents in the water are not significantly affected by either the silicate body underneath or the low-conductivity ice above. The current density for such a layer

![Figure 2](image_url)

Figure 2.- Maximum TE-mode current densities as a function of surface material and temperature.
is several orders of magnitude larger than that of the solids, shown in figure 2. The current density in the liquid as a function of layer thickness is shown in figure 3, peaking at $7 \times 10^{-5} \text{ Am}^{-1}$ for a thin layer. The bottom of the water layer is at $R = 1469$ km. The top of the water layer is covered by ice sufficient to make the total radius 1569 km, but the presence of the ice does not alter the current density shown. The temperature of the water is assumed to be 271 K. In this range, the current density is almost directly proportional to the salinity of the liquid water.

![Figure 3. Maximum TE-mode current density within a salt-water layer, of salinity 0.035, as a function of the layer thickness.](image)

The geometry for the TM mode is shown in figure 4. The steady-field component, $B$, points in the $z$-direction (southward) at Europa, whereas Europa's velocity, $V$, with respect to Jupiter's corotating magnetosphere is in the negative $y(-y)$-direction (i.e., opposed to the direction of Europa's orbital velocity). An electric field equal to $V \times B$ drives the current inside Europa in the $-x$-direction (i.e., outward from Jupiter). The direction of current flow is modified by variations in conductivity within Europa. If the satellite can be assumed to be spherically symmetric, the current will be cylindrically symmetric about the $x$-axis (symbolized by the vector $I$ in the figure). Since Europa is synchronously rotating, the sub-Jupiter point is a relatively fixed position on
Europa's surface. A colatitude angle $\theta$ can be defined as the arc between a point $P$ on the surface and the sub-Jupiter point, as shown in the figure. The vertical current density varies as $\cos \theta$ and the horizontal current density varies as $\sin \theta$. The current densities discussed here are the maximum values, i.e., the vertical current density at $\theta = 0$ and the horizontal current density at $\theta = \pi/2$.

The potential which drives the TM-mode current is the product of Europa's diameter, the magnetic field, and the relative velocity of Europa with respect to the corotating Jovian-magnetic-field lines. The current resulting from this potential drop must pass through Europa and then be completed by the external circuit, which leaves Europa in a direction nearly parallel to the field lines. Closure is through the distributed impedance of the Alfvén wings (as discussed by Neubauer, 1980), modified for some configurations by the Jovian ionosphere. The current in the interior could be bypassed by an ionosphere of Europa; however, no atmosphere or ionosphere in the vicinity of Europa has been reported. The current is limited by the total impedance. In this discussion we first consider the case in which the current is limited only by Europa's impedance. Then we consider the results of the further restriction of current by the external impedance.

The current in the silicate part of the body is insignificant in comparison with possible currents in the ice or liquid layers discussed here. The satellite is modeled as an insulating core below 1469 km, with a liquid layer at a temperature of 271 K topped by an ice layer of sufficient thickness to make the total radius 1569 km. The temperature in the ice is assumed to drop linearly with distance from 271 K to a surface temperature of 90 K. Tidal heating in the ice layer would modify
the shape of this profile, but not its general character. Since the ice layer controls the current, variations in temperature and salinity of the liquid have almost no effect on the current density. The resulting maximum TM-mode current densities are plotted in figure 5, against the water layer thickness, $D_W$, and the ice layer thickness, $D_I$. $I_H$, water, is the horizontal current density in the water, essentially independent of depth. $I_H$, ice, is the maximum horizontal current density in the ice, calculated for the interior of the ice layer, and is seen to be negligible in comparison to that of the water. $I_V$ is the vertical current density at the ice-water interface.

An upper limit to the total current through Europa can be estimated by comparison with Io. For the Io current system, the current was apparently limited by the conduction of the external circuit through the flux tubes (Colburn, 1980). Neubauer (1980) has calculated the conduction for Europa to be less by a factor of 3
resulting from lower plasma densities. The driving potential for Europa is less than that of Io by a factor of 2. Therefore, the upper limit for the current in Europa can be considered to be a factor of 6 less than that observed at Io. A further reason for considering this to be an extreme upper limit is that the magnetic field of such a current through Europa might exceed that of Jupiter and thus deflect the plasma and reduce the flow. We take the upper limit here to be

\[ 2.3 \times 10^5 \text{A}, \]

following Neubauer (1980). The corresponding maximum current density is

\[ 3.0 \times 10^{-8}. \]

Since the current densities in figure 5 are much lower than the limit discussed here, they are not limited by the external circuit.

ENHANCEMENT OF CURRENT DENSITIES BY CONCENTRATION IN CRACKS

It has been suggested that an ice layer overlaying a liquid ocean might have occasional vertical cracks (Cassen et al., 1979; Squyres et al., 1983). The cracks would be temporary because the liquid filling them would be subjected to freezing temperatures and to a vacuum that would make it tend to boil furiously at the surface. In the electrical context, an ice layer with an ensemble of vertical cracks would have an anisotropic conductivity—the vertical conductivity would be approximated by the product of the liquid conductivity and the fractional area of the cracks and the horizontal conductivity would be very nearly that of the ice (assuming there are no continuous meridional paths of appreciable length on a planetary scale).

A mathematical model can be computed for this anisotropic case. The coordinate system chosen here is a spherical coordinate system using \( r, \theta, \) and \( \phi \), with the applied electric field in the direction \( \theta = \pi \). All of the quantities are independent of \( \phi \) and direction is designated by the subscripts \( H \) for horizontal and \( V \) for vertical; \( H \) is in the direction of \( +\theta \), and \( V \) is in the direction of \(-r\).

The components of current density \( I_V \) and \( I_H \) are determined by \( \nabla \cdot I = 0 \), from which

\[ \frac{d}{dr} I_V = \frac{2(I_H - I_V)}{r} \]  \hspace{1cm} (3)

and by \( \nabla \times (I/\sigma) = 0 \), from which

\[ \frac{d}{dr} I_H = \frac{I_H}{\sigma_H} \left( \frac{dc_H}{dr} \right) + \frac{I_V \sigma_H}{r \sigma} - \frac{I_H}{r} \]  \hspace{1cm} (4)

Since currents in the silicate portion of the body are negligible, integration starts at \( r = 1469 \text{ km} \) with \( I_V = 0 \) and \( I_H = 1 \). After integration to \( r = 1569 \text{ km} \), all of the currents are scaled to satisfy \( \nabla \times E = 0 \) at the surface, i.e., \( I_H = \sigma_H V_0/r \).
The conductivity tensor is determined from $\sigma_w$ and $\sigma_i$, the conductivities of liquid and ice, respectively, and a factor $A_C$, which is the fraction of the surface that is the open-crack area. As an example, if the total crack area over a 1-km$^2$ surface is 1 m$^2$, then $A_C = 1 \cdot 10^{-6}$.

Then for small $A_C$ and $\sigma_i \ll \sigma_w$

$$\sigma_v = A_C\sigma_w + (1 - A_C)\sigma_i \quad (5)$$

$$\sigma_H = \sigma_i \quad (6)$$

The radial variation of conductivity is approximated by assuming a linear temperature drop for the ice from 271 K at the inner surface to 90 K at the outer surface, which determines $\sigma_i$. The water is assumed to have the same conductivity and temperature throughout.

By varying the fractional crack area, we calculate the horizontal current density in the water at the ice-water interface, $I_H$, assuming no current limitation by the external circuit. The curves of $I_H$ are shown in figure 6 for three ice-layer thicknesses $D_i = 10$, 50, and 90 km. The abscissa is the proportional surface area of the cracks. The vertical current density at the surface $I_V$ is also plotted. Values of $I_H$ and $I_V$ are denoted in the figure corresponding to the

![Figure 6.- Maximum TM-mode current densities in a system with a salt-water layer covered by a cracked ice layer.](image)
nominal upper limit for total current in Europa, $2.3 \times 10^5$ A. In this case, the maximum $I_V$ is $3.0 \times 10^{-8}$ Am$^{-2}$, shown on the figure as a horizontal line. The intersection of this line with each curve for $I_V$ determines the fractional crack area for the corresponding ice thickness. For example, the point P in the figure indicates that the fractional crack area for a thickness of 90 km is $6 \times 10^{-9}$. The corresponding $I_H$ for this thickness is found on the $I_H$ curve ($D_I = 90$); it has the same abscissa as point P. Thus the three circled points on the figure show the $I_H$ for each thickness when the total current is at the nominal upper limit.

The variation of maximum $I_H$ with ice thickness is better understood by means of an analytic approximation. Since $\sigma_H$ of the ice is several orders of magnitude less than $A_C \times \sigma_W$ over the range of interest, $\sigma_H$ of the ice can be approximated as zero, whereupon the model has an analytic solution. In this solution we define $D_I$ as the ice layer thickness, $D_W$ as the water layer thickness, and three radii: $b = 1469$ km, $a = 1569$ km, and $s = 1469$ km + $D_W$. In the water ($b < r < s$), the potential is given as

$$V = M \left( \frac{r}{b^3} + \frac{1}{2r^2} \right)$$

and in the ice ($s < r < a$), as

$$V = P + Q \left( \frac{1}{s} - \frac{1}{r} \right)$$

with constants $P$, $Q$, and $M$ determined by the following boundary conditions: $V = V_0$ (i.e., 82 kV) at $r = a$ and $V$ and $I_V$ are continuous at $r = s$. Then

$$A_C = I_HS \left( \frac{1}{b^3} - \frac{1}{s^3} \right) \left( \frac{1}{s} - \frac{1}{a} \right) / \left[ \sigma_W \left( \frac{1}{b^3} + \frac{1}{2s^3} \right) \left( V_0 - \frac{sI_HS}{\sigma_W} \right) \right]$$

$$I_V = s^2I_HS \left( \frac{1}{b^3} - \frac{1}{s^3} \right) / \left[ a^2 \left( \frac{1}{b^3} + \frac{1}{2s^3} \right) \right]$$

or, rewriting in terms of the water and ice thicknesses $D_W$ and $D_I$, respectively,

$$A_C = I_HS \left( \frac{1}{b^2} + \frac{1}{bs} + \frac{1}{s^2} \right) / \left[ \sigma_Wab \left( \frac{1}{b^3} + \frac{1}{2s^3} \right) \left( V_0 - \frac{sI_HS}{\sigma_W} \right) \right]$$

$$I_V = I_HS \left( \frac{1}{b^2} + \frac{1}{bs} + \frac{1}{s^2} \right) / \left[ a^2b \left( \frac{1}{b^3} + \frac{1}{2s^3} \right) \right]$$

Here, $I_HS$ is the horizontal current density in the liquid at its upper surface, and $I_V$ is the vertical current density at $r = a$. Equation (11) has been written with $A_C$ on the left side for algebraic simplicity, but $A_C$ is the
independent variable and the equation shows implicitly the dependence of $I_{HS}$ on $A_C$. The dependence of $I_{V0}$ on $A_C$ follows from equation (12). The linear relation in figure 6 between $A_C$, $I_{HS}$, and $I_{V0}$ is confirmed for the regime in which the potential across the ice ($V_0 - s l_{HS}/\sigma_w$) is essentially equal to $V_0$. The asymptotic limit for $I_{HS}$ suggested at the top of figure 6 is reached when the potential across the ice falls to zero. The near identity of the two curves for $I_{HS}$ at ice layer thicknesses of 10 km and 90 km is evident because the product $D_{W}D_{I}$ is the same for both thicknesses, whereas $s$, a function of the widths, never varies more than 7%.

The maximum horizontal current density is determined by the analytic solution under the constraint that the total current not exceed $2.3 \times 10^5$ A; it is plotted as a function of liquid layer thickness in figure 7. The corresponding crack area which will achieve this current density is also shown. The agreement between figures 6 and 7 is excellent for current densities as a function of liquid layer thickness, whereas the required crack area for any given thickness of figure 7 determined by the analytic solution is greater than that of figure 6 by 30% to 60%. The difference is attributed to the neglect of ice conductivity in the analytic model. Thus,

![Graph showing horizontal current density and relative crack area as a function of water layer thickness](image-url)

Figure 7.- Horizontal current density and relative crack area as a function of water layer thickness for the system of figure 5.
in figure 7, the upper \( A_C \) curve is plotted for the analytic case and the lower \( A_C \) curve is plotted for the more realistic case in which the conductivity of ice is appreciable. The latter curve is constructed by translating the analytic curve vertically to fit the three \( I_H \) points found in figure 6. Because the dependence of the current density \( I_H \) on water layer thickness is nearly the same in both cases, it is shown as a single curve.

It can be seen from figure 7 that \( I_H \) will be limited to about \( 2 \times 10^{-5} \, \text{A m}^{-2} \). The relative crack area for which this can be obtained is approximately \( 1 \times 10^{-8} \), or \( 1 \, \text{m}^2 \) per \( 100 \, \text{km}^2 \) of surface area. In this range, conduction through the ice is very small in comparison with that through the cracks; consequently the estimate of current density is not sensitive to accurate knowledge of the low-temperature conductivity of ice or the linearity of the temperature gradient through the ice. Since in the more general case, the abscissas in figures 6 and 7 are the product of the fractional crack area and liquid conductivity, any revision upward in the estimate of salinity (proportional to conductivity) would cause a lowering of the calculated fractional-crack area by the same factor.

ELECTROLYSIS AND HEATING FOR LIQUID-FILLED CRACKS

The presence of Europa in Jupiter's corotating magnetosphere implies a potential, so if the electrical conductivity is sufficient, a current should flow through the satellite. The magnitude of such a current would depend very strongly on the composition of Europa's upper layers; the current would be very small unless a layer of water were present on a global scale. Among the consequences of such a current flow, two in particular appear to be of interest and to warrant calculation: the possibility of the electrolysis of \( \text{H}_2\text{O} \) and the amount and distribution of electrically generated heat.

As an extreme limit, if a continuous current of \( 2.3 \times 10^5 \, \text{A} \) were to electrolyze water in a single cell, hydrogen would be produced at the rate of \( 7.5 \times 10^4 \, \text{kg yr}^{-1} \) and oxygen at the rate of \( 6.0 \times 10^5 \, \text{kg yr}^{-1} \). The 100-km layer of \( \text{H}_2\text{O} \) contains \( 10^{16} \, \text{kg} \) of hydrogen, so that significant water loss would occur only after approximately \( 10^9 \, \text{yr} \). If several cells were connected in series, the rate could be correspondingly much greater; but we do not consider this possibility likely. The energy storage for a single cell under these conditions is \( 10^{13} \, \text{J yr}^{-1} \), while the available energy is \( 10^{18} \, \text{J yr}^{-1} \).

The possibility of electrolysis depends on the electrical-conduction mechanism. Laboratory experiments show that conduction in ice occurs principally through the migration of protons. In one experiment, when the electrodes were made of a porous black metal saturated with hydrogen, protons were provided at the anode by ionization of hydrogen, whereas at the cathode, protons were neutralized to produce hydrogen. The current decreased over the course of time as the anode became more depleted, and the experimenter postulated that eventually the protons would be supplied by natural ionization of \( \text{H}_2\text{O} \) (Hobbs, 1974). Thus, oxygen production by
At Europa the interfaces considered likely are (1) between the surface of Europa and the tenuous atmosphere and (2) between ice and seawater. The atmosphere is ionized gas, which is electrically conducting, consisting largely of protons and electrons, so that hydrogen plasma electrodes similar to Auvert's could be maintained. The protons supplied by the plasma could carry the charge through the ice and cause proton depletion at the anode and concentration at the cathode, while the external circuit could provide the electrons needed for charge neutrality. However, appreciable concentrations of plasma would not be expected to occur since the plasma corotates with Jupiter's field rather than being tied to Europa, and general turbulence and thermal effects would maintain a relatively uniform plasma concentration over time, providing a reservoir of protons. Thus, we cannot expect either hydrogen or oxygen to accumulate at the icy surface. If there were liquid water at the surface, chemical interactions between the solvated ions and plasma could possibly allow hydrogen and oxygen generation, but this interface would last only a short time before freezing over at the prevailing low surface temperature.

At the water-ice boundaries, it is questionable whether gaseous hydrogen or oxygen would be generated. The protons leaving the ice could dissolve in the water as H\textsuperscript{+} ions. At the other boundary, where protons need to enter the ice, H\textsuperscript{+} ions (which are always present in water) could leave the water. Thus, in the absence of metallic electrodes, electrolysis would probably not occur.

Drobyshchevski (1980, 1982) has proposed a method called "volume electrolysis" whereby unipolar generator currents in icy planets such as Europa could produce large quantities of hydrogen and oxygen in the interior, which could subsequently be ignited upon meteoric impact. He suggested that over time, the buildup of the gases could be large enough that the explosions could be of near-planetary proportions and remove sufficient material to change the satellite mass significantly. If the current were limited to a value commensurate with that observed at Io (Drobyshchevski used 10\textsuperscript{7} A), the amount of gas buildup discussed in his model could occur only if the current were to pass through numerous cells in series to multiply the gas production. Drobyshchevski proposes that inhomogeneities in the ice would provide these cells; although it is not obvious how such a series of cells could be produced, the current could pass around such cells rather than being forced through them. In any event, if the electric field were similar to that now present at Europa, ohmic losses in the ice should reduce the current by many orders of magnitude from that which he proposed.

The total available electrical power averaged over the surface is only 10\textsuperscript{-3} Wm\textsuperscript{-2}, which is insignificant when compared with the 50 Wm\textsuperscript{-2} of solar energy arriving at the surface. In a discussion of possible electrical currents in Io, Gold (1979) suggested a positive-feedback mechanism in which any tendency for currents to concentrate in a local area would be enhanced by localized electrical heating which would reduce the resistivity and, in turn, draw more current. Under
proper conditions, the same process might occur at Europa. However, this possibility must be weighed against the extra heat losses that such a concentration would cause. We consider first whether such a concentration could maintain itself if created by fissures in the ice. If a crack occurred that exposed liquid at Europa's surface, some of the energy removed by boiling might be provided by electrical heating, although heat losses would be so great that an open, liquid surface would tend to freeze rapidly. When water boils in a vacuum at 273 K, the flux of escaping water is about 0.5 kg m\(^{-2}\) sec\(^{-1}\), as determined from a one-dimensional momentum-transport calculation.

The details of this calculation are as follows. In the absence of another gas, water will boil at 273 K if the pressure of the vapor above the liquid surface is 610 kg m\(^{-1}\) sec\(^{-2}\), as given in vapor pressure tables. The average speed of water molecules at this temperature is 615 m sec\(^{-1}\) (since the velocity \(v\) is defined by \(v^2 = \frac{3}{2} K T/m\), where \(K\) is Boltzmann's constant and \(m\) the mass of a molecule). The momentum blocked by a wall of the vapor container is half the pressure, or 305 kg m\(^{-1}\) sec\(^{-2}\). Thus if the wall were removed, a rough approximation is that a flux of 0.5 kg m\(^{-2}\) sec\(^{-1}\) would be maintained by the boiling process. The back pressure of the vapor would reduce this flux, but the number here represents a first order approximation.

The heat of vaporization for this flux is \(1.3\times10^6\) W m\(^{-2}\), and the power lost by radiation is negligible in comparison. Again, for a current comparable to that at Io, the available power for the whole satellite is \(4\times10^{10}\) W, which is enough to maintain by electrical heating only \(10^4\) m\(^2\) of exposed water surface under the most optimum conditions. For such a concentration of current, the plasma surrounding Europa would have to be able to conduct the current to the cracks with low loss, which would require unusually high plasma conductivity. Hence, it appears that an exposed liquid surface could not be maintained by electrical heating.

A situation in which electrical heating might impede the closure of a crack involves the conditions just after an initial ice layer has formed, with a column mass sufficient to contain the vapor pressure of the water, i.e., a thickness of approximately 0.5 m. If the radiative heat loss to space and the heat loss by horizontal conduction could be provided electrically, further freezing would not occur. This condition would require that the ice remain at a relatively high temperature (near 273 K), for if it were to cool significantly, the electrical conductivity would become very low and would shut off the available power. The heat loss by radiation from a 273-K ice surface is \(-300\) Wm\(^{-2}\). With the ice-column resistance of \(1.7\times10^5\) ohms m\(^{-2}\), this heating rate could be produced by a current of 0.04 Am\(^{-2}\), thus maintaining a thin crack-surface of 5 km\(^2\) at constant thickness. The voltage drop across such a plug would be only 10% of the total available for this case, so that salt-water conductivity would not limit current flow. The heat loss to the surrounding ice would be less than the surface loss for horizontal thermal gradients, in which the entire temperature drop of about 180 K occurs over a distance of a few meters or more. Such cracks would also have to be maintained simultaneously in both hemispheres if a current were to flow. It is assumed here that the
cracks had been created by tidal action or by the pinching effect described by Gold (1979) (if such a mechanism can operate at the temperatures of Europa's surface).

CONCLUSIONS

Electrical currents through Europa would be expected to be orders of magnitude larger than those through Io because of the lack of a significant ionosphere and the relatively high conductivity of ice in comparison with silicates. However, the currents are still shown to be quite small for a solid satellite. If a liquid water layer were to exist on Europa of a salinity comparable to that of the Earth's oceans, a current density of $10^{-5}$ Am$^{-2}$ should flow in the TE mode, since the flow depends only on the strength of the alternating magnetic field and the conductivity of the water, both of which are known. However, electrical heating would be negligible at this current density level and electrolysis would not occur because the complete circuit is in the liquid which contains no electrodes. On the other hand, the current density in the TM mode depends on a larger number of factors of which only the $E = V \times B$ potential is known. The magnitude of the current density depends on the conductivity in the total circuit comprising the liquid, the ice, the flux tubes in the magnetosphere, and a connection somewhere in the neighborhood of Jupiter's ionosphere; but this conductivity could be much less than in the corresponding circuit at Io. It is thus quite possible that no significant TM-mode currents can occur. It is shown for an ice layer at least 4 km thick, the current density in the liquid will be restricted to $10^{-10}$ Am$^{-2}$ or less. It is also shown that if cracks occurred in the ice layer that exposed $10^{-9}$ to $10^{-8}$ of the Europa surface to the liquid layer, average current densities as high as $2 \times 10^{-5}$ Am$^{-2}$ could be reached. If electrolysis were to occur under these conditions, with a total current of $2.3 \times 10^5$ A, then $7 \times 10^5$ kg yr$^{-1}$ of water could be electrolyzed with an annual energy storage of $10^{13}$ J. However, it is not certain that the interfaces between the liquid, ice, and external plasma provide suitable electrodes for electrolysis. The electrical heating from TM-current levels discussed here is inconsequential in comparison with other energy sources on a planetary scale, and is shown to be significant only when concentrated at an ice layer approximately 1 m thick, such as might occur after the freezing over of a crack in an ice layer. The Galileo spacecraft mission to Jupiter will make several close fly-bys of Europa and should permit the measurement of any significant current through the Europa flux tubes.
REFERENCES


Model calculations of electrical currents expected to flow in Europa, which were summarized in a previous publication (Colburn and Reynolds, 1985) are presented in detail. The rationale that determined the choice of parameters is given, as well as the derivation of the equations defining the model, for clarification of the proposed current systems.

Electrical currents should flow in the Galilean satellite, Europa, because it is located in Jupiter's corotating magnetosphere. The possible magnitudes of these currents are calculated by assuming that Europa is a differentiated body consisting of an outer H₂O layer and a silicate core. Two types of models are considered here: one in which the water is completely frozen and a second in which there is an intermediate liquid layer. For the transverse electric mode (eddy currents), the calculated current density in a liquid layer is approximately 10⁻⁵ Am⁻². For the transverse magnetic mode (unipolar generator), the calculated current density in the liquid is severely constrained by the ice layer to a range of only 10⁻¹⁰ to 10⁻¹¹ Am⁻², for a total H₂O thickness of 100 km, provided that neither layer is less than 4 km thick. The current density is less for a completely frozen H₂O layer. If transient cracks were to appear in the ice layer, thereby exposing liquid, the calculated current density could rise to a range of 10⁻⁶ to 10⁻⁵ Am⁻², depending on layer thicknesses, which would require an exposed area of 10⁻⁵ to 10⁻⁸ of the Europa surface. The corresponding total current of 2.3×10⁵ A could in 1 yr electrolyze 7×10⁵ kg of water (and more if the cells were in series), and thereby store up to 10¹⁸ J of energy, but it is not clear how electrolysis can take place in the absence of suitable electrodes. Electrical heating would be significant only if the ice-layer thickness were on the order of 1 m, such as might occur if an exposed liquid surface were to freeze over; the heating under this condition could hinder the thickening of the ice layer.