SIZE-DENSITY RELATIONS IN DARK CLOUDS: NON-LTE EFFECTS

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One of the major goals of molecular astronomy has been to understand the physics and dynamics of dense interstellar clouds. Because the interpretation of observations of giant molecular clouds is complicated by their very complex structure and the dynamical effects of star formation, a number of studies have concentrated on dark clouds (Martin and Barrett 1978; Myers and Benson 1983; Arquilla and Goldsmith 1985). Leung, Kutner and Mead (1982) (hereafter LKM) and Myers (1983), in studies of CO and NH₃ emission, concluded that dark clouds exhibit significant correlations between linewidth and cloud radius of the form \( \Delta v \propto R^{0.5} \) and between mean density and radius of the form \( n \propto R^{-1} \), as originally suggested by Larson (1981). This result suggests that these objects are in virial equilibrium. However, the mean densities inferred from the CO data of LKM are based on an LTE analysis of their \(^{13}\)CO data. At the very low mean densities inferred by LKM for the larger clouds in their sample, the assumption of LTE becomes very questionable. As most of the range in \( R \) in the density-size correlation comes from the clouds observed in CO, it seems worthwhile to examine how non-LTE effects will influence the derived densities.

The basis of the LTE analysis is the assumption that the excitation temperatures of the \(^{12}\)CO and \(^{13}\)CO J=1-0 transitions are identical. Then the difference in observed antenna temperatures \( T_R \) is due solely to the smaller optical depth of the \(^{13}\)CO transition. The optical depth is then converted to a column density. To obtain mean densities, LKM used their LTE column densities to obtain cloud masses \( \mathbf{M} \), then defined the mean density as the volume-averaged density of a sphere of radius \( R \) and mass \( \mathbf{M} \).

For clouds with constant power-law density profiles of the form \( n(r) = n_0 (r/r_0)^a \), the volume-averaged density depends only on the density at the cloud boundary, and for a fixed boundary density is independent of \( n_0 \) and \( r_0 \). Clouds which are not strongly centrally condensed will have densities which are everywhere similar to the mean density, while clouds with steep profiles may have central densities one or two orders of magnitude higher than the mean. For unresolved clouds, the mean density probably is an appropriate parameter to use in considering the excitation.

One way to assess the validity of LTE-derived densities is to construct cloud models and then to interpret them in the same way as the observed data. Figure 1 shows a plot of \( n(H_2) \) versus cloud radius for homogeneous models of the dark clouds in the sample studied by LKM. The level populations were obtained assuming statistical equilibrium, using the Sobolev approximation for the radiative transfer. The kinetic temperature was derived by assuming that the \(^{12}\)CO J=1-0 transition is thermalized in all the clouds. No correction has been made for the coupling between the beam and the source. The dark line is a least-squares best fit to the model density-radius relation; the slope is -0.51. This is considerably shallower than the slope of -1.3 derived by Myers using the data of LKM. The shallower slope is a direct result of non-LTE conditions in the cloud models: higher densities are required to increase the optical depth to compensate for the sub-thermal excitation of the \(^{13}\)CO J=1-0 line. Microturbulent models of inhomogeneous clouds of varying central concentration with the linewidth-size and mean density-size relations found by Myers also show sub-thermal excitation of the \(^{13}\)CO line in the larger clouds, with the result that LTE analysis considerably underestimates the actual column density.

A more general approach which doesn't require detailed modeling of the clouds is to consider whether the observed \( T_R^{^{13}\text{CO}} / T_R^{^{12}\text{CO}} \) ratios in the clouds studied by LKM are in the range where the LTE-derived optical depths (and hence column densities) can be seriously in error due to sub-thermal excitation of the \(^{13}\)CO molecule. Figure 2 shows the ratio of \( T_R^{^{13}\text{CO}} / T_R^{^{12}\text{CO}} \) for the 1-0 transition as a function of \(^{13}\)CO optical depth for varying values of the ratio of excitation temperatures of the two transitions, for \( T_{\text{EX}}^{^{12}\text{CO}} = 10 \, \text{K} \). The curves are labeled...
with $T_{\mathrm{EX}}(^{13}\mathrm{CO})/T_{\mathrm{EX}}(^{12}\mathrm{CO})$. The ratio of antenna temperatures from LKM are in the range 0.2 - 0.8. It is clear from the figure that errors of an order of magnitude can be made by assuming LTE in situations where it is not appropriate.

The shallower relation between mean density and cloud size inferred from the non-LTE analysis has important implications for the structure and stability of dark clouds. In particular, the conclusion that they are in virial equilibrium may be incorrect. The mean densities in the larger clouds may be high enough to make them unstable to collapse.

REFERENCES

MODELS OF LKM DARK CLOUDS: DENSITY-SIZE RELATION

![Figure 1.](image1)

![Figure 2.](image2)