CRATERING MECHANICS

B.A. Ivanov

Main concepts and theoretical models which are used for studying the mechanics of cratering are discussed. Numerical two-dimensional calculations are made of explosions near a surface and high-speed impact. Models are given for the motion of a medium during cratering. Data from laboratory modeling are given. The effect of gravitational force and scales of cratering phenomena is analyzed.
Introduction

One can define mechanics of cratering as a discipline studying the formation of an indentation (crater or funnel) on the surface of a solid deformable body as a result of high-speed collision or explosion near the surface with regard for the resulting shock waves, high-speed plastic deformation of the destroyed materials, scattering of the ejecta and other concomitant phenomena.

The mechanics of cratering thus applied is an applied discipline using results of the physics of shock waves and high pressures, mechanics of a solid deformable body, hydro gas dynamics, etc. At the same time, the mechanics of cratering becomes a basic section of science in relation to planetology, which is related to gradual recognition of the role of high-speed impact processes in the formation of planetary bodies of the solar system and its subsequent geological history of the solid crust of these bodies.

The large role of high-speed impact processes in cosmogony is naturally manifest in one of the most promising models for the formation of the solar system through the evolution of a protoplanetary cloud and accretion growth of planetary bodies. O. Yu. Schmidt [1, 2] laid the foundations for a similar theory. K. P. Florenskiy [1] and V. S. Safronov [1] were perhaps the first to make a more detailed examination of the significance of high-speed cratering.

In addition to cosmogonic applications, mechanics of explosive cratering are very important in studying young (in a geological...
sense) craters, which, as shown by space research of the last 20 years, are the main landscape details on the surface of such planetary bodies as the Moon, Mercury, Mars, and the satellites of Jupiter and Saturn. As an example of the need for using the mechanics of cratering, we will indicate the question of the depth from which material is ejected during formation for example, of lunar craters. This is important to know in studying the lunar soil samples sent to Earth in planning future space expeditions, since collection of ejecta from craters of varying size and principles allows us to study the deep cross-section of the planetary crust.

It is also extremely urgent to investigate the meteorite craters found on Earth. About 100 meteorite craters from 10 meters to 100 km in size (see, for example, the monograph of V. L. Masaytis, A. N. Danilin, et al. [1]) have been found by now on the Earth's surface. We note that the world's largest meteorite crater, the Popigay, 100 km in diameter is in the USSR (V. L. Masaytis, M. V. Mikhaylov and T. V. Selivanovskaya [1]).

Another more practical aspect of the mechanics of cratering is related to developing principles for meteorite protection of spacecraft. These questions are presented fairly completely in the Russian translated book edited by R. Kinslow [1]. Finally, cratering is a particular case of an explosion for ejection using chemical explosives or nuclear explosions. Explosive crater formation was discussed, for example, in the monograph of V. N. Rodionov, et al. [1] and in the survey of V. V. Adushkin, et al. [1] from this viewpoint.

The main feature of the explosion near the surface of the ground, which brings it closer to the impact of a fast body is transmission of energy of the explosion products into the ground which is rapid as compared to the total time of funnel formation. With a fairly deep explosion, the time for expansion of the explosion products during which they complete work above the ground is comparable to the total time for ground ejection. This is graphically illustrated in the
experiments described by A. N. Romashov [1]. It is precisely the short transmission of energy with contact explosion and high-speed impact and the inertial nature of the ground motion emerging in this case that makes it possible to isolate an individual class of explosive processes which can be called cratering, distinguishing it from explosion for ejection.

According to tradition, in the Russian language there are two terms, "voronka" (crater) and "krater" (crater) which refer respectively to the cases of explosive explosions and nuclear explosions and to high-speed collision. The English language has adopted a single term, crater.

1. Main Concepts and Theoretical Models

Explosive cratering is a complicated process whose study requires especial procedures and methods to describe different stages of the phenomenon. Conventional division into the successive stages is characteristic for early theoretical works by K. P. Stanyukovich and V. V. Fedynskiy [1] and K. P. Stanyukovich [1] which made a separate discussion of the propagation of shock waves and motion proper of the material of a target resulting in cratering.

Later, D. E. Gault, W. L. Quaide and V. R. Oberbeck [1] suggested separating the following stages of cratering during high-speed impact: impact wave stage, stage of ejecta and stage of subsequent change in the crater shape.

In the case of an explosion near the surface of the ground, the aforementioned stages are preceded by energy release in the explosive charge (detonation of a chemical explosive or nuclear reaction during atomic or thermonuclear explosion). In the case of a nuclear explosion, energy at the early stages is transferred by short-wave radiation, at the same time defining yet another stage in explosive cratering, the stage of radiation processes.
Because of the change in characteristic parameters of explosive cratering in the broad range of variables, description of the successive stages of the process requires the use of different theoretical approaches. Thus, the early stage of a strong explosion can be described using equations of radiation dynamics. For the subsequent stage of a strong explosion and for the early stage of a high-speed impact, equations of hydrodynamics can be used which are closed by equations of state of material. At the late stages of cratering, during which the main mass of material is ejected, we need to take into consideration the elastoplastic properties of the material during intensive shear deformations.

C. P. Knowles and H. L. Brode [1] have described the results of calculating the radiation stage of the powerful nuclear contact explosion. According to this description, in time less than a microsecond, the energy of the nuclear reaction is formed of short-wave radiation which penetrates the environment around the charge and heats the ground to temperature on the order of $10^7$ K, while the air above the ground is heated to a temperature on the order of $5 \times 10^6$ K. After this, intensive re-radiation of energy from the ground into the air begins. In the final analysis, only a small (about 8%) quantity of energy released during the explosion remained in the ground.

H. F. Cooper, H. L. Brode and G. G. Leigh [1] have recalled experiments to determine the effectiveness of energy transmission into the ground during a contact nuclear explosion. The experiments were conducted with explosion of small charges to the bottom of underground cavities. It was noted that the effectiveness of the contact nuclear explosion for the intensity of shock waves with low energy density in the source in relation to a camouflet explosion is 10%. For high powered nuclear explosions with high energy density in the source, it is recommended that a coefficient of effectiveness in relation to underground explosion be used which is approximately equal to 5%.

We note that if radiation processes are not taken into consideration, the effectiveness of the mechanical effect on the ground is also
reduced with a rise in energy concentration of the source. G. P. Schneyer [1] made a numerical calculation of shock waves emerging during explosive release of energy in a source on the boundary of a half space filled with condensed material. According to the calculation results, similar shock waves at distances much greater than the initial dimensions of the source could be created by charges, whose energy $E_s$ and volume $V_s$ are linked by the correlation

$$E_s \cdot V_s^{0.233} = \text{const.} \tag{1}$$

By introducing volumetric density of the source $E_s/V_s$, this condition can be transformed into

$$E_i = \text{const} \cdot (E_s/V_s)^{1/3}. \tag{2}$$

from which it is graphically apparent that with an increase in energy density of the source, in order to maintain the mechanical effect on the ground the total explosion energy must be increased. Thus, with an increase in $E_s/V_s$ 10-fold, the total energy needs to be increased 1.5-fold. We stress that a similar increase was obtained especially in the framework of hydrodynamic equations without the use of radiation dynamics methods.

There is an interesting parallel here between the dependences of the effectiveness of the mechanical effect on energy density of the source during an explosion and a high-speed impact. In the latter case, the density of energy contained in the high-speed impact is determined by the collision velocity $v$. The density of kinetic energy per unit of mass equals $v^2/2$.

The problem of comparing high-speed impact in a one-dimensional case was resolved in the famous work of Ya. B. Zel'dovich [1] which showed that the effectiveness of the mechanical effect of a high-speed impact is determined not by the impact energy (which would occur in the case of a simple energy similarity), but by a certain combination
of energy and impulse of the impact object.

The results of a theoretical analysis of a concentrated impact on the surface of an ideal gas are collected in the monograph of Ya. B. Zel'dovich and Yu. P. Rayzer [1]. Numerous results of numerical calculations and analytical discussions are presented in the work edited by R. Kinslow [1].

J. K. Dienes and J. M. Walsh [1] conducted a series of numerical calculations on a high-speed impact on aluminum, whose properties were described by an equation of state presented below (see formulas (6) and (7)). According to these calculations, the analog of the mechanical effect is observed for impact objects of mass $M$ and velocity $v$ under conditions

$$Mv^{1.74} = \text{const.}$$

Reducing condition (3) to the appearance of (2) we find

$$Mv^2 = \text{const} \cdot (v^2)^{1/7.7}.$$ 

It follows from (4) that with an increase in the density of kinetic energy $(v^2/2)$ 10-fold (approximate tripling of the velocity) in order to preserve the parameters of the mechanical effect it is necessary to increase the total kinetic energy of the impact object $(Mv^2/2)$ approximately 1.3-fold. This situation is similar to the case of increasing energy density during contact explosion.

It should be noted that the values of the exponents in expressions (1) and (3) depend on the properties of the target material, although as the similar discussion shows for an ideal gas with different values $\gamma = c_p/c_v$, the exponent changes relatively equally with a change in the properties of the material.

Currently the most powerful method for studying the high-speed impact and contact explosion is numerical computer modeling. On the
assumption of axial symmetry, a two-dimensional problem is solved for explosion on a surface or perpendicular high-speed collision. A survey of the use of numerical methods for calculating explosion at high-speed impact was recently published by C. P. Knowles and H. L. Brode [1]. Weighing the advantages and the shortcomings of the Lagrange and Euler calculation plans, they note that currently the most convenient method of calculation is the use of hybrid grids. In this case the near zone to the energy source is described in a Euler presentation, while the peripheral zone of the crater-forming flow is divided by a Lagrange grid. In the Euler presentation, one of the most powerful methods of calculation is the method of large particles proposed by O. M. Belotserkovskiy and Yu. M. Davydov [1].

After selecting the calculation plan, the next stage in numerical calculation is specification of the equation of state of the material. If we do not consider the equation of state of an ideal gas which is natural for test problems, the first equation of state of rock used to calculate a contact nuclear explosion (H. L. Brode and R. L. Bjork [1]) and high-speed impact (R. L. Bjork [1]) was an equation of the type

\[ p = 0.425 \rho e + 0.113 \eta^{1/2} e + 5.30 \rho e^{1/2} + 0.707 \rho e^2 (10^6 + e), \]

where \( e \) is the specific internal energy of the material, GJ/m\(^3\)(10\(^{10}\) erg/g), \( \eta \) is the ratio of density \( \rho \) to the initial value \( \rho_0 \), \( p \) is pressure, GPa (10\(^{10}\) dyne/cm\(^2\)). In order to obtain numerical values of the coefficients in formula (5), data were used for tufa (\( \rho_0 = 1.7 \) g/cm\(^3\)).

It is easy to see that formula (5) does not have an ideal gas continuation (\( p \to e^{1/2} \eta \) instead of \( p \to e \eta \) for an ideal gas).

It should be stated that with fairly large volume of computer memory, one can abandon the analytical specification of the equation of state and use the tabular method to specify the link between the thermodynamic quantities and their subsequent interpolation. This allows us to use more cumbersome, but more accurate approaches to
S. V. Bobrovskiy, V. M. Gogolev, B. V. Zamyshlyayev and V. P. Lozhkina [1] constructed an equation of state for granite which used the data of the Thomas-Fermi method for SiO₂. The use of the hypothesis on thermodynamic equilibrium made it possible to calculate the position of the two-phase region. The overall appearance of the impact adiabatic curve, boundaries of the two-phase area and isotropic line of granite load are shown in Figure la. A similar equation of state / was constructed by S. V. Bobrovskiy, V. M. Gogolev, B. V. Zamyshlyayev, V. P. Lozhkina and V. V. Rasskazov [1] for shale (Figure lb). The equation of state of calcite CaCO₃ with regard for its thermal breakdown calculated by S. V. Bobrovskiy, V. M. Gogolev, B. V. Zamyshlyayev, V. P. Lozhkina and V. V. Rasskazov [2] is of especial interest from the viewpoint of discussing the transformation of sedimentary rocks during meteorite cratering on the Earth (see section 6).

One of the most widespread equations of state used in numerical two-dimensional calculations in the United States is the equation of state of J. H. Tillotson [1] initially suggested for metals. This equation of state has the following construction:

For density \( \rho > \rho_0 \)

\[
p = \left[ a + b \left( \frac{e}{(e_0 \eta^2)} + 1 \right) \right] e \rho + A \mu + B \mu^2.
\] (6)

where \( p \) is the pressure, \( e \) the specific internal energy, \( \eta = \rho / \rho_0, \mu = \eta - 1 \) for \( \rho > \rho_0 \),

\[
p = a e \rho + \left( b e \rho / (e_0 \eta^2 + 1) \right) + A \mu \exp \left[ -\beta (\nu - 1) \right] \exp \left[ -\alpha (\nu - 1)^2 \right],
\] (7)

where \( V = 1 / \rho \).

Here the quantity \( (a + b) \) means the Gruneisen coefficient with zero pressure, \( A \) is the modulus of volumetric compression. The constants \( e_0 \) and \( B \) guarantee that the Thomas-Fermi model joins with the experimental data for impact wave compression; the quantity \( a \) takes into consideration the magnitude of the Gruneisen coefficient of
electron gas at high pressures, when equation (6) actually looks like $p = a e^p$.

The constants $\alpha$ and $\beta$ in equation (7) are selected to guarantee with high degrees of expansion a transition to polytropic equation of state with indicator of the polytropic curve equal to $(\alpha + 1)$. Equation (7) guarantees a smooth transition from gas to condensed substance and guarantees continuity of pressure and sound velocity on the isotropic lines of load.

The constants of equations (6) and (7) for some materials used in the numerical two-dimensional calculations are presented in Table 1.

Insofar as essentially all the minerals and rocks consisting of them during impact compression experience polymorphous phase transitions with compaction (a classic example of this is the transition of quartz into coesite and stishovite) efforts have been made to develop equations of state of rocks with regard for this transition.

J. D. O'Keefe and T. J. Ahrens [2] have constructed an equation of state of lunar rock (gabbroid anorthosite) modeled by a mixture of plagioclase and pyroxene. In the area of mixture of high and low pressure phases, the correlations of activity are fulfilled

$$V = f_p V_h(p, e) + (1 - f_p) V_l(p, e),$$
$$e = f_p e_h(p, e) + (1 - f_p) e_l(p, e),$$

where the indexes "B" and "H" refer respectively to the high and low pressure phases. For thermodynamic parameters of phases, correlations in form (6) and (7) are used, whose constants are presented in Table 1. The equilibrium concentration of the high pressure phase $f_p$ is computed from the condition of minimum free energy.

The equation for the dynamics of formation of a high pressure phase looks like


TABLE 1. CONSTANTS OF EQUATION OF STATE OF TILLOTSON FOR CERTAIN MATERIALS

<table>
<thead>
<tr>
<th>Вещество</th>
<th>( \rho_0 ) g/cm(^3)</th>
<th>( \rho / \rho_0 )</th>
<th>( a )</th>
<th>( \varepsilon )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Железо (( \sigma ))</td>
<td>7.83</td>
<td>0.5</td>
<td>1.5</td>
<td></td>
</tr>
<tr>
<td>Аллюминий (( \gamma ))</td>
<td>2.70</td>
<td>0.5</td>
<td>1.63</td>
<td></td>
</tr>
<tr>
<td>Габброидный анизтозит</td>
<td>2.94</td>
<td>0.5</td>
<td>0.11</td>
<td></td>
</tr>
<tr>
<td>Габброидный анизтозит: фаза низкого давления</td>
<td>2.966</td>
<td>0.5</td>
<td>0.14</td>
<td></td>
</tr>
<tr>
<td>Фаза высокого давления</td>
<td>3.955</td>
<td>0.5</td>
<td>0.1</td>
<td></td>
</tr>
<tr>
<td>Известняк (( \delta ))</td>
<td>2.70</td>
<td>0.5</td>
<td>0.6</td>
<td></td>
</tr>
</tbody>
</table>

Key:
- a. material
- b. g/cm\(^3\)
- c. bar
- d. cm\(^3\)/g
- e. iron
- f. aluminum
- g. gabbroid anorthosite
- h. gabbroid anorthosite: low pressure phase
- i. high pressure phase
- j. limestone

1. J. D. O'Keefe and T. J. Ahrens [1].

\[
\frac{dI}{dt} = (I_0 - I)/\varepsilon. \quad (9)
\]
The magnitude of the temporal constant of transition $\tau$ is assumed to be equal to 0.25 $\mu$sec. Increment in internal energy with transition to a high pressure phase is assumed to be equal to $1.3 \times 10^{10}$ erg/g. The results of the calculation using this equation of state (T. J. Ahrens and J. D. O'Keefe [1]) will be discussed below.

S. S. Grigoryan, L. S. Yevterev, B. V. Zamyshlyayev and S. G. Krivosheyev [1] used a somewhat different approach; they employed the equation of state of quartz in Mie-Gruneisen form, after presenting it in the form of a set of equations of state of stishovite and normal quartz with transitional two-phase area corresponding to numerous experimental data for shock wave compression of quartz. The isotropic lines of load in the two-phase area are constructed by the principle of additivity using the isotropic line of load of quartz and stishovite on the assumption of an unchanged ("frozen") correlation between the fractions of the quartz and stishovite selected according to the attained state on the impact adiabatic curve.

Recently numerical calculations have been made more frequently to reproduce specific large-scale or laboratory experiments, therefore we need to have equations of state not only of natural, but artificial materials used in the laboratory. An example of this material could be modeling clay widely used to model explosions in high-speed impacts. J. M. Thomsen, M. G. Austin, S. F. Ruhl, et al. [1] used the following equation of state of modeling clay for numerical modeling:

\begin{equation}
\rho = 2.8 \mu + 40.7 \mu^2 - 36.0 \mu^3 + 1.7 \varepsilon_0 / \rho_0.
\end{equation}

(10)

with \( \rho > \rho_0 \)

\begin{equation}
\rho = 1.7 \varepsilon / \rho_0.
\end{equation}

(11)

with \( \rho < \rho_0 \)

where \( \rho \) is the pressure, GPa (10 kbar), \( \varepsilon \) is specific internal energy, GJ/m$^3$ (10$^{10}$ erg/g), \( \mu = \rho / \rho_0 - 1 \), \( \rho_0 = 1.69 \) g/cm$^3$. 

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In order to reduce pressure on the shock wave front below several hundred kilowatts, it becomes significant to calculate the strength of the target material. The first calculations of this type were made from metals, and the model of plastic flow was mainly used for rheological description of the target material. In the framework of this model, the target strength is described by stress of plastic flow \( Y \) which can be introduced by a specific condition of type

\[
s_{ij}s_{ij} = 2Y^2, \quad (12)
\]

where \( s_{ij} \) is the deviator of the stress tensor.

If the quantity \( Y \) is assumed to be independent of the magnitude of comprehensive pressure \( p \), then this model is called the Mises model. If a relationship of the following type is assumed

\[
Y = Y_0 + kp, \quad (13)
\]

then this model is called the Coulomb model, and the quantity \( Y_0 \) is called cohesion.

It is often assumed for some materials (for example, for rocks) that when a certain critical comprehensive pressure \( p_k \) is reached, the quantity \( Y \) ceases to grow:

\[
Y = \begin{cases} Y_0 + kp & \text{with } p < p_k \\ Y_0 + kp_k = Y_c & \text{with } p > p_k \end{cases}, \quad (14)
\]

This formula is called the Coulomb-Mises model (C. P. Knowles and H. L. Brode [1]).

Of course the models listed above are the simplest approximations of the available experimental data and the general elastoplastic model (see the survey of V. N. Nikolayevskiy, I. A. Sizov and L. D. Lifshits [1]), but it is precisely because of their simplicity that they are often used for specific calculations of numerical two-dimensional problems.
Rheological models which take into consideration the deformation history of the material is very promising. One of these models ("cap model") was described by C. P. Knowles and H. L. Brode [1]. Recently P. F. Korotkov [1] specified a model for gradually destroyed rock and published some results of calculating a one-dimensional, spherically symmetrical problem based on this model. "Gradual" destruction means that as the material is deformed, the relationship of the creep limit, generally similar to model (14) will change, and with inelastic deformation on the order of 1%, there will be a conversion of rock into a loose medium with zero cohesion.

The main methods of calculating the change in strength of the medium with explosive cratering are currently calculation of the decreased strength because of heating (with decrease in it to zero with melting temperature) and because of the crushing effect of the main shock wave, and this effect is taken into consideration simply through the amplitude of pressure on the wave front.


\[ Y = (Y_0 + Y_1 \eta + Y_2 \eta^2)(1 - e/e_i), \] (15)

where \( \eta = \rho/\rho_0 - 1 \), \( e_i \) is the specific internal energy at melting point. According to experiments with planar waves of loading, they selected the values \( Y_1 = 42 \text{ kbar} \) and \( Y_2 = 72 \text{ kbar} \) (the values of shear strength with zero pressure \( Y_0 \) were 0.75 kbar for "soft" aluminum and 2.39 kbar for "hard" aluminum).

J. D. O'Keefe and T. J. Ahrens [1] used the Mises model, hypothesizing for gabbroid anorthosite \( Y = 25 \text{ kbar} \) with decrease to zero at melting point. J. D. O'Keefe and T. J. Ahrens [2] subsequently reduced the hypothetical strength and adopted it in the form (15) with \( Y_0 = 2.7 \text{ kbar} \), \( Y_1 = 338 \text{ kbar} \) and \( Y_2 = 901 \text{ kbar} \). Insofar as J. D. O'Keefe and T. J. Ahrens [1, 2] only examine the initial stage of the impact and
focused on the parameters of the shock wave, the strength properties of the target have little influence on their findings.

J. G. Trulio [1] in calculating a contact nuclear explosion used the Coulomb-Mises type model. For granite, the strength value for shear was: with pressure 0.1 kbar 0.13 kbar, with pressure 1 kbar, 0.4 kbar, and reached the maximum value 17 kbar with pressure 56.3 kbar. These values were selected from the results of testing granite from the site of the "Piledriver" explosion. G. W. Ullrich, D. J. Roddy and G. Simmons [1] used the Coulomb-Mises model to calculate the crater of a contact explosion of a TNT charge weighing 20 T "Mixed Company II" conducted on sedimentary origin rock. In the calculation, the medium was assumed to consist of horizontal layers, whose shear strength was described in form (14) with the following constants: for the upper loose layer of soil (0 - 0.5 m for depth) it was assumed that $Y_0 = 6.9 \times 10^{-4}$ kbar, $k = 0.466$, $Y_K = 0.0517$ kbar; for the second layer (0.5 - 1.8 m) $Y_0 = 3.5 \times 10^{-3}$ kbar, $k = 0.7$, $Y_K = 7.59$ kbar; for the third layer (1.8 - 3.5 m) $Y_0 = 0.069$ kbar, $k = 1.0$, $Y_K = 7.59$ kbar, for the fourth and fifth layers (depth over 3.5 m) $Y_0 = 0.0517$ kbar, $k = 0.75$, $Y_K = 2.07$ kbar.

R. P. Swift [1] used a more complicated model, one of the modifications of the "cap model" to calculate a contact nuclear explosion. In this case, the creep surface was thought to be dependent on the loading history. For the initial material, this surface had an appearance similar to (14):

$$V \sqrt{I_2(s_{ij})} = A - C \exp(BI_1)$$  \hspace{1cm} (16)

where $I_2(s_{ij}) = \frac{1}{2}s_{ij}s_{ij}$ is the second invariant of the stress tensor deviator, while $I_1 = \sigma_{ii} = -3p$ is the first invariant of the stress tensor. In this case cohesion with zero comprehensive pressure $p(I_1 = 0)$ is $A - C$, and with infinite pressure, the strength approaches the maximum value equal to $A$. 

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Decrease in cohesion during deformation is expressed through the hydrostatic pressure \( p_m \) which is the maximum attained for the examined moment in time.

\[
A-C = f(p_m)(A-C_0),
\]

where \( A-C_0 \) is cohesion of the original material.

The function \( f(p_m) \) which is included in (17) is described through the pressure value \( p_1 \) with onset of pulverization of rock and \( p_2 \) with complete pulverization of rock during compression. (The concepts "onset of pulverization" and "complete pulverization" are not pinpointed by the author.) Introducing the degree of loss of cohesion during rock destruction

\[
\delta = 1 - (A-C)/(A-C_0),
\]

where \( (A-C_0) \) is the initial cohesion, while \( (A-C)' \) is cohesion of the destroyed material, R. P. Swift [1] defines the destruction surface in the form

\[
\begin{align*}
\delta V^* I_2 = & \begin{cases} 
A-C_0 \exp(BI_1) \text{ with } p_m < p_1; \\
A-|C_0 - \delta(A-C_0)(p_m - p_1)/p_1| \exp(BI_1) \text{ with } p_1 < p_m < p_2; \\
A-|C_0 + \delta(A-C_0) \exp(BI_1) \text{ with } p_m > p_2.
\end{cases}
\end{align*}
\]

Selection of the model parameters in the described variant of calculation was made as follows. For cohesion of the destroyed material, the value \( 2 \times 10^{-3} \) kbar was adopted, the lower limit of the destroying pressure \( p_1 \) was considered to be 2.5-fold greater than the lithostatic pressure \( \rho g h \), the upper limit \( p_2 \) equalled \( p_2 = p_1 + 0.01 \) kbar.

Similar calculation of rock destruction during its deformation increased the calculated parameters of the crater roughly 1.5-fold as compared to the calculation variant with constant destruction surface (16).
We note that calculation of gradual destruction of material expressed as a decrease in cohesion through the magnitude of mean pressures is fairly artificial. As it seems, it is more natural to view the degree of destruction as a function of inelastic deformation, as was done in the aforementioned work of P. F. Korotkov [1] (see also the survey of V. N. Nikolayevskiy, L. D. Livshits and I. A. Sizov [1]).

A more determined model of destruction, constructed in terms of growth of brittle fractures and their joining with the formation of fragments was used by D. R. Curran, et al. [1] for numerical calculation of a high-speed impact and contact explosion on the surface of a uniform block of quartzite. The experimental parameters were selected so that brittle failure played the main role, for which a mathematical model was constructed. Numerical calculation satisfactorily reproduced configuration of the destruction zone observed in the experiments, and even with definite accuracy the granulometric composition of the formed fragments.

Despite the significant influence from calculation of the change of strength properties of the material during intensive shear deformation, many calculations continued to be made using the simplest model of Mises. J. B. Bryan, D. E. Burton, M. E. Cunningham and L. A. Lettis [1] used this model to calculate the formation of the Arizona Meteorite Crater, considering the target to be "limestone" with constant shear strength equal to 0.2 kbar. The Mises criterion is more substantiated for laboratory media of the modeling clay type. J. M. Thomsen, M. G. Austin, S. F. Ruhl, et al. [1] for modeling clay used values of shear strength equal to 1.5 bar with temperature 17.5°C and 0.5 bar at 32°C. In experiments, this reduced strength corresponded to tripling of the crater volume.

2. Numerical Two-Dimensional Calculations of Explosion Near a Surface and High-Speed Impact

There are different purposes for stating numerical two-dimensional calculations of a contact explosion and a meteorite impact. These
could be, for example, development and demonstration of the efficiency of a numerical method as such (N. V. Gusev [1], P. F. Korotkov and D. A. Sudakov [1]), calculation of specific large-scale (D. L. Orphal [1, 2]; G. W. Ullrich, D. J. Roddy and G. Simmons [1]) or laboratory (J. N. Thomsen, M. G. Austin, S. F. Ruhl, et al. [1]) experiments, evaluation of quantities important for studying meteorite craters on Earth and planets, such as the mass of ejecta beyond the limits of the gravitational sphere (J. D. O'Keefe and T. J. Ahrens [3]) or the quantity of material evaporated and melted during explosive crater formation (J. D. O'Keefe and T. J. Ahrens [4]).

The first two-dimensional numerical calculation of a contact explosion was made in 1960 by H. L. Brode and R. L. Bjork [1]. In the last 10 years, the efforts of researchers have been aimed at pinpointing the calculated results by selecting more adequate models for behavior of the medium in studying the influence of parameters of the source on the effectiveness of the mechanical effect of an explosion.

S. S. Grigoryan and L. S. Yevterev [1] studied different methods for energy release in the source and showed that the release of energy only in the form of heat of compressed gas or in the form of heat and kinetic energy of the vapors of the source have a comparatively weak influence on the overall mechanical effectiveness of the effect of an explosion. G. P. Schneyer [1] came to similar conclusions.

D. L. Orphal [1] calculated the development of the Johnie-Boy nuclear explosion. This explosion which took place at the Nevada test site, United States in 1962, was an explosion of a nuclear charge with power of 0.5 kt of TNT at depth 0.585 m in the alluvium. Because this explosion was the only one conducted with a shallow depression, it was the object of intensive theoretical and experimental modeling. D. L. Orphal [1] presented the charge in the form of a rotation ellipsoid with vertical axis 57 cm long and horizontal axis 90 cm long. Release at the source of energy 0.5 kt (1 kt is equivalent to $4.2 \times 10^{19}$ erg) resulted in the emergence of alluvium vapor with pressure 45.3 Mbar and
density of internal energy $9.06 \times 10^{13}$ erg/cm². With the passage of
time, the energy of the source is transferred into the ground and
used to excite a shock wave in the ground and to scatter the source
vapors and ground evaporated in the shock wave. The scattering of
high-temperature vapors reduces the energy remaining in the ground
and subsequently was used to form the crater proper. The maximum quan-
tity of energy transferred into the ground in calculation was 0.15 kT
for approximately 30% of the complete explosion energy. However beyond
the hemisphere of radius 3.6 m with center on the free surface and the
explosion epicenter only about 6% of the total explosion energy was
transferred. Beyond the hemisphere of radius 5.2 m, the quantity of
energy did not exceed 3%. The calculation was stopped within 0.8 sec
when the calculated crater had reached the final dimensions: volume $9.6 \times 10^3$ m³, radius 21 m, depth 14 m. The experimental data yields
respectively $6 \times 10^3$ m³, 22 m and 12 m.

We note that the final depth of the crater was attained approxi-
mately by 0.11 sec after detonation, that is in a time approximating
8-fold shorter than the time for attaining the final radius of the
 crater.

R. M. Schmidt [2] has reported calculations of the Johnie Boy
explosion which have not been published in periodicals which compared
the nuclear explosion and the explosion of ordinary explosive, and by
selecting the weight and the penetration of the explosive charge condi-
tions for the best reproduction of the nuclear explosion were selected.
Based on this analysis, the Mine Throw explosion was made. The charge
was made of ammonium nitrate with liquid fuel of diesel type (ANLF) and
weighed 120 T.

As reported, similar calculations were made for a charge made of
PETN. From the viewpoint of reproducing the velocity field of the
nuclear explosion the best was a spherical charge of PETN with radius
188 cm with center of gravity at depth 120 cm. The weight of this
charge was 49.3 T, the energy released during the explosion was $2.8
\times 10^{18}$ erg or 13.4% of the nuclear explosion energy. A similar
comparison made it possible to conduct centrifugal modeling of the explosion of a similar explosive charge. These results will be discussed in section 5.

J. G. Trulio [1] has reported some results of calculating a megaton nuclear explosion in granite at depth about 4.5 m. In this work the primary attention was focused on studying the features of the velocity field and its evolution in time. Using ballistic extrapolation, evaluations were obtained for dimensions of the crater and the volume of the ejecta. The radius of the crater was evaluated as 137 - 160 m, the weight of the ejecta as 5.7 MT.

In the already mentioned publication of R. P. Swift [1] an evaluation of crater radius about 115 m was obtained for a crater of a contact nuclear explosion with energy 5 MT (details of calculating the initial phase of the explosion and distribution of energy are not reported).

G. W. Ullrich, D. J. Roddy and G. Simmons [1] conducted numerical modeling of a contact explosion Mixed Company II (spherical charge 20 T of TNT standing on the surface of sedimentary rock). The strength properties of the rock adopted in the calculation are presented in section 1. The calculations show the good reproduction of the oscillation parameters of the ground at distances 15 - 30 m from the epicenter both for maximum mass velocity in the wave, and for duration of the positive oscillation phase. A distinguishing feature of this work is the attempt to make a parametric study of the influence of the rheological model of the ground on reproduction in the model of the interesting feature of an experimental crater, formation in its center of an elevation (hill) whose peak was approximately 1 m above the original surface of the ground. This study showed that the most significant for the formation of a central elevation is dispersion (dilatancy) of rock in the cratering flow and modification in gravitational field of certain intermediate craters surrounded by destroyed material with reduced strength.
C. P. Knowles and H. L. Brode [1] summarized the data on the calculated dimensions of nuclear explosion craters.

There are a lot more publications on cratering with high-velocity impact.

The first calculations of this type were made in the early 1960's both to model the process of forming large terrestrial meteorite craters (R. L. Bjork [1]), and to develop principles for designing antimeteorite protection for spacecraft (R. L. Bjork, K. N. Kreyenhagen and M. Wagner [1]). Recently published calculations can be conventionally divided into three trends: study of the influence of initial velocity on mechanical effectiveness of the impact process, description of collisions with selection of different types of equations of state and analysis of the overall pattern of cratering in order to apply calculated results of geology and planetology.

After the basic theoretical solution to the problem of comparing impact with varying velocity by Ya. B. Zel'dovich [1] and extension of this solution to a broader class of problems (see survey of W. J. Rae [1]) a number of reports were made on results of numerical two-dimensional calculations for specific collision conditions.

Using the equation of state of Tillotson [1] for aluminum (see Table 1), J. K. Dienes and J. M. Walsh [1, 2] studied impacts with different velocity for aluminum. Comparison of these results showed that a convenient form for their analysis which was suggested by W. J. Rae [1] is introduction of the characteristic pressure scale in the form $\rho c^2$, where $\rho$ is the density of the target material, $c$ is the "volumetric" velocity of sound equal to $\sqrt{c_l^2 - 4/3 c_t^2}$ ($c_l$ and $c_t$ are the velocities of the longitudinal and transverse elastic waves), and the characteristic scale of the impact process

$$L = (1/2)^{1/3} (M/\rho)^{1/3} (\sigma/c)^{0.13}. \quad (19)$$
where \( M \) is the weight of the impact object, \( \rho \) is the density of the target material, \( v \) is the velocity of collision. The exponent with \( \frac{v}{c} \) is a function of the equation of state. We note that if the effectiveness of the effect on the target was determined only by kinetic energy of the impact object, then the characteristic length would be determined by an expression of the type

\[
L \sim M^{1/3} v^{2/3},
\]

that is, the exponent with velocity would equal 0.67, and not 0.58 as in (19). Decrease in the effectiveness of the effect on the target with rise in the velocity of the impact object has already been discussed in section 1.

Calculation of a high-velocity impact on rock in principle does not differ at all from calculations of impacts on metals, if the equation of state proposed for describing the properties of rocks does not differ from the "metal" equation of state. B. A. Ivanov [5] compared the results of calculations for metals (J. K. Dienes and J. M. Walsh [1], W. J. Rae [1]) and rocks (R. L. Bjork [1], J. D. O'Keefe and T. J. Ahrens [1]) for damping of pressure on the front of a shock wave \( p \) with depth \( z \) along the axis of symmetry directed into the depth of the target (Figure 2). In this case the relative coordinates \( \frac{p}{\rho c^2}, \frac{z}{L} \) were used, where \( L \) is the characteristic linear scale of the impact event (19).

It is apparent from Figure 2 that in the relative coordinates at the stage of equivalence, the mass of the target material encompassed by the shock wave is much greater than the mass of the impact object. For targets with equation of state of the "metal" type, the data presented in Figure 2 can be generalized in the form

\[
\frac{p}{\rho c^2} = A \left( \frac{z}{L} \right)^{-25},
\]

where the coefficients \( A \). Change in the A coefficient from 2 to 4 in coordinates \( \frac{p}{\rho c^2} - \frac{z}{L} \) yields a band which includes essentially all the experimental data (dotted line in Figure 2).
Introduction into the numerical calculations of equations of state of rocks taking into consideration the polymorphous phase transitions significantly complicates generalization of the calculated data in a form similar to (20). As shown by numerical calculations of a high-speed impact (J. D. O'Keefe and T. J. Ahrens [2], T. J. Ahrens and J. D. O'Keefe [1]) and underground explosion (S. S. Grigoryan, L. S.}
Yevterev, B. V. Zamyshlyayev and S. G. Krivosheyev [1]), consideration for the phase transitions with compaction increases the velocity of damping of the shock wave, and possibly, leads to a significant dependence of the law of damping on the velocity of collision (T. J. Ahrens and J. D. O'Keefe [1]).

In the example of calculating a one-dimensional problem, S. S. Grigoryan, L. S. Yevterev, B. V. Zamyshlyayev and S. G. Krivosheyev [1] showed that introduction into the calculation of explosion of granite of a transition of quartz into stishovite results in a change in the mass velocity epure. At distances corresponding to five radii of the initial band, the amplitude of pressure in the shock wave diminishes approximately in half as compared to the variants in which the phase transition is not taken into consideration.

T. J. Ahrens and J. D. O'Keefe [1], by using the equation of state for lunar rock (gabbroid anorthosite) taking into consideration the polymorphic phase transition with increase in density calculated damping of a shock wave for cases of impact of iron and stone meteorites on the surface of a half space filled with gabbroid anorthosite. The rock meteorite was considered to consist of the same material as the target. The velocity of the meteorites was specified as equal to 5, 15 and 45 km/sec. Insofar as the material undergoing polymorphous phase transition, during impact compression experiences a change in its physical properties, use in this case of reduction for impact velocity according to formula (19) encounters definite difficulties. Although J. D. O'Keefe and T. J. Ahrens [4] attempted to get around this difficulty and proposed using a formula of type (19), substituting into it the velocity of sound for the phase of high pressure of the target material, this avenue is extremely nonexact from the viewpoint of shock wave mechanics and high-velocity impact. Instead, B. A. Ivanov [7] proposed using as the first approximation in this case reduction for the hypothesis of energy similarity. In fact, the characteristic scale of the impact event with change in collision velocity 10-fold (for example, from 5 to 50 km/sec) according to (19) will change 3.8-fold, and with
use of the hypothesis of energy similarity (19') will change 4.64-fold, i.e., the difference in the scales of the process will not exceed 20%. Considering that from the viewpoint of applying calculated data to natural large-scale phenomena, this accuracy is sufficient, data on damping of shock waves under the center of an impact can be compared in coordinates pressure $p$, reduced depth from the initial surface $z/K^{1/3}$, where $K = Mv^2/2$ is the kinetic energy of a falling body (Figure 3). As follows from Figure 3, the calculated points for impacts on gabbroid anorthosite with velocity greater than 15 km/sec correspond satisfactorily to each other in coordinates $p - z/K^{1/3}$, and in the area of pressure greater than 100 kbar, agree poorly with the calculation data for the "metal" equation of state without consideration for the phase transitions previously presented in Figure 2.

Figure 3. Damping of Shock Wave Under Point of High-Velocity Impact Depending on Depth Relative for Energy. Impact of an iron meteorite on gabbroid anorthosite with velocity (1) 45 km/sec, (2) 15 km/sec, (3) 5 km/sec; impact of anorthosite missile (Continued on next page)
Figure 3. Continued:
for anorthosite with velocity (4) 45 km/sec, (5) 15 km/sec, (6) 5 km/sec, calculation with regard for phase transitions in iron and gabbroid anorthosite (T. J. Ahrens and J. D. O'Keefe [1]). Calculation without consideration for phase transitions: (7) iron-anorthosite with 15 km/sec (J. D. O'Keefe and T. J. Ahrens [1]), (8) iron-tufa 30 km/sec, (R. L. Bjork [1]), aluminum-aluminum with 80.4 km/sec (9), 24.3 km/sec (10) and 7.3 km/sec (11) (J. K. Dienes and J. M. Walsh [1]). The arrows designate the approximate levels of pressures sufficient for melting dense crystalline rocks during unloading.

A surprising result which follows from the findings by T. J. Ahrens and J. D. O'Keefe [1] is the severe increase in damping of a shock wave which is easily noticeable for cases 3, 5 and 6 in Figure 3, corresponding to cases of impact of iron meteorite with velocity 5 km/sec and impact of a rock meteorite with velocities 15 and 5 km/sec. This discrepancy both with the remaining calculated data of these authors, and the results previously published have not yet been explained.

All the other data for rocks shown in Figure 3 for estimates can be generalized in the form

\[ p = B(z/K^{1/3})^{-2.7}, \]

where \( p \) is pressure on the front of the shock wave (kbar) under the center of impact at depth \( z \) (km), \( K \) is the kinetic energy of the impact object (MT TNT). The constant \( P \) "on the average" can be assumed to be equal to 0.2, change in \( B \) from 0.1 to 0.3 in coordinates \( p - z/K^{1/3} \) is a band which includes essentially all component points as shown in Figure 3.

After adopting as the characteristic level of pressure in the shock wave necessary for melting dense crystalline rocks with discharge beyond the shock wave a quantity on the order of 600 kbar, according to the data presented in Figure 3 one can evaluate the maximum depth of occurrence \( z_m \) of rocks which could melt during meteorite impact (arrows in Figure 3):
J. D. O'Keefe and T. J. Ahrens [4] in the example of the equation of state of gabbroid anorthosite calculated quantity of evaporated and molten material formed during meteorite impact with velocities from 5 to 15 km/sec. These data can be considered typical for all dense crystalline rocks.

With low velocities of impact, where pressures in the shock wave sufficient for melting and evaporating the rocks are reached at distances comparable to the size of the falling body, the actual fact that melting and evaporation processes appear depends only on the impact velocity with specific properties of the materials of the impact object on the target. The influence of impact velocity can be characterized by initial pressure $p_0$ defined from impact adiabatic curves of the meteorite and target materials in approximation of the one-dimensional collision of flat surfaces. Table 2 presents evaluations of the collision velocities, which when exceeded result in melting and evaporation of the target and meteorite.

If the initial pressure of collision significantly exceeds the corresponding threshold values, the quantity of evaporated or molten material is mainly determined by the kinetic energy of the meteorite. For collision velocity greater than 15 km/sec, the data for the quantity of melt which were obtained by J. D. O'Keefe and T. J. Ahrens [4] can be generalized in the form

$$m_p \approx 0.4 K,$$  \hspace{1cm} (23)

where $m_p$ is the mass of the melt, $MT$, $K$ is kinetic energy of the meteorite expressed as the weight of the equivalent trotyl charge, $MT$ TNT.

With impact velocity over 30 km/sec, one can present the mass of evaporated material in a similar form
\( m_0 \approx 0.05 \) \( K \). \(^{(24)}\)

**TABLE 2.** INITIAL PRESSURES \( P \) AND CORRESPONDING COLLISION VELOCITIES MINIMUM NECESSARY FOR BEGINNING OF MELTING AND EVAPORATION OF DENSE CRYSTALLINE ROCKS (in the example of gabbroid anorthosite from the data of J. D. O'Keefe and T. J. Ahrens [4])

<table>
<thead>
<tr>
<th>( v ), km/sec</th>
<th>( P_0 ), Mbar</th>
<th>Change in Condition of Material at Impact Point After Discharge</th>
<th>Meteorite</th>
<th>Target</th>
</tr>
</thead>
<tbody>
<tr>
<td>Impact of stone meteorite:</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>0.5</td>
<td>Melting</td>
<td>Melting</td>
<td></td>
</tr>
<tr>
<td>7.5</td>
<td>1.0</td>
<td>Beginning of evaporation</td>
<td>Beginning of evaporation</td>
<td></td>
</tr>
<tr>
<td>22</td>
<td>5.9</td>
<td>Complete evap.</td>
<td>Complete evap.</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Impact of iron meteorite:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3.5</td>
<td>0.5</td>
<td>Destruction</td>
<td>Melting</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>1.0</td>
<td>Destruction</td>
<td>Beginning of evaporation</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>2.2</td>
<td>Beginning of melt.</td>
<td>Partial evap.</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>2.6</td>
<td>Complete melting</td>
<td>Partial evap.</td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>4.2</td>
<td>Beginning of evap.</td>
<td>Partial evap.</td>
<td></td>
</tr>
<tr>
<td>17</td>
<td>5.9</td>
<td>Partial evap.</td>
<td>Complete evap.</td>
<td></td>
</tr>
<tr>
<td>30</td>
<td>16.8</td>
<td>Complete evap.</td>
<td>Complete evap.</td>
<td></td>
</tr>
</tbody>
</table>

D. L. Orphal, W. F. Borden, S. A. Larson and P. H. Schultz [1] studied the spatial-temporal sequence for the formation and movement of evaporated and molten material at early stages of impact of an iron meteorite on gabbroid anorthosite with velocities 5 and 15.8 km/sec. According to these calculations, with velocity 5 km/sec, the mass of a target equal to 4\% of the mass of a meteorite \( M \) was exposed to complete melting, and 3\% \( M \) to partial melting.

With impact velocity 15.8 km/sec, the mass of rock equal to 3.6 \( M \) is partially evaporated, mass 7.6 \( M \) is exposed to complete melting, and 2.8 \( M \) is exposed to partial melting. Comparison of the calculated values of the quantity of rock melt and the observed volumes of solidified melt in terrestrial meteorite craters will be made in section 5.
Evaluation of the quantity of material evaporated during meteorite impact was important, in particular, for verifying one of the hypotheses for the emergence of the Moon, according to which condensation of vapor formed during gigantic impact on the surface of the protoearth could provide the necessary quantity of material to form the Moon. Consequently S. M. Rigden and T. J. Ahrens [1] focused on the dependence of the quantity of formed impact vapor on the initial target temperature. They compared the magnitudes for cold silicate (modeled by quartz) crust of the Earth and for molten Earth modeled by molten quartz with temperature about 2000°C. It was noted that with impact velocities 11 - 15 km/sec, mass of material equal to 10 - 15% of the mass of the impact object is evaporated. However with velocity 25 km/sec, impact in the molten quartz of a body with weight 20% of the weight of the modern Moon could result in evaporation of mass equal to the mass of the Moon. Of course, there are still questions related to the angular momentum of the Moon, etc. (see Ye. P. Ruskol [1]).

The question of the correlation between the mass of a falling body and the mass of ejecta leaving the gravitational sphere of the planetary body-target is very important from the viewpoint of cosmogeny. This correlation determines whether this body will increase or decrease its mass as a result of meteorite bombardment. If the calculated data which were obtained by J. D. O'Keefe and T. J. Ahrens [3] are generalized, then it turns out that with second cosmic velocity $v_e$ less than 1 km/sec, mass $M_e$ which is ejected beyond the limits of the gravitational sphere mainly depends on the kinetic energy of the impact object $K$. The approximate correlation between $M_e$ and $K$ with velocity of impact greater than 5 km/sec looks like

$$M_e/K \approx 400/v_e \text{ m/sec},$$

where $M_e$ is measured in MT, while $K$ is measured in a weight of trotyl charge equivalent for energy, MT of TNT.
With second cosmic velocity $v_e > 1$ km/sec, mass $M_e$ also depends on the velocity of the meteorite. With specific value $v_e$, with a rise in collision velocity $v$ increases and the ratio of the mass $M_e$ to the mass of the impact object $M$. For boundary velocity of collision $v_1$ which guarantees equality $M_e = M$, the data which were obtained by J. D. O'Keefe and T. J. Ahrens [3] can be generalized in the form of a correlation

$$v_1 \approx 9.4 \left(v_e - 0.3 \right), \quad v_e > 1.$$  \hspace{1cm} (26)

where $v_1$ and $v_e$ are expressed in km/sec.

With fixed $v_e$ and with $v > v_1$, the mass of material $M_e$ ejected beyond gravitation limits exceeds the mass of the impact object $M$, with $v < v_1 \ M_e < M$. One should, of course, note that these assessments were made without consideration for the influence of atmosphere in which deceleration unconditionally will reduce the quantity of material ejected beyond the gravitational sphere during meteorite impact.

Assessment (26) can be illustrated by quantitative data after using the values of second cosmic velocity for some modern planetary bodies of the solar system (without consideration for influence of the atmosphere): for Earth $v_e = 11.2$ km/sec and $v_1 \ 100$ km/sec, for Mars $v_e = 5.09$ km/sec and $v_1 \approx 45$ km/sec, for Mercury $v_e = 4.15$ km/sec, and $v_1 \approx 36$ km/sec, for the Moon $v_e = 2.38$ km/sec and $v_1 \approx 20$ km/sec.

It is apparent from these assessments that such large bodies as the Earth are capable of increasing their mass during meteorite bombardment even if there is no atmosphere. Smaller bodies (for example, the Moon) when hit by high-speed meteorites will lose more mass than they obtain in the form of falling bodies. For these bodies the balance of mass depends to a great degree on the spectrum of velocities of the falling bodies. For even smaller bodies, the presence of an atmosphere (if only temporary) could possibly be the main mechanism for retaining material during growth because of the falling on the surface of other bodies.

30
But with retention in the gravitational field and in the atmosphere of material of ejecta, their thermochemical properties and conversion could have significance. It is apparent from Table 2 that pressure in the shock wave in which there is complete evaporation of material is several times greater than the pressure which when released initiates evaporation. In a broad range of pressures, the thermodynamic paths for discharge of material pass through the area of existence of the melt in the saturated vapor. As a result of discharge along these isotropic lines, the material will be in a partially evaporated state.

The importance of partial evaporation and subsequent condensation of a shock wave are determined, for example, by the possible formation through this avenue of one of the puzzling geological objects, tektites (see, for example, the book of P. V. Florenskiy and A. I. Dabizhi [1]).

Basic questions of the thermodynamics of the condensation of material evaporated during meteorite impact have been discussed in the monograph of Ya. B. Zel'dovich and Yu. P. Rayzer [1]. The two-dimensional numerical calculations of B. A. Ivanov, P. F. Korotkov and D. A. Sudakov [1] demonstrated the rapid emergence of conditions for breakdown of a melt and saturated vapor in the case of high-speed impact on silicate rock. According to these calculations, evaporation and condensation in the main mass of the cloud of the impact vapor precede in time discharge of the main quantity of pulverized cold material during cratering.

E. M. Jones and M. T. Sandford [1] have illustrated the effect of the finiteness of the terrestrial atmosphere on the parameters of a very powerful (500 MT) nuclear explosion on the surface of the Earth. These data can be used for the case of superhigh-speed collision on the Earth's surface.

On the whole one should note that by now a lot more attention has been focused on movement of condensed material during high-speed impact cratering than the spatial-temporal history of small masses.
of material heated to great temperatures.

Results of numerical two-dimensional calculations of crater-forming flow of target material emerging as a result of high-speed impact which have been published in the last 3 - 4 years refer both to the reproduction of large-scale natural events, and to mathematical modeling of laboratory experiments, which makes it possible to verify the suggestions made in the calculated methods.

Numerical two-dimensional calculations which were made by J. M. Thomsen, M. G. Austin, S. F. Ruhl, et al. [1], J. M. Thomsen, M. G. Austin and P. H. Schultz [1] and M. G. Austin, J. M. Thomsen, S. F. Ruhl, et al. [1] cover mathematical modeling of a high-speed impact in a widespread laboratory material, modeling clay. The equation of state of modeling clay used in the calculations looks like (10) and (11) (see section 1). The strength properties of modeling clay have been described by the Mises criterion (12) with shear strength 50 kPa (0.5 kg/cm²). The main calculation variant discussed impact of a sphere made of aluminium 2024 weighing 0.3 g with velocity 6 km/sec. The kinetic energy of the sphere corresponded to approximately 5.4 x 10¹¹ erg (1.3 g TNT). Both the early stage (up to 18 μsec after impact, J. M. Thomsen, M. G. Austin, S. F. Ruhl, et al. [1]) and the late moments (up to 0.6 msec after the impact, M. G. Austin, J. M. Thomsen, S. F. Ruhl, et al. [1]) of cratering were calculated.

Comparison of the findings with the experiment showed high reliability of the adopted method of mathematical modeling. In this case it was possible to study details of flow inaccessible to the experimenters. The calculations thus showed that by the 18th microsecond after the impact, impact of the base, the percentage of kinetic energy contained in the target material and subsequent flow traced up to 600 sec occurs by inertia.

Using the same mathematical models for modeling clay, M. G. Austin and J. M. Thomsen [1] reproduced experiments on centrifugal modeling
of a contact explosion using a charge of PETN with acceleration \(10 g_0\) and \(517 g_0\) (\(g_0 = 9.81 \text{ m/sec}^2\) is the normal acceleration of the gravitational force on the surface of the Earth). It is shown that the gravitational force acts only on the latest stage of cratering where the reserve of kinetic energy in the crater-forming flow becomes comparable to the initial energy of the material in the gravitational field at depth on the order of the depth of the final crater.

P. H. Schultz, et al. [1] reported the results of calculating impact of an iron meteorite on gabbroid anorthosite with velocities 5 and 15.8 km/sec. In this case primary attention was focused on the change in the shape of the crater during its formation. It was noted that about 50% of the crater volume is formed because of ejection of the target material. The calculated density in the layer of ejecta falling around the crater corresponds to decrease in the thickness of the heap proportionally to \(r^{-4}\) (where \(r\) is the distance from the crater center) near the crater and proportional to \(r^{-2.5}\) at distances several diameters from the crater. Level of pressure of the shock wave averaged for mass was also calculated, where ejecta falling at a specific distance was exposed to its influence at an early stage of cratering. For example, with impact with velocity 5 km/sec, the ejecta falling 2 km from the center of the impact (on an atmosphere-free body) was exposed to the effect of a shock wave with pressure (on the average) \(\approx 4.5\) kbar, at distance 20 km, 23 kbar. For impact with velocity 15.8 km/sec, these pressures for ejecta at distance 2 km are 32 kbar, at distance 20 km 118 kbar.

Similar data are important, for example, for interpreting results of studying samples of lunar soil, whose surface layers are material reprocessed by numerous impacts.

One of the best preserved terrestrial meteorite craters is the Arizona Meteorite Crater about 1.2 km in diameter. Similar studies by geophysical and geological methods, including a large volume of drilling (see D. J. Roddy, J. H. Boyce, et al. [1] made it a favorite
object of mathematical modeling. The magnitude of kinetic energy necessary for the calculations is usually determined using extrapolation data on craters of slightly buried nuclear explosions and generally is 4 - 5 MT of TNT. The pieces of meteorite iron found around the Arizona crater allow the majority of researchers to view it as the impact of an iron meteorite on the surface of the half space filled with porous sedimentary rock.

D. J. Roddy, S. H. Schuster, et al. [1] reported calculating impact of an iron meteorite with energy 3.8 MT at velocities 15 and 25 km/sec on the surface of a stratified half space. The properties of the alternating layers of limestone and sandstone were specified according to the available geological data. The maximum dimensions of the crater were attained within 0.3 sec after the impact.

In their subsequent work, D. J. Roddy, K. N. Kreyenhagen and S. H. Schuster [1] compare the calculated fields of velocities obtained for cases of high-speed impact and explosion of a spherical charge of TNT weighing 100 T, located on the surface of rocky ground. The second calculated variant reproduces the explosive experiment Middle Gust III. The comparison made with regard for the difference in scales for 180 msec after impact and 25 msec after the explosion showed high degree of similarity of the velocity field, which permits a more substantiated approach to the long-discussed problem of modeling high-velocity impact cratering by contact explosions of normal explosives.

A group of researchers headed by J. B. Bryan and D. E. Burton set themselves a similar goal, that of determining similarity between a high-velocity impact and a slightly buried nuclear explosion. In the first work of this cycle, J. B. Bryan, D. E. Burton, M. E. Cunningham and L. A. Lettis [1] reported developing a technique for numerical modeling of cratering which includes three stages: numerical two-dimensional calculation of impact-wave stage and formation of velocity field in the target, ballistic extrapolation of the inertial motion at the late stage determining the intermediate shape of the crater and
distribution of ejecta around it, and calculation of subsequent collapse of the intermediate crater in the gravitational field in order to obtain its final dimensions. Despite the evident shortcomings, this method is currently one of the logically finished methods of calculating cratering. In criticizing these methods one should take into consideration that there are no intelligent alternatives.

At the early stage, a calculation was made of the impact of an iron meteorite with energy 4.5 MT with velocity 15 km/sec on the surface of limestone described by equation of state of Tillotson (see Table 1). In order to model the strength properties of limestone, the shear modulus with magnitude $350$ kbar and Mises criterion with shear strength $0.2$ kbar were used.

The numerical two-dimensional calculation on the program of the SOIL type was made to time 0.5 sec after impact. By this moment in time the shape of the growing crater was close to a hemisphere (depth of the crater depression approximately $250$ m, radius at the level of the initial surface was about $200$ m). At this moment in time, the calculation was stopped and the next analysis of the velocity field was made. The cells in which the magnitude and direction of velocity guarantee that the level of initial main surface would be reached in the case of free motion in the gravitational field were considered to be ejected and placed in a position corresponding to free ballistic scattering. The other cells were assumed to be fixed.

The profile of the layer of ejecta was then modified to attain conditions of stability on the assumption that the angle of rest equals $35^\circ$. The profile of the crater proper attained as a result of ballistic extrapolation was smoothed by replacement by hyperbola determined by the condition of preserving the volume of the crater.

The described procedure, although it contains internal contradiction, permits bringing the calculation to determination of crater dimensions.
J. B. Bryan, D. E. Burton and L. A. Lettis [1] used this program to make calculations for a slightly buried nuclear explosion which was modeled by a sphere of evaporated limestone with internal energy 4.5 MT placed at depth 86 m (relative depth 5.2 m/kg\(^{1/3}\)). The calculations showed similarity of the velocity fields with high-speed impact and explosion and the calculated parameters of the explosion crater and impact crater (radii differ by 2%, volume 10%).

The more detailed study which was made by J. B. Bryan, D. E. Burton, L. A. Lettis, L. K. Morris and W. E. Johnson [1] showed that the magnitude of burial of the explosive source reproducing (with equal total energy) the velocity field (and this means the dimension of the crater as well) of a high-speed impact depends on the velocity of the impact. With impact velocity 2 km/sec, the relative equivalent depth of a nuclear explosion is 11.6 m/kT\(^{1/3}\), with 15 km/sec 5.2 m/kT\(^{1/3}\), with 20 km/sec 4.0 m/kT\(^{1/3}\), with 25 km/sec 3.2 m/kT\(^{1/3}\). We note that previously based on a comparison of experimental data for the scattering dynamics of ground with impact cratering of dry sand, B.A. Ivanov [2] came to a similar conclusion about the decrease in relative depth of explosions reproducing the impact, with a rise in impact velocity.

Concluding this section it is appropriate to dwell on the question of the shape of the craters and funnels formed during high-speed impact and explosion near the surface of the ground. It is common knowledge that the ratio of the radius to depth of these funnels and craters changes in a broad range of absolute dimensions relatively weakly and is usually a magnitude of 2 - 3. An exception is the flooded ground in which the funnels creep in the gravitational field (A. J. Piekutowski [1]). At the same time, the impact craters and the explosive funnels in metals and artificial media of the modeling clay type are close in shape to a hemisphere (A. T. Bazilevskiy and B. A. Ivanov [1], R. M. Schmidt and K. A. Holsapple [1]).

If we compare data on the shape of craters and funnels obtained using two-dimensional numerical calculations, then it is found that
the hemispherical shape of the craters appears in the calculations using the strength criterion of Mises, the shear strength of the medium which does not depend on pressure (J. K. Dienes and J. M. Walsh [1], J. B. Bryan, D. E. Burton, M. E. Cunningham and L. A. Lettis [1], J. M. Thomsen, M. G. Austin, S. F. Ruhl, P. H. Schultz and D. L. Orphal [1] and others). This agrees completely with the nature of the shear strength of metals and modeling clay (which, it is true, can be viewed as a very viscous liquid). At the same time, the use of the Coulomb model in the calculations results in a two-stage growth of the crater. At the first stage which ends when the crater reaches the maximum depth, its shape is close to hemispherical. At the second stage with constant depth, only the radius of the crater rises (D. L. Orphal [1, 2], C. W. Ullrich, D. J. Roddy and G. Simmons [1], R. P. Swift [1] and others).

In this case the shape of the craters and the funnels in the hard and soft ground is reproduced much better by numerical calculations.

We can draw a conclusion from here that one of the most important reasons for the creation of funnels and craters with ratio of the radius to depth on the order of 2 - 3 is the dependence of shear strength of rocks on the surrounding pressure; in mathematical modeling it is usually described by a Coulomb type model (13). One can think that use of these models is one of the necessary conditions for reproducing the correct shape of the funnels and craters in two-dimensional calculations. It should be noted that the dimensions and shape of the craters and the funnels are often the only parameters which can confirm correctness of the models in the calculation methods.

It has already been noted above that destruction of rocks during their deformation decreases the role of cohesion, and correspondingly, raises the role of dry friction, determining the dependence of shear strength on pressure. This circumstance was evidently the reason for the long-noted similarity between craters and funnels in sand and similar soft ground and in real large-scale rock massifs (see sections 5, 6). The definitive role of dry friction should be manifest in the dependence of the effectiveness of cratering on the scale of
the phenomenon. This question will be discussed in more detail in section 5. We only note that the experimental data obtained with different accelerations of the gravitational force (section 5), like the observation data on meteorite craters on planets of the solar system (section 6) indicate similarity of shape of the craters and funnels (ratio of radius to depth on the order of $2 - 3$) which differ in absolute dimensions at least $10^5$-fold.

3. Models of Motion of a Medium During Cratering

As shown in section 2, currently numerical two-dimensional calculations make it possible to obtain fairly detailed information about the propagation of shock waves during explosion and high-speed impact. However, for a number of reasons, it is fairly difficult to use the numerical methods to calculate motion of a medium at the late stages of cratering, when the area encompassed by a shock wave becomes much greater than the characteristic dimensions of the forming crater. One of these reasons is the significant decrease in the rates of movement of material towards the end of cratering which makes it necessary to have a large number of steps in time with preservation of dimension in the cells, since the step in time in the majority of currently developed programs is limited by the Curant criterion: the size of the step in time cannot exceed the time for propagation of sound perturbations through the calculated cell. However in this circumstance one can see an escape from the situation: the small size of the rates of movement of the medium as compared to the rate of propagation of sound perturbations makes it possible to ignore details in the wave processes and to examine the late stage of cratering in an approximation of the incompressible medium. Generally this should be motion of the medium which has strength properties. In the cases of rocks it is possibly necessary to take into consideration the dilatancy effects, and to understand the "incompressibility" of the medium as low significance of the wave processes.

The works of K. P. Stanyukovich [1, 2] initiated a similar approach to describing meteorite cratering. He viewed this
process as "explosion" of a meteorite which initially penetrated the
ground to a certain depth. According to this model, the ground after
the explosion begins to move on radii-vectors traveling from the buried
point of explosion. However it subsequently became clear that a high-
speed impact to a great degree is similar to a contact or shallow
explosion. This was experimentally demonstrated by V. R. Oberbeck [1].

The contact and shallow explosions are distinguished from explo-
sion for ejection by the much smaller role of the piston effect of the
detonation products in the explosion cavity. In the case of explosion
near the surface, the main movements of the medium during cratering
are completed by inertia because of kinetic energy transferred
into the ground at the early stage of the explosion.

Models of cratering which take into consideration the cumulative
experience of experimental and theoretical study of cratering during
contact explosion (B. A. Ivanov [2, 4] and D. E. Maxwell [1]) have been
suggested for simplified analysis of this inertia motion of the medium
during cratering independently in the USSR and the United States. The
following hypotheses were the basis for the model.

We will introduce the polar coordinates $R, \theta$, in this case $\theta = 0$
is directed deep into the target perpendicularly to the plane of its
free surface. After explosive release of energy, a shock wave which
involves material in motion is propagated through the medium to the
surface of the target at point $R = 0$. Beyond the wave, a zone is
formed of inertial moment of the medium in which the following hypo-
theses are adopted for the field of velocities. If we present the
vector of the flow velocity at a certain point of the examined region
in the form of a vector sum of radial $v_R$ and tangential $v$ component,
then one can approximately consider that the quantity $v_R$ does not
depend on the angle $\theta$ and is a diminishing exponential function only
of the distance from the coordinate center:

\[ v_R \sim R^{-N}. \] (27)
The condition of medium incompressibility in spherical coordinates looks like

\[
\text{div}\mathbf{v} = \frac{1}{R} \frac{\partial}{\partial R} (R^2 v_R) + \frac{1}{\sin \theta} \frac{\partial}{\partial \theta} (v_\theta \sin \theta) = 0. \tag{28}
\]

Substitution of (27) into (28) leads to an equation of the dependence of the tangential component \(v_\theta\) on \(R\) and \(\theta\), whose solution looks like

\[
v_\theta = (N - 2) v_R (1 - \cos \theta) / \sin \theta. \tag{29}
\]

Thus, in postulating relationship \(v_R (R)\) in form (27) it follows from the condition of incompressibility (28) that \(v\) depends on the distance \(R\), in the same way as \(v_R\), and with specific \(R\), increases smoothly from the zero value on the axis of symmetry \(\theta = 0\) to maximum value on the free surface.

In specifying the field of velocities in the form (27) and (29), the trajectories of particle motion of the medium during cratering look like

\[
R = R_1 \left( \frac{1 - \cos \theta}{1 - \cos \theta_1} \right)^{1/(N - 2)}, \tag{30}
\]

where \(R_1\) and \(\theta_1\) are the coordinates of the point fixing this trajectory.

It is apparent from expression (30) that the shape of the trajectories does not depend on time, and thus, they are stationary current lines. This circumstance allows us to abandon the postulating relationship (27) for the whole stream as a flow and to base the model on the assumption that the medium moves on trajectories of type (30).

Figure 4 shows trajectories of type (30) as compared to data of experiments and numerical two-dimensional calculations for cases of cratering in aluminum, water and sand. It is apparent that the described plan provides a true transmission of the main qualitative characteristics of the crater ring flow.
Figure 4. Motion Trajectories of Individual Points of the Target During Cratering as Shown in Cylindrical Coordinates $R, z$. The values of the coordinates for each point are normed for the magnitude of the initial distance of the specific point to the beginning of coordinates $r_0$. The black and light circles designate points with different $r_0$ moving on close trajectories, the solid curve lines show trajectories described by formula (30) with $N = 3$.

Key:

a. comparison with numerical two-dimensional calculation for aluminum (J. K. Dienes and J. M. Walsh [1])
b,c. comparison with experimental data for contact explosion in water (b) and wet sand (c) (B. A. Ivanov [5])
Recently works have been published in which the postulates and applicability of the aforementioned simple model of cratering were comprehensively verified by comparison with the results of experiments and numerical two-dimensional calculations. D. L. Orphal [2] thus showed that the field of velocities calculated for a shallow nuclear explosion Johnie Boy can be described in the form (27) and (29) with $N = 2.7$. J. M. Thomsen, M. G. Austin, S. F. Ruhl, et al. [1] made a numerical calculation of the impact of an aluminum sphere weighing 0.3 g with velocity 6 km/sec in modeling clay (shear strength 50 kPa, density 1.7 g/cm³) and compare the results of the calculation both with experimental data confirming the adequacy of the calculation and using the model of the velocity field (27) and (30). The comparisons show that approximation of radial velocities in the zone of developed cratering flow in the form (27) results in a quantity $N$ which depends weakly on the angle $\theta$. Thus, by the moment in time 18 sec after impact, the quantity $N$ changes from 1.8 with $\theta = 0^\circ$ to 2.4 with $\theta = 70^\circ$.

In their subsequent work, M. G. Austin, J. M. Thomsen, S. F. Ruhl, et al. [1] continued calculation of the same high-speed impact from 18 msec to 0.6 msec. It was found that from the viewpoint of correspondence to the velocity field (27) and (29), the flow becomes stable within 0.2 msec after impact; with $60^\circ < \theta < 90^\circ$ $N \approx 3.85$, and with $60^\circ < \theta < 90^\circ$ $N \approx 3.9$. The hemispherical shape of the crater made it possible to establish that the best adjustment occurs on the assumption that the center of the polar coordinates is located 0.6 cm below the impact point, which agrees with the similarity of the high-speed impact and shallow explosion which was repeatedly mentioned above. The publications of J. M. Thomsen, M. G. Austin and P. H. Schultz [1] and M. G. Austin, J. M. Thomsen, D. L. Orphal, W. F. Borden, S. A. Larsen and P. H. Schultz [1] covers a similar adjustment of flow in the form of (27) and (29). The latter publication discusses a case of high-speed impact in gabbroid anorthosite.

half space with velocity 5 km/sec. In this case the values of the N quantities which correspond best in formulas (27) and (29) were 2.0 ± 0.1 for angles 0° < θ < 10° and 2.7 ± 0.1 for angles 45° < θ < 90°.

For definition for the entire flow, that is with 0° < θ < 90°, N = 2.6 ± 0.1.

We note that all the estimates made above have meaning for times not long after impact, the shape of the growing crater is still close to a hemisphere, therefore one can use the original formulation of the presented model and consider the radial velocity v_R to be independent of angle zero.

In the case where the shape of the growing crater differs from hemispherical, direct adjustment of the parameters of the simple cratering model in the form (27) and (29) for the calculated velocity fields can result in erroneous conclusions. In fact, expression (27) can be written in the form

\[ v_R = v_a (a/R)^N, \]  

(27')

where a is the radius of the hemispherical crater at a certain moment in time, while v_a = da/dt is the rate of growth in the crater radius in time. If parameter N in (27') is adjusted by processing velocity fields calculated in the two-dimensional problem in the form of an exponential dependence on the distance along a certain beam determined by the selection of the polar angle θ, then this will mean that with increase in the distance R_θ we will examine the velocity of points lying on different trajectories (30) which can be presented in the form

\[ R = a_0 [(1 - \cos \theta)/(1 - \cos \theta_a)]^{1/(N-1)} \]  

(30')

where θ_a is the polar angle of point a_θ on the surface of the growing crater through which the selected trajectory passes. For this reason, the requirement for the possibility of specifying the velocity field in the form (27'), (30) means a requirement of independence v_a on θ.
and this means requirement for preserving the hemispherical shape of the growing crater.

As already noted, one can construct a simple model of cratering after postulating an analytical expression for trajectories (30) or (30'). In this case correlations (27) and (29) will be fulfilled along each trajectory, but along beams \( \theta = \text{const} \), the velocities could change randomly depending on the shape of the cavity of the growing crater.

Based on a model field of velocities (27), (29) and trajectories (30), B. I. Ivanov [4] and B. A. Ivanov and L. I. Komissarov [1] made simple analytical assessments of such parameters of cratering as maximum depth of excavation of ejecta depending on the range of their scattering and change with distance in the thickness of the ejecta layer. S. K. Croft [1] used the same model to examine cratering as a whole, and in particular, noted that the boundary observed in the experiment Preria Flat (explosion of a spherical charge of TNT weighing 500 T on the surface of the ground) between the ejected ground and the ground pressed under the bottom of the crater is described well by a curve of type (30) with \( N = 2.7 \).

The simple model of cratering which is kinematic in its construction and based on the field of velocities (27), (29) and trajectories (30) which can be considered the impact form of generalization of the results of experiments and numerical two-dimensional calculations could also be used for quantitative assessments of cratering dynamics. In fact, a field of velocities \( v_R, v_\theta \) (27) and (30) make it possible to define the tensor of deformation velocities, and after making hypotheses which are normal in the mechanics of soils regarding the applicability of certain hypotheses of plastic flow, to link the tensor of stresses with the tensor of deformation velocities. The possibility of determining the power of plastic work follows from here, and using the law of preservation of energy one can determine the rate of decrease in kinetic energy because of completion of the plastic work. This
allows us to find a law for growth in the dimensions of the crater in
time and the finite dimensions of the crater attained at the moment of
complete exhaustion of kinetic energy of flow. The initial conditions
for this problem could be two-dimensional numerical calculations of
the initial stage of high-speed impact or contact explosion.

J. D. O'Keefe and T. J. Ahrens [6] used a similar approach in a
somewhat simplified form to evaluate the influence of the strength of
the medium of the gravitational force on the law of growth in time and
the maximum magnitude of crater depth in media whose strength is de-
scribed both by the Mises condition (constant magnitude of shear strength),
and by a combination of Coulomb and Mises conditions (14) (see section
1).

Other simplified models have been suggested for evaluating differ-
ent parameters of cratering. W. R. Seebaugh [1] has reported a model
of formation and motion of ejecta with regard for their deceleration
in the atmosphere. The model of formation of fragments determining
their granulometric composition and its variations with a change in
the initial velocity of ejecta is mainly based on experimental data
obtained in large-scale explosive experiments on rocky ground. In
particular, based on these data which are for the most part unpublished,
the granulometric composition of the fragments during contact explosions
on basalt and granite is generalized in the following form

\[
dM/dx = 0.5M_0(x_m x)^{-0.5},
\]

(31)

where \(dM\) is the mass of the fragments whose sides \(x\) are contained in
the interval \(dx\), \(M_0\) is the total mass of the ejecta, \(x_m\) is the maximum
size of a fragment in this event. Mass \(m_m\) of the fragment of maximum
size is related to \(M_0\) by the correlation

\[
m_m = \pi \rho x_m^2 \frac{3}{8} \approx 6.6 \cdot 10^{-5} M_0^{0.5}.
\]

(32)
After specifying the granulometric composition of fragments and the usual velocities of scattering, an examination is made of the ballistic scattering of these fragments in the terrestrial atmosphere, and the aerodynamic deceleration is taken into consideration on the assumption that motion of individual fragments through fixed air is independent. Previously scattering of fragments was discussed in a similar approximation with underground (A. E. Sherwood [1]) and contact (B. A. Ivanov [1]) explosions. However these publications did not take into consideration the influence of air compressibility (Mach number) on the coefficient of resistance. W. R. Seebaugh [1] in generalizing different data proposes the following evaluation for the coefficient of resistance $c_D$

$$c_D = 0.6 + 36/R_e$$ (33)

for all values of the Mach number $M \leq 0.4$ and any Reynolds numbers $R_e$ or for $R_e \leq 60$ and any $M$. With $M \geq 1.2$ and $R_e \geq 60$, the coefficient of resistance is assumed to be constant and equal to $c_D = 1.2$, while for $0.4 < M < 1.2$, the coefficient of resistance is determined by linear interpolation between the value (33) for $M = 0.4$ and $c_D = 1.2$ with $M = 1.2$.

Calculations on this model are in reasonable agreement with the available data on the distribution of the mass of ejecta around craters.

M. E. Tauber, D. B. Kirk and B. E. Gault [1] made a numerical study of the trajectories of fragments in the form of a sphere and cube rotating in flight for cases of the atmospheres of the Earth, Venus and Mars. It was found that in ranges of fragment dimensions 1 - 100 m, initial velocities of ejection 0.1 - 1 km/sec at angles from $10^\circ$ to $60^\circ$ to the horizon, the calculated data are subject to fairly simple analytical approximation. If the ballistic coefficient is introduced in the form

$$\beta = c_D A \rho \mu m.$$ (34)
where $c_D$ is the coefficient of resistance with initial conditions of ejection, $A$ is the area of the cross section sample (assuming to be equal to $a^2$ for a cube rotating in flight with length of the rib $a$ and $\pi a^2$ for the sphere of the radius $a$), $\rho$ is the atmospheric density, $m$ is the mass of the fragment, then the scattering range in atmosphere $L$ can be defined after solving the transcendental equation

$$\exp(\beta L) - (1 + 0.5\beta L) \beta L = 1.$$  \text{(35)}

where $L_v$ is the scattering range in a vacuum with the same initial condition. One can similarly generalize data for the ratio of the tangent of the angle of incidence $\tan \theta_1$ to the tangent of the angle of projection $\tan \theta_0$:

$$\tan \theta_1/\tan \theta_0 = 1 + (2/\beta L_v) [1 - \exp(\beta L)]$$  \text{(36)}

and for the ratio of the rate of fall $v_1$ to the rate of projection $v_0$:

$$v_1/v_0 = (\cos \theta_0/\cos \theta_1) \exp(-0.5\beta L).$$  \text{(37)}

We recall that the scattering range in a vacuum for a planar surface in the gravitational field with acceleration $g$

$$L_v = \left(\frac{v_0^2 \sin 2\theta_1}{g}\right).$$

Using these data, M. Settle [1] evaluated the volume of ejecta falling within the crater because of deceleration of them by the atmosphere. According to this evaluation, with an increase in the diameter of the terrestrial meteorite crater from 10 to 100 km, the percentage of mass to the ejecta decelerated by the Earth's atmosphere within the boundary of the crater rises from 10% to 13%, while the greatest size of the decelerated particles increases from 1 m to 7 m. A similar evaluation for the Arizona crater 1.2 km in diameter results in a percentage of fragments decelerated within the crater equal to 5%, while the percentage of material evaluated from geological data which could be
interpreted as ejecta decelerated by the atmosphere is about 4%. The mean size of the decelerated fragments is estimated as 1.7 cm. This mechanism must be very important for forming meteorite craters on Venus which has a very dense atmosphere. M. Settle [1] based on his estimates believes that with crater diameter over 10 km, over 50% of all the ejecta cannot fly beyond the crater limits. In this case the decelerated fragments could be up to 10 m and larger.

As already noted, in discussing meteorite cratering, it is very important to evaluate the scattering range of material which at the early stage is exposed to the effect of a shock wave of definite intensity. M. G. Austin and B. R. Hawke [1, 2] attempted to make this evaluation in the simplest hypothesis that scattering of material beyond the shock wave occurs with rate of splitting off, that is with double mass velocity of material in a shock wave. B. A. Ivanov [6] suggested an evaluation based on the simple use of a cratering model (27) - (30), described in the beginning of this section. He assumed that after passage of the shock wave, the target material immediately begins to move on trajectories that are described by formula (30). An evaluation showed the slight increase in percentage of material that is exposed to the strong impact with distance at which the eject was located after scattering. This agrees qualitatively with the conclusions that are based on the results of numerical two-dimensional modeling (see P. H. Schultz et al., [1]).

4. Some Data of Laboratory Modeling

If we disregard the numerous studies covering explosion for ejection, perhaps one of the first experiments that was set up especially for modeling was the work of A. Wegener [1] who is more famous as the author of the hypothesis on moving continents. Translated into Russian in 1923, this work is now accessible to the majority of readers, probably only in the 1975 English translation.

A. Wegener used cement dust as the target material, while the "impact object" was a clump of cement dust that was ejected onto the
target by a tablespoon. This fairly primitive experiment already intuitively saw some features of the modern methods of experimental modeling of large-scale explosive cratering, in particular, the use of loose material to model the behavior of real rocks in large scales, the relative independence of the overall pattern of cratering at the late stages from the velocity of the impact object, and the use of layered targets to evaluate the possibility of determining their properties from the observed features of meteorite crater structure.

The technique suggested by A. Wegener [1] was used in the 1950's by P. F. Sabaneyev [1] who conducted a series of experiments with great diversity in the properties of the target material and with collision velocities from 1 to 8 m/sec.

A large amount of experimental material on laboratory modeling of cratering was examined in the surveys of A. T. Bazilevskiy and B. A. Ivanov [1] and B. A. Ivanov [7]. Below are the recently published data on laboratory modeling of explosive and impact cratering; if necessary, materials included in previously published surveys are added.

The most popular material for laboratory studies of cratering is dry sand which was previously used successfully to model the movement of large masses of rock during explosion for ejection by M.A. Sadovskiy et al. [1] (see also V. N. Rodionov et al. [1]). Essentially devoid of cohesion, dry sand permits study on a laboratory scale of the influence of the gravitational force on cratering. This property allowed it to be used very successfully in centrifugal modeling, whose data will be discussed in the next section.

The overall pattern of cratering can be presented in the following form. After explosion or collision, an impact wave begins to spread from the point of energy release, putting the ground into motion. After the shock wave, as a result of the spread of acoustic perturbations which travel from the free surface of the target, a certain velocity field is formed. Its development in time results
in movement of the target material which is partially ejected upwards and to the side of the energy release site, and is partially forced in under the bottom of the forming funnel (crater). The correlation between the volumes of ejected and forced-in ground depends on the mechanical properties of the target material.

B. I. Ivanov [3 - 5] and A. J. Piekutowski [1, 2] used the following technique to observe cratering during a contact explosion. One of the walls of a box filled with sand was made of organic glass. An explosive charge was detonated either on a steel plug of small diameter inserted into the opening in a transparent wall (B. A. Ivanov [4, 5]), or directly on the surface of the wall (A. J. Piekutowski [1, 2]). This made it possible to observe as it were "in cross section" the formation of a crater and the movement of the ground in this case. The experiments demonstrated that the development of a contact explosion crater can be divided into two stages. At the first stage, the growing crater has a shape close to the hemisphere segment. When the crater reaches the maximum depth, its indentation ends; in this case, the growth of the crater radius continues for a time that is several fold greater than the time for achieving the maximum depth of the crater. At late stages of growth of a crater in dry sand, its edges cave in, as a result of which the radius rises and the depth diminishes.

Growth of the crater is accompanied by ejection of material. The angle at which the ejection velocity vector is directed changes very slowly with distance from the center of the explosion to the point of ejection and is from 40° - 45° at distances 0.3 - 0.6 \( R_B \) (\( R_B \) is the radius of the final crater), diminishing to 30° - 35° at great distances. The magnitude of the ejection velocity diminishes rapidly with an increase in \( r \). With \( 0.3 < r/R_B < 0.9 \), velocity \( v \) is proportional to \( r^{-m} \). The magnitude of the exponent can change slightly depending on the type of ground and the burial of the charge. For dense, moist sand \( m = 3 \) (B. A. Ivanov [3]), for dry, dense sand with burial of the charge by one radius \( m = 2.5 \) (processing of data presented in the work of A. J. Piekutowski [2]). Insofar as the range of ballistic scattering (in a vacuum) with constant angle
of ejection is proportional to $v^2$, the scattering range of the ejecta ejected at different distances $r$ from the center of the explosion diminishes with a rise in $r$ as $r^{-5} - r^{-6}$.

The material which scatters at different velocities forms during flight a characteristic "plume" of ejecta which resembles an overturned truncated cone.

A detailed experimental study which confirmed the similarity of material ejection during high-speed impact and contact and shallow explosions was conducted by V. R. Oberbeck [1]. B. A. Ivanov [4] compared numerous data on the development in time of a plume of ejecta during explosions at different (shallow) depth and during high-speed impact and showed that the described main features of scattering of ground are common for cratering in dry sand, regardless of the type of source of energy release. It should be noted that the plume of ejecta is not a cohesive formation, but is a geometric region for the position of the ejecta material, moving on almost parallel trajectories. The angle at which the material is ejected during cratering, and the angle at which the external envelope of the ejecta plume is inclined therefore do not coincide, and the angle of the ejecta is always smaller than the angle of the plume inclination. This obvious circumstance is often ignored and the aforementioned angles are made identical. For example, W. K. Hartmann [1, 2] calls the angle of inclination of the plume, the angle of ejecta, which exaggerates the real magnitude of the latter 1.5-2-fold.

The experimentally measured angles of ejecta and the fact of their almost constant magnitude contradict the early models of cratering which are based on the model of buried explosion (see, for example, K. P. Stanyukovich [1]). These models result in ejection angles onto the edge of the crater up to several degrees to the horizon. Because of this discrepancy, misunderstandings could arise in discussing the semi-empirical models which are based on experimental data, from the viewpoint of early views (see, for example, D. E. Rehfuss et al. [1]).
As already noted in the previous sections, increase in the collision rate results in an increase in the losses for irreversible heating of the material near the impact point. This reduces the percentage of kinetic energy used to form the crater. The survey of A. T. Bazilevskiy and B. A. Ivanov [1] noted that with an increase in pressure emerging during high-speed impact into sand from 40 to 450 kbar, the adjusted final radius of the crater diminishes from 23.5 cm/g$^{1/3}$ to 13.1 cm/g$^{1/3}$ (here the kinetic energy of the impact object is expressed in equivalent weight of a TNT charge, 1 g of TNT = 4.2 x 10$^{10}$ erg). D. E. Gault and J. A. Wedekind [1] made a systematic study of the dimensions of crater dimensions in dry sand with change in velocity of the impact object from 0.4 to 8 km/sec. It was found that for an aluminum impact object of constant mass with velocity lower than 1.5 km/sec, the diameter of the crater D depends on the velocity of impact v as

$$D \sim v^{0.234}. \quad (36)$$

With velocities $v > 1.5$ km/sec, the relationship alters its appearance:

$$D \sim v^{0.362}. \quad (37)$$

We recall that if the size of the crater depended only on the energy of the impact object, this relationship would look like $D \sim v^{2/3}$. It is important to note that the data of numerical two-dimensional calculations of high-speed collision of an iron meteorite with target made of sedimentary rock, limestone (J. B. Bryan, D. E. Burton, L. A. Lettis, L. K. Morris and W. E. Johnson [1]) by the technique described in section 2 result in the following relationship

$$D \sim v^{0.49}. \quad (38)$$
with constant mass of the impact object in the range of velocities 2 - 25 km/sec. It is obvious that the exponent in the expressions of type (36) - (38) must depend on the equation of state of the material of the target and impact object. This interesting and important question has not yet been fully investigated.

In addition to the influence of velocity, the dependence of high-speed impact cratering on the angle at which the impact object strikes the target is also of great interest. This question was discussed back when the hypothesis of meteorite origin of lunar craters was emerging, and the a priori hypothesis that craters will be very elliptical in oblique impacts was considered to be a strong argument against the meteorite hypothesis, since it is obvious that meteorites on the average penetrated with equal probability into the surface at any angle, while the lunar craters are mainly round. Because there is no axial symmetry, mathematical modeling of an oblique impact is still impossible, therefore experiments in this case play a decisive role.

D. E. Gault and J. A. Wedekind [2] made a detailed study of an oblique impact into dry sand, granite and cement dust with angles of incidence to 1° with collision velocities up to 7 km/sec. It was found that the ratio of the crater size in dry sand along the trajectory of incidence $D_B$ and transverse to it $D_n$ with velocities of the impact object from 0.05 to 7.2 km/sec remains constant and equals one all the way to angles of incidence equal to 15°. With angles of incidence less than 15°, the ratio $D_n/D_B$ begins to rise rapidly. For granite with velocities of aluminum impact objects from 2 to 7 km/sec, the roundness of the crater ($D_B/D_n = 1$) is preserved up to angles of incidence equal to 30°. In cement dust with angles of incidence from 30° to 5° there is even a slight elongation of the crater in a direction transverse to the trajectory of incidence ($D_B/D_n \approx 0.8$).

The ratio of crater depth to diameter with angles of incidence to 15° remains constant (granite, cement dust) or is slightly reduced (approximately by 10 - 20% in the case of dry sand). The deepest point of a crater during high-speed impact is
shifted along the trajectory in the direction of the impact object flight. The effectiveness of cratering defined as the ratio of the crater volume during oblique impact to the crater volume during normal (perpendicular) impact depends on the collision angle \( \alpha \) either as \( \sin^2 \alpha \) (granite) or as \( \sin \alpha \) (sand, cement dust). The distribution of ejecta around the crater also depends weakly on the angle of trajectory inclination with \( \alpha < 20^\circ \) and with smaller angles adopts characteristic outlines of "butterfly wings" directed to the left and right from the impact trajectory. When there is a narrow, long lobe of ejecta extended along the trajectory on the movement path of the impact object, the ricochet of the impact object (or its fragments) occurs at angles of incidence less than 15°, and is easily noticeable at angles less than 10°. As shown by experiments, the greatest differences during oblique impact are observed for ejecta from the near zone of impact where the impulse of the impact object is comparable to the impulse of the scattering ejecta. During growth of the crater, the mass of ejected material rises, there is an increase in the number of movements in them, and on this background, the initial trend of the impact object impulse becomes unnoticeable. From here a conclusion is drawn that one should search for traces of oblique meteorite impacts on the Moon and planets in the arrangement of ejecta from the zone nearest to the impact. An indicator of this zone could be, for example, melting of rocks forming during impact - wave compression.

The discovery of meteorite craters on Mars and the satellites of Jupiter and Saturn led to attempts at modeling using nontraditional target materials.

Many meteorite craters of Mars have unusual structure in the covering of the ejecta, indicating the liquid-like flow of ejecta after the end of the ballistic scattering phase. Observation of these craters which have been called "rampart craters" because of the presence of swells on the anterior front of the streams of flowing ejecta made it possible to advance a hypothesis that there is water in the upper layers of the Martian crust in the form of permafrost.
According to this hypothesis, heating during impact cratering results in the melting of ice and the formation of flowing ejecta. Experiments on the formation of impact craters in frozen water-saturated sand with impact velocities from 2 to 6 km/sec were made by S. K. Croft [2]. Insofar as in laboratory experiments with ground that has cohesion, it is very difficult to satisfy all conditions of scalar modeling, in these experiments because of the great (in relation to the crater dimensions) range of scattering of the material from the zone of ice melting, no flowing ejecta were formed.

Another attempt to model this phenomenon was made in experiments on high-speed impact on targets made of viscous liquid (of the liquid mud type). D. E. Gault and R. Greely [1] conducted these experiments with impact velocity about 2 km/sec. The formation of individual fragments of flowing ejecta was observed; the ejecta appeared as a result of the oblique incidence of cakes of ejecta with subsequent spread along the surface in a radial direction from the center. On the anterior fronts of these seepages there were characteristic swells which are very similar to swells of ejecta of Martian craters. The growth of a crater in these experiments passed through the stage of formation of a certain intermediate cavity which then flowed in, forming a depression with slight central elevation.

R. Greely et al. [1] made a fairly extensive systematic study of cratering in viscous targets. The viscosity varied from 100 to 1000 P, the intensity of the shear stresses varied from 50 to 2000 dyne/cm² and the impact velocity varied from 0.5 to 5 km/sec.

Great complexities are noted in presenting the experimental data depending on the properties of the target material.

Experiments on targets made of real rocks have very limited value because of the great differences in the strength properties of the laboratory samples and the large rock massifs in nature. The scalar effects of rock strength have been previously discussed in detail in the surveys of V. N. Nikolayevskiy, L. D. Livshits and

D.R. Curran et al. [1] conducted thoroughly controlled experiments on impact and explosive cratering in a target made of quartzite with complete collection of all the formed fragments, which made it possible to obtain reliable information about the granulometric composition of the ejecta. These data are especially interesting as compared to the results of the calculations which on the basis of the model of fracture growth permit a theoretical evaluation of the granulometric composition of fragments.

W.K. Hartmann [1] published interesting data on impact of a missile weighing 166 g with velocity 90 m/sec. on samples of cemented gravel. The initial grains of gravel had a fairly narrow distribution by mass of individual particles with maximum in the region 0.25 - 0.5 g. Study of the granulometric composition of the fragments indicated the formation of a broad spectrum of fragmentary masses that had a maximum in the region of initial particle mass, and the appearance of fragments of conglomerate which have mass distribution in the form

\[ N_m \]

where \( N_m \) is the number of fragments of the conglomerate with mass greater than the specified magnitude \( m \).

Despite the advantage of studying the granulometric composition of fragments, on the whole one should take into consideration that according to the data for camouflet explosions, granulometric distribution of fragments could depend on the magnitude of shear deformation, and consequently, on the distance at which a certain part of the fragments is formed from the center of the explosion.

I.A. Sizov and V. M. Tsvetkov [1] demonstrated that most of the rock during an underground explosion is crushed not in the front
of the shock wave, but in the plastic flow after it. Depending on
the degree of deformation, the appearance of the granulometric
distribution of fragments by sizes changes from log-normal near
the explosion (which is interpreted as multiple crushing of fragments
during expansion of the explosion cavity) to Rosin–Rammler distri-
bution far from the explosion. The latter distribution is inter-
preted as the result of single crushing of the medium (V. M.
Kuznetsov [1]).

We note in conclusion that progress in creating accelerators
which accelerate significant masses to velocities of over 10 km/sec
(see, for example, the work of V. V. Sil’vestrov [1]) affords new
opportunities in laboratory study of high-speed cratering. Another
method of creating highly concentrated sources of explosive type
could be the use of strong-current relativistic beams of charged
particles; their production is developing actively because of work
on impulse initiation of a thermonuclear reaction. For example,
B.A. Demidov and A. I. Martynov [1] have reported measuring shock
waves that emerge in a metal target under the influence of a rela-
tivistic electron beam. The pressure in the explosion source reached
3 Mbar. This affords the possibility of experimental study of
the essentially important question of the influence of energy con-
centration in the source on the effectiveness of the mechanical
action of a contact explosion,
5. Effect of Gravitational Force and Scales of Phenomenon on Cratering

Because of space flights in the last 10 years it has become possible to compare meteorite cratering on planetary bodies with different acceleration of the gravitational force on the surface. Consequently, there has been greater interest in studying the effect of the gravitational force on cratering, previously stimulated mainly by the idea of laboratory modeling of large-scale explosions using change in effective acceleration of free fall by using centrifugal forces and different types of linear accelerators.

General principles of a method proposed in the USSR for centrifugal modeling of large-scale geotechnical processes have been described by G. I. Pokrovskiy and I. S. Fedorov [1]. General questions of the theory of similarity as applied to modeling explosive processes were developed by L. I. Sedov [1]. The case of buried explosion for ejection was discussed most completely in the previous publications. In this case the relative effect of the gravitational force on the formation of a crater could be evaluated by comparing the energy of explosion $E$ and potential energy in the gravitational field of the ejected mass of ground. The characteristic altitude determining the reserve of potential energy in the case of an explosion for ejection is considered to be the depth of placement of the charge $z$. Then for similarity of two explosive events, in addition to definite restrictions for the properties of the medium, it is necessary to fulfill the correlation

$$E/\rho g z^4 = \text{const},$$

where $\rho$ is the density of the ground, $g$ is acceleration of free fall. If (39) is fulfilled, craters of different explosions will be similar, while their linear dimensions (depth $H$ and radius $R$) will be directly proportional to the depth of placement $z$. Then, for example, radii of craters which are similar in sense (39) will change with a change in the energy of explosion and the gravitational force as

$$B \sim (gE)^{1/4}.$$

(40)
The possibility follows from here of comparing the event of varying scale (energy) under the influence of different fields of mass forces (characterized by acceleration of free fall) to adequate comparison of physical-mechanical properties of the ground.

A similar approach to contact explosion \((z = 0)\) encounters certain difficulties, since there is no previously specified linear scale of depth, and this means the scale of potential energy.

This difficulty is usually avoided (see, for example, A. T. Bazilevskiy and B. A. Ivanov [1], R. M. Schmidt [1]) by introducing a linear scale related to explosion energy. If we initially examine the case of a charge of explosive with density \(\delta\) of radius \(r_0\), then one can introduce the concept of potential energy of mass equal to the mass of the charge elevated to height \(r_0\). This energy is proportional to \(gr_0^4\). If we classify it with energy released during explosion of this charge \(E\), proportional to \(Q\delta r_0^3\) (\(q\) is the specific energy release per unit of weight) then the obtained dimensionless parameter

\[
gr_0/Q
\]

is convenient for reducing the data of a contact explosion. In order to reduce the parameter (41) to a more normal appearance, usually \(r_0\) is expressed through complete explosion energy. Then it acquires the appearance

\[
g_{r_0^1/3} Q^{1/3} \delta
\]

In the case of sources of explosion which differ from normal explosives (nuclear explosion, high-speed impact) one can use the following argument: at the early stage of a nuclear explosion and high-speed impact, such high flow velocities of the medium are realized that one can ignore the influence on them of the gravitational force. Within a certain time, the characteristic density of energy is reduced to the level of energy density in a trinitrotoluene charge, and
the cratering process will further be exposed to the same effect of the gravitational force as during a trotyl explosion. It follows from here that the parameter

\[ gE^{1/3} \tag{43} \]

could be a universal parameter of scale for comparing the processes of explosive cratering, of course, with similar processes of an early stage of energy transmission in the ground.

In the case where the ground has strength, part of the energy is spent for deforming the ground. If the losses for work against the gravitational force are low, ejection of the ground and its heating in the case of deformation are the main mechanisms which stop the cratering flow of the target material. With strength properties of the ground that are independent of scale, this results in the well known principle of energy similarity, according to which the dimensions of the crater are proportional to the size of the charge, or what is the same, the cube root of the explosion energy.

Insofar as the effect of the gravitational force depends on the effective vertical scale of the event, its role with an increase in explosion energy rises more, therefore generally one can isolate ranges of the dominating value of the ground strength proper and the predominant role of the gravitational force which is realized both in the form of transition of kinetic energy of the cratering flow into potential energy, and to the influence of lithostatic pressure on the ground strength.

Based on the semiempirical model, B. A. Ivanov [2] showed that during contact explosion, the relative radius of the crater in the range of dominance of gravitation will depend on the explosion energy and on the gravitational force as

\[ \frac{R}{E^{1/3}} \sim (gE^{1/3})^{\alpha}, \tag{44} \]

\[ -\]
capable of bearing a payload of up to 250 kg each (R. A. Schmidt and K. A. Holsapple [1]). The arrangement of the centrifuge makes it possible to have stereoscopic photography, which in addition to the possibility of rotating the containers in a vertical plane allows work both with cohesive and loose materials. A distinguishing feature of the American research is the attempt to generalize data for cases using different explosives.

E. S. Gaffney [1] published data on centrifugal modeling of contact explosions in alluvium and in dry sand. In this case he compared the effective size of the crater and the effective explosion scale. The effective method was suggested previously by R. M. Schmidt [1]. As the effective volume of the crater the following quantity is used

$$\pi_V = \frac{\rho V}{q},$$

(46)

where $\rho$ is the density of the ground, $V$ is the volume of the crater, $q$ is the weight of the explosive charge. The following quantity was used as the effective scale

$$g^3q[\delta U^6]_{\text{TNT}}/(\delta U^6)_{\text{exp}},$$

(47)

where $\delta$ is the explosive density, while $U$ is the Chapman-Jouget velocity respectively for TNT in any other explosive. In these coordinates points corresponding both to experimental data for centrifugal modeling and for large-scale full-scale explosions agree well.

E. S. Gaffney [1] showed that with $g = g_0$, explosions of TNT charges weighing about 10 – 100 T on dry sand will already result in craters whose dimensions are equal to or less than dimensions of craters of natural ground with finite strength.

R. M. Schmidt [2] later abandoned the use of the Chapman-Jouget velocity to characterize specific energy capacitance of the explosive in an expression for an effective scale (47) and suggested using an expression
depended on the gravitational force as

\[ D(\text{cm}) = 16 \left( \frac{g}{g_0} \right)^{-0.165}. \]

A characteristic feature of the craters formed with different gravitational force was their similarity, with a change in the ratio \( \left( \frac{g}{g_0} \right) \) from 1.0 to 0.073, the ratio of the depth of craters \( H \) to their diameter on the level of the initial surface \( H/D = 0.24 \pm 0.1 \), ratio of \( H \) to diameter of the crest of the swell \( H/D_r = 0.19 \pm 0.1 \), and the ratio of \( D_r/D = 1.25 \pm 0.1 \).

It is appropriate here to note the work of M. G. Austin and J. M. Thomsen [1] who used numerical two-dimensional calculations of contact explosion of a PETN charge on modeling clay to show that the cratering flows with different acceleration of the gravitational force begin to differ from each other only in the very end, where the dimensions of the growing crater are close to the final. One can draw the conclusion from here that similarity of craters in dry sand means similarity of cratering flows in this case.

Both past (V. V. Viktorov and R. D. Stepanov [1]), and recent (S. B. Barsanayev, et al. [1]) experiments have confirmed the basic possibility of modeling large-scale explosions for ejection using change in effective mass forces which is attained either by using linear accelerators which accelerate or decelerate the specific acceleration systems containing a container with ground for modeling, or using rotation of the container in a centrifuge.

American researchers have recently also focused attention on the possibility of centrifugal modeling, and the Boeing company has constructed a geoengineering centrifuge capable of creating centrifugal acceleration in testing containers to 600 \( g_0 \), where \( g_0 = 9.81 \text{ m/sec}^2 \) is the acceleration of the free fall on the surface of the Earth. The centrifuge consists of a rotor with radius of the arm about 140 cm, where rotating containers are attached to the opposite ends and are
where on the basis of experimental data, it was assumed. Later the same relationship was obtained using the simple model of cratering described in section 3 (B. A. Ivanov, L. I. Komissarova [1], B. A. Ivanov [4]). According to this model, these exponential relationships (44) are a consequence of a type of approximate self-modeling in the velocity field of the cratering flow.

Exponential relationship of type (44) on the whole agrees well with the empirical data. We will initially discuss experiments with dry sand which can be viewed as an ideally flowing medium that does not have cohesion. In this case the influence of the gravitational force on the size of the crater should be manifest already in the randomly small scale of the experiment.

S. W. Johnson, et al. [1] conducted a series of experiments on contact and buried explosions in dry sand with accelerations of the gravitational force from 1 \( g_0 \) to 0.17 \( g_0 \). The dependence of the crater radius on the gravitational force can be described in the form

\[ R \sim g^{-n} \]

where \( n = 0.111 \) for zero penetration and smooth increase in the value \( n \) with increase in penetration.

D. E. Gault and J. A. Wedekind [1] studied the formation of a crater in dry sand with impact of plastic and aluminum missiles with velocities of the impact from 0.4 to 8 km/sec. The target was placed in a carriage falling with specific acceleration from 1 \( g_0 \) to 0.073 \( g_0 \). The dimensions of the craters with specific acceleration were determined from data of stereoscopic high speed filming.

According to these data with constant parameters of impact (aluminum impact object 3.18 mm in diameter with velocity 6.64 km/sec, which corresponds to kinetic energy approximately equivalent to 0.25 g TNT), the diameter of the crater for the level of the original surface
which by construction coincides with the previously obtained expression (42) and expresses the linear scale as the ratio of gravitational energy of the charge elevated to the height of its radius, to the energy of explosion. In other words, expressions of type (41, 42, 48) show the altitude expressed in radii of the charge to which a mass equal to the mass of the charge can be elevated in the gravitational field because of total explosion energy.

R. M. Schmidt and K. A. Holsapple [1] analyze the data obtained on the centrifuge of the Boeing firm with contact explosions on the surface of dry sand. To reduce the volume they use expression (46) and the radius and depth were reduced respectively by formulas

\[ \pi_R = R \left( \frac{\rho}{q} \right)^{1/3}, \]

\[ \pi_H = H \left( \frac{\rho}{q} \right)^{1/3}. \]

K. A. Holsapple and R. M. Schmidt [2] recently discussed in detail questions of selecting the system of variables describing a contact explosion. In designations (46), (48) - (50) the experimental data look like

\[ \pi_\nu = 0.194 \pi_2^{-0.482 \pm 0.05}, \]

\[ \pi_R = 0.765 \pi_2^{-0.19 \pm 0.03}, \]

\[ \pi_H = 0.154 \pi_2^{-0.16 \pm 0.14}. \]
in the range $10^{-8} < \eta_2 < 2 \cdot 10^{-4}$. If dimensionless expressions are transformed to normal variables, considering the values of the constant for PETN ($\rho = 1.8 \text{ g/cm}^3$, $Q = 5.4 \times 10^{10} \text{ erg/g}$) and density of sand $\rho = 1.8 \text{ g/cm}^3$, then these relationships adopt the appearance

\begin{align*}
V(M^3) &= 60(g/g_0)^{-0.472}q^{0.842}(T), \\
R(M) &= 5.33(g/g_0)^{-0.189}q^{0.286}(T), \\
H(M) &= 1.15(g/g_0)^{-0.164}q^{0.279}(T).
\end{align*}

Thus, based on the system of data on cratering in noncohesive media, one can consider the correlations between the size of the crater and the weight of the charge to be experimentally proven and theoretically substantiated, considering the effect of the gravitational force. For the more frequently used characteristic of a crater (its radius) the following correlation is correct

\begin{equation}
R \sim g^{-\frac{1-n}{3}},
\end{equation}

where the quantity $n$ is in limits from 0.111 to 0.165. With constant $g$, this corresponds to a relationship of the type $R \sim g^{-\frac{1-n}{3}M}$, where $M = \frac{3}{(1 - n)}$ is in limits 3.4 - 3.6. The previously presented generalizations of data on large-scale explosions near the surface (see, for example, R. B. Vaile [1]) lead to the value $M = 3.4$, which agrees completely with results of model experiments with variable $g$.

The use of a similar approach to cratering in ground with finite cohesion should answer the important question about the scales in which one can ignore the gravitational force and in which it needs to be considered.
For example, according to the summary of data presented in the survey of A. T. Bazilevskiy and V. A. Ivanov [1], contact explosions of soft ground are subordinate to the law of geometric similarity at least in the range $100 \text{ kg} < q < 1 \text{ kT}$. Extrapolation of these data towards charges of greater weight requires discussing the boundary for the beginning of the influence of the gravitational force.

K. A. Holsapple and R. M. Schmidt [1] have suggested a system of variables which describe cratering during contact explosion in a fairly general case of specifying the strength properties of the medium in the form of a Mohr-Coulomb model. The employed system of variables looks like: density of ground $\rho$, cohesion $c$, angle of internal friction $\phi$, weight of explosive charge $q$, specific energy release of the explosive $Q$, density of the explosive $\delta$, acceleration of the gravitational force $g$. These variables are united into the following groups of dimensionless variables: scale of explosion $\pi_1 = \frac{g}{Q \delta^{1/3}}$, ratio of densities $\pi_3 = \frac{\rho}{\delta}$, parameter of internal friction $\pi_5 = \frac{g \varphi}{\delta}$.

The effectiveness of cratering can be expressed through the parameter $\pi_v = \frac{\rho V}{q}$. Then K. A. Holsapple and R. M. Schmidt [1] made a simplified examination of the balance of energy which according to their concept was formed from work against the strength of forces and the gravitational forces. For craters of a certain radius $R$ which have a similar shape, a general expression is written for the necessary energy of cratering which comprises a fraction of $\eta$ of the total explosion energy $E$

$$\eta E = k_1 c R^3 + k_2 g R^4 \tan \varphi + k_3 g R^4.$$  

Based on a similar examination, the following parameter is introduced which depends on the strength properties of the medium

$$\pi_2' = \frac{c}{pQ (\tan \varphi + 0.1)} + \frac{g}{Q \delta^{1/3}}. \quad (55)$$
whose substitution into expressions (51) - (53) results in formulas for the dependence of the parameters of the contact explosion craters and cohesive ground. For example, for ground with angle of internal friction $\phi = 35^\circ$ (tg $\phi = 0.7$) the expression is obtained

$$\rho V/q = 0.174 \left[ \frac{c}{\rho Q} + \frac{g}{Q} \left( \frac{q}{\delta} \right)^{1/3} (\log \varphi + 0.1) \right]^{-0.472},$$

which satisfactorily describes the experimental data of both laboratory experiments and full-scale experiments.

From expressions (55) and (56) it is easy to evaluate the boundary values of the weight of the charge $q_{KP}$ and the radius of the crater $R_{KP}$ which separate the ranges of insignificance and the importance of considering the gravitational force. It is evident that this boundary can be conditionally made with

$$\frac{g}{Q} \left( \frac{q_{KP}}{\delta} \right)^{1/3} = \frac{c}{\rho Q (\log \varphi + 0.1)}.$$ 

We substitute here the density of a typical explosive $\delta = 1.6 \text{ g/cm}^3$, $Q = 4.2 \times 10^{10} \text{ erg/g}$ and parameters of typical soft ground $\varphi = 0.3$, $\rho = 2 \text{ g/cm}^3$, with $\phi = 30^\circ$. Then

$$q_{KP}(r) \approx 680 \frac{c}{(g/\delta)^{1/3}} \text{ kg-f/cm}^2 \quad (57)$$

In this case the parameter

$$\pi'_{2KP} = \frac{2c}{(\log \varphi + 0.1)^2 Q} \approx 5.2 \times 10^{-4} c \text{ kg-f/cm}^2$$

then, substituting the value $\pi'_{2KP}$ into (52) and solving (51) in relation to the crater radius $R$, we obtain

$$\bar{R}_{KP}(m) \approx 40 (g/\delta)^{-1} c^{0.341} \text{ kg-f/cm}^2 \quad (58)$$

If for typical soft ground we adopt the magnitude of cohesion $c = 1 \text{ kg-f/cm}^2$, then for terrestrial conditions of the critical weight
of the explosive charge will be about 700 T, while the critical radius of the crater will be about 40 m.

For rocky ground, evaluation of the critical parameters of the explosion is difficult because of indefiniteness in the quantity \( c \), which, as already noted, differs in large scale from the quantities measured on laboratory samples. If for the evaluation we adopt the values of cohesion used in numerical calculations (J. G. Trulio [1], G. W. Ullrich, D. J. Roddy and G. Simmons [1]) for rocky ground at the site of conducting the full-scale experiments as equal to \( 50 - 100 / 108 \) kg-f/cm\(^2\), then the critical radii of the craters separating the strength of gravitational ranges of the scale of phenomena for terrestrial conditions according to [58] are

\[
R_{\text{cr}} \approx 1000 \div 2000 \text{ m.} \tag{59}
\]

However significant loss of cohesion because of destruction of rocky ground during intensive shear deformation hypothesized in the more realistic models of the behavior of rocks (R. P. Swift [1], P. F. Korotkov [1], see section 1) eliminates the necessary confidence in evaluation (59). Moreover one should note that the actual derivation of correlation (55) and adjustment of the numerical coefficients were carried out by K. A. Holsapple and R. M. Schmidt [1] based on experimental material on soft ground with initially low magnitudes of cohesion.

J. D. O'Keefe and T. J. Ahrens [5, 6] came to similar conclusions in constructing the model for growth of the depth of a meteorite crater based on the simple model of cratering described in section 3. In their first work, J. D. O'Keefe and T. J. Ahrens [5] used the model of shear strength of ground of Coulomb-Mises (14) (see section 1). According to the calculations there are three ranges of crater sizes: range I (smallest sizes) in which there is a dominance of cohesion of material and the role of the gravitational force is insignificant; range II (intermediate sizes) in which there is a dominance of shear strength depending on the lithostatic pressure; range III in which there is a
dominance of the limiting shear strength and the influence of the gravitational force is manifest only in the form of the transition of kinetic energy of the substance into potential reserved during elevation in the gravitational field. Cases were examined (for the designations see section 1, formula (14)):

1) \( Y_0 = Y_k = 0.1 \text{ kg-f/cm}^2 \), the gravitational force dominates in the entire range of crater depths from 1 m to 300 km (the calculations here and further were made for lunar value of gravitational force);

2) \( Y_0 = Y_k = 10 \text{ kg-f/cm}^2 \), the strength mode of geometric similarity for craters with depth less than 1 km. According to formula (58) for this case \( R_{kp} = 1700 \text{ m} \), while the depth which can be adopted as \( 1/2 R_{kp} \), \( H \approx 800 \text{ m} \);

3) \( Y_0 = Y_k = 2.7 \text{ kbar} \), dominance of strength and independence of the gravitational force to depth \( H \) on the order 35 km;

4) \( Y_0 = 0, Y_k = 2.7 \text{ kbar} \), dominance of strength to \( H = 20 \text{ m} \) depending on the gravitational force of range \( H \) 20 m to 3 km;

5) \( Y_0 = 10 \text{ kg-f/cm}^2 = 0.01 \text{ kbar}, Y_k = 2.7 \text{ kbar} \). Strength mode to \( H' \approx H' = 90 \text{ m} \) depending on the gravitational force of range \( H \) from 90 m to 3 km.

We thus see that depending on the proposed type of destruction surface, the boundaries of different ranges shift considerably, and the most unrealistic case (3) results in laws for a crater of almost 100 km in size which do not depend on the gravitational force.

In their next work, J. D. O'Keefe and T. J. Ahrens [6] modified the strength model by a method of the type used by R. P. Swift [1], cohesion \( Y_0 \) in the model was made dependent on pressure in a shock wave:

\[
Y_0 = (Y_1 - Y_2)(p_2/p)^\alpha + Y_2
\]

with \( p > p_2 \), where \( p_2 \) is the minimum destroying cohesion of the material. It was assumed for specific calculations \( p_2 = 10 \text{ kbar}, \alpha = 1, Y_2 = 10^{-7} \text{ kbar} \) (essentially zero), \( Y_1 = 10^{-2} \text{ kbar} \) or 2.7 kbar, with preservation of the previous magnitude of maximum shear strength \( Y_k = 2.7 \text{ kbar} \). In the variant with \( Y_1 = 10^{-2} \text{ kbar} = 10 \text{ kg-f/cm}^2 \), the effective boundary
between the strength and gravitational ranges could be considered the depth of a crater on the order of 80 m, which is almost 20-fold lower than the case of preserving the magnitude of cohesion constant.

These, although very approximate evaluations show that consideration of the loss of cohesion with gradual destruction of rocks could be decisive in determining the boundaries of the ranges of dependence and independence of crater and funnel sizes on the magnitude of the gravitational force.

Returning to the experiments on centrifugal modeling on the geo-engineering centrifuge of the Boeing firm, one should also note the two works of R. M. Schmidt [2, 3].

R. M. Schmidt [2], based on concepts described in detail above reproduced under laboratory conditions the crater of a shallow nuclear explosion Johnie Boy. Numerical two-dimensional calculations for the nuclear explosion were used to select the charge of the chemical explosive whose explosion reproduced at a definite stage the velocity field of the nuclear explosion. The full-scale weight of the model charge of PETN approximately equalled 50 T. Explosion of a similar full-scale charge was modeled by explosion in a centrifuge of a PETN charge weighing 120 g with magnitude of centrifugal acceleration 345 g₀. The model ground was alluvium screened through sieve #14 (diameter of the hole 1.41 mm by American standard ASTM), with moisture content 4.1%. At the opposite ends of the centrifuge rotor two model explosions were simultaneously conducted, with depth of the center of gravity of the charge 0.36 cm and 0.84 cm. The correspondence between the model and the full-scale craters for these depths by volume was: -22% and +16%, by radius: -1% and +13%, by depth: -16% and +2%, by ratio of diameter to depth: +19% and +11%.

In another work, R. M. Schmidt [3] describes experiments on high-speed impact in sand for which a small light-gas gun rotating together with the centrifuge was designed. The velocity of the impact object
was 2 km/sec. Based on these experiments, K. A. Holsapple [1] attempted to construct a system of variables to describe the link between the parameters of the impact object and the dimensions of a formed crater. Assume that $E^*$ is the energy of the impact object, $U$ is its velocity, $\delta^*$ is the density. Then the density of internal energy in the impact object equals $(1/2) U^2$, while the mass is $M = 2 E^*/U^2$. The ratio of crater mass $\rho V$ to mass of the source (46) in the case of impact, according to the logic of K. A. Holsapple [1] adopted the appearance

$$\pi_V^* = \frac{\rho V}{M} = \rho V U^2 / 2E^*. $$ (61)

The effective scale of impact is written in the form (42)

$$\pi_e^* = \frac{q}{(0.5 U^2)^{1/2}} \left( \frac{E^*}{\delta^*} \right)^{1/3} \frac{3.22 \pi a}{U^2} .$$ (62)

where $a$ is the radius of the impact object on the assumption that its shape is spherical. Then the experimental data of R. M. Schmidt [3] for impacts with velocity $U \sim 2$ km/sec can be presented using (61) and (62) in the form

$$\pi_e^* = 0.25 (\pi_V^*)^{-0.501}.$$ (63)

for the case of noncohesive medium, dry sand.

Based on correlation (63), R. M. Schmidt [3] makes a bold attempt from laboratory data of centrifugal modeling with relatively low-velocity impact objects and target made of dry sand to evaluate the energy of formation of the Arizona Meteorite Crater and arrives at numbers which differ significantly from the generally accepted value 4 - 5 MT of TNT for velocity of impact object 15 - 20 km/sec. According to R. M. Schmidt [3], the energy of the meteorite to form the Arizona crater for a meteorite with hypothetical velocity 15 km/sec was $(34 \pm 6)$ MT, and with velocity of the impact object 25 km/sec, it was $(51 \pm 10)$ MT, which is roughly an order greater than the estimates made by extrapolation of data from shallow nuclear explosions. In order to explain
the reasons for this discrepancy, an additional analysis of the work of K. A. Holsapple [1] and R. M. Schmidt [3] is required of course. We will merely note that with constant gravitational force and mass of the impact object, the set of relationships (61) - (63) results in a dependence of the crater volume and the velocity of the impact object in the form

\[ V \sim U^{1.02}. \]  \hspace{1cm} (64)

that is, the volume of the crater essentially depends only on the impact object impulse. The laboratory data of D. E. Gault and J. A. Wedekind [1] result in the relationship (37)

\[ V \sim D^3 \sim U^{1.29} \]  \hspace{1cm} (65)

with velocities of impact in sand 1.5 - 8 km/sec.

On the other hand, calculations of high-speed impact on metals and rocks for the equation of state in the form Tillotson (see section 111) and their conversion to gravitational craters (B. A. Ivanov [5]) show that

\[ V \sim U^{1.5}. \]  \hspace{1cm} (66)


\[ V \sim D^3 \sim U^{1.26} \]  \hspace{1cm} (67)

for a target whose properties have been described from data for limestone.

Comparison of the expressions presented above (64) - (67) indicate the importance of the type of equation of state determining the dependence of the effectiveness of cratering on the velocity of the impact.
object. Therefore transfer of data on dry sand for great velocities of impact could result in an exaggeration of the required energy as compared to the data on denser rocks.

As already noted in section 2, comparison of the calculations of damping of impact waves shows the permissibility of energy estimates without consideration of the first approximation for the effect of the velocity of the impact object.

M. R. Dence, R. A. F. Grieve and P. B. Robertson [1] have suggested a formula for linking the energy of a meteorite $K$ with the diameter of a terrestrial meteorite crater, based on extrapolation of data for shallow nuclear explosions:

$$D(\text{KM}) \approx 2 \times 10^{-5} K^{1/3.4} \text{[J]}.$$  

Introducing into this expression dependence on the gravitational force in the form (54), B. A. Ivanov [7] obtained a formula:

$$D(\text{KM}) \approx 0.3 \left(\frac{g}{g_0}\right)^{-0.12} K^{1/3.4},$$

where $K$ is expressed in MT of TNT. Solving this expression for $K$, we obtain a convenient formula for evaluating the energy of cratering for the diameter of a crater:

$$K(\text{MT TNT}) \approx 2.4 \left(\frac{g}{g_0}\right)^{0.14} D^{3.4}(\text{KM}).$$  \hspace{1cm} (68)

This formula can be verified for terrestrial conditions, using data on volumes of solidified melt found by geologists in meteorite craters (see, for example, V. L. Masaytis and M. F. Mashchak [1], W. C. Phinney and C. H. Simonds [1]). Using estimate (23) (see section 2) based on numerical two-dimensional calculations for dense rock, and evaluation (68) we obtain a link between the mass of melt formed during impact of the meteorite, and diameter of the crater:
Evaluation (69) which is converted to the volume of impact melt on the assumption of density 3 g/cm³, is presented in Table 3 for some terrestrial meteorite craters. The quite reliable coincidence of the observed calculated data based on joint use of numerical calculations and data on nuclear explosions shows the applicability of evaluation (68).

<table>
<thead>
<tr>
<th>Crater</th>
<th>D, km</th>
<th>Volume of Melt, km³</th>
<th>Maximum Depth of Melting: Evaluation (79) zₘ, km</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brent, Canada</td>
<td>3.8</td>
<td>0.03</td>
<td>0.29</td>
</tr>
<tr>
<td>Mistastin, Canada</td>
<td>20</td>
<td>8.5</td>
<td>12¹</td>
</tr>
<tr>
<td>Boltysh, USSR</td>
<td>22</td>
<td>12</td>
<td>2.0</td>
</tr>
<tr>
<td>West Clearwater, Canada</td>
<td>32</td>
<td>43</td>
<td>From 34 to 50¹</td>
</tr>
<tr>
<td>Maniaguagan, Canada</td>
<td>65</td>
<td>470</td>
<td>6.6</td>
</tr>
<tr>
<td>Popigay, USSR³</td>
<td>75 or 100</td>
<td>From 800 to 1750²</td>
<td>9</td>
</tr>
</tbody>
</table>

¹ W. C. Phinney and C. H. Simonds [1]
² V. L. Masaytis and M. S. Mashak [1]
³ The Popigay crater was formed in a stratified target and has diameter for the rock base 75 km, for the upper sedimentary mantle about 100 km (V. L. Masaytis, M. V. Mikaylov and T. V. Selivanovskaya [1]).

We note that according to (69), the mass of melt in the lunar craters should be approximately half that in terrestrial craters of equal size.

Using evaluation (68) one can also assess the depth of melting of rocks zₘ under the center of impact, substituting (68) into (22) (see section 2):
Thus, by using estimates of type (68) one can make a number of interesting estimates which can be used to discuss the structure of meteorite craters on Earth and other planetary bodies of the solar system.

6. Structure of Meteorite Craters

As established by studying space photographs of the Moon, Mercury, Mars and other planetary bodies of the solar system, the entire diversity in meteorite craters on their surfaces can be classified by using two variables, scale and geological age. Insofar as the main characteristics of the crater for remote studies could only be its external appearance, one can speak of definite morphological classes of meteorite craters, distinguishing the craters within one morphological class for relative geological age. Below we will discuss only the data for the most geologically fresh craters, whose structure is complicated to the least degree by subsequent geological processes.

Meteorite craters are customarily divided by structure into two types: simple dish-shaped and complex craters including craters with central elevations (hillock), craters with annular elevations and multi-ring basins.

These types and classes of morphological craters are common for the Moon, Mercury and Mars (see, for example, the work of K. P. Florenskiy, A. T. Bazilevskiy and N. N. Grebennik [1]), for satellites of Jupiter, Ganymede and Callisto (see, for example, S. K. Croft [3]),
for satellites of Saturn, Dione, Rhea and Mimas (A. T. Bazilevskiy [1]), and of course, for the Earth (V. L. Masaytis, A. N. Danilin, et al. [1], M. R. Dence, R. A. F. Grieve and P. B. Robertson [1]). On each of the planetary bodies one can isolate boundary values for the diameters of the craters which separate fairly distinctly the ranges of different morphological types of craters. Sometimes the dependence of the boundary diameters on the type of target can be isolated. For example, for the Earth dimensions of simple dish-shaped craters of the Arizona type do not exceed in diameter 4 km in the case of a target made of dense crystalline rocks and 2 - 2.5 km in the case of less dense sedimentary rocks (see, for example, M. R. Dence and R. A. F. Grieve [1]).

Simple craters differ from complex because of the basically different dependence of the depth of the crater \( H \) on the diameter of the crater \( D \). (Generally photogeological interpretation of space photographs measures the diameter of the crater \( D \) on the crest of the swell; below we will designate as \( D_a \) the diameter of the crater on the original surface).

For example, for the Moon according to data cited by R. J. Pike [1], for simple craters

\[
H = 0.196 D^{1.01} \tag{71}
\]

with \( D < 15 \) km, and for complex craters \( (D > 15 \) km)

\[
H = 1.04 D^{0.301} \tag{72}
\]

We will indicate for comparison that dependence of depth in relation to the level of the initial surface \( H_a \) on diameter \( D_a \) looks like (R. J. Pike [2]): for simple craters \( (D < 15 \) km)

\[
H_a = 0.225 D_a^{1.91} \tag{73}
\]
for complex craters \( D < 15 \text{ km} \)

\[
H_a = 0.748D^{0.304}. \quad (74)
\]

All the dimensions in formulas (71) - (74) are given in km.

The most detailed study of craters of the Moon provides the grounds for more detailed comparison of the cratering conditions of different regions. For example, R. J. Pike [6] noted that complex craters in lunar continental regions formed by the most ancient rocks that were exposed to meteorite bombardment to a strong degree are 12 - 18% deeper than craters on the surface of lunar "seas" covered with a layer of younger basalts and destroyed by meteorite impact to a lesser degree. Instead of correlation (72) for complex craters, R. J. Pike [6] proposes the relationship \((D > 15 \text{ km})\)

\[
H = 1.028D^{0.317} \quad (72')
\]

for continental regions and

\[
H = 0.819D^{0.341} \quad (72'')
\]

for sea regions.

Relationships \( H(D) \) for Mercury and Mars can be expressed in the same way. For example, according to the latest data which were published by R. J. Pike [4] for simple Martian craters \((D < 4 - 6 \text{ km})\)

\[
H = 0.162D^{0.935}. \quad (75)
\]

\[
\cdot H = 0.343D^{0.135} \text{ with } D > 4 - 6 \text{ km.} \quad (76)
\]

For simple craters of Mercury with \( D < 10 \text{ km} \) (M. C. Malin and D. Dzurisin
for complex craters \( D > 10 \, \text{km} \)

\[
H = 0.176 D^{1.960}
\]  

(77)

\[
H = 0.910 D^{0.260}
\]  

(78)

Comparison of correlations \( H(D) \) for simple craters of Mars, the Moon and Mercury \((71), (75), (77)\) demonstrate first, similarity of simple craters on each planet, and second, their common proximity to each other in relation to \( H/D = 0.2 \) for the Moon, 0.18 for Mercury and 0.16 for Mars.

With diameters greater than the critical, the depth of craters rises as \( D^{0.3-0.4} \) and does not exceed 5 - 6 km on all planets.

The most critical parameter of planetary bodies of terrestrial type is approximately inversely proportional to the gravitational force \((R. J. Pike [3])\): complication of the crater shape begins on the Moon with \( D = 15 \, \text{km} \) (gravitational force about 1/6 of the terrestrial), on Mars with \( D \approx 4 - 6 \, \text{km} \), on Mercury with \( D \approx 10 \, \text{km} \) (gravitational force in both cases is about 1/3 of the terrestrial) and on Earth with \( D \approx 2 - 4 \, \text{km} \). The ice-covered satellites of Jupiter and Saturn, as shown by A. T. Bazilevskiy \([1]\) are not subordinate to the terrestrial sequence, complicated craters on them appear with much smaller diameters than could be assumed by extrapolating data regarding the critical diameters on planetary bodies of the terrestrial group for lower values of the gravitational force. For example, S. K. Croft \([3]\) indicates that on Jupiter's satellites of Ganymede and Callisto, simple craters with an increase in diameter disappear earlier than on the Moon, although the gravitational force on the surface of these bodies equals the lunar.

Study of the structure of terrestrial meteorite craters of different type could yield rich information for interpreting the available data on craters of other planets. Recently a whole series of
generalizing works were published both on individual craters (for example, V. L. Masaytis, M. V. Mikhaylov and T. V. Selivanovskaya [1], P. V. Florenskiy and A. I. Dabizha [1]), and for whole regions of the Earth (A. A. Val'ter and V. A. Ryadenko [1], V. L. Masaytis, A. N. Danilin, et al. [1], M. R. Dence, R. A. F. Grieve and P. B. Robertson [1]) which make it possible to present fairly completely the law governing the change in the structure of terrestrial craters with a growth in their size.

We will dwell in somewhat more detail on one question, the depth of complex terrestrial meteorite craters. As shown by the aforementioned geological descriptions of the studied craters, in the range of their diameters for 3 - 4 km to approximately 50 - 100 km, in the center of the crater there is mandatorily a central elevation which, it is assumed, in its origin is similar to central hillocks on other planetary bodies. Geological data on craters in targets with horizontally stratified structure make it possible to compare the depth of occurrence of individual layers beyond the crater and under its bottom. These data show that the residual shifts in rocks on the bottom of the crater are directed upwards. The amplitude and elevation $\Delta z$ for the most well-studied craters are: in the Steinheim crater, FRG, diameter $D = 4$ km, $\Delta z \approx 0.13$ m (W. Reiff [1]), in the Flynn creek crater, United States $D = 4$ km, $\Delta z \approx 0.4$ km (D. J. Roddy [1]), in the Decaturville crater, United States $D = 6$ km, $\Delta z \approx 0.3 \pm 0.5$ km (T. W. Offield and H. A. Rohn [1]), in the Sierra Madre crater, United States $D \approx 13$ km, $\Delta z \approx 1$ km, in the Gosses Bluff crater, Australia $D \approx 20$ km, $\Delta z \approx 2.5 \pm 3$ km (M. R. Dence, R. A. F. Grieve and P. B. Robertson [1]).

The volume of impact-molten material observed in the crater (Table 3) could be another source of estimates regarding vertical movement of the crater bottom. If we assume that initially the boundary of the melting zone coincided with the isobaric curve of pressure in the impact wave sufficient for melting rocks during discharge (400 - 600 kbar for typical crystalline rocks), then for simplicity, assuming a
hemisphere as the shape of the isobaric curve, one can obtain an evaluation for the maximum depth of rock melting under the impact point

\[ z_M \approx (3V_{M_{\text{isob}}} / 2\pi)^{1/3}, \]  

(79)

where \( V_{M_{\text{isob}}} \) is the volume of impact melt in evaluating from geological data (see Table 3).

Insofar as the central elevations in the terrestrial meteorite craters consist of rocks that were not exposed to melting, while the current depth of complex craters is much less than the evaluations of the depth of the melting zone, the quantity \( z_M \) (79) can be considered an evaluation of the amplitude of elevation of crater bottom during its formation.

Data on craters in stratified targets and evaluation based on the volume of the melt can be summarized in the form of an empirical law according to which the magnitude of residual shape of rocks upwards in relation to their original position in the terrestrial meteorite craters with diameter from 4 to 100 km is approximately

\[ \Delta z \sim D/10. \]  

(80)

Without dwelling on numerous speculative models that are repeatedly advanced to explain elevation, we will indicate that under laboratory conditions significant movement upwards of crater material after the formation of an intermediate crater was observed only in water-saturated sand, which is a clear manifestation of the gravitational modification of the intermediate crater (A. J. Piekutowski [1]), and in an extremely exotic medium made of finite cut rubber powder, where motion of material upwards had distinct nature of elastic springiness which was recorded by high-speed filming (B. A. Ivanov and A. T. Bazilevskiy [1]).

Under field conditions, large-scale contact explosions sometimes result in the formation of central elevations and (or) displacement of
the ground under the crater upwards in relation to the original occurrence (see, for example, G. H. S. Jones [1, 2], D. J. Roddy [2]).

Generally complication of the shape of contact explosion craters is related to the geology of the experimental areas, and mainly, the presence of water near the surface. J. Wisotski [1] has summarized some geological parameters of areas for conducting the latest contact explosions in the United States.

Explosions at the Pacific Ocean test site of the United States located on Bikini and Eniwetak atolls are especially clear examples of the decrease in relative depth of the craters of large-scale explosions in a water-saturated medium. Here the ratio of depth to the radius of the crater with radius up to 1 km reaches 0.02 (L. J. Vortman [1, 2]). Some data on the geological study of craters of contact nuclear explosions are cited by J. Vizgirda and T. J. Ahrens [1]. According to the data of drilling with geochemical study of a core sample and determination of the natural $\gamma$-radioactivity of rocks, the true depth of the crater of the 17 kT nuclear explosion Cactus was $18 \pm 2$ m.

Transfer of experimentally obtained data regarding the decrease in relative depth of experimental craters under definite geological conditions to meteorite craters of the Earth encounter significant difficulties because of the lack of clarity regarding both the properties and the methods of describing the properties of large masses of rocks. This was attempted in the series of publications of H. J. Melosh and W. C. McKinnon [1, 2] in the example of using the traditional theory of ground plasticity.

The main idea of H. J. melosh [1, 3] is the hypothesis that by the end of the stage of material ejection a certain intermediate crater of simple dish shape is formed whose stability in the gravitational field depends on the crater dimensions. In his first work, H. J. Melosh [1] studied the limiting problem of stability in the gravitational field of a plastic half-space with cylindrical or parabolic depression. The strength properties of the material were described by cohesion c, ratio
of the depth of the depression to its radius based on data regarding simple craters on the Moon and planets (71), (73), (75), (77) was selected equal to \( \frac{H}{D} = 0.2 \). In solving the ideally plastic program with axial symmetry, a standard assumption was made that the intermediate main stress equals the maximum main stress.

The result of the calculations was ranges of ratio \( \frac{p_g H}{c} \) in which different types of areas of sliding lines are realized. With \( \frac{p_g H}{c} < 5 \), the craters with ratio \( \frac{H}{D} = 0.2 \) are stable. In the range \( \frac{5}{2} < \frac{p_g H}{c} < 7 \) there is collapse of the crater edges which is interpreted as forming terraces on the edges. With \( \frac{7}{2} < \frac{p_g H}{c} \leq 16 \), destruction develops on the bottom of the crater which has the possibility of moving upwards during collapse of the crater edges.

Comparison of this model with the data on lunar craters indicated that the general course of change in crater structure could be described by this model, if we assume cohesion of the lunar ground \( C = 20 - 30 \, \text{kg} \cdot \text{f/cm}^2 \).

As already noted, the boundaries of morphological types of craters on different planets change in the first approximation inversely proportional to the gravitational force, therefore this evaluation of cohesion must be applicable to all planetary bodies of the terrestrial group.

In developing the model of collapse, W. C. McKinnon [1] examined the same problems for a medium with condition of maximum equilibrium of Mohr–Coulomb. It was found that with preservation of the condition \( \frac{p_g H}{c} > 5 \) for the beginning of collapse of the crater edges, spread of the boundary of creeping towards the center of the crater fundamentally depends on the magnitude of the angle of internal friction \( \phi \). If with \( \phi = 2^\circ \), the pattern of destruction is qualitatively maintained the same as in the work of H. J. Melosh [1], then already with \( \phi = 5^\circ \), the craters with \( \frac{H}{D} = 0.2 \) on the whole are stable in the entire range of crater dimensions, assuming only the formation of
terraces on the slopes with $\rho g H/c > 5.5$.

Such low angles of internal friction needed for reproduction of the sequence of change in morphological forms of meteorite craters in the framework of the traditional theory of plasticity with an increase in their size casts doubt on this approach and require additional studies.

The authors of this model, H. J. Melosh and W. C. McKinnon [2] verified it by testing for stability of craters in clay with salt on the influence of centrifugal accelerations in the already mentioned geotechnical centrifuge of the Boeing firm. Cohesion of the model material was 0.05 - 0.2 kg-f/cm², acceleration up to 100 - 150 $g_0$. The experiments confirmed the model calculations and showed that with $\rho g H/c < 5$ the craters are stable, while with $\rho g H/c > 5.5$ they creep until they reach an equilibrium value $\rho g H/c = 5.1$.

H. J. Melosh and W. C. McKinnon [2] noted an analogy between the anomalous behavior of large-scale landslides on Earth and creeping craters on the Moon. In both cases description of the processes in the classic mechanics of ground requires assuming unusual properties of the medium. Examining the data on giant avalanches and landslides, S. S. Grigoryan [2] used a new model according to which friction in the rock seams during loading above a definite magnitude ceases to depend on pressure according to the Coulomb law and becomes equal to shearing strength of the weakest of the rubbing materials. In turn, H. J. Melosh [2] proposed a model of acoustic thinning of pulverized rock streams because of some perturbations generally during their high-speed deformation. This model which is interesting in idea cannot yet be confirmed by estimates because we do not know such parameters as, for example, the Q factor of large masses of moving rock.

One can thus state that the class of mechanics of rocks does not so much explain the observational data, as it finds in them problems for its own self-development. We hope that this process will be
fruitful both for mechanics and for planetology.

S. K. Croff [4] focused on the fact that even in the absence of very developed collapse of slopes one can assume a considerable elevation of the bottom of simple craters of the Arizona type because, as we have seen, of reserves of elastic energy. R. M. Schmidt and K. A. Holsapple [2] made an experimental study in the centrifuge on the formation of an explosive crater with high accelerations. Explosion of a contact charge of PETN in modeling clay created two craters, one with acceleration $10 \, g_0$, the other with acceleration $50 \, g_0$. After this, the first crater was exposed to the effect of acceleration $500 \, g_0$, which modifies it. The result of the dual successive process "explosion + increased gravitational force" was almost accurate reproduction of the explosion crater with increased gravitational force. The authors claim that on the cross section of the sample that experiences the dual effect, no traces of intermediate stage of the process were visible, a crater formed after explosion with $10 \, g_0$, which indicates the high degree of reversibility of the process of deformation in modeling clay during explosive cratering and subsequent gravitational modification. At the same time, this circumstance indicates the basic difference between deformation in a plastic model medium and irreversibly destroyed rocks which reduces the information content of similar experiments from the viewpoint of understanding the natural large-scale processes.

In addition the observational data indicate the change in structure of even simple craters with a rise in their side. D. L. Orphal [3] compared the thickness of a layer of crushed shifted material (allo- genetic breccia) $d_B$ on the bottom of several craters of contact explosions and meteorite craters of dish-shape and obtained the correlation

$$d_B \approx 0.11 \, D^{1.318}$$

($d_B$ and $D$ are in km) which indicates that the differences between the depth of the true and the visible craters increases with a rise in the
dimensions of the crater. In meteorite craters Lonar ($D = 1.83$ km) in basalt and Arizona ($D = 1.2$ km) and sedimentary rocks, the thickness of layers of allogenic breccia according to the data of drilling is respectively 220 and 175 m (about 50% of the total depth of the true crater).

The presence in the crater of a layer of crushed rock, as well as formation under the crater of a zone of crushing and fracturing because of thinning of the destroyed rocks result in the existence of gravitational anomalies related to craters. These anomalies are small in amplitude (up to 50 mgal, 1 gal = $1$ cm/sec$^2$) but are one of the reliable methods of searching for the buried meteorite craters.

Like the shape of craters, the gravitational anomalies related to them are complicated as the diameter of the structure rises from simple circular negative anomalies to complex sign-variable zone of perturbations which, however, preserves a certain central symmetry (A. I. Dabizha, V. V. Fedynskiy [1], A. I. Dabizha, B. A. Ivanov [1]).

The amplitude of negative gravitational anomalies with diameter up to 4 - 10 km rises together with a rise in dimensions of the crater indicating preservation of definite similarity in the structure of the subcrater zone of thinning (R. J. Pike [5]).

For craters with diameter above 30 km, the amplitude of anomalies is essentially the same and does not go beyond 20 - 30 mgal. For zones of destruction of meteorite craters on Earth, the value of the thinning magnitude $\Delta \rho \approx 0.1$ g/cm$^3$ (see, for example H. D. Ackerman, R. H. Godson and J. A. Watkins [1], J. Pohl [1]). These anomalies correspond to thickness of the layer of thin rocks 5 - 7 km. The dependence which is observed for large terrestrial craters of the "defective" mass $\Delta M$ computed from that magnitude of the gravitational anomaly on the diameter of the crater is related to the maximum depth of the thinning zone (A. I. Dabizha and V. A. Fedynskiy [1]): in simple craters $\Delta M \sim D^2$, this corresponds to similarity of the thinning zones, in large complex
craters $\Delta M \sim D^2$, indicating that the zone of thinning with an increase in diameter rises only to the side, preserving approximately constant thickness.

The reason for constant thickness of the thin layer could be suppression of dilatancy of deep rocks because of lithostatic pressure and temperature (V. M. Nikolayevskiy [1]). Pinpointing of the data on gravitational anomalies of geologically fresh meteorite craters (see, for example, J. Dvorak and R. J. Phillips [1]) will create, one hopes, the basis for comparing the reaction of a crust of two planetary bodies differing in internal structure to the same mechanical effect, high-velocity meteorite impact.
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Translation: Teoriya udara: nekotorye obshchiye printsipy i metod rascheta v eylerovykh koordi-
Translation: Teoriya protsessov kratero-obrazovaniya (obzor), in Udar, vzryv i razrusheniye, Moscow, Mir, 1981, 8-42.


Main concepts and theoretical models which are used for studying the mechanics of cratering are discussed. Numerical two-dimensional calculations are made of explosions near a surface and high-speed impact. Models are given for the motion of a medium during cratering. Data from laboratory modeling are given. The effect of gravitational force and scales of cratering phenomena is analyzed.