PASSIVE DAMPING AUGMENTATION FOR
FLEXIBLE STRUCTURES

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FIRST NASA/DOD
CSI TECHNOLOGY CONFERENCE
Norfolk, Virginia
November 18-21, 1986
INTRODUCTION

Many proposed future large space structure designs, including the NASA Space Station, may need to incorporate active and/or passive damping mechanisms in order to meet pointing, slewing, or microgravity acceleration requirements. Methods for implementing active and passive damping have been the subject of many studies which have indicated the merits of passive damping, either in itself or in concert with active damping.

Incorporation of passive damping for vibration suppression in the design of large space structures offers many benefits. Passive dampers require no power source, are inherently stable, and are potentially simple and reliable. Properly designed passive damping treatments can greatly reduce the settling time in transient response problems and reduce the peaks of steady-state response problems. The existence of small amounts of passive damping in an active control system can reduce active control effort such as actuator force, stroke, bandwidth, and system penalties such as the number of actuators, added mass, cost, and on-board power and microprocessing needs. Passive damping devices also provide an increased safety margin for all active control systems.

Passive damping can be added to a structure through a variety of mechanisms including constrained layer treatments, impact/friction joints, discrete viscous dampers, electromagnetic devices, fluidic devices, and tuned-mass dampers. Each of these damping treatments performs best for certain classes of damping problems. The tuned-mass damper is especially well-suited for damping large structures which are characterized by low, highly distributed, strain energy, e.g., the NASA Space Station. Although initially dubbed a large flexible space structure, the NASA I0C Station response to orbiter docking exhibits small loads and only a few inches of deflection over the distance of a baseball field. The corresponding low strains may not be enough to efficiently "work" a distributed damping material, or a damping material or device placed in the load path. The advantage of the tuned-mass damper is that it is "tuned" to draw energy from the main structure to a mechanism which works the damping material or damper. Some disadvantages of the tuned-mass damper (also termed vibration absorber) are that it adds nonstructural mass, typically provides only modest damping levels, and does not compensate for changes in the plant dynamics.

This work was supported by the National Aeronautics and Space Administration, Langley Research Center, Contract No. NAS1-17760, Harold G. Bush, Technical Monitor.
Because of the variety of different Space Station disturbances and the continuing evolution of the IOC design in the Phase B program it was decided to model the transient disturbances using an impulse input. This simplification is justified by the relatively short duration of the transient pulses in comparison with the long periods of the dominant structural modes. Changes in the design evolution of the Space Station could easily alter the mix of modes which are excited. Because the impulse input excites all modes, some of the dependence of the study conclusions on a particular evolutionary configuration is removed.

The absorber parameters which are varied in this study are the mass $M_2$, the stiffness $k_2$, and the damper strength, $c$. Because absorber performance increases with increasing absorber mass (until a saturation point is reached), the absorber mass is selected a priori based on the available mass budget. In the analysis for the chart below, the absorber mass is assumed to be 2% of the plant modal mass. The value of $\delta$ which maximizes the absorber/structure interaction for a mass ratio of 2% is 0.98. The chart shows the impulse response envelopes (which connect the peaks of the amplitude of the sinusoidal transient response) as a function of the nondimensional damper coefficient $\mu$. The chart shows that too high a value of $\mu$ "locks" the damper, restraining the motion across the damper and resulting in sub-optimal performance. Similarly, too low a value of $\mu$ provides too little damping. A value of $\mu$ between .086 and .11 seems desirable for this mass ratio.
COMPARISON OF ABSORBER PERFORMANCE WITH STRUCTURAL DAMPING

The effect of inherent structural damping on absorber performance is analyzed using the two-DOF system shown in the chart. A non-zero value for $c_1$ is used to represent the modal damping of the original plant mode. The settling times to 20% of the peak impulse response are plotted vs. the existing structural damping for the system with and without an absorber. The results show that a 2% modal mass absorber can significantly improve the settling time of systems with less than about 5% inherent structural damping. Beyond the 5% level, the structure itself is dissipating energy so well that the absorber has little effect. Examination of the chart yields that for a 2% modal mass absorber, the response time to 20% peak is equivalent to that for the same plant without an absorber but with a structural damping level of 6%.

Similar investigations examined the effect of inherent structural damping on the applicability of the absorber tuning laws that were developed. The results indicate that the tuning laws dictate an optimal design for systems with small amounts of inherent structural damping (around 2%). Above the 2% level, the surface contours of the optimization cost function start to flatten out, indicating a reduced sensitivity of the system performance to the tuning of the absorber.

\[ c_1 = 2M_1\omega t_{ST} \]
MULTI-DOF MULTIMODE ABSORBER DESIGN

Tuning laws were developed to account for the effects of absorber placement and the use of multiple absorbers to damp the same mode. Note that if the absorber is not placed at the maxima of the mode which is to be damped, the effective modal mass ratio ($\beta_{\text{eff}}$) is reduced. This implies both reduced performance and the use of different optimal physical parameters for the absorber, based on $\beta_{\text{eff}}$.

The equations and chart below illustrate the cost function used in the optimization, which basically minimizes the area under the impulse response curve. This cost function was selected after comparing the impulse response performance of several other penalty functions. A unique feature of this cost function is that for the impulse response case, the value of $J$ can be expressed solely as a function of $\beta$ and the existing structural damping. The good fit of the equation with the exact solution illustrates this unique feature.

EFFECT OF PLACEMENT: 

$$\beta_{\text{eff}} = \frac{M_A \phi_A^2}{M_1 \phi_1} \text{ Normalized So Peak} = 1.0$$

FOR n ABSORBERS: 

$$M_i = \frac{M}{n}, c_i = \frac{c}{n}, k_i = \frac{k}{n}$$

$$J = \int_0^1 |x(t)| \exp(-\zeta \omega_n t) \sin \omega_d t \, dt, \quad J = \frac{1}{\zeta \omega_n \omega_d} \text{ TERMS IN } \omega_n, \omega_d$$

BUT $t = 1/2 \sqrt{\frac{\beta}{\beta + 2}}$

COMPARISON OF AREA EQUATION AND EXACT SOLUTION

LEGEND

$$\int_0^1 |x(t)| \, dt = 2.57 \phi_1 \phi_j \sqrt{\frac{\beta + \beta}{\beta}} \quad \text{(ACCELERATION)}$$

$$\int_0^1 x(t) \, dt = \phi_1 \phi_j \sqrt{\frac{\beta + \beta}{\beta}} + 1.2 \quad \text{(DISPLACEMENT)}$$

- EXACT SOLUTION

$$\zeta_{\text{ST}} = 0.005 \quad \phi_1 \phi_j = 1.0$$
UNCOPLED MODE ABSORBER MASS OPTIMIZATION FOR MULTI-DOF, MULTIMODE SYSTEMS

The fact that the performance index for an absorber tuned to a particular mode can be described as a function of \( \beta \) suggests that an uncoupled optimization can be conducted to determine the optimal allocation of the mass budget among several absorbers, each tuned to a particular mode. The procedure outlined below generalizes the performance index to include several modes. The cost function minimizes the sum of the areas under the impulse responses of all the modes. An important assumption is that the absorber on any one mode does not couple with the absorber on another mode. The amount of absorber cross-coupling depends on the spatial location of the absorbers and the frequency separation of the modes. The chart below illustrates the process whereby the cost function is minimized subject to the constraint of the total available absorber mass budget. Once the absorber mass attached to each primary mode is calculated, classical tuning laws can be used to calculate the absorber stiffness parameters, based on the value of \( \beta \).

ASSUME AN ABSORBER OF MASS \( \beta_i \) (TO BE DETERMINED) ON EVERY PRIMARY MODE

\[
\text{Min } J_T = J(\beta_1) + J(\beta_2) \ldots + J(\beta_n) = \sum_{i=1}^{n} J_i
\]

MODE 1  MODE 2  MODE n

SUBJECT TO CONSTRAINT

\[
M_{A_T} = \beta_1M_1 + \beta_2M_2 + \ldots + \beta_nM_n
\]

WHERE

\[
J(\beta_i) = \phi_1 \phi_j \sqrt{\frac{2 + \beta_i}{\beta_i} + 1.2} \quad \text{(DISPLACEMENT)}
\]

\[
J(\beta_i) = 2.57 \phi_1 \phi_j \sqrt{\frac{2 + \beta_i}{\beta_i}} \quad \text{(ACCELERATION)}
\]

INVOKE SIMPLE CONSTRAINED MULTIVARIATE SOLUTION:

\[
\frac{\alpha J_T}{\alpha \beta_i} - \frac{\alpha J_T}{\alpha \beta_i} = 0 \quad (i = 2, \ldots, n)
\]

\[
M_{A_T} - (\beta_1M_1 + \beta_2M_2 + \ldots + \beta_nM_n) = 0
\]

WHICH YIELDS \( n \) NONLINEAR EQUATIONS IN \( n \) UNKNOWNS

WHICH CAN BE SOLVED FOR \( \beta_i \)

ONCE \( \beta_i \) ARE KNOWN, USE CLASSICAL TUNING LAWS TO FIND \( k_{Ai}, c_{Ai} \) FOR ABSORBERS
CONTROL DESIGN PROCESS

The key concepts that allow application of feedback control techniques to absorber design are the placement of the design problem in a linear format, and the recasting of the combined structure-absorber dynamic equations in a feedback canonical form. This linear form is useful because most of the control-theoretic results apply to linear systems and the linear format greatly simplifies analysis and design. The feedback canonical form allows expression of the absorber parameters as controller gains and provides a convenient method for the evaluation of absorber performance. This formulation also provides needed visibility into the absorber design process.

CONVERT NONLINEAR DESIGN PROBLEM TO LINEAR PROBLEM – DO NOT LINEARIZE

- OPTIMAL SOLUTION REQUIRES ABSORBER MASS MAXIMIZATION
  - SET ABSORBER MASS AT MAXIMUM
  - ELIMINATE AS A VARIABLE

- ELIMINATE PLACEMENT AS A VARIABLE BY CHOOSING LOCATIONS OUTSIDE THE CONTROL DESIGN PROCESS
  - ASCERTAIN TROUBLESOME MODES
  - PICK LOCATIONS OF HIGHEST MODAL GAIN FOR ABSORBER ATTACHMENT

- PERFORM DESIGN WITH WELL-KNOWN READILY AVAILABLE CONTROL ALGORITHMS
  - LET ALGORITHMS DETERMINE ABSORBER FREQUENCY
  - ROOT LOCUS
  - OPTIMAL OUTPUT FEEDBACK
The dynamic equations for the absorber and the system may be formulated as shown. These equations take the form of coupled second-order differential equations. System I denotes the main system, or the structure to be damped. System II denotes the absorber dynamics for the coupled equations. The main system variables and parameters are denoted by the subscripts 1, and the absorber variables and parameters by the subscripts 2. The symbol, P, represents an external force applied to the system.

\[ \frac{P}{M_1} = x_1 + \frac{C_2}{M_1} \dot{x}_1 + \frac{K_1 + K_2}{M_1} x_1 - \frac{C_2}{M_1} \dot{x}_1 - \frac{K_2}{M_1} x_{11} \quad \text{SYSTEM I (MAIN)} \]

\[ 0 = \dot{x}_{11} + \frac{C_2}{M_2} \dot{x}_{11} + \frac{K_2}{M_2} x_{11} - \frac{C_2}{M_2} \dot{x}_{11} - \frac{K_2}{M_2} x_1 \quad \text{SYSTEM II (ABSORBER)} \]

**IN MATRIX NOTATION (SYSTEM I)**

\[
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2
\end{bmatrix}
= \begin{bmatrix}
0 & -\frac{(K_1 + K_2)}{M_1} \\
\frac{C_2}{M_1} & -\frac{C_2}{M_1}
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2
\end{bmatrix}
+ \begin{bmatrix}
0 & 0 \\
1 & 1
\end{bmatrix}
\begin{bmatrix}
P \\
y_2
\end{bmatrix};
\begin{bmatrix}
y_1 \\
y_2
\end{bmatrix}
= \begin{bmatrix}
\frac{K_2}{M_1} & \frac{C_2}{M_1}
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2
\end{bmatrix}
\]

**SYSTEM II**

\[
\begin{bmatrix}
\dot{x}_3 \\
\dot{x}_4
\end{bmatrix}
= \begin{bmatrix}
0 & 1 \\
-\frac{K_2}{M_2} & -\frac{C_2}{M_2}
\end{bmatrix}
\begin{bmatrix}
x_3 \\
x_4
\end{bmatrix}
+ \begin{bmatrix}
0 \\
1
\end{bmatrix}
\begin{bmatrix}
y_1 \\
y_2
\end{bmatrix};
\begin{bmatrix}
y_1 \\
y_2
\end{bmatrix}
= \begin{bmatrix}
\frac{K_2}{M_2} & \frac{C_2}{M_2}
\end{bmatrix}
\begin{bmatrix}
x_3 \\
x_4
\end{bmatrix}
\]
This diagram demonstrates the multipath nature of the control problem. The control design gain, K, appears in two inner feedback loops, and forms the coupling matrix between the two systems. Although these gains seem to be independent and appear in different system loops, in actuality they are identical parameters that appear simultaneously. This implies that parametric adjustment in one loop yields simultaneous adjustment in every loop containing that parameter. This property makes design difficult and illustrates a basic design limitation of the absorber. The outer feedback loop is positive in nature. Positive feedback loops are generally avoided in practice because of reduced stability margins that can cause system-wide instability. However, stability constraints are not a concern in this design process, for the entire system is guaranteed to remain stable as the passive nature of the absorber guarantees stability. The system dynamic equations are inherently stable for all physically realizable parameter values.

\[ G_1 = \text{MAIN SYSTEM (STRUCTURE)} \]

\[ G_2 = \text{ABSORBER SYSTEM} \]

\[ K = \text{ABSORBER COUPLING PARAMETERS} \]

\[ \{\text{SPRING CONSTANTS AND DAMPER VALUES} = (k_2 + sc_2)\} \]

\[ P = \text{DISTURBANCE INPUT} \]

\[ Y_1 = \text{SYSTEM 1 OUTPUT (DEFLECTION)} \]

\[ Y_2 = \text{SYSTEM 2 OUTPUT (DEFLECTION)} \]
This system has the structure of a simple output feedback control system entailing a single feedback loop, and may be used to synthesize system gains corresponding to absorber parameters. This feedback formulation provides insight to the ability of the absorber to affect system eigenvalues. It should be emphasized that $G_2$, the transfer function associated with absorber, has the functional form $1/s^2$ and corresponds to the dynamics of the absorber mass without the spring and damper attached. The remaining dynamic elements of the absorber are associated with the feedback loop. The transfer function, $G_1$, is associated with the structure and has the functional form $1/(s^2 + \omega_0^2)$ and corresponds to a structural vibration mode. The total system may be viewed as a rigid body mode and a vibration mode that are coupled by an external feedback loop, $K$.

\[
G_1 = \frac{1}{M_1} \left( \frac{1}{s^2 + \omega_0^2} \right) \quad \omega_0^2 = \frac{M_1}{k_1}
\]

\[
G_2 = \frac{1}{M_2} \frac{1}{s^2}
\]

\[
K = k_2 + sc_2
\]
The pole-zero constellation and associated root-locus plot are shown. The poles are indicated by X's and the zeros are indicated by O's. The pole frequency at $\omega_0$ corresponds to the vibration mode of the structure with no absorber attached. A double pole occurs at the origin and corresponds to the absorber mass dynamics. The zeros occur as a result of absorber action and are located at $\pm j\omega_0/(1 + \beta)$ where $\beta = M_1/M_2$ is the ratio of the absorber mass to structural mass. A zero also occurs on the real axis at $-k_2/c_2$ where $k_2$ is the absorber spring constant and $c_2$ is the damper value. Zero placement strongly affects the locus behavior, because the closed-loop system poles tend to migrate toward the open-loop system zeros.
COST FUNCTIONAL FOR OPTIMIZATION

The goal of the optimization problem is to minimize some performance index which penalizes the response-displacement, velocity, or acceleration. The most common performance index applied in linear optimal control theory is the linear quadratic regulator cost functional. The positive semi-definite matrix $Q$ and positive definite matrix $R$ describe the weighting of the state and control variables in the performance index.

- **LQR COST**: \[ J = \int_0^\infty (\eta^T Q \eta + u^T R u) \, dt \]

- **MAY PENALIZE DISPLACEMENT, VELOCITY, OR ACCELERATION**

- **KEEPS FINITE ELEMENT MODEL 'HONEST'**

- **OUTPUT FEEDBACK FORM** ($u = -FC\eta$)

\[
J = \int_0^\infty \eta^T(Q - C^T F^T R F) \eta \]
The optimization techniques described in the previous sections are applied to example vibration damping problems on the NASA dual keel configuration Space Station. Two example cases are considered which evaluate the capabilities of the uncoupled dynamic optimization and the parameter optimization algorithms: (1) micro-g acceleration response at the lab module, and (2) pointing response at a location on the lower payload boom. The disturbance input for both cases is a unit impulse at the habitation module. The force input at this location simulates either a shuttle docking or a crew motion disturbance, depending on the strength of the impulse. The inherent structural damping is assumed to be 0.5%.

The chart below illustrates the IOC configuration as of January 1986 which is modeled using a finite element code. Two absorbers are employed in each of the example problems, located at the maxima of the two most prominent modes in the transient response. The response locations corresponding to the two examples are also shown.

REFERENCE CONFIGURATION 5m DUAL KEEL

![Diagram of Space Station Configuration]

- **A_p** = Absorber location for pointing example
- **A_A** = Absorber location for micro-g example
MICRO-ACCELERATION EXAMPLE RESPONSE

The chart below compares the acceleration responses at the lab module due to a unit impulse input. A total absorber weight budget of 77.2 lbs is assumed. Both techniques result in improved damping performance in comparison with the open loop case which has no absorber and 0.5% structural damping. The performance of the system tuned using the parameter optimization technique is slightly preferable. Further examination of the problem reveals that the absorbers are highly cross-coupled in this example, explaining the sub-optimal result obtained using the uncoupled dynamic optimization. The more general parameter optimization technique takes into account the effects of absorber cross-coupling.

TRANSIENT RESPONSE - PROBLEM 1

![Chart 1: Open Loop Response](image1)

![Chart 2: Response with Gradient Optimization Results](image2)

![Chart 3: Response with Uncoupled Mode Optimization](image3)
PAYLOAD POINTING EXAMPLE RESPONSE

The chart below compares the pointing responses at the payload boom due to a unit impulse input. A total absorber weight budget of 2,316 lbs is assumed. Again, both techniques result in improved damping performance in comparison with the open loop case. The performance of the system tuned using the uncoupled dynamic optimization is slightly preferable. Further examination of the problem reveals that the absorbers are only lightly cross-coupled in this problem. The sub-optimal result obtained using the parameter optimization technique is attributed to the fact that the averaged initial conditions on the absorbers contributed to the cost function.

TRANSIENT RESPONSE - PROBLEM 2

[Graph showing transient response with labels for each graph]

OPEN LOOP RESPONSE

RESPONSE WITH GRADIENT OPTIMIZATION RESULTS

RESPONSE WITH UNCOUPLED OPTIMIZATION

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CONCLUSIONS

The potential damping performance gains achieved through the use of tuned-mass dampers on lightly damped structures merits the further study of the hardware issues associated with these devices.

- DEVELOPED OPTIMIZATION TECHNIQUES FOR MULTI-DOF-MULTI-MODE ABSORBER TUNING
  - SIMPLE UNCOUPLED MASS OPTIMIZATION IS PREFERABLE FOR LIGHTLY COUPLED MODES
  - NONLINEAR OUTPUT FEEDBACK OPTIMIZATION IS PREFERABLE FOR COUPLED MODES
    - USES COUPLING EFFECTS TO ADVANTAGE
    - USES UNCOUPLED MASS OPTIMIZATION TO OPTIMIZE ABSORBER MASSES AND PROVIDE INITIALIZATION FOR NONLINEAR OPTIMIZATION
REFERENCES


