ANALYSIS OF SILICON STRESS/STRAIN RELATIONSHIPS

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O. Dillon

Dislocation Density Contour Plot

\[ T = 1412 - 110.74X + 3.5X^2 \]

UNIT OF X AND Y = CM, Z = 1 PER CM²

Dislocation density contour plot for the parabolic thermal profile of Eq. (4-8), 8x7 cm ribbon and an initial dislocation density = 0.5 cm⁻².
Dislocation Density Along $Y = 0$ (Centerline)

$T=1412-110.74X+3.5X^2$

LINE 1 FOR WIDTH=8 CM  LINE 2 FOR WIDTH=7 CM
LINE 3 FOR WIDTH=6 CM  LINE 4 FOR WIDTH=4 CM

Dislocation density along the centerline of the ribbon for the parabolic thermal profile of Eq. (4-8), initial dislocation density = $0.5 \text{ cm}^{-2}$, and width = 8, 7, 6, 4 cm.
Final Dislocation Density Along the Ribbon Width for Westinghuse Profile

LENGTH = 12 CM, WIDTH = 3.5 CM
STAR FJR NO = 13 /CM××2
DIAMOND FOR NO = 5 /CM××2
SQUARE FOR NO = 1 /CM××2
Dislocation Density Contour Plot

Westinghouse Profile, N0 = 0.25/CMx2, R/T = 0
UNIT OF X AND Y = CM, Z = 1 PER CMx2

LEGEND: Z

- - - - - - - 10
- - - - - - - 392
- - - - - - - 774
- - - - - - - 1156
- - - - - - - 1538
- - - - - - - 1920
- - - - - - - 2302
- - - - - - - 2684

- - - - - - - 201
- - - - - - - 583
- - - - - - - 965
- - - - - - - 1347
- - - - - - - 1729
- - - - - - - 2111
- - - - - - - 2493
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Dislocation Density Contour Plot

WESTINGHOUSE PROFILE. \( \mu_0 = 0.375 / \text{cm} \times \text{m} \times 2 \)
UNIT OF \( X \) AND \( Y = \text{cm} \), \( Z = 1 / \text{cm} \times \text{m} \times 2 \).
\( R/T = 1.6667 \) M

LEGEND: \( Z \)

- \( 130 \)  \( 440 \)
- \( 750 \)  \( 1060 \)
- \( 1370 \)  \( 1680 \)
- \( 1990 \)  \( 2300 \)
- \( 2610 \)  \( 2920 \)
- \( 3230 \)  \( 3540 \)
- \( 3850 \)  \( 4160 \)

Note Low Densities

with Dendrites

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Effective Plastic Strain Rate

WESTINGHOUSE PROFILE, NO=0.25/CM××2
WIDTH = 6 CM, LENGTH = 8 CM, A/T=0.
UNIT OF X AND Y=CM, Z=10××-5 PER SEC
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Effective Plastic Strain Rate

WESTINGHOUSE PROFILE, NO.: 0.375/CM mm²
WIDTH = 6 CM, LENGTH = 8 CM, A/T = 1.6667 MM
UNIT OF X AND Y = CM, Z = 10-MM PER SEC

Plastic Strain Rate

\[
\begin{array}{c}
\text{X} \\
\text{Y} \\
\text{Z}
\end{array}
\]

\[
\begin{array}{c}
0.41 \\
0.28 \\
0.14 \\
0.00
\end{array}
\]

\[
\begin{array}{c}
8.00 \\
5.39 \\
2.67 \\
0.00
\end{array}
\]

\[
\begin{array}{c}
1 \\
2 \\
3 \\
4
\end{array}
\]

outside
ADVANCED SILICON SHEET

Residual Stress XX Along Ribbon Width for Westinghouse Profile

LENGTH = 12 CM, WIDTH = 3.5 CM
STAR FOR NO = 13 /CM × 2
DIAMOND FOR NO = 5 /CM × 2
SQUARE FOR NO = 1 /CM × 2

No = 13/cm²
No = 5/cm²
No = 1/cm²
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Deflection Shape

Original page is of poor quality

Half-width (c) = 1.0, length = 8.0
Diameter of dendrites is 0.0 inches
Critical thickness = 0.00526 inches

\[ T(x) \]

Diagram shows a 3D plot with axes labeled as follows:
- X: 0.0 to 6.0
- Y: -1.0 to 1.0
- Z: -0.010 to 0.010

The diagram illustrates the deflection shape with specific coordinates and dimensions.
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Deflection Shape

HALF-WIDTH (C) = 1.0, LENGTH = 8.0
DIAMETER OF DENDRITES IS 0.2 INCHES
CRITICAL THICKNESS = 0.057120 INCHES

T(x) = Parabolic

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Topics

1 Dislocation Motion

A) Problem Formulation
B) Calculation of Forces
C) Tracking the motion of a single Dislocation

2 Dislocation Multiplication & Density

A) Three methods of calculations - based on resolved shear stresses on each slip system
B) Dislocation density by averaging the \(|\text{shear stresses}|\) in 0.5 cm. widths of ribbon. (starting at \(x = 0.2\) cm.)
### Possible Dislocations in the Silicon Crystal

<table>
<thead>
<tr>
<th>Burgers Vector</th>
<th>Tangent Vector</th>
<th>Slip Plane</th>
<th>Type of Dislocation</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt;-1 0 -1&gt;</td>
<td>&lt;-1 0 -1&gt;</td>
<td>(-1 -1 1), Left, Screw</td>
<td></td>
</tr>
<tr>
<td></td>
<td>&lt;0 1 1&gt;</td>
<td>, Left, 60' (-120') ✓</td>
<td></td>
</tr>
<tr>
<td></td>
<td>&lt;-1 -1 0&gt;</td>
<td>, Left, 60' (+120') ✓</td>
<td></td>
</tr>
<tr>
<td></td>
<td>&lt;-1 -2 -1&gt;</td>
<td>, Left, Edge</td>
<td></td>
</tr>
<tr>
<td></td>
<td>&lt;0 1 1&gt;</td>
<td>, Left, Screw</td>
<td></td>
</tr>
<tr>
<td></td>
<td>&lt;-1 -1 0&gt;</td>
<td>, Left, 60' (+120') ✓</td>
<td></td>
</tr>
<tr>
<td></td>
<td>&lt;-1 -1 0&gt;</td>
<td>, Left, 60' (-120') ✓</td>
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<tr>
<td></td>
<td>&lt;0 1 1&gt;</td>
<td>, Left, Edge</td>
<td></td>
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<tr>
<td></td>
<td>&lt;-1 1 0&gt;</td>
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<td>, Right, Edge</td>
<td></td>
</tr>
<tr>
<td></td>
<td>&lt;0 -1 -1&gt;</td>
<td>, Transv., Screw</td>
<td></td>
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<tr>
<td></td>
<td>&lt;0 -1 1&gt;</td>
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<td></td>
<td>&lt;-1 -2 -1&gt;</td>
<td>, Transv., Edge</td>
<td></td>
</tr>
</tbody>
</table>

1. Surface of the ribbon is (1 1 1) plane.
2. Growth direction to the melt is <-2 -1 -1>.
3. For the motion of the dislocations, 60' dislocations that have
   -120 degree with the Burgers vector will be chosen because these
   60' dislocations may multiply themselves more than +120' type 60'
   dislocations as many investigators observed.
Assumptions

1. The density of dislocation at the liquid solid interface is uniform.

2. The pulling rate of the ribbon is 3cm/min.

3. Dislocations can move only in active slip systems that have their resolved shear stress higher than 95% of the most active slip system that has maximum Schmid Factor.

4. Average velocity of the dislocations in the presence of other dislocations is almost same as the velocity of isolated dislocation.

   Equivalent to low dislocation density.

5. The velocity equation proposed by K.Sumino

   \[ V = V_0 \tau \exp(-E/kT) \]

   where \( E \) is 2.2 eV for 60' and 2.35 eV for screw,
   
   \( V_0 \) is 0.035 for screw and 0.01 m\(^3\)/MN.sec for 60'.

   is still valid at high temperature like around melting temperature.
Motion of Dislocation

UNIT OF AXES ARE CM
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Motion of the Dislocations Emerging to the Surface

possible 60° dislocations emerging to the surface

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<th>Burger's Vector</th>
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<th>Plane</th>
<th>Motion</th>
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<tr>
<td>a/2(-1 0 -1)</td>
<td>(0 1 1)</td>
<td>(-1 -1 1) left strong</td>
<td></td>
</tr>
<tr>
<td>a/2(-1 1 0)</td>
<td>(0 -1 -1)</td>
<td>(-1 -1 1) *</td>
<td></td>
</tr>
<tr>
<td>a/2(-1 1 0)</td>
<td>(1 0 1)</td>
<td>(-1 -1 1) split</td>
<td></td>
</tr>
<tr>
<td>a/2(0 1 1)</td>
<td>(-1 0 -1)</td>
<td>(-1 -1 1) right weak</td>
<td></td>
</tr>
<tr>
<td>a/2(1 0 -1)</td>
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<td>(-1 0 -1)</td>
<td>(1 -1 -1) left weak</td>
<td></td>
</tr>
</tbody>
</table>

*; These are forced into the liquid
Calculation of the Density of Dislocations

From the K. Sumino's equation of dislocation multiplication

\[ dN_m = K \cdot K_0 \cdot N_{m1} \cdot (T_a - G \cdot b \cdot \text{SQRT}(N_{m2})/\beta \cdot (m+\lambda) \cdot \exp(-Q/kT) \cdot dt \]  \(\text{(A)}\)

where \(K, K_0, b, \beta, m, \lambda, k, Q\) are constants given by K. Sumino

- \(N_{m1}\) : dislocation density
- \(N_{m1} \propto\) source density
- \(N_{m2}\) is the density controlling the back stress

\(T_a\) : applied stresses
\(T\) : temperature
\(G\) : shear modulus
\(t\) : time

Three possible ways of application of the equation (A)

1. \(N_{m1}\) and \(N_{m2}\) are total density of dislocations

2. \(N_{m1}\) is the partial density of dislocations on each slip system and

   \(N_{m2}\) is the total density of dislocations.

3. Both \(N_m\)'s are partial densities of dislocations on each slip system
Method Calculating Dislocation Density

\( N_{m1} \quad N_{m2} \)

\begin{align*}
(1) \quad L \rightarrow RSS_i & \quad \text{total density } N_{m1} = N_{m2} \\
& \quad dN_m = dN_{m1} = dN_{m2}
\end{align*}

\begin{align*}
(2) \quad L \rightarrow RSS_i & \quad \text{total density } \sum_i (N_{m1})_i = N_{m2} \\
& \quad dN_{m2} = \sum_i (dN_m)_i
\end{align*}

\begin{align*}
(3) \quad L \rightarrow RSS_i & \quad \text{total density } \sum_i (N_{m1})_i = \sum_i (N_{m2})_i \\
& \quad (dN_{m1})_i = (dN_{m2})_i
\end{align*}
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Total Density of Dislocation Using YI, YTOT and DGEAR

Initial Total Density is 90/M^2 Lambda is 1.0

Legend: Dens
- 8.5915E+17
- 6.0140E+18
- 8.5915E+18
- 1.1169E+19
- 1.3746E+19

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Total Density of Dislocation Using YI, YI and DGEAR

INITIAL TOTAL DENSITY IS 90/μM². LAMDA IS 1.0
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Averaging the $|\tau a|$ in Calculating Density

cm

$X$

1 2 3

$Y \text{ (cm)}$

center edge

column 1; *, column 2; , column 3; □

$\text{liq-sol. interface} @ 0.2 \text{ cm}$
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Nine Slip Systems

<table>
<thead>
<tr>
<th>burger's vector</th>
<th>slip plane</th>
<th>type of plane</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a/2 \begin{pmatrix} 1 &amp; 0 &amp; 1 \end{pmatrix}$</td>
<td>$\begin{pmatrix} -1 &amp; -1 &amp; 1 \end{pmatrix}$</td>
<td>left</td>
</tr>
<tr>
<td>$a/2 \begin{pmatrix} 0 &amp; 1 &amp; 1 \end{pmatrix}$</td>
<td>$\begin{pmatrix} -1 &amp; -1 &amp; 1 \end{pmatrix}$</td>
<td>left</td>
</tr>
<tr>
<td>$a/2 \begin{pmatrix} -1 &amp; 1 &amp; 0 \end{pmatrix}$</td>
<td>$\begin{pmatrix} -1 &amp; -1 &amp; 1 \end{pmatrix}$</td>
<td>left</td>
</tr>
<tr>
<td>$a/2 \begin{pmatrix} 1 &amp; 0 &amp; -1 \end{pmatrix}$</td>
<td>$\begin{pmatrix} -1 &amp; 1 &amp; -1 \end{pmatrix}$</td>
<td>right</td>
</tr>
<tr>
<td>$a/2 \begin{pmatrix} 1 &amp; 1 &amp; 0 \end{pmatrix}$</td>
<td>$\begin{pmatrix} -1 &amp; 1 &amp; -1 \end{pmatrix}$</td>
<td>right</td>
</tr>
<tr>
<td>$a/2 \begin{pmatrix} 0 &amp; 1 &amp; 1 \end{pmatrix}$</td>
<td>$\begin{pmatrix} -1 &amp; 1 &amp; -1 \end{pmatrix}$</td>
<td>right</td>
</tr>
<tr>
<td>$a/2 \begin{pmatrix} 0 &amp; -1 &amp; 1 \end{pmatrix}$</td>
<td>$\begin{pmatrix} 1 &amp; -1 &amp; -1 \end{pmatrix}$</td>
<td>transverse</td>
</tr>
<tr>
<td>$a/2 \begin{pmatrix} 1 &amp; 0 &amp; 1 \end{pmatrix}$</td>
<td>$\begin{pmatrix} 1 &amp; -1 &amp; -1 \end{pmatrix}$</td>
<td>transverse</td>
</tr>
<tr>
<td>$a/2 \begin{pmatrix} 1 &amp; 1 &amp; 0 \end{pmatrix}$</td>
<td>$\begin{pmatrix} 1 &amp; -1 &amp; -1 \end{pmatrix}$</td>
<td>transverse</td>
</tr>
</tbody>
</table>
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Density of Dislocations When NO is 1 at x = 0.2 cm

UNIT OF X IS CM, Z IS DENSITY IN 1/M**2
Density of Dislocations When NO is 1 at $x = 0.2$ cm

UNIT OF X IS CM, Z IS DENSITY IN $1/m^{**2}$
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Density of Dislocations When NO is 1 at x = 0.2 cm

UNIT OF X IS CM, Z IS DENSITY IN 1/M**2

FIG. 111

(111) —— (111)

T = 0.225 — STAR, T = 0.75 — DIAMOND, T = 1.275 — ROUND, USING AVERAGE STRESS
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Dislocation Multiplication by Stress Averaging Over 0.5 cm

### Dislocation Type

<table>
<thead>
<tr>
<th>Type</th>
<th>Multiplication Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>L</td>
<td>Center</td>
</tr>
<tr>
<td>[101]</td>
<td>750</td>
</tr>
<tr>
<td>[011]</td>
<td>2.5</td>
</tr>
<tr>
<td>[110]</td>
<td>23</td>
</tr>
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<td>[110]</td>
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</tr>
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<td>[011]</td>
<td>12</td>
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<td>[011]</td>
<td>1</td>
</tr>
<tr>
<td>[101]</td>
<td>1</td>
</tr>
<tr>
<td>[110]</td>
<td>2</td>
</tr>
<tr>
<td>[011]</td>
<td>1.5</td>
</tr>
</tbody>
</table>

Calculations started at x=0.2 cm.
Dislocation Distribution in Web Dendrite Ribbon

(a) Top view of etched web ribbon

(b) Magnified view of dislocation distribution

(c) 30° angle marker

100 microns
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Cz Temperature Dependence

![Graph showing the temperature dependence of R. S. Stress (MPa) with different strain rates.](image)

Cz Strain Rate Dependence

![Graph showing the strain rate dependence of R. S. Stress (MPa) at different temperatures.](image)
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Cz Temperature Dependence

Web Ribbon: Strain Rate Dependence

STRAIN RATE (1/sec)
Web Ribbon: Temperature Dependence

R. S. STRESS (0.2% offset/ε_2, ε_y) (MPa)

1E 1

1E 0

0.65 0.7 0.75 0.8 0.85

1000/T (°deg. K)

Strain rate = 1E-04
Strain rate = 1E-05

Sumino N = 2 x 10^4 m^{-1}

Δε = 1.2 x 10^{-5} sec