

# ANALYSIS OF SILICON STRESS/STRAIN RELATIONSHIPS

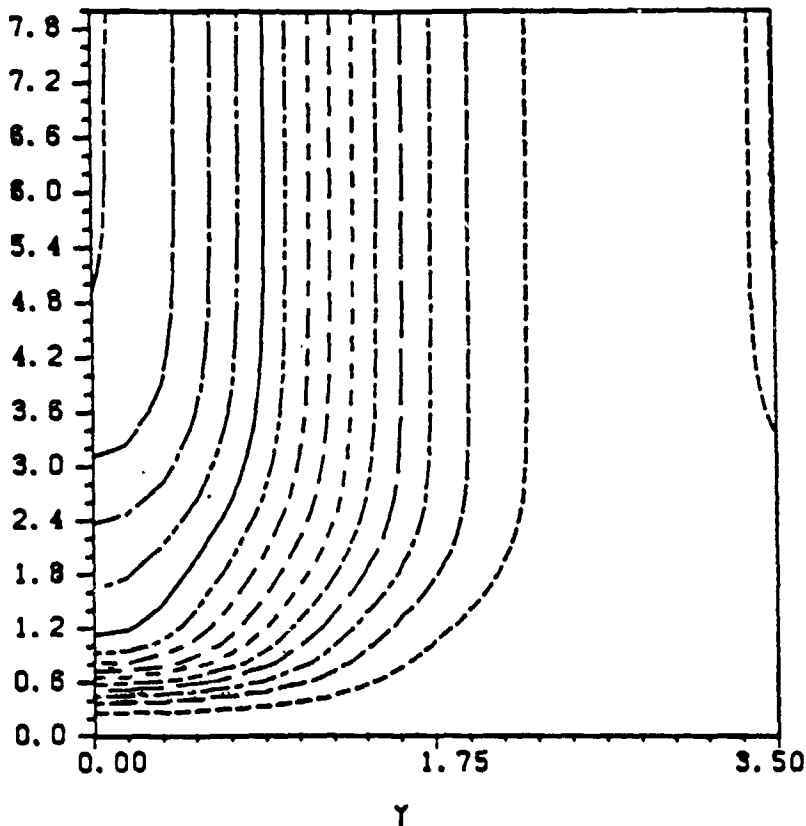
UNIVERSITY OF KENTUCKY

O. Dillon

## Dislocation Density Contour Plot

$$T = 1412 - 110.74X + 3.5X^2$$

UNIT OF X AND Y=CM, Z=1 PER CM



LEGEND: Z

-----	0.5	-----	9.6
-----	18.	-----	27.
-----	36.	-----	46.
-----	55.	-----	64.
-----	73.	-----	82.
-----	91.	-----	100.
-----	109.	-----	118.
-----	127.		

Dislocation density contour plot for the parabolic thermal profile of Eq.(4-8), 8x7 cm ribbon and an initial dislocation density = 0.5 cm<sup>-2</sup>.



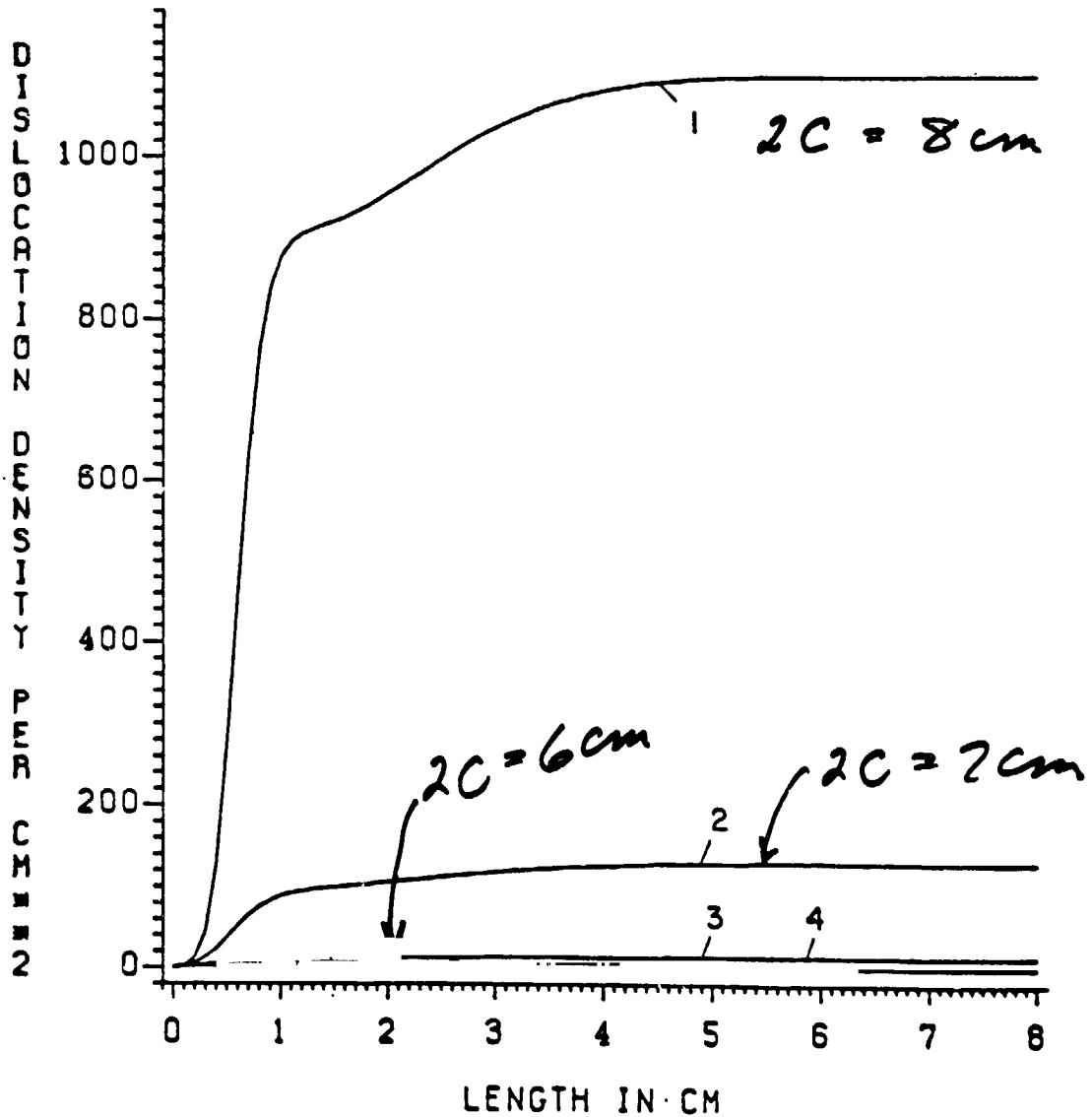
ADVANCED SILICON SHEET

Dislocation Density Along  $Y = 0$  (Centerline)

$$T = 1412 - 110.74 \times X + 3.5 \times X^2$$

LINE 1 FOR WIDTH=8 CM  
LINE 3 FOR WIDTH=6 CM

LINE 2 FOR WIDTH=7 CM  
LINE 4 FOR WIDTH=4 CM

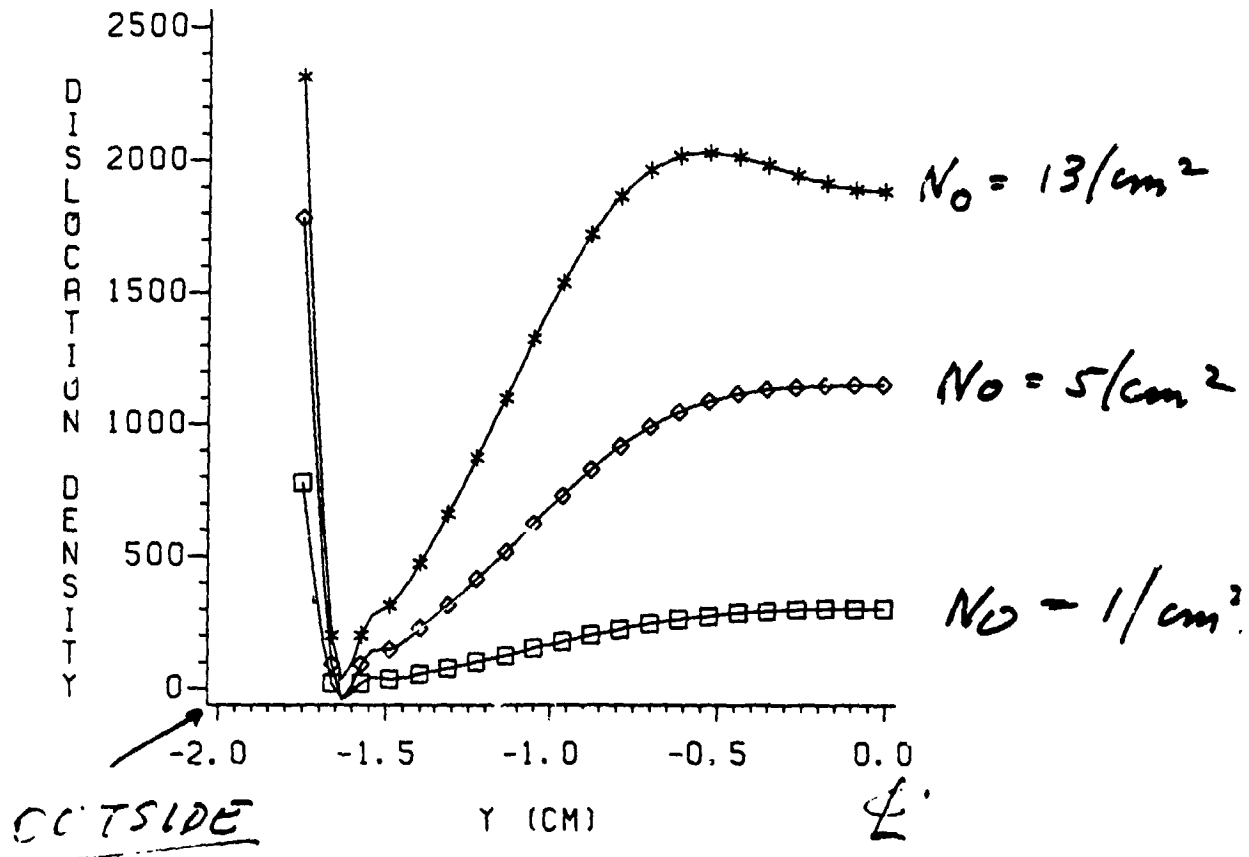


Dislocation density along the centerline of the ribbon for the parabolic thermal profile of Eq. (4-8), initial dislocation density =  $0.5\text{ cm}^{-2}$ , and width = 8, 7, 6, 4 cm.

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Final Dislocation Density Along the Ribbon  
Width for Westinghuse Profile

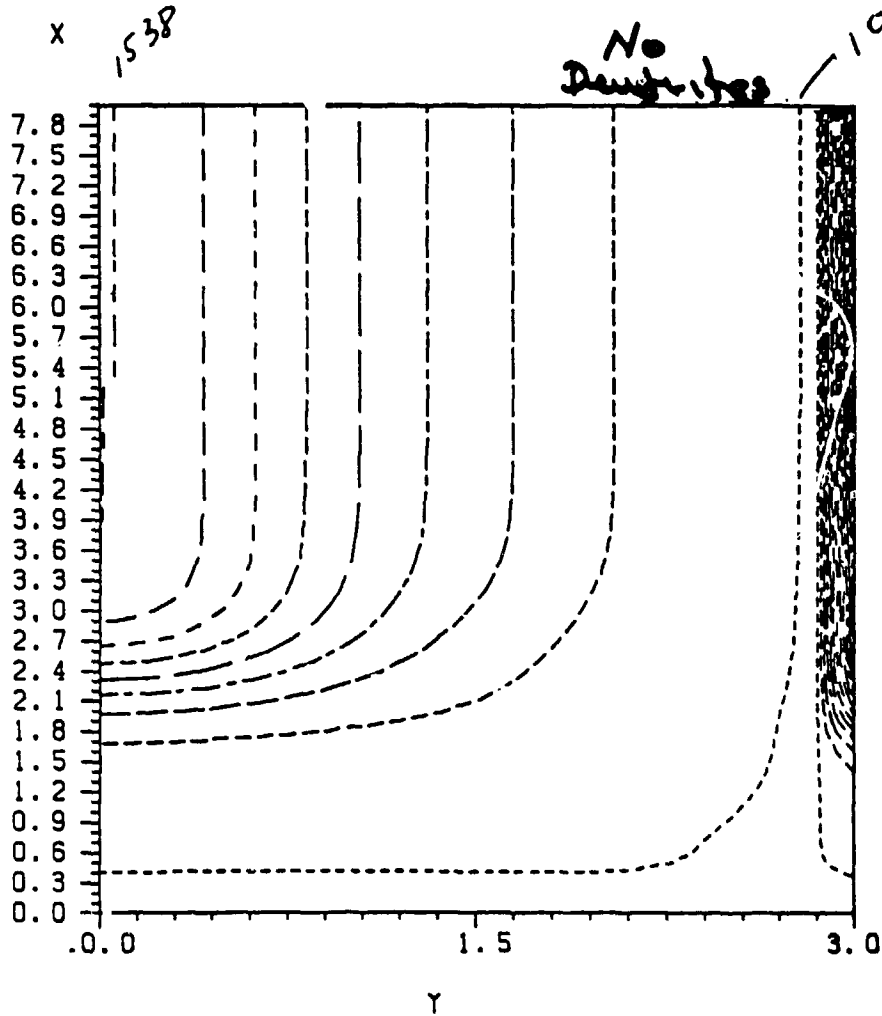
LENGTH = 12 CM, WIDTH = 3.5 CM  
STAR FOR NO = 13 /CM\*\*2  
DIAMOND FOR NO = 5 /CM\*\*2  
SQUARE FOR NO = 1 /CM\*\*2



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Dislocation Density Contour Plot

WESTINGHOUSE PROFILE, NO=0.25/CM<sup>2</sup>, R/T=0  
 UNIT OF X AND Y=CM, Z=1 PER CM<sup>2</sup>



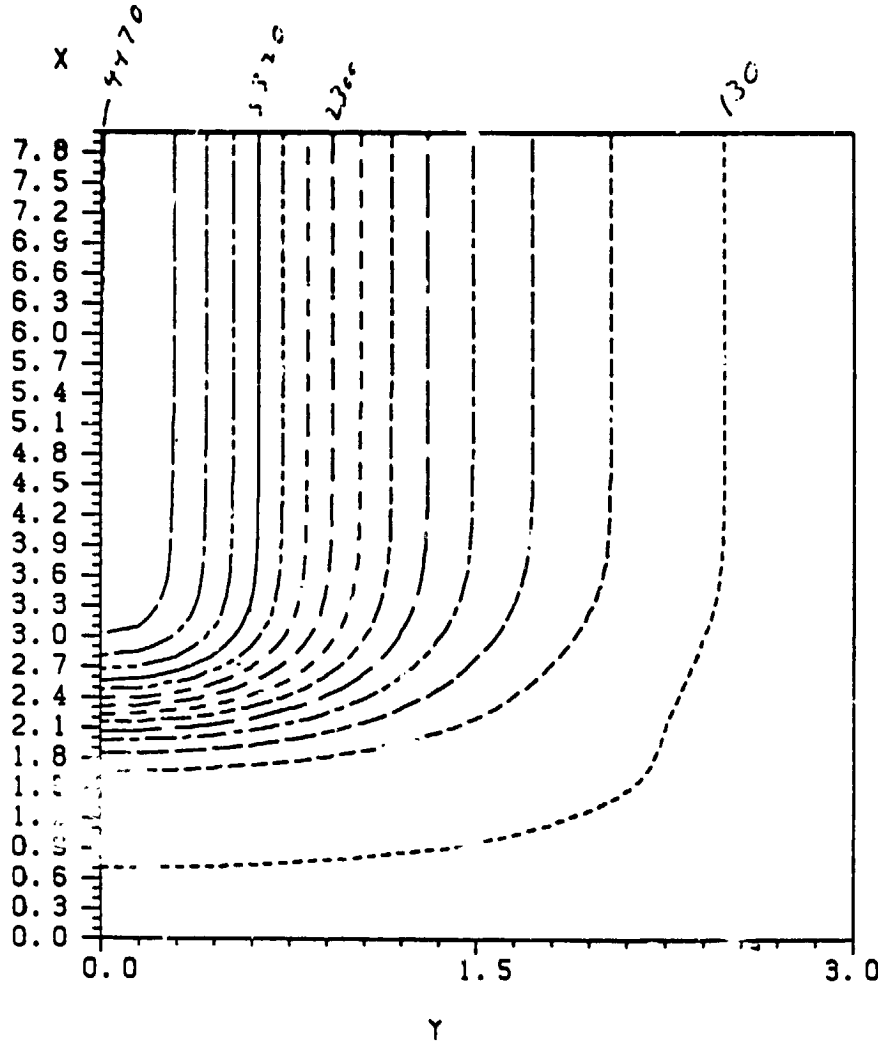
LEGEND: Z

-----	10	-----	201
-----	392	-----	583
-----	774	-----	965
-----	1156	-----	1347
-----	1538	-----	1729
-----	1920	-----	2111
-----	2302	-----	2493
-----	2684		

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Dislocation Density Contour Plot

WESTINGHOUSE PROFILE, NO=0.375/CM<sup>2</sup>  
 UNIT OF X AND Y=CM, Z=1 /CM<sup>2</sup>, A/T=1.6667 M



LEGEND: Z

-----	130	-----	440
-----	750	-----	1060
-----	1370	-----	1680
-----	1990	-----	2300
-----	2610	-----	2920
-----	3230	-----	3540
-----	3850	-----	4160
-----	4470		

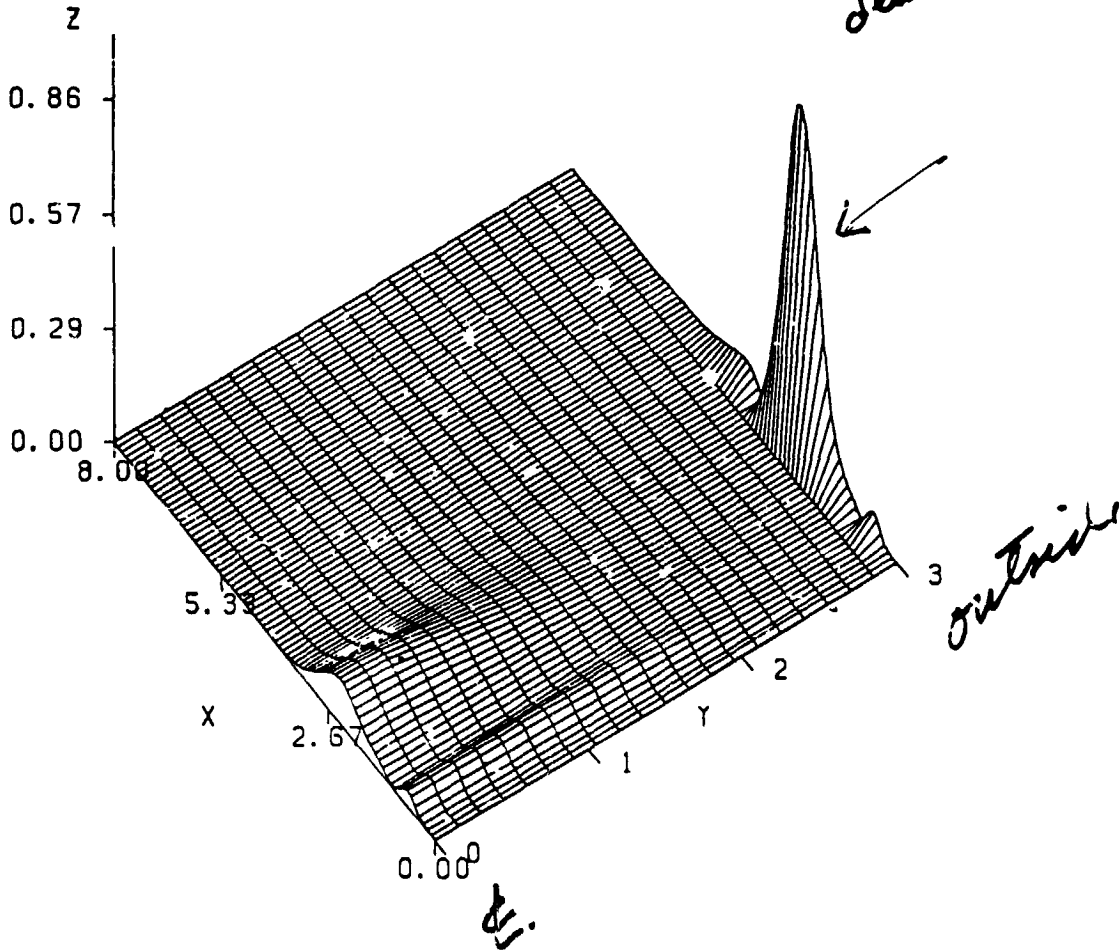
*Note  
 Low  
 Densities*

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Effective Plastic Strain Rate

WESTINGHOUSE PROFILE, NO=0.25/CM<sup>2</sup>  
WIDTH = 6 CM, LENGTH = 8 CM, R/T=0.  
UNIT OF X AND Y=CM, Z=10<sup>-5</sup> PER SEC

PLASTIC STRAIN RATE

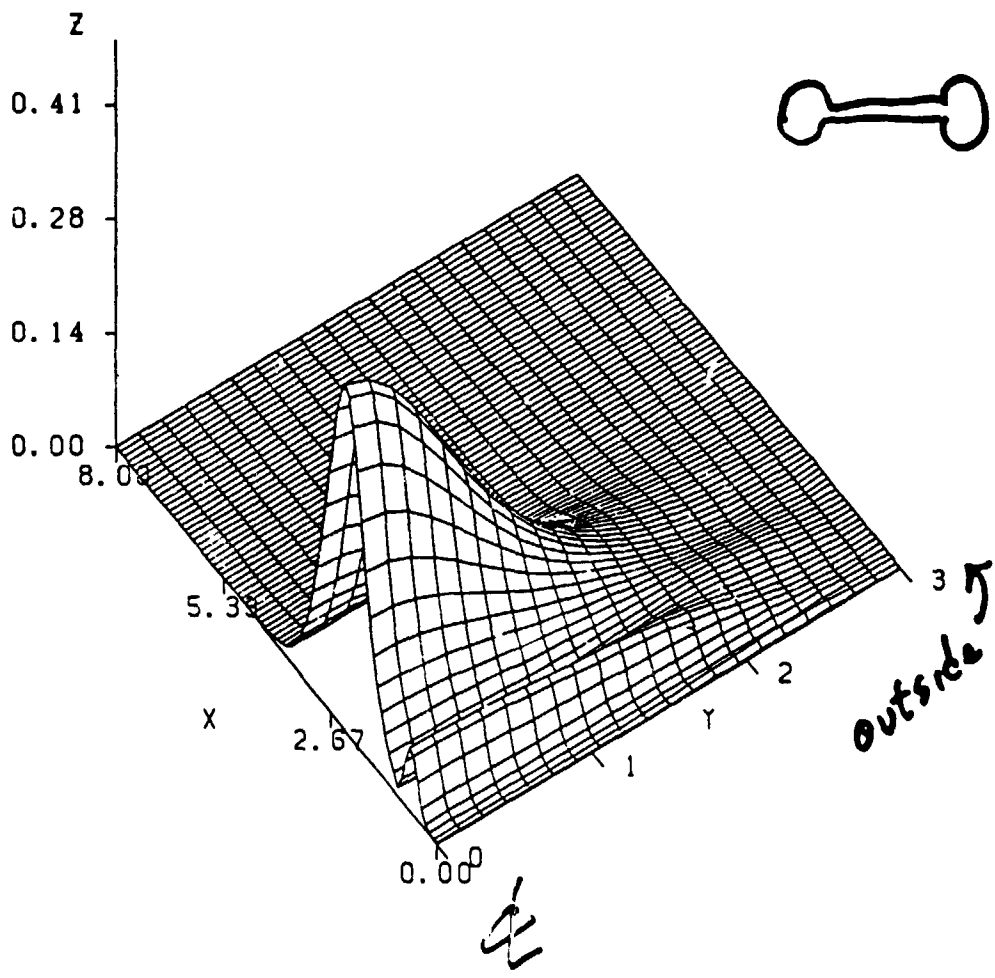


ADVANCED SILICON SHEET

Effective Plastic Strain Rate

WESTINGHOUSE PROFILE, NO=0.375/CM $\times$ 2  
WIDTH = 6 CM, LENGTH = 8 CM, R/T=1.6667 MM  
UNIT OF X AND Y=CM Z=10 $\times$ 10<sup>-5</sup> PER SEC

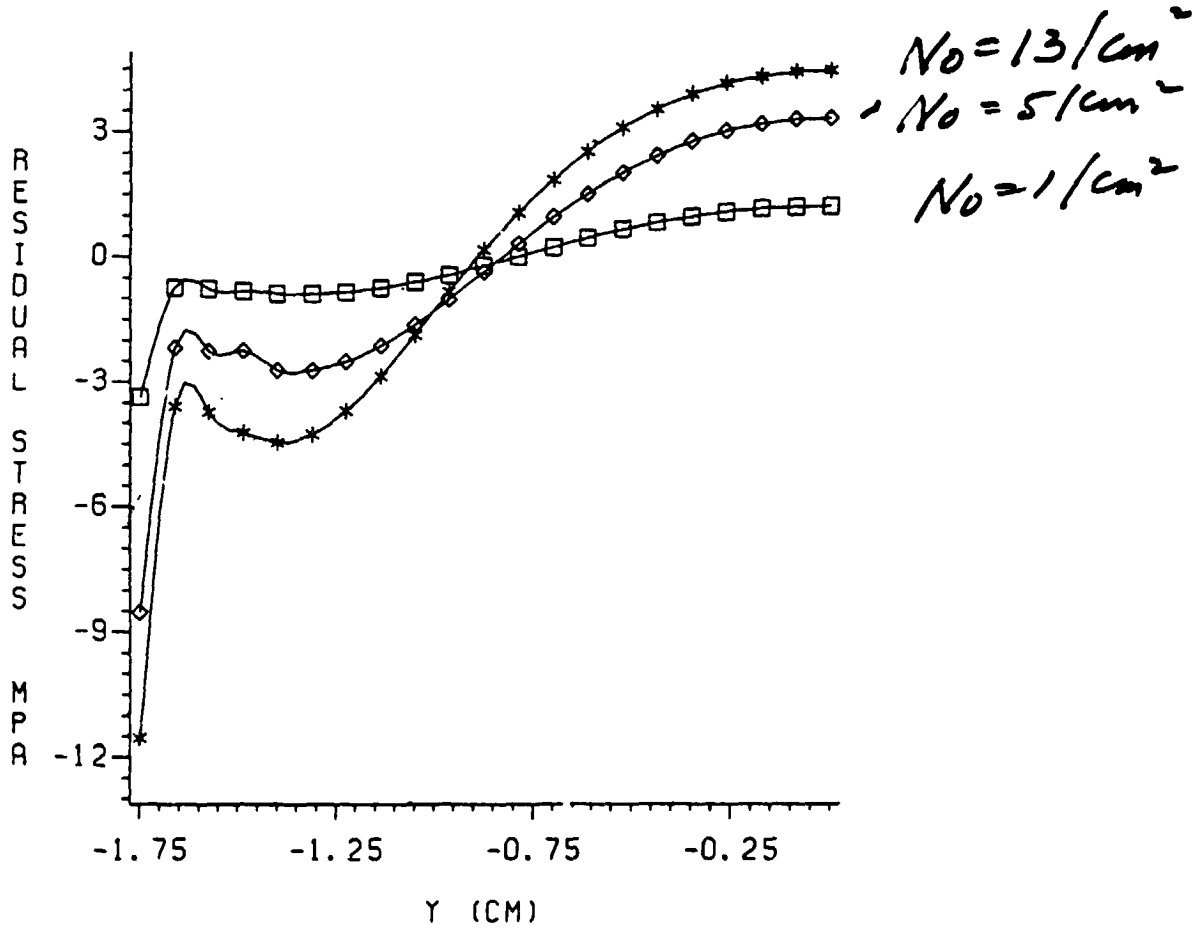
PLASTIC STRAIN RATE



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Residual Stress XX Along Ribbon  
Width for Westinghouse Profile

LENGTH = 12 CM. WIDTH = 3.5 CM  
STAR FOR NO = 13 /CM\*\*2  
DIAMOND FOR NO = 5 /CM\*\*2  
SQUARE FOR NO = 1 /CM\*\*2

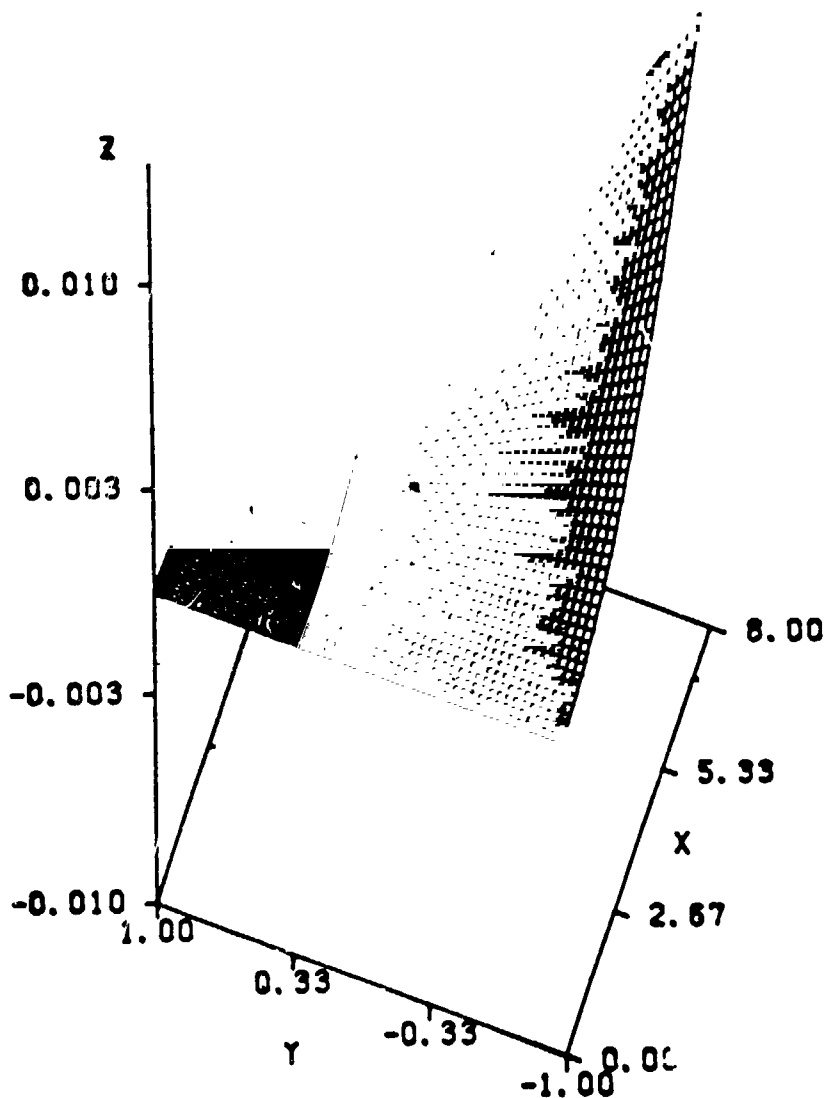




Deflection Shape

HALF-WIDTH (C)=1.0 LENGTH=8.0  
DIAMETER OF DENDRITES IS 0.0 INCHES  
CRITICAL THICKNESS = 0.00526 INCHES

$T(x) = \text{parabolic}$

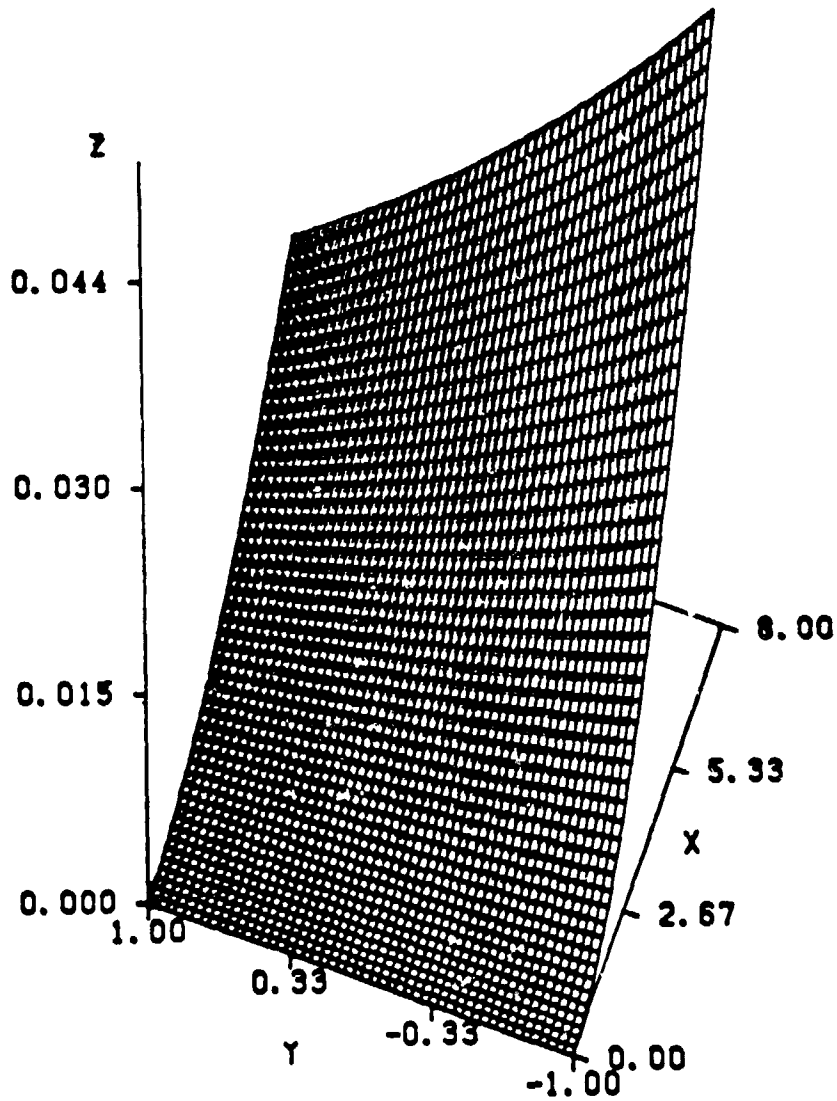


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Deflection Shape

HALF-WIDTH (C)=1., LENGTH=8.0  
DIAMETER OF DENDRITES IS 0.2 INCHES  
CRITICAL THICKNESS = 0.057120 INCHES

$T(x) = \text{parabolic}$



Topics

# 1 Dislocation Motion

- A) Problem Formulation
- B) Calculation of Forces
- C) Tracking the motion of a single Dislocation

# 2 Dislocation Multiplication & Density

- A) Three methods of calculations - based on resolved shear stresses on each slip system
- B) Dislocation density by averaging the | shear stresses | in 0.5 cm. widths of ribbon. (starting at  $x = 0.2$  cm.)

Possible Dislocations in the Silicon Crystal

burgers vector	tangent vector	slip plane	type of dislocation
<-1 0 -1>	<-1 0 -1>	(-1 -1 1)	left, screw
=	<0 1 1>	=	left, 60' (-120') ✓
=	<1 -1 0>	=	left, 60' (+120')
=	<1 -2 -1>	=	left, edge
<0 1 1>	<0 1 1>	=	left, screw
=	<1 -1 0>	=	left, 60' (+120')
=	<-1 0 -1>	=	left, 60' (-120') ✓
=	<-2 1 -1>	=	left, edge
<-1 1 0>	<-1 1 0>	=	left, screw
=	<0 -1 -1>	=	left, 60' (-120') ✓
=	<1 0 1>	=	left, 60' (+120') ✓
=	<-1 -1 2>	=	left, edge
<1 0 -1>	<1 0 -1>	(-1 1 -1)	right, screw
=	<-1 -1 0>	=	right, 60' (-120') ✓
=	<0 1 1>	=	right, 60' (+120') ✓
=	<-1 -2 -1>	=	right, edge
<0 -1 -1>	<0 -1 -1>	=	right, screw
=	<1 1 0>	=	right, 60' (-120') ✓
=	<-1 0 1>	=	right, 60' (+120')
=	<-2 -1 1>	=	right, edge
<1 1 0>	<1 1 0>	=	right, screw
=	<-1 0 1>	=	right, 60' (+120')
=	<0 -1 -1>	=	right, 60' (-120') ✓
=	<-1 1 -2>	=	right, edge
<0 -1 1>	<0 -1 1>	(-1 1 -1)	transv., screw
=	<1 1 0>	=	transv., 60' (+120') ✓
=	<-1 0 -1>	=	transv., 60' (-120') ✓
=	<2 1 1>	=	transv., edge
<1 1 0>	<1 1 0>	=	transv., screw
=	<0 -1 1>	=	transv., 60' (+120')
=	<-1 0 -1>	=	transv., 60' (-120') ✓
=	<1 -1 2>	=	transv., edge
<1 0 1>	<1 0 1>	=	transv., screw
=	<-1 -1 0>	=	transv., 60' (-120') ✓
=	<0 1 -1>	=	transv., 60' (+120')
=	<-1 -2 1>	=	transv., edge

- (1). Surface of the ribbon is (1 1 1) plane.
- (2). Growth direction to the melt is <-2 -1 -1>.
- (3). For the motion of the dislocations, 60' dislocations that have -120 degree with the burger's vector will be chosen because these 60' dislocations may multiply themselves more than +120' type 60' dislocations as many investigators observed.

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Assumptions

- (1) The density of dislocation at the liquid solid interface is uniform
- ✦ (2) The pulling rate of the ribbon is 3cm/min.
- ✦ (3) Dislocations can move only in active slip systems that have their resolved shear stress higher than 95% of the most active slip system that has maximum Schmid Factor.
- (4) Average velocity of the dislocations in the presence of other dislocations is almost same as the velocity of isolated dislocation. Equivalent to low dislocation density.
- ✦ (5) The velocity equation proposed by K.Sumino

$$V = V_0 \tau \exp(-E/kT)$$

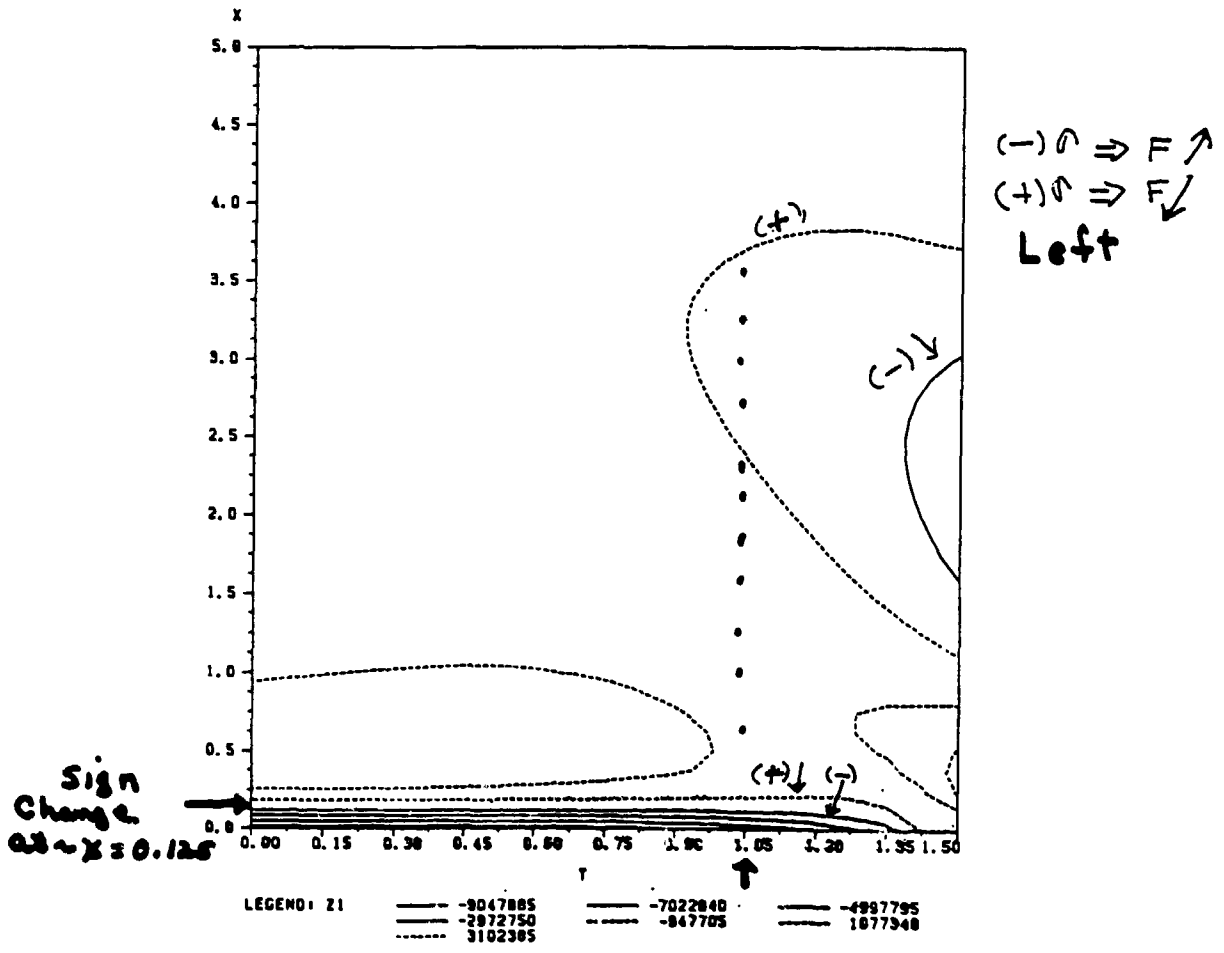
where E is 2.2 eV for 60' and 2.35 eV for screw,

$V_0$  is 0.035 for screw and 0.01 m<sup>3</sup> / MN.sec for 60'.

is still valid at high temperature like around melting temperature.

Resolved Shear Stress

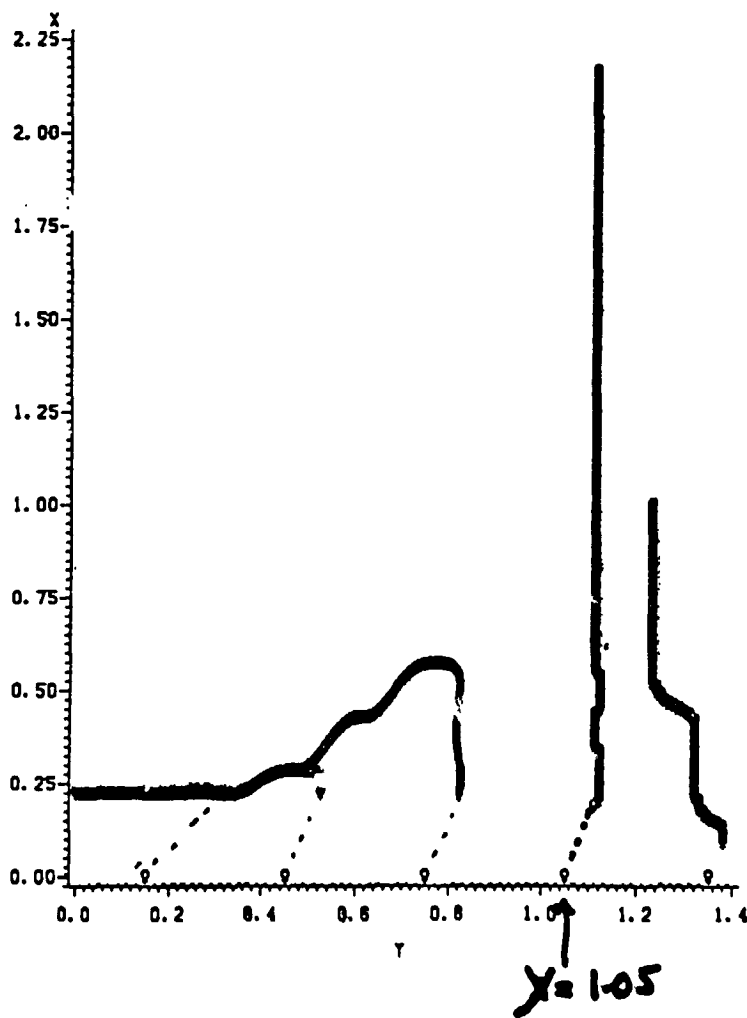
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Motion of Dislocation

$B = R/2(-1 \ 1 \ 0)$ ,  $T = (1 \ 0 \ 1)$ ,  $M = (-1 \ -1 \ 1)$



UNIT OF AXES ARE CM

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Motion of the Dislocations Emerging to the Surface

possible 60° dislocations emerging to the surface

burger's vector	tangent vecto:	plane	motion
$a/2(-1 \ 0 \ -1)$	(0 1 1)	(-1 -1 1)	left strong
$a/2(-1 \ 1 \ 0)$	(0 -1 -1)	(-1 -1 1)	*
$a/2(-1 \ 1 \ 0)$	(1 0 1)	(-1 -1 1)	split
$a/2(0 \ 1 \ 1)$	(-1 0 -1)	(-1 -1 1)	right weak
$a/2(1 \ 0 \ -1)$	(0 1 1)	(-1 1 -1)	right strong
$a/2(1 \ 0 \ -1)$	(-1 -1 0)	(-1 1 -1)	*
$a/2(1 \ 1 \ 0)$	(0 -1 -1)	(-1 1 -1)	*
$a/2(0 \ -1 \ -1)$	(1 1 0)	(-1 1 -1)	right weak
$a/2(0 \ -1 \ 1)$	(-1 0 -1)	(1 -1 -1)	right strong
$a/2(0 \ -1 \ 1)$	(-1 -1 0)	(1 -1 -1)	left strong
$a/2(1 \ 0 \ 1)$	(-1 -1 0)	(1 -1 -1)	left weak
$a/2(1 \ 1 \ 0)$	(-1 0 -1)	(1 -1 -1)	left weak

\* ; These are forced into the liquid



Calculation of the Density of Dislocations

From the K. Sumino's equation of dislocation multiplication

$$\Rightarrow dN_m = K K_0 N_{m1} (T_a - G b \text{SQRT}(N_{m2}) / \beta)^{m+\lambda} \exp(-Q/kT) dt \text{ ---(A)}$$

where K, K<sub>0</sub>, b, β, m, λ, k, Q are constants given by K. Sumino

N<sub>m</sub>'s ; dislocation density

N<sub>m1</sub> α source density

N<sub>m2</sub> is the density controlling the back stress

T<sub>a</sub> ; applied stresses

T ; temperature

G ; shear modulus

t ; time

Three possible ways of application of the equation (A)

(1) N<sub>m1</sub> and N<sub>m2</sub> are total density of dislocations

(2) N<sub>m1</sub> is the partial density of dislocations on each slip system and

N<sub>m2</sub> is the total density of dislocations

(3) Both N<sub>m</sub>'s are partial densities of dislocations on each slip system

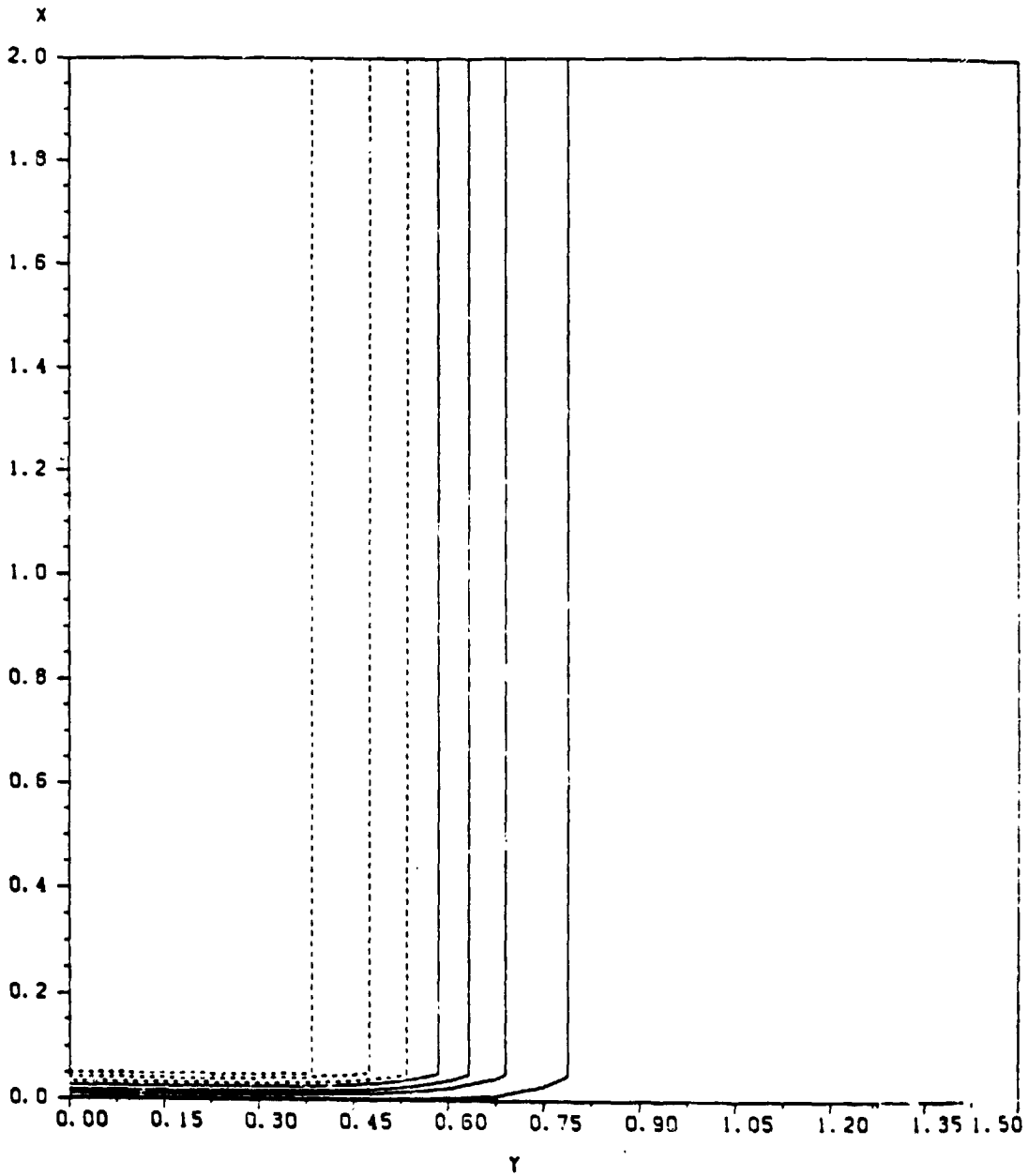
Method Calculating Dislocation Density

	$Nm_1$	$Nm_2$	
	-----	-----	
(1)	$\perp \rightarrow RSS_i$	$\parallel$	total density = $Nm_1 = Nm_2$ $dNm = dNm_1 = dNm_2$
(2)	$\perp \rightarrow RSS_i$	$\parallel$	total density = $\sum_i^q (Nm_1)_i = Nm_2$ $dNm_2 = \sum_i^q (dNm)_i$
(3)	$\perp \rightarrow RSS_i$	$\parallel$	total density = $\sum_i^q (Nm_1)_i = \sum_i^q (Nm_2)_i$ $(dNm_1)_i = (dNm_2)_i$

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Total Density of Dislocation Using YI, YTOT and DGEAR

INITIAL TOTAL DENSITY IS 90/M\*\*2 LAMDA IS 1.0



LEGEND: DENS

—  
- - -  
.....  
- . - .

0.5915E+17  
6.0140E+18  
1.1169E+19  
1.6324E+19

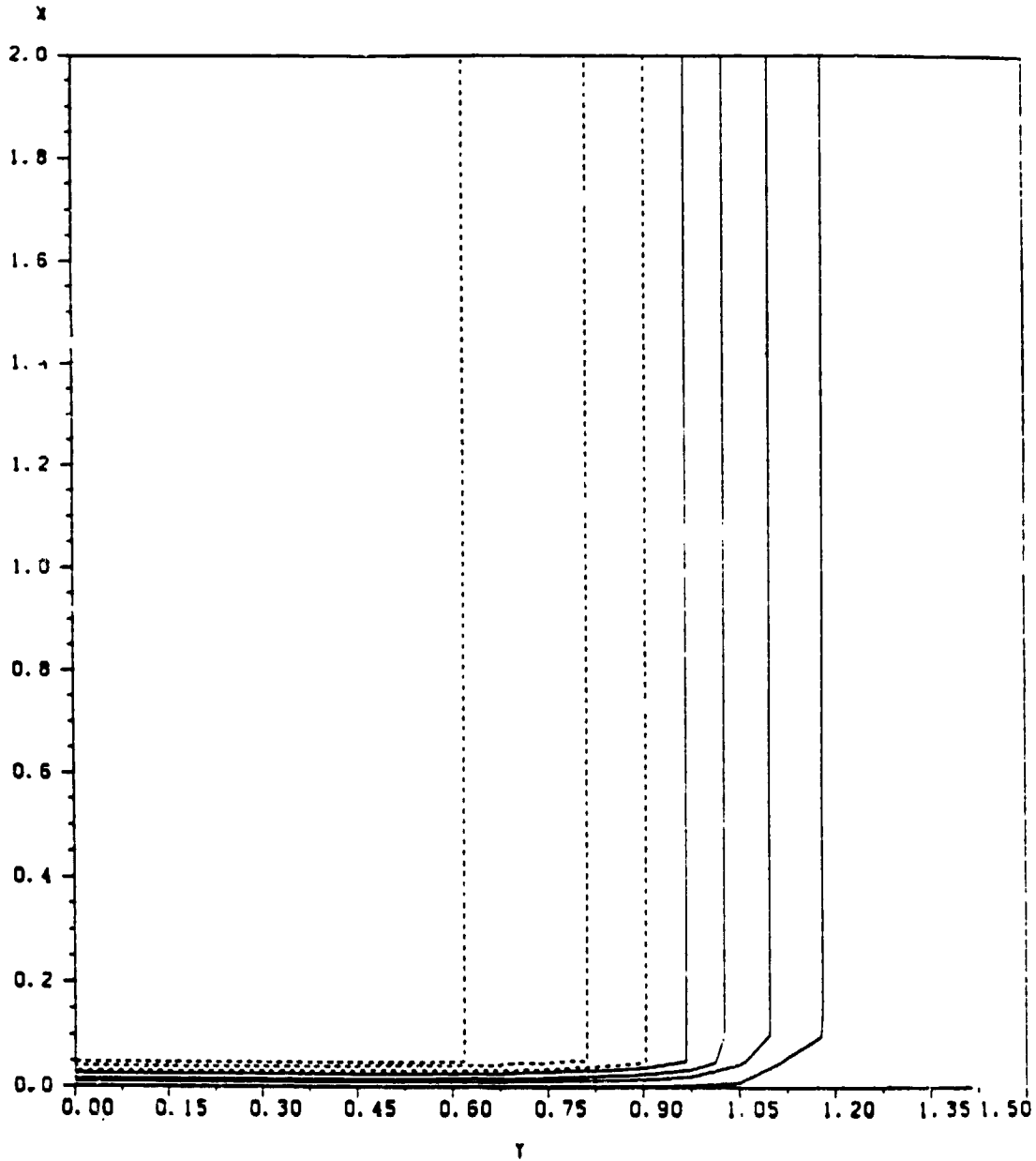
—  
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3.4366E+18  
8.5915E+18  
1.3746E+19

ADVANCED SILICON SHEET

Total Density of Dislocation Using YI, YI and DGEAR

INITIAL TOTAL DENSITY IS  $90/M^2$  .LAMOR IS 1.0



LEGEND: DENS8

—  
- - -  
.....

7.7220E+11  
5.4054E+12  
1.0090E+13

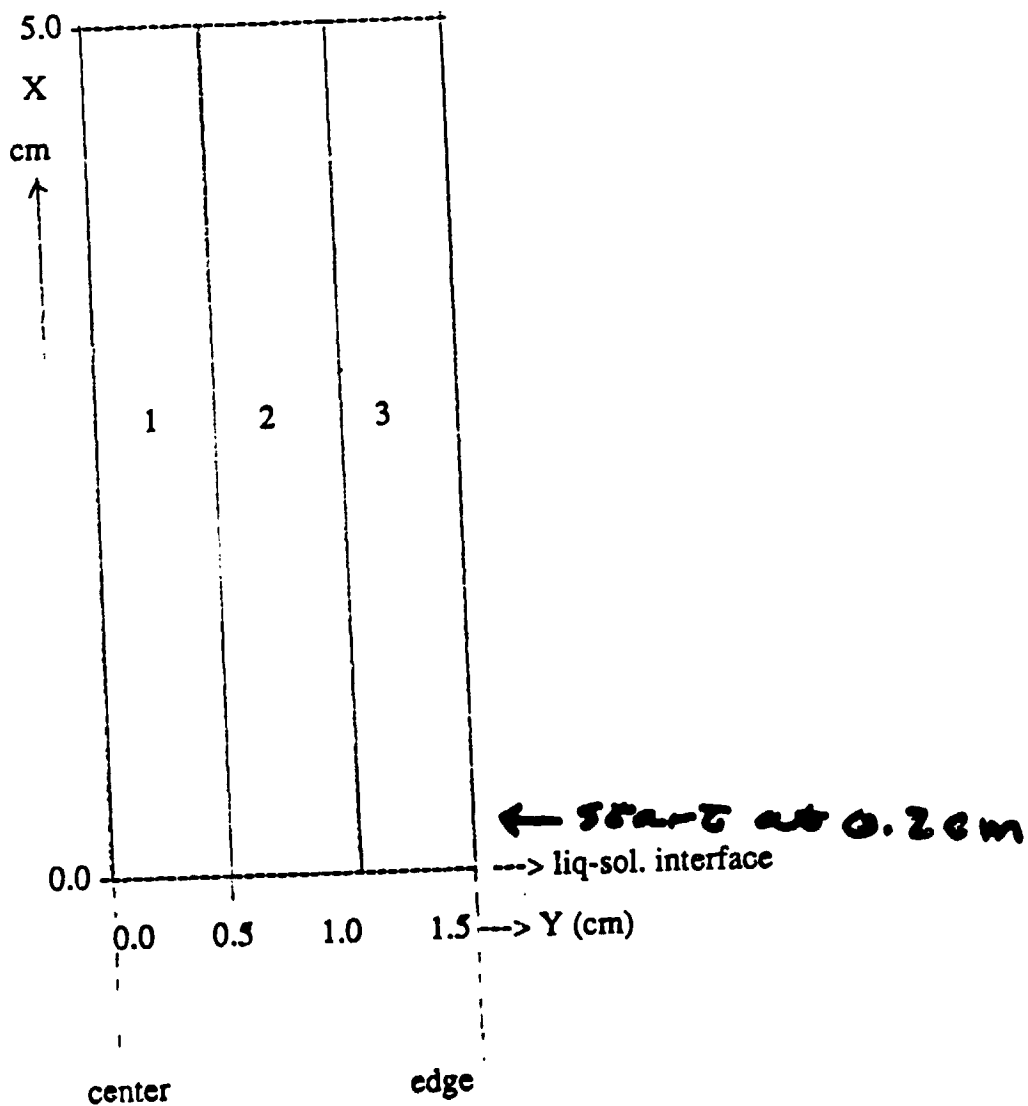
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- - -  
.....

3.0888E+12  
7.7220E+12  
1.9455E+13

C-4

# ADVANCED SILICON SHEET

## Averaging the $|\tau_a|$ in Calculating Density



column 1 ; \* , column 2 ; , ♦ column 3 ; □

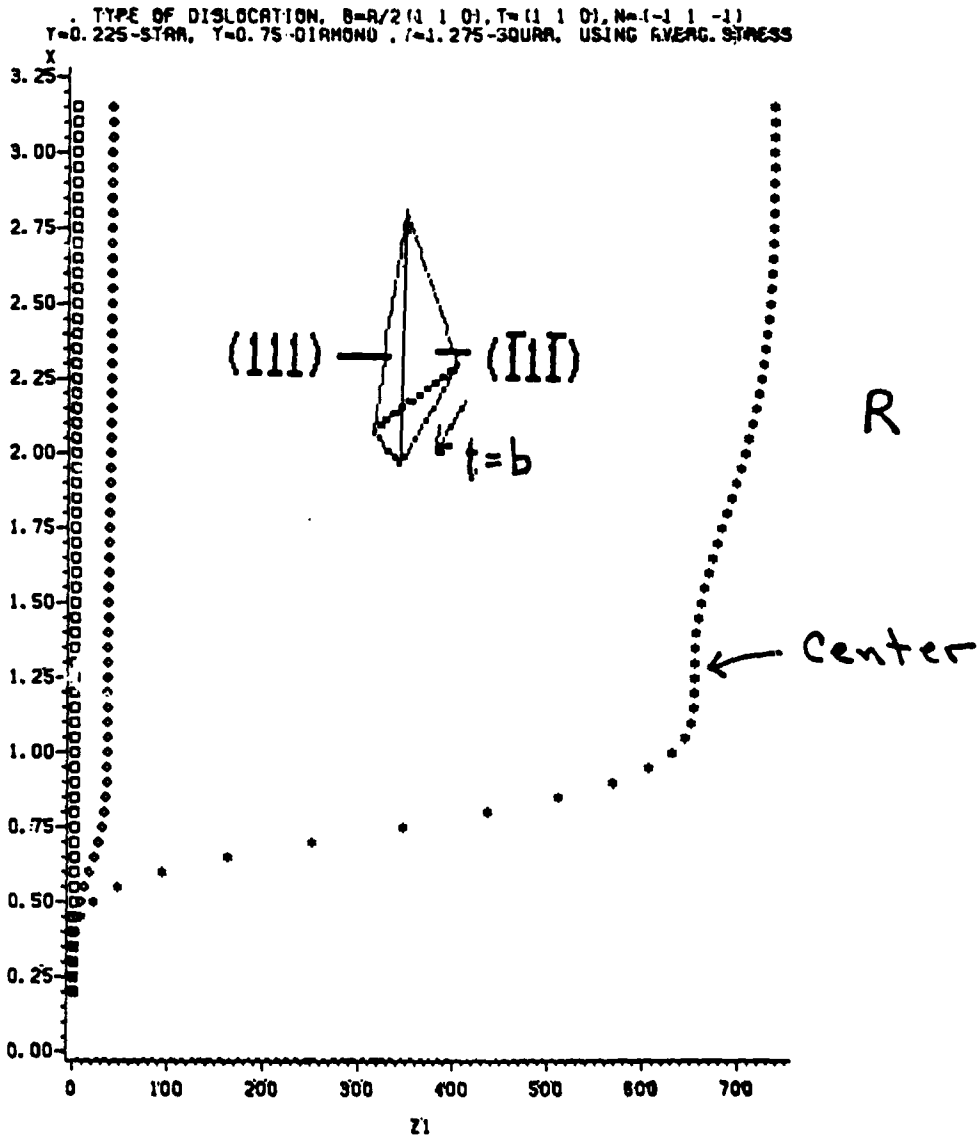
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Nine Slip Systems

burger's vector	slip plane	type of plane
$a/2 (1 0 1)$	$(-1 -1 1)$	left
$a/2 (0 1 1)$	$(-1 -1 1)$	left
$a/2 (-1 1 0)$	$(-1 -1 1)$	left
$a/2 (1 0 -1)$	$(-1 1 -1)$	right
$a/2 (1 1 0)$	$(-1 1 -1)$	right
$a/2 (0 1 1)$	$(-1 1 -1)$	right
$a/2 (0 -1 1)$	$(1 -1 -1)$	transverse
$a/2 (1 0 1)$	$(1 -1 -1)$	transverse
$a/2 (1 1 0)$	$(1 -1 -1)$	transverse

ADVANCED SILICON SHEET

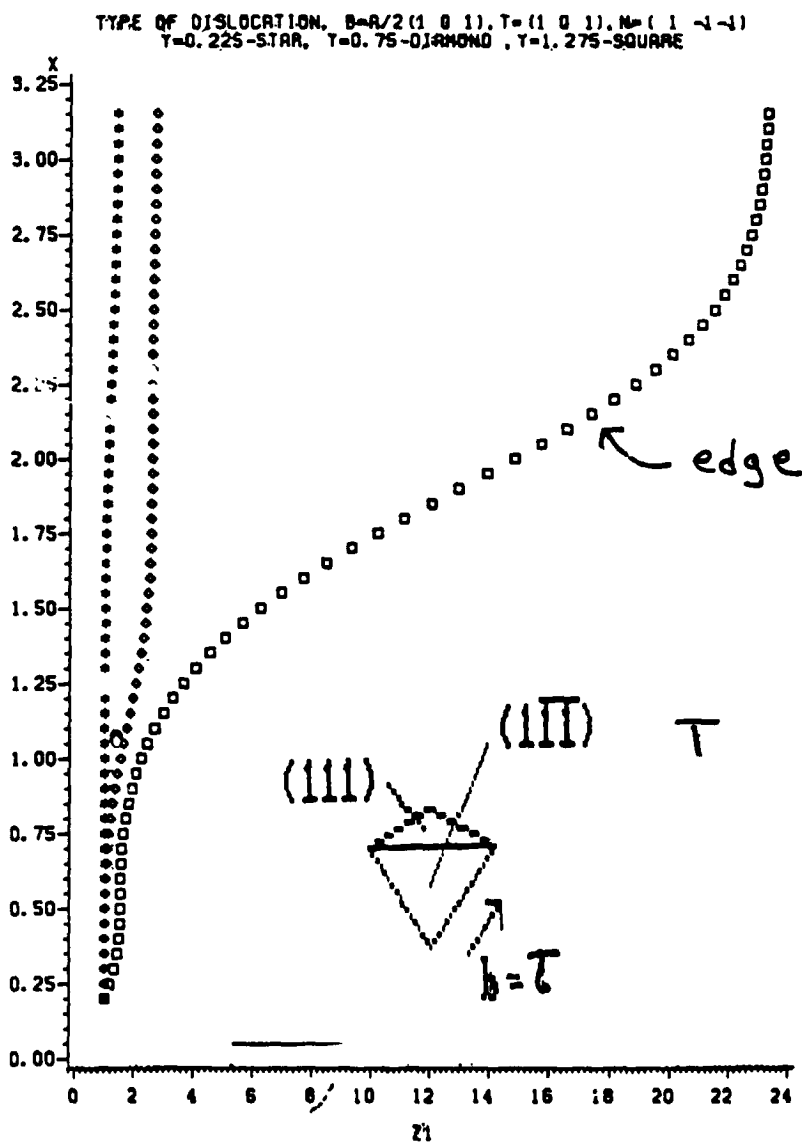
Density of Dislocations When NO is 1 at  $x = 0.2$  cm



UNIT OF X IS CM, Z IS DENSITY IN  $1/M^{**2}$

ADVANCED SILICON SHEET

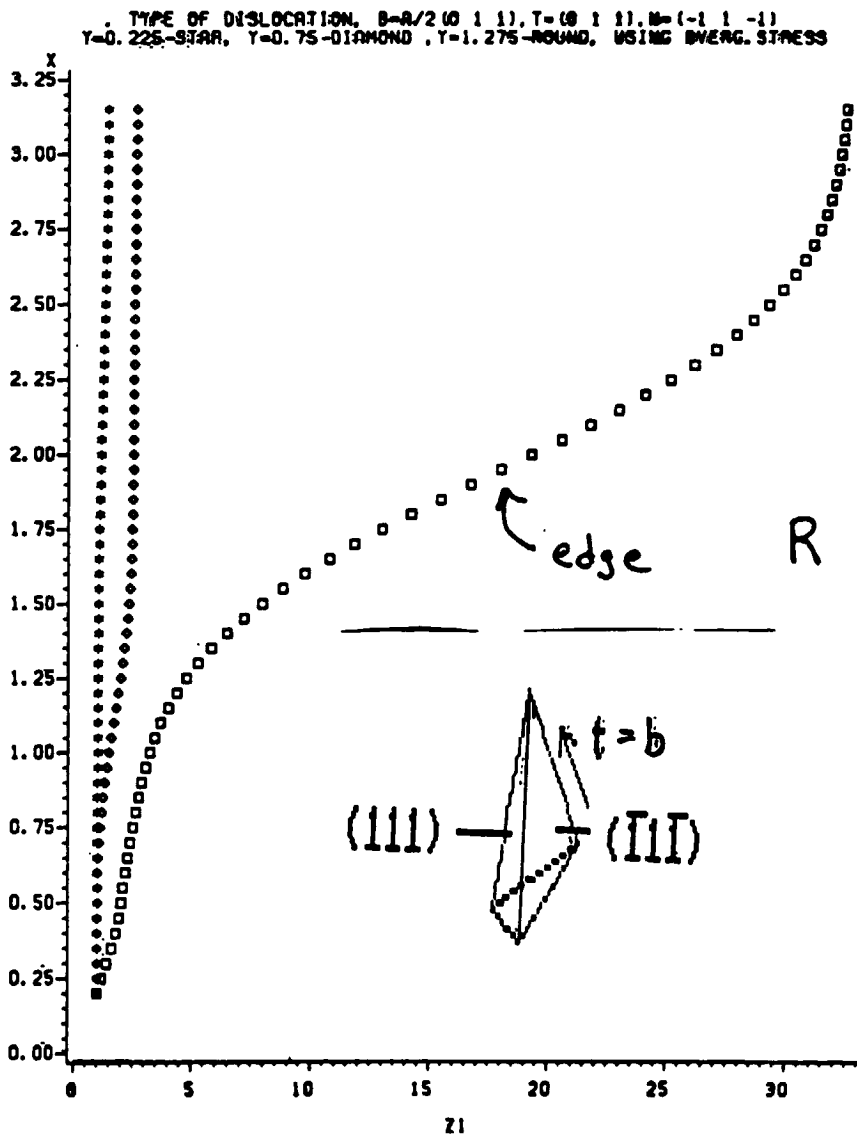
Density of Dislocations When NO is 1 at  $x = 0.2$  cm





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Density of Dislocations When NO is 1 at  $x = 0.2$  cm



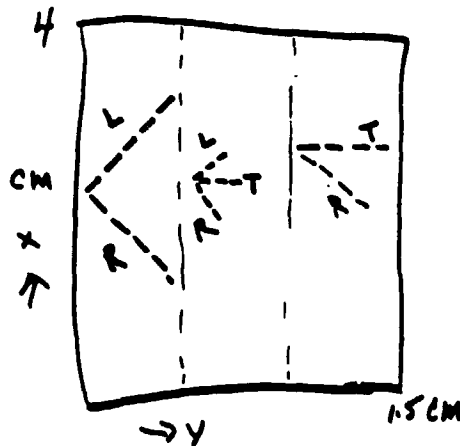
UNIT OF X IS CM, Z IS DENSITY IN  $1/M^{**2}$

ADVANCED SILICON SHEET

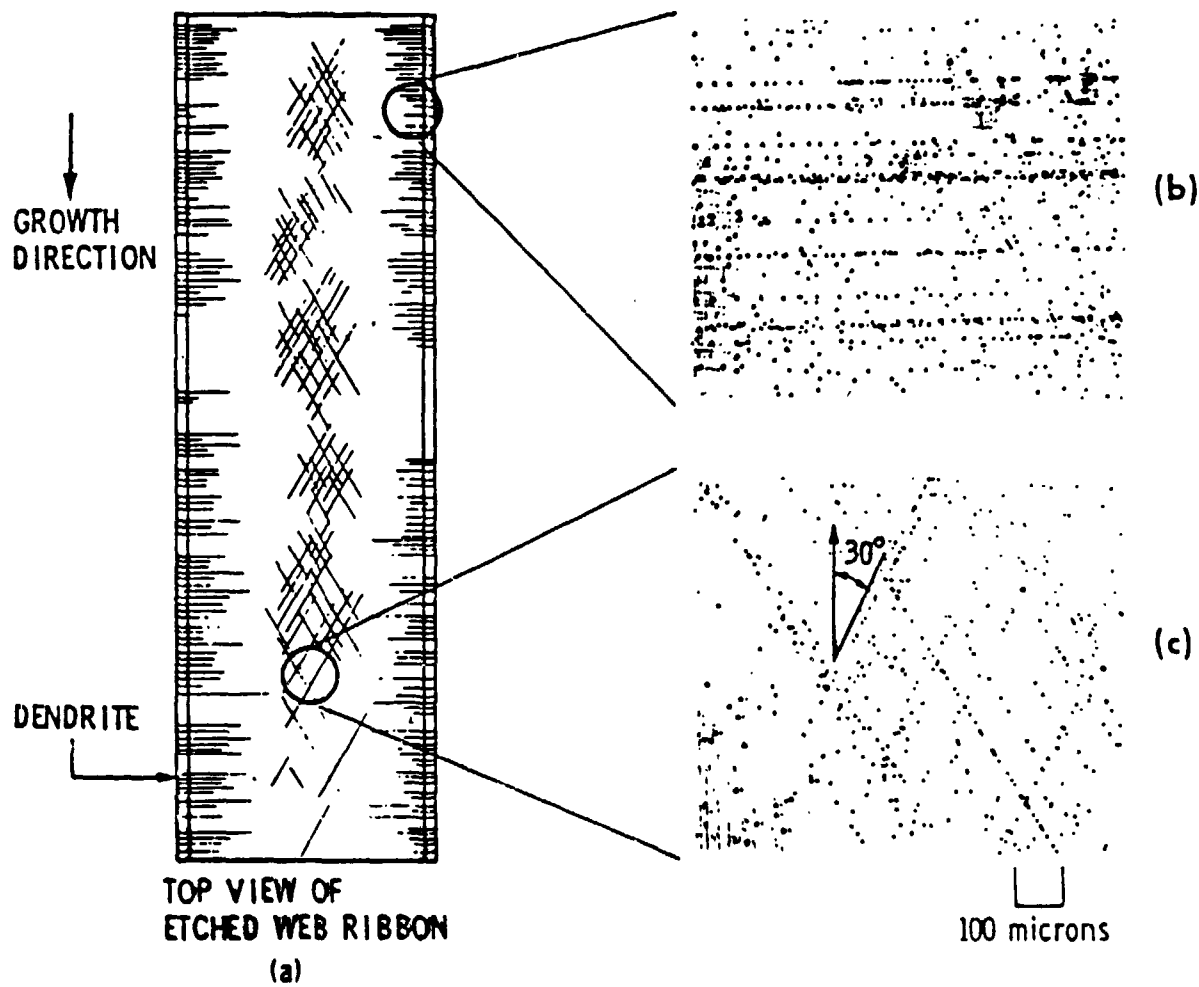
Dislocation Multiplication by Stress Averaging Over 0.5 cm

Dislocation Type			Multiplier Factor		
<u>b</u>	<u>N</u>	<u>Type</u>	<u>Center</u>	<u>Mid-Width</u>	<u>Edge</u>
[101]	[111]	L	750	50	1
[011]	[111]	L	2.5	3.5	5
[110]	[111]	L	23	1	2
[110]	[111]	R	750	50	1
[101]	[111]	R	12	1	50
[011]	[111]	R	1	2.5	35
[101]	[111]	T	1	3	24
[110]	[111]	T	2	3	10
[011]	[111]	T	1.5	66	37

Calculations started at  $x=0.2$  cm.

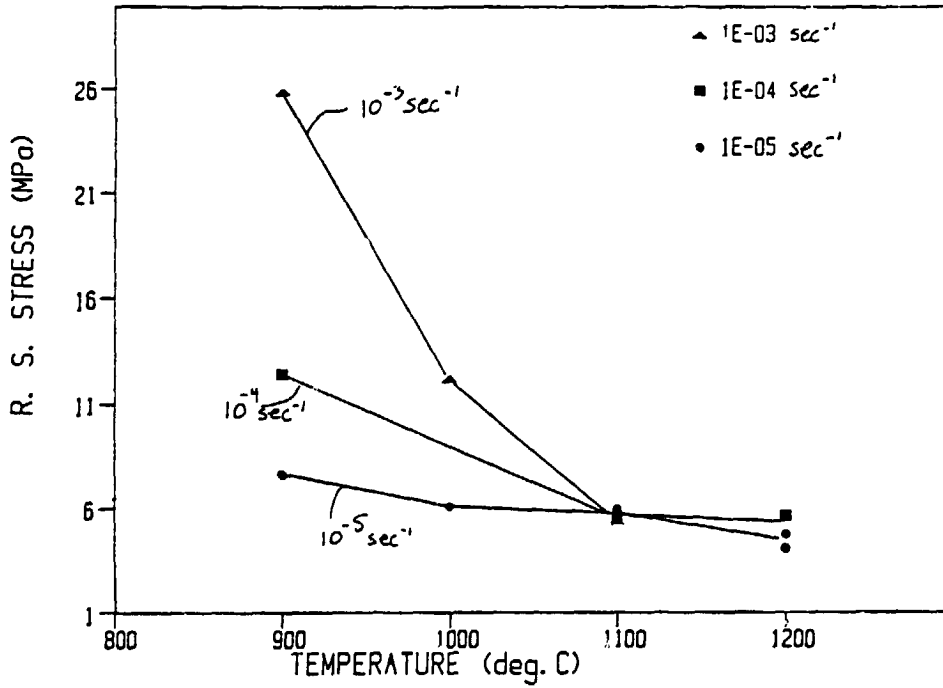


Dislocation Distribution in Web Dendrite Ribbon

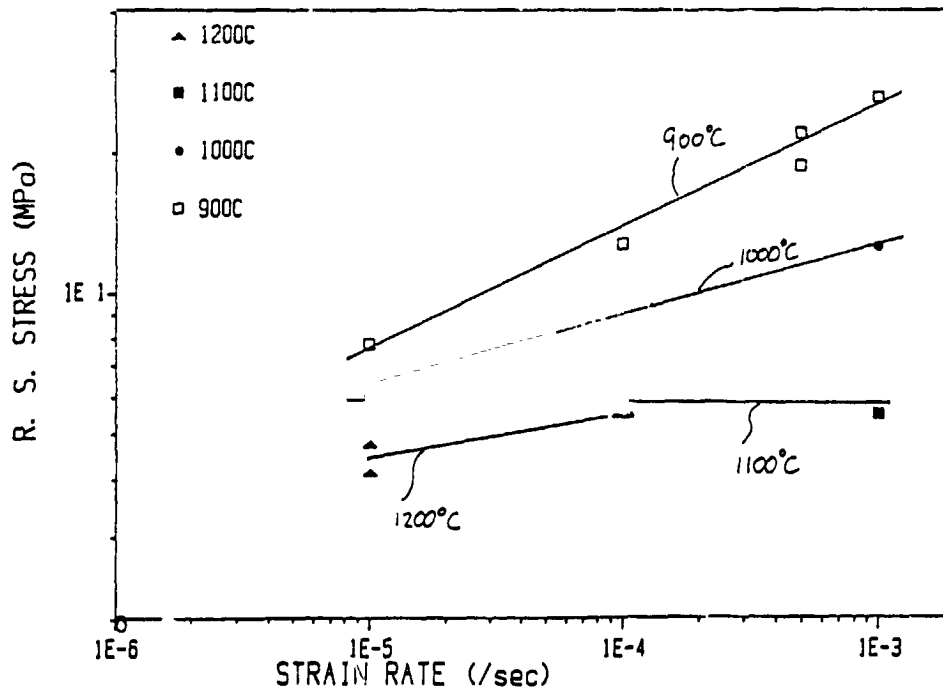


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Cz Temperature Dependence

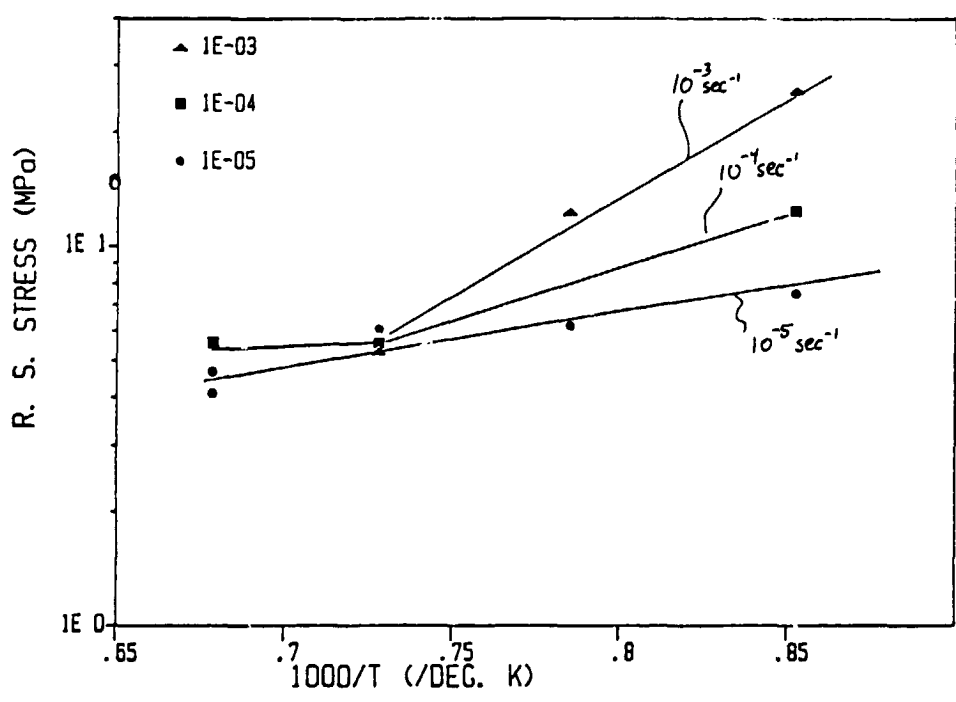


Cz Strain Rate Dependence

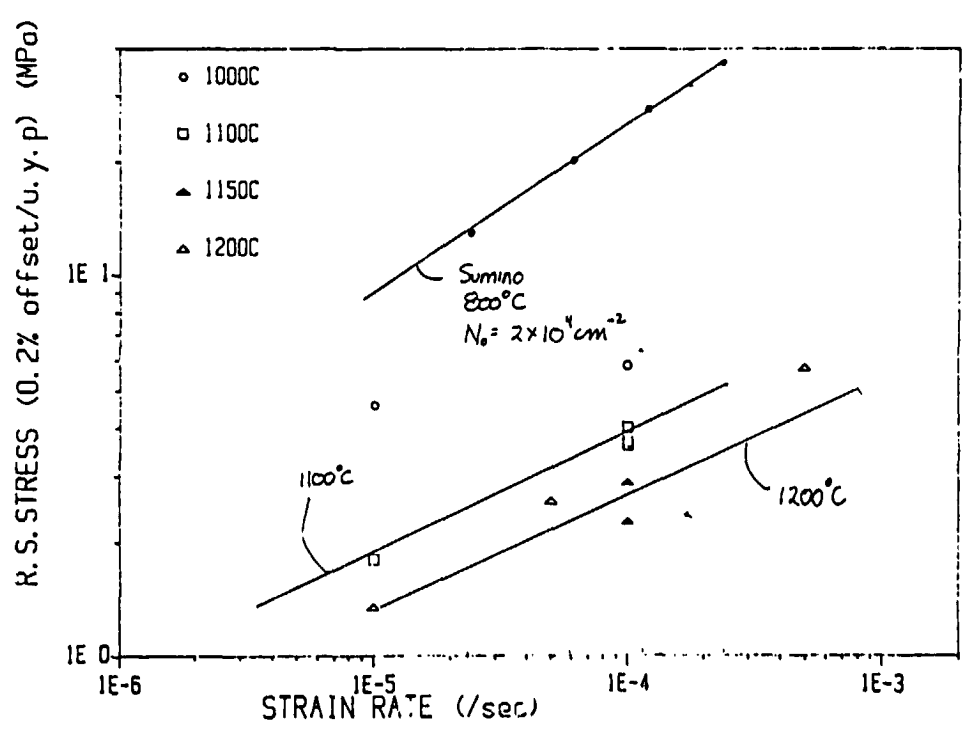


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Cz Temperature Dependence



Web Ribbon: Strain Rate Dependence



Web Ribbon: Temperature Dependence

