Dislocation density contour plot for the parabolic thermal profile of Eq. (4-8), 8x7 cm ribbon and an initial dislocation density = 0.5 cm⁻².
Dislocation Density Along Y = 0 (Centerline)

\[ T = 1412 - 110.74X + 3.5X^2 \]

Line 1 for width = 8 cm
Line 2 for width = 7 cm
Line 3 for width = 6 cm
Line 4 for width = 4 cm

Dislocation density along the centerline of the ribbon for the parabolic thermal profile of Eq. (4-8), initial dislocation density = 0.5 cm\(^{-2}\), and width = 8, 7, 6, 4 cm.
Final Dislocation Density Along the Ribbon
Width for Westinghust Profile

LENGTH = 12 CM, WIDTH = 3.5 CM
STAR FJR NO = 13 /CM××2
DIAMOND FOR NO = 5 /CM××2
SQUARE FOR NO = 1 /CM××2
Dislocation Density Contour Plot

WESTINGHOUSE PROFILE, NO = 0.25/CM*2, R/T = 0
UNIT OF X AND Y = CM, Z = 1 PER CM*2

LEGEND: Z

- - - - - - - - - - - - - - - - 10  - - - - - - - - - - - - - - - - 201
- - - - - - - - - - - - - - - - 39  - - - - - - - - - - - - - - - - 583
- - - - - - - - - - - - - - - - 774 - - - - - - - - - - - - - - - - 965
- - - - - - - - - - - - - - - - 1156 - - - - - - - - - - - - - - - - 1347
- - - - - - - - - - - - - - - - 1538 - - - - - - - - - - - - - - - - 1729
- - - - - - - - - - - - - - - - 1920 - - - - - - - - - - - - - - - - 2111
- - - - - - - - - - - - - - - - 2302 - - - - - - - - - - - - - - - - 2493
- - - - - - - - - - - - - - - - 2684
Dislocation Density Contour Plot

WESTINGHOUSE PROFILE. H0 = 0.375/CH×mm²
UNIT OF X AND Y = CH, Z = 1/CH×mm², A/T = 1.6667 M

Legend: Z

<table>
<thead>
<tr>
<th>Z</th>
<th>130</th>
<th>750</th>
<th>1370</th>
<th>1990</th>
<th>2610</th>
<th>3230</th>
<th>3850</th>
<th>4470</th>
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<tbody>
<tr>
<td>Note</td>
<td>Low</td>
<td>Density</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

with Dendrites

259
ADVANCED SILICON SHEET

Effective Plastic Strain Rate

WESTINGHOUSE PROFILE, NO=0.25/CM×M2
WIDTH = 6 CM, LENGTH = 8 CM, A/T=0.
UNIT OF X AND Y=CM, Z=10××-5 PER SEC

PLASTIC STRAIN R.A.TE

Z

0.86

0.57

0.29

0.00

X

Y

0.00

2.6

5.3

1

2

3

D

No dehydrates

D
ADVANCED SILICON SHEET

Effective Plastic Strain Rate

WESTINGHOUSE PROFILE, NO. 0.375/CM××2
WIDTH = 6 CM, LENGTH = 8 CM, R/T = 1.6667 MM
UNIT OF X AND Y = CM Z = 10××-5 PER SEC

PLASTIC STRAIN RATE
ADVANCED SILICON SHEET

Residual Stress XX Along Ribbon Width for Westinghouse Profile

LENGTH = 12 CM, WIDTH = 3.5 CM
STAR FOR NO = 13 /CM²
DIAMOND FOR NO = 5 /CM²
SQUARE FOR NO = 1 /CM²

No = 13 /cm²
No = 5 /cm²
No = 1 /cm²
Half-width (c) = 1.0, Length = 8.0
Diameter of dendrites is 0.0 inches
Critical thickness = 0.00526 inches

\[ T(x) = \text{parabola} \]
ADVANCED SILICON SHEET

Deflection Shape

HALF-WIDTH (C) = 1.0, LENGTH = 6.0
DIAMETER OF DENDRITES IS 0.2 INCHES
CRITICAL THICKNESS = 0.057120 INCHES

T(x) = PARABOLIC

![Graph showing deflection shape with coordinates and dimensions labeled]
Topics

1 Dislocation Motion

A) Problem Formulation
B) Calculation of Forces
C) Tracking the motion of a single Dislocation

2 Dislocation Multiplication & Density

A) Three methods of calculations - based on resolved shear stresses on each slip system

B) Dislocation density by averaging the |shear stresses| in 0.5 cm. widths of ribbon, (starting at x = 0.2 cm.)
## Possible Dislocations in the Silicon Crystal

<table>
<thead>
<tr>
<th>Burgers vector</th>
<th>Tangent vector</th>
<th>Slip plane</th>
<th>Type of dislocation</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt;-1 0 -1&gt;</td>
<td>&lt;-1 0 -1&gt;</td>
<td>(-1 -1 1)_left, screw</td>
<td></td>
</tr>
<tr>
<td>=</td>
<td>&lt;0 1 1&gt;</td>
<td>_left, 60' (-120') _</td>
<td></td>
</tr>
<tr>
<td>=</td>
<td>&lt;1 -1 0&gt;</td>
<td>_left, 60' (+120') _</td>
<td></td>
</tr>
<tr>
<td>=</td>
<td>&lt;1 -2 -1&gt;</td>
<td>_left, edge _</td>
<td></td>
</tr>
<tr>
<td>&lt;-0 1 1&gt;</td>
<td>&lt;-1 0 1&gt;</td>
<td>_left, screw _</td>
<td></td>
</tr>
<tr>
<td>=</td>
<td>&lt;1 -1 0&gt;</td>
<td>_left, 60' (+120') _</td>
<td></td>
</tr>
<tr>
<td>=</td>
<td>&lt;1 -0 -1&gt;</td>
<td>_left, 60' (-120') _</td>
<td></td>
</tr>
<tr>
<td>=</td>
<td>&lt;-2 1 -1&gt;</td>
<td>_left, edge _</td>
<td></td>
</tr>
<tr>
<td>&lt;-1 1 0&gt;</td>
<td>&lt;-1 1 0&gt;</td>
<td>_left, screw _</td>
<td></td>
</tr>
<tr>
<td>=</td>
<td>&lt;-0 -1 -1&gt;</td>
<td>_left, 60' (-120') _</td>
<td></td>
</tr>
<tr>
<td>=</td>
<td>&lt;1 0 1&gt;</td>
<td>_left, 60' (+120') _</td>
<td></td>
</tr>
<tr>
<td>=</td>
<td>&lt;-1 1 -2&gt;</td>
<td>_left, edge _</td>
<td></td>
</tr>
<tr>
<td>&lt;1 0 -1&gt;</td>
<td>&lt;1 0 -1&gt;</td>
<td>(-1 1 -1)_right, screw</td>
<td></td>
</tr>
<tr>
<td>=</td>
<td>&lt;-1 -1 0&gt;</td>
<td>_right, 60' (-120') _</td>
<td></td>
</tr>
<tr>
<td>=</td>
<td>&lt;0 1 1&gt;</td>
<td>_right, 60' (+120') _</td>
<td></td>
</tr>
<tr>
<td>=</td>
<td>&lt;-1 -2 -1&gt;</td>
<td>_right, edge _</td>
<td></td>
</tr>
<tr>
<td>&lt;0 -1 -1&gt;</td>
<td>&lt;0 -1 -1&gt;</td>
<td>_right, screw _</td>
<td></td>
</tr>
<tr>
<td>=</td>
<td>&lt;1 1 0&gt;</td>
<td>_right, 60' (-120') _</td>
<td></td>
</tr>
<tr>
<td>=</td>
<td>&lt;1 0 1&gt;</td>
<td>_right, 60' (+120') _</td>
<td></td>
</tr>
<tr>
<td>=</td>
<td>&lt;-2 -1 -1&gt;</td>
<td>_right, edge _</td>
<td></td>
</tr>
<tr>
<td>&lt;1 1 0&gt;</td>
<td>&lt;1 1 0&gt;</td>
<td>_right, screw _</td>
<td></td>
</tr>
<tr>
<td>=</td>
<td>&lt;-1 0 1&gt;</td>
<td>_right, 60' (+120') _</td>
<td></td>
</tr>
<tr>
<td>=</td>
<td>&lt;-0 -1 -1&gt;</td>
<td>_right, 60' (-120') _</td>
<td></td>
</tr>
<tr>
<td>=</td>
<td>&lt;-1 1 -2&gt;</td>
<td>_right, edge _</td>
<td></td>
</tr>
<tr>
<td>&lt;0 -1 1&gt;</td>
<td>&lt;0 -1 1&gt;</td>
<td>(-1 1 -1)_transv., screw</td>
<td></td>
</tr>
<tr>
<td>=</td>
<td>&lt;1 1 0&gt;</td>
<td>_transv., 60' (+120') _</td>
<td></td>
</tr>
<tr>
<td>=</td>
<td>&lt;-1 0 -1&gt;</td>
<td>_transv., 60' (-120') _</td>
<td></td>
</tr>
<tr>
<td>=</td>
<td>&lt;-2 1 1&gt;</td>
<td>_transv., edge _</td>
<td></td>
</tr>
<tr>
<td>&lt;1 1 0&gt;</td>
<td>&lt;1 1 0&gt;</td>
<td>_transv., screw _</td>
<td></td>
</tr>
<tr>
<td>=</td>
<td>&lt;-0 -1 1&gt;</td>
<td>_transv., 60' (+120') _</td>
<td></td>
</tr>
<tr>
<td>=</td>
<td>&lt;-1 0 -1&gt;</td>
<td>_transv., 60' (-120') _</td>
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</tr>
<tr>
<td>=</td>
<td>&lt;-1 1 -2&gt;</td>
<td>_transv., edge _</td>
<td></td>
</tr>
<tr>
<td>&lt;1 0 1&gt;</td>
<td>&lt;1 0 1&gt;</td>
<td>_transv., screw _</td>
<td></td>
</tr>
<tr>
<td>=</td>
<td>&lt;-1 -1 0&gt;</td>
<td>_transv., 60' (-120') _</td>
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</tr>
<tr>
<td>=</td>
<td>&lt;-1 -2 -1&gt;</td>
<td>_transv., edge _</td>
<td></td>
</tr>
</tbody>
</table>

1. Surface of the ribbon is (1 1 1) plane.
2. Growth direction to the melt is <-2 -1 -1>.
3. For the motion of the dislocations, 60° dislocations that have -120 degree with the burger's vector will be chosen because these 60° dislocations may multiply themselves more than +120° type 60° dislocations as many investigators observed.
Assumptions

(1) The density of dislocation at the liquid solid interface is uniform.

(2) The pulling rate of the ribbon is 3cm/min.

(3) Dislocations can move only in active slip systems that have their resolved shear stress higher than 95% of the most active slip system that has maximum Schmid Factor.

(4) Average velocity of the dislocations in the presence of other dislocations is almost same as the velocity of isolated dislocation. Equivalent to low dislocation density.

(5) The velocity equation proposed by K.Sumino

\[ V = V_o \tau \exp\left(-\frac{E}{kT}\right) \]

where \( E \) is 2.2 eV for 60' and 2.35 eV for screw,
\[ V_o = 0.035 \text{ for screw and } 0.01 \text{ m}^3/\text{MN.sec for 60'.} \]

is still valid at high temperature like around melting temperature.
Motion of Dislocation

$B=A/2 \begin{bmatrix} -1 & 1 & 0 \end{bmatrix}, T=\begin{bmatrix} 1 & 0 & 1 \end{bmatrix}, W=\begin{bmatrix} -1 & -1 & 1 \end{bmatrix}$

UNIT OF AXES ARE CM
ADVANCED SILICON SHEET

Motion of the Dislocations Emerging to the Surface

possible 60° dislocations emerging to the surface

<table>
<thead>
<tr>
<th>burger's vector</th>
<th>tangent vector</th>
<th>plane</th>
<th>motion</th>
</tr>
</thead>
<tbody>
<tr>
<td>a/2(-1 0 -1)</td>
<td>( 0 1 1)</td>
<td>(-1 -1 1) left strong</td>
<td></td>
</tr>
<tr>
<td>a/2(-1 1 0)</td>
<td>( 0 -1 -1)</td>
<td>(-1 -1 1) *</td>
<td></td>
</tr>
<tr>
<td>a/2(-1 1 0)</td>
<td>( 1 0 1)</td>
<td>(-1 -1 1) split</td>
<td></td>
</tr>
<tr>
<td>a/2( 0 1 1)</td>
<td>(-1 0 -1)</td>
<td>(-1 -1 1) right weak</td>
<td></td>
</tr>
<tr>
<td>a/2( 1 0 -1)</td>
<td>( 0 1 1)</td>
<td>(-1 1 -1) right strong</td>
<td></td>
</tr>
<tr>
<td>a/2( 1 0 -1)</td>
<td>(-1 -1 0)</td>
<td>(-1 1 -1) *</td>
<td></td>
</tr>
<tr>
<td>a/2( 1 1 0)</td>
<td>( 0 -1 -1)</td>
<td>(-1 1 -1) *</td>
<td></td>
</tr>
<tr>
<td>a/2( 0 -1 -1)</td>
<td>( 1 1 -0)</td>
<td>(-1 1 -1) right weak</td>
<td></td>
</tr>
<tr>
<td>a/2( 0 -1 1)</td>
<td>(-1 0 -1)</td>
<td>( 1 -1 -1) right strong</td>
<td></td>
</tr>
<tr>
<td>a/2( 0 -1 1)</td>
<td>(-1 -1 0)</td>
<td>( 1 -1 -1) left strong</td>
<td></td>
</tr>
<tr>
<td>a/2( 1 0 1)</td>
<td>(-1 -1 0)</td>
<td>( 1 -1 -1) left weak</td>
<td></td>
</tr>
<tr>
<td>a/2( 1 1 0)</td>
<td>(-1 0 -1)</td>
<td>( 1 -1 -1) left weak</td>
<td></td>
</tr>
</tbody>
</table>

* : These are forced into the liquid

270
Calculation of the Density of Dislocations

From the K. Sumino's equation of dislocation multiplication

$$dN_m = K K_0 N_{m1} (T_a - G b \sqrt{N_{m2}}/ \beta )^{(m + \lambda )} \exp(-Q/kT) \, dt \quad \text{---(A)}$$

where $K, K_0, b, \beta, m, \lambda, k, Q$ are constants given by K. Sumino

$N_m$'s; dislocation density

$N_{m1} \alpha$ source density

$N_{m2}$ is the density controlling the back stress

$T_a$; applied stresses

$T$; temperature

$G$; shear modulus

$t$; time

Three possible ways of application of the equation (A)

(1) $N_{m1}$ and $N_{m2}$ are total density of dislocations

(2) $N_{m1}$ is the partial density of dislocations on each slip system and

$N_{m2}$ is the total density of dislocations.

(3) Both $N_m$'s are partial densities of dislocations on each slip system
ADVANCED SILICON SHEET

Method Calculating Dislocation Density

\[ \text{total density} = N_m = N_m \]
\[ dN_m = dN_m = dN_m \]

(1) \[ \text{total density} = (N_m)_1 = N_m \]
\[ dN_m = (dN_m)_1 \]

(2) \[ \text{total density} = \sum (N_m)_i = N_m \]
\[ dN_m = \sum (dN_m)_i \]

(3) \[ \text{total density} = \sum (N_m)_i = \sum (N_m)_i \]
\[ (dN_m)_i = (dN_m)_i \]
ADVANCED SILICON SHEET

Total Density of Dislocation Using YI, YTOT and DGEAR

INITIAL TOTAL DENSITY IS 90/M*2 LAMDA IS 1.0

LEGEND: DENS

- 8.5915E+17
- - 6.0149E+18
- 1.1169E+19
- 1.6324E+19
- 3.4366E+18
- 8.5915E+18
- 1.3746E+19
Total Density of Dislocation Using YI, YI and DGEAR

INITIAL TOTAL DENSITY IS 90/\(\text{m}^2\) . LAMDA IS 1.0

LEGEND: DENSB

- 7.7220E+11
- 5.4054E+12
- 3.0888E+12
- 7.2200E+12
- 3.0888E+12
ADVANCED SILICON SHEET

Averaging the $|\tau a|$ in Calculating Density

$\text{cm}$

$0.0 \quad 0.5 \quad 1.0 \quad 1.5 \rightarrow Y \text{ (cm)}$

center edge

column 1 ; * , column 2 ; , column 3 ; □
ADVANCED SILICON SHEET

Nine Slip Systems

<table>
<thead>
<tr>
<th>Burger's Vector</th>
<th>Slip Plane</th>
<th>Type of Plane</th>
</tr>
</thead>
<tbody>
<tr>
<td>a/2 (1 0 1)</td>
<td>(-1 -1 1)</td>
<td>left</td>
</tr>
<tr>
<td>a/2 (0 1 1)</td>
<td>(-1 -1 1)</td>
<td>left</td>
</tr>
<tr>
<td>a/2 (-1 1 0)</td>
<td>(-1 -1 1)</td>
<td>left</td>
</tr>
<tr>
<td>a/2 (1 0 -1)</td>
<td>(-1 1 -1)</td>
<td>right</td>
</tr>
<tr>
<td>a/2 (1 1 0)</td>
<td>(-1 1 -1)</td>
<td>right</td>
</tr>
<tr>
<td>a/2 (0 1 1)</td>
<td>(-1 1 -1)</td>
<td>right</td>
</tr>
<tr>
<td>a/2 (0 -1 1)</td>
<td>(1 1 -1)</td>
<td>transverse</td>
</tr>
<tr>
<td>a/2 (1 0 1)</td>
<td>(1 1 -1)</td>
<td>transverse</td>
</tr>
<tr>
<td>a/2 (1 1 0)</td>
<td>(1 1 -1)</td>
<td>transverse</td>
</tr>
</tbody>
</table>
Density of Dislocations When NO is 1 at \( x = 0.2 \text{ cm} \)

UNIT OF \( X \) IS CM, \( Z \) IS DENSITY IN \( 1/\text{M}^2 \)
Density of Dislocations When NO is 1 at \( x = 0.2 \text{ cm} \)

Type of dislocation: \( b = a/2 \langle 101 \rangle, T = \langle 101 \rangle, N = (1 -1 -1) \)

\( T = 0.225 \text{-star}, T = 0.75 \text{-diamond}, T = 1.275 \text{-square} \)

UNIT OF \( X \) IS CM, \( Z \) IS DENSITY IN \( 1/\text{m}^2 \)
ADVANCED SILICON SHEET

Density of Dislocations When NO is 1 at x = 0.2 cm

UNIT OF X IS CM, Z IS DENSITY IN 1/M**2
Dislocation Multiplication by Stress Averaging Over 0.5 cm

<table>
<thead>
<tr>
<th>Dislocation Type</th>
<th>Multiplication Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Center</td>
</tr>
<tr>
<td>[101] [{1}1]</td>
<td>750</td>
</tr>
<tr>
<td>[0{1}1]</td>
<td>750</td>
</tr>
<tr>
<td>[1{1}0]</td>
<td>750</td>
</tr>
<tr>
<td>[1{1}1]</td>
<td>12</td>
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<tr>
<td>[0{1}1]</td>
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</tr>
<tr>
<td>[10{1}1]</td>
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</tr>
<tr>
<td>[1{1}0]</td>
<td>2</td>
</tr>
<tr>
<td>[0{1}1]</td>
<td>1.5</td>
</tr>
</tbody>
</table>

Calculations started at \( x = 0.2 \) cm.
Dislocation Distribution in Web Dendrite Ribbon

GROWTH DIRECTION

TOP VIEW OF ETCHED WEB RIBBON

(a)

(b)

(c)

100 microns

DENDRITE
ADVANCED SILICON SHEET

Cz Temperature Dependence

Cz Strain Rate Dependence
ADVANCED SILICON SHEET

Cz Temperature Dependence

Web Ribbon: Strain Rate Dependence
Web Ribbon: Temperature Dependence

- STRAIN RATE = 1E-04
- STRAIN RATE = 1E-05

Summed: $N = 2 \times 10^4 \text{m}^{-1}$
$Z = 1.2 \times 10^5 \text{sec}$