On Seasonal Variations of Mars' Gravitational Field

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INTRODUCTION

It is well-known that Mars has seasons like the Earth. However, the Mars' colder atmospheric temperature (in average some 70°K lower than the Earth's) and its predominantly carbon dioxide (CO$_2$) atmosphere (more than 95%) conspire to create a large exchange of CO$_2$ mass between the Martian atmosphere and polar caps. The mass exchange occurs in seasonal cycles: a great quantity of CO$_2$ condenses to form (or add to) the polar caps in winter and sublimes into the atmosphere in summer. The mass involved is at least $8 \times 10^{15}$ kg, which is about 25% of the total mass of the Martian atmosphere, or equivalent to a cap of solid CO$_2$ extending to 45° latitude changing its thickness by some 20 cm (Hess et al., 1979). In contrast, the Earth's atmosphere only varies seasonally by about $1 \times 10^{15}$ kg (or about 0.02% of the total), primarily due to variation of water vapor content (Trenberth, 1981).

The effect of this CO$_2$ mass redistribution on Mars' rotation has been discussed by Reasenberg and King (1979) and Cazenave and Balmino (1981). The present paper is an attempt to address quantitatively this CO$_2$-exchange effect on the Martian gravitational field. Qualitatively, the gravitational effect of any global-scale mass redistribution is largely decided by the ratio of the net redistributed mass to the total mass of the planet. For the Martian CO$_2$-exchange effect, the ratio is $8 \times 10^{15}/6.4 \times 10^{23}$, or about $10^{-8}$. No geological or geophysical phenomenon on Earth is known to produce a relative mass redistribution of nearly that size in a year. The post-glacial rebound of the mantle, the strongest southern oscillation / El Niño events in the atmosphere and the ocean, and the greatest earthquakes are all at least one to two orders of magnitudes smaller. Since even these geophysical phenomena are already being observed gravitationally by Earth-orbiting satellites (e.g., Rubincam, 1984; Gross and Chao, 1985), we can expect the CO$_2$-exchange effect on Mars to be detectable from future Mars orbiter missions. If so, the observed gravitational field changes can provide gross constraints on the meteorological models for Mars.
GENERAL FORMULATION

Let the coordinate system be such that the origin is at Mars' center of mass, and the x, y, and z axes define the 0°E Meridian, the 90°E Meridian, and the North Pole, respectively. We express the gravitational potential $U$ of a planet at an external field point $r_0$ through spherical harmonic expansion (e.g., Balmino, 1981):

$$U(r_0) = \frac{GM}{r_0} \left[ 1 + \left( \frac{R}{r_0} \right)^2 C_2^0 \cdot \mathbf{Y}_2^0(\Omega_0) \right] + T(r_0) \quad (1)$$

where $\Omega$ is an abbreviation for the co-latitude $\theta$ and east longitude $\lambda$, $(r_0, \theta_0, \lambda_0) = (r_0, \Omega_0)$ is the spherical coordinate of $r_0$, $G$ is the gravitational constant, $M$ the total mass, and $R$ the mean radius of the planet. The spherical harmonics in this paper are defined as

$$Y_{lm}(\Omega) = \left[ (2-\delta_{m0})(2l+1)(1-m)!/(1+m)! \right]^{1/2} P_{lm}(\cos\theta) e^{im\lambda} \quad (2)$$

where $l$ and $m$ are respectively the degree and order, $\delta$ is the Kronecker delta function, $P_{lm}$ is the associated Legendre function. We shall later use the index $(l,m)$ to specify a particular harmonic component. The normalization in (2) is such that

$$\int Y_{l_1}^{*m_1}(\Omega) Y_{l_1}^{*m_1}(\Omega) \, d\Omega = 4\pi (2-\delta_{m0}) \delta_{l_1} \delta_{m_1} \quad (3)$$

where the asterisk denotes complex conjugation, $d\Omega$ is an element of solid angle, and the integration is over the unit sphere.

Neglecting higher order terms, the first two terms in equation (1) represent the potential the planet would have assumed in hydrostatic equilibrium (indicated by the superscript $H$) under rotation. That constitutes the reference spheroid. Thus the remaining term $T(r_0)$ is the anomalous potential relative to the reference spheroid. It can be expressed as

$$T(r_0) = \frac{GM}{r_0} \text{Re} \left[ \sum_{l=2}^{\infty} \sum_{m=0}^{l} \left( \frac{R}{r_0} \right)^l A_{lm} Y_{lm}^*(\Omega_0) \right] \quad (4)$$

where Re denotes the real value. The normalization in (3) is chosen such that the harmonic coefficient $A_{lm}$ in (4), here referred to as the complex Stokes coefficient, is directly related to the ordinary (normalized) Stokes coefficients $C_{lm}$ and $S_{lm}$ (c.f. Kaula, 1966) simply through

$$A_{lm} = C_{lm} + iS_{lm} . \quad (5)$$

The geoid of a planet is the equipotential surface on which $T(r_0)=0$. Its departure from the reference spheroid is the geoid height (c.f. Figure 3).
By comparing equations (1) and (4) with the multipole expansion of \( U \), one can relate the complex Stokes coefficients to the density distribution \( \rho(r) \) of the planet (e.g., Chao & Gross, 1986):

\[
A_{1m} = \frac{1}{(2l+1) MR^l} \int \rho(r) \ r^l \ Y_{1m}(\theta) \ dV
\]  

(6)

where \( dV \) is the volume element, and the integration is over the entire volume \( V \) of the planet.

Now suppose a time-varying change \( \Delta \rho(r,t) \) is imposed upon \( \rho(r) \). In an Eulerian description, the resultant change in the complex Stokes coefficient follows immediately from equation (6):

\[
\Delta A_{1m} = \frac{1}{(2l+1) MR^l} \int \Delta \rho(r) \ r^l \ Y_{1m}(\theta) \ dV
\]  

(7)

where the integration is now over the changing volume of the planet. In the following we shall translate these changes into equivalent changes in the geoid height \( N \) using Bruns formula \( N = T/g \), where \( g = GM/R^2 \) is the surface gravitational acceleration (e.g., Rapp, 1975). This, in the present case, gives the root-mean-square (rms) amplitude of the \( (1,m) \) component of the geoid height change:

\[
\Delta N_{1m}(t) = R \ | \Delta A_{1m}(t) |
\]  

(8)

\( \Delta N_{1m}(t) \) (in millimeters) provides a comfortable measure of the effect of \( \Delta \rho(r,t) \) on the geoid.
CO₂ MASS EXCHANGE ON MARS

Equation (7) will now be applied to the CO₂ mass exchange between the atmosphere and polar caps on Mars. First, we notice that this mass exchange occurs in a thin veneer on the Martian surface (only a few tens of kilometers thick), and any radial movement is negligible in the presence of the global-scale lateral movement. Therefore, to a good approximation we can take \( r = R \) and use surface density \( \Delta \sigma(R, t) \) in place of the volume density \( \Delta \rho(r, t) \). Equation (7) then reduces to

\[
\Delta A_{1m}(t) = \frac{R^2}{(2l+1)M} \int \Delta \sigma(R, t) \ Y_{1m}(\Omega) \ d\Omega
\]

(9)

where the integration is over the entire surface of the planet.

The simplicity of equation (9) is deceiving because to obtain the true \( \Delta \sigma(R, t) \) one should in general take into consideration the gravitational interaction between the shifting mass and the rest of the planet, or, in the present case, the effect of the elastic yielding of the solid Mars due to the loading and unloading of the CO₂ mass. However, as shown below, this interaction is small on Mars so that the solid Mars can be treated as a rigid body.

The relevant parameter is the load Love number \( k'_l \) of degree 1 and how it compares to unity. Assuming a homogeneous planet,

\[
k'_2 = \frac{-1}{1 + \left( \frac{19 \bar{\mu}}{2 \bar{p} g R} \right)}
\]

(10)

(Munk & MacDonald, 1960). Here \( \bar{\rho} \) and \( \bar{\mu} \) are the average density and rigidity, respectively, and \( \bar{g} = 3.9 \times 10^3 \) kg m⁻³. If we assume that Mars has the same average rigidity as the Earth, \( \bar{\mu} \approx 1.45 \times 10^{11} \) kg m⁻¹ s⁻¹, then \( |k'_2| = 0.037 \), or about one eighth of the Earth's (Lambeck, 1980). This, in fact, may be an upper limit on \( |k'_l| \) since Mars, being a colder planet internally, might have a higher rigidity than the Earth's. Likewise the other \( |k'_1| \ll 1 \).

We thus conclude that the elastic yielding of the solid Mars can be safely neglected. We shall also neglect the self-gravitation within the CO₂ mass itself. The resultant errors are relatively small compared with those introduced by the simplifications we shall make in our model below. In the rigid solid Mars approximation, we can extend equation (9) to encompass \( l=1 \) (\( m=0, \pm 1 \)) terms. These terms simply give the relative shift in the center of mass of the solid part of Mars due to the CO₂ variations.

Now let us follow the seasonal journey of the CO₂ mass and examine separately the contributions to (9) of two phases: (i) the waxing and waning of the solid CO₂ in polar caps, and (ii) the geographical distribution of the gaseous CO₂ in the atmosphere. The two phenomena, of course, accompany each other inasmuch as the total CO₂ mass is constant. The total CO₂-exchange effect is simply the sum of the two contributions.
Fig. 1. A cross section of Mars showing schematically two solid CO$_2$ polar caps.
Waxing and Waning of Polar Caps

First let us examine the simple geometry where the seasonal condensation and sublimation occur uniformly over the polar caps, and the caps are circular and symmetrical with respect to the poles (Figure 1). The surface density \( \Delta \sigma (\varphi, t) = \Delta \sigma (t) \) is thus a constant of position on the caps which extend to a constant angle \( \xi \) from the poles.

For \( m \neq 0 \), equation (9) gives \( \Delta A_{1m} = 0 \), an expected result in light of the axial symmetry under the present assumptions. For \( m = 0 \), after some simple algebra, equation (9) leads to

\[
\Delta A_{10} (t) = \Delta C_{10} (t)
\]

\[
= \left[ \frac{1}{\sqrt{2l+1}} \right] \int \frac{1}{\cos \mu} \left[ P_1 (\mu) \frac{d \Delta M(t)}{d \mu} \cos \mu \right] n + (-1)^{l} \left[ \frac{\text{same}}{s} \right]
\]

(11)

where \( P_1 \) is the Legendre function of degree 1, \( \Delta M(t) = 2\pi R^2 (1-\cos \xi) \Delta \sigma (t) \) is the total \( \text{CO}_2 \) mass accumulated on the cap at any given moment, and the subscripts \( n \) and \( s \) indicate the northern and southern hemispheres, respectively. Due to the eccentricity of the Mars' orbit the southern \( \text{CO}_2 \) polar cap is considerably more massive and extensive than its northern counterpart (Hess et al., 1979). At the peak of its formation, the angular extent \( \xi_s \) actually varies with longitude between 40° and 50°, according to Viking orbiter observations (Briggs et al., 1977). For simplicity we shall take as an average \( \xi_s = 45 \)°. The value of \( \xi_n \) is less certain. The variation of the \( \text{CO}_2 \) mass in the polar caps, \( \Delta M(t) \), can be estimated from the atmospheric pressure recordings of the two Viking landers (Hess et al., 1979, 1980). It is shown schematically in Figure 2, where the zero level corresponds to the minimum polar cap \( \text{CO}_2 \) (or, equivalently, maximum atmospheric \( \text{CO}_2 \)). Here an annual period (669 sôls, or 687 Earth days) and a semi-annual period dominate. Although it is not possible from these records to separate the contributions from the northern and southern caps, each cap is responsible for essentially half a (Martian) year. It is interesting to note from equation (11) that the variation in the Stokes coefficient, \( \Delta C_{10} \), will follow the sum of the contributions from both caps if \( l \) is even, and the difference if \( l \) is odd. Thus, under the present simplification, if we can observe two varying \( C_{10}(t) \), one for an even \( l \) and one for an odd \( l \), it is in principle possible to solve for \( \Delta M_n(t) \) and \( \Delta M_s(t) \) using equation (11), thus providing two important constraints on meteorological models for Mars.

It is also evident from equation (11) that the amplitude becomes progressively smaller as \( l \) increases. Here we shall only consider a few lowest degree terms by evaluating their maximum amplitudes which occur at the peak of formation of the southern cap (\( t=t_0 \), see Figure 2). Thus, substituting into equation (11) the value for \( \Delta M, \Delta M_s(t_0) = 8 \times 10^{15} \text{kg}, \) and \( \xi_s=45 \)°, we get \( \Delta C_{10}(t_0) \) for \( l=1-5 \), listed in Table 1. Table 1 also shows the corresponding amplitude of change in the more familiar zonal \( J_1 \) coefficients which are related to \( C_{10} \) by \( J_1 = -\sqrt{2l+1} C_{10} \), as well as the rms amplitude.
Fig. 2. $\text{CO}_2$ mass change in the course of one Martian year where the zero time ($t=0$) corresponds to the winter solstice (adopted from Hess et al., 1980). The upper curve represents the formation of the solid $\text{CO}_2$ polar caps, and the lower curve represents the corresponding loss of atmospheric $\text{CO}_2$. 

8 x $10^{15}$ 

$\Delta m_S$

$\Delta m_N$

$\Delta m$

Atmosphere

Time (sol)
in the corresponding geoid height change as given by equation (5) using 
R=3390 km. The largest change is in the l=1 term, which corresponds to a 
maximum shift of \( \sqrt{2(l+1)} \Delta N_{10} = 36 \text{ mm} \) by the solid planet with respect to the 
center of mass at \( t=t_0 \). The yearly peak-to-peak variation taking into 
account both northern and southern caps is about 52 mm. The most geodeti-
cally interesting change, however, is in \( J_2 \) which is the second largest 
term: \( \Delta J_2 = -7.5 \times 10^{-9} \). The nominal value of \( J_2 \) has been well determined 
(e.g., Reasenberg et al., 1975) to be \( J_2 = 1.956 \times 10^{-3} \). The maximum 
amplitude of the relative seasonal change is therefore \( \Delta J_2/J_2 = 3.8 \text{ ppm} \) 
(parts per million). In comparison, this is two orders of magnitude larger 
than the observed secular change in the Earth's \( J_2 \) (presumably due to the 
post-glacial rebound of the mantle) which is found to be only 0.024 ppm per 
year (Rubincam, 1984).

The values in Table 1 are obtained under the simplification of a uniform 
condensation/sublimation of solid CO\(_2\) over the entire polar caps (Figure 2). 
The real distribution, however, is probably more concentrated toward the 
pole. The latter will yield somewhat larger changes in \( C_{10}(t) \). Hence Table 1 
is only a lower bound on \( AC_{10}(t_0) \). For example, the extreme case where all 
the condensed CO\(_2\) is concentrated at the pole \( \kappa=0^\circ \) gives, according to 
equation (11), a \( \Delta J_2(t_0) \) of \( -12.5 \times 10^{-9} \) which is some 60% larger than the 
uniform case. A more realistic, "layered cake" model where the CO\(_2\) layer 
between \( \kappa=0^\circ \) and \( 22.5^\circ \) is twice as thick as that between \( \kappa=22.5^\circ \) and \( 45^\circ \) 
yields a \( \Delta J_2(t_0) \) of \( -8.7 \times 10^{-9} \).

Table 1. Maximum gravitational effect of the changing polar cap CO\(_2\) 
(equation 11).

<table>
<thead>
<tr>
<th>( l )</th>
<th>( AC_{10} \times 10^9 )</th>
<th>( \Delta J_1 \times 10^9 )</th>
<th>( \Delta N_{10} \text{ (mm)} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-6.1</td>
<td>10.6</td>
<td>21</td>
</tr>
<tr>
<td>2</td>
<td>3.4</td>
<td>-7.5</td>
<td>11</td>
</tr>
<tr>
<td>3</td>
<td>-1.5</td>
<td>4.0</td>
<td>5.0</td>
</tr>
<tr>
<td>4</td>
<td>0.31</td>
<td>-0.94</td>
<td>1.1</td>
</tr>
<tr>
<td>5</td>
<td>0.30</td>
<td>-1.0</td>
<td>1.0</td>
</tr>
</tbody>
</table>
Geographical Distribution of the Atmosphere

The Martian surface topography will influence the way the changing atmospheric CO$_2$ mass distributes itself geographically; that, in turn, will produce changes in the gravitational field. For simplicity, we here assume that the atmosphere is always in hydrostatic equilibrium. Physically, this requires an instantaneous redistribution of CO$_2$ mass over the globe. This is permissible since we are only concerned with seasonal periods which are much longer than the time scale for Martian atmospheric circulation.

In equilibrium, any isopycnic (constant density) surface of the atmosphere will follow an equipotential surface of the gravitational field. In a hypothetical case where the surface topography is itself a geoid (for example, if the entire planet were covered by an ocean), the geographic distribution of the atmosphere would be practically uniform in the sense that its surface density $\sigma(Q,t)$ would be independent of geographic location $Q$, changing only with time $t$. In such a case, the changing atmosphere has no gravitational effect (except on the total mass which, of course, is compensated by the waxing and waning of polar caps). In reality, the Martian surface topography is not a geoid. In fact, Mars, unlike the Earth, has a relatively large surface relief, so much so that a substantial fraction of the atmosphere can "feel" the topography. The atmospheric surface density $\sigma$ will be smaller where the topography protrudes the geoid (e.g. over a high plateau), and larger otherwise (e.g. over a low basin).

Thus, it is the departure of the true topography from the geoid that determines how $\sigma(Q)$ is to deviate from being uniform. This "effective topography", denoted by $h(Q)$ (see Figure 3), can be written in terms of spherical harmonics as

$$h(Q) = R \Re \left[ \sum_{l=1}^{\infty} \sum_{m=0}^{l} H_{lm} Y_{lm}^*(Q) \right].$$

(12)

The coefficient $H_{lm}$, using equation (4) and Bruns formula, is

$$H_{lm} = H'_{lm} - A_{lm}$$

(13)

where $H'_{lm}$ is the complex harmonic coefficient of the true topography $h'_{l}(Q)$ (relative to the reference spheroid):

$$h'(Q) = R \Re \left[ \sum_{l=1}^{\infty} \sum_{m=0}^{l} H'_{lm} Y_{lm}^*(Q) \right].$$

(14)

Notice that in equations (12) and (14) we included first-degree ($l=1$) harmonics. They represent a displacement of the center of figure from the center of mass (Bills and Ferrari, 1978).
Fig. 3. Schematic diagram showing the distribution of atmosphere as specified by the scale height, in relation to the topography, the geoid, and the reference spheroid. N: geoid height, h': true topography, h: effective topography, z: vertical height from surface.

Table 2. Maximum gravitational effect of the changing atmospheric CO₂ (equation 20).

<table>
<thead>
<tr>
<th>(l,m)</th>
<th>(ΔC₁₀ₘ, ΔS₁₀ₘ) (x10⁹)</th>
<th>ΔJ₁ (x10⁹)</th>
<th>ΔN₁₀ₘ (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1,0)</td>
<td>(-0.48, --)</td>
<td>0.84</td>
<td>1.6</td>
</tr>
<tr>
<td>(1,1)</td>
<td>(-0.021, 0.51)</td>
<td>--</td>
<td>1.7</td>
</tr>
<tr>
<td>(2,0)</td>
<td>(-0.73, --)</td>
<td>1.6</td>
<td>2.5</td>
</tr>
<tr>
<td>(2,1)</td>
<td>(0.11, -0.16)</td>
<td>--</td>
<td>0.67</td>
</tr>
<tr>
<td>(2,2)</td>
<td>(-0.31, -0.25)</td>
<td>--</td>
<td>1.4</td>
</tr>
<tr>
<td>(3,0)</td>
<td>(0.046, --)</td>
<td>-0.12</td>
<td>0.16</td>
</tr>
<tr>
<td>(3,3)</td>
<td>(0.085, -0.25)</td>
<td>--</td>
<td>0.88</td>
</tr>
</tbody>
</table>
From $h(n)$ we can obtain $Aa(n)$ for the atmospheric $C_2O$ as follows. The lower atmosphere is where virtually all the atmospheric mass resides. The equilibrium density variation there can be approximated by the isothermal profile

$$\rho(z,n) = \rho_0 \exp\left[-(z+h(n))/Z\right]$$

where $z$ is the vertical height from the surface, and $\rho_0$ is the atmospheric density on the geoid $z=-h(n)$. $Z$ is the atmospheric scale height (see Figure 3) and is given by

$$Z = kT/\nu g$$

where $k$ is the Boltzmann constant ($1.38 \times 10^{-23}$ J $\cdot$K$^{-1}$), $T$ is the temperature of the lower atmosphere, $\nu$ is the molecular weight of $C_2O$ ($7.3 \times 10^{-26}$ kg), and the surface gravity $g=3.7$ m s$^{-2}$.

The temperature $T$, and hence the scale height $Z$ are of course functions of time and position. For simplicity we shall take $T$ to be the spatially averaged atmospheric temperature over the globe at any given time. By doing so we are essentially disregarding the effect on $Z$ due to diurnal fluctuations in the temperature. This is permissible because these fluctuations average out on a seasonal time scale and has little bearing in the present study. However, $T$ is still a function of the time of the year because of the eccentricity of the Mars' orbit relative to the Sun. In particular, $T$ will be highest near perihelion (late southern spring) and lowest near aphelion (late southern fall). A brief examination of Figure 2 reveals that at the times under consideration, namely the times when the amount of the atmosphere is at its maximum ($t=0$) and minimum ($t=t_0$), the globally averaged $T$ should be about the same and somewhat higher than the yearly mean value. Therefore we can take $T$ as a constant for which our best judgment is $T=220^\circ$K, corresponding to a nominal $Z=11$ km according to equation (16). For Viking lander observations during landing at Northern mid-latitudes in summer, the reader is referred to Seiff and Kirk, 1977. We should mention that we have considered the surface values only. In reality, the temperature decreases with increasing altitude fairly rapidly, so that the "effective" scale height is somewhat smaller than 11 km. In other words, 11 km is probably an upper bound on $Z$.

Under all the simplifications made so far, the change in surface density of the atmospheric $C_2O$ is

$$\Delta \sigma(n,t) = \int_0^\infty \Delta \rho(z,n,t) \, dz = \Delta \rho_0(t) \, Z \exp[-h(n)/Z].$$

To proceed, we notice that, although the true Martian topography ($h'$) is at places comparable to or even exceeds the atmospheric scale height $Z$, the effective topography ($h$) is generally much smaller. In fact, the rms amplitudes $R|H_{1,m}|$ of the individual $(1,m)$ harmonic component of $h$ are all small compared with $Z$ (the largest term is only about 0.1Z, see Bills and Ferrari, 1978). This is a consequence of the high correlation between the topography and the gravitational field of Mars (e.g., Bills & Ferrari, 1978), especia-
lly in low-degree harmonics on which we shall concentrate. Therefore we can approximate \( \exp[-h(n)/Z] \) by substituting the expression (12) for \( h(n) \) and retaining the first-order terms in a Taylor expansion

\[
\exp[-h(n)/Z] = 1 - \frac{R}{Z} \Re \left[ \sum_{l=1}^{\infty} \sum_{m=0}^{l} H_{lm} Y_{lm}(n) \right] \quad (18)
\]

Substituting this expression into equation (17) and then into (9) leads to

\[
\Delta A_{lm}(t) = \frac{\Delta M(t)}{4\pi(2l+1)M} \left[ 1 - \frac{R}{Z} \Re \left[ \sum_{l'=1}^{\infty} \sum_{m'=0}^{l'} H_{l'm'} Y_{l'm'}(n) \right] \right] Y_{lm}(n) \mathrm{d}\Omega \quad (19)
\]

where \( \Delta M(t) = 4\pi R^2 Z \Delta \rho_0(t) \) is the total CO\(_2\) mass added to the atmosphere. Using the orthogonality (3) of the spherical harmonics, equation (19) reduces to

\[
\Delta A_{lm}(t) = -\frac{2-\delta_{m0}}{2l+1} \frac{R H_{lm}}{Z} \frac{\Delta M(t)}{M} \quad (20)
\]

as long as \( l,m \neq 0 \). Equation (20) is the final formula for the gravitational effect due to the changing atmospheric CO\(_2\) mass. Note that this effect separates naturally into spherical harmonics, in the sense that the \((l,m)\) component of the change in the complex Stokes coefficient, \( \Delta A_{lm} \), is directly and solely proportional to the \((l,m)\) component of the effective topography, \( R H_{lm} \). This further justifies a posteriori the approximation in equation (18). The atmospheric CO\(_2\) mass change \( \Delta M(t) = -[\Delta M_n(t) + \Delta M_s(t)] \) is shown in Figure 2. The \( H_{lm} \) coefficients can be obtained, according to equation (13), from the \( H_{lm} \) estimates published by Bills and Ferrari (1978) and the \( A_{lm} \) estimates by Christensen and Balmino (1979) and Balmino et al. (1982). Note that the \( S_{lm} \) values in these studies should change sign to conform to our present definition of the (east) longitude \( \lambda \).

Table 2 presents the maximum amplitude of \( \Delta A_{lm}(t) \), which occurs at \( t=t_0 \) when \( \Delta M(t_0) = -8 \times 10^{15} \) kg (Figure 2), for some most prominent low-degree harmonics:

The first-degree \((l=1)\) harmonics gives the change in the relative displacement of the center of figure from the center of mass (Bills and Ferrari, 1978). This displacement is found to be 2.4 mm in the direction \((92^\circ E, 44^\circ N)\) using our longitude convention.

The change in the \((2,0)\) harmonic, and hence in \( J_2 \), is again the largest term, this time reflecting the solid Mars' large deviation from a hydrostatic equilibrium state. It is out of phase with the polar cap contribution, decreasing the amplitude by nearly one quarter. The reason these two changes in \( J_2 \) are out of phase is due to Mars' excess topographic oblate-
The polar regions are topographically low. Hence the excess atmosphere there decreases when the caps grow, increasing $J_2$. Note that the change in the $(3,0)$ harmonic, and hence in $J_3$, is also out of phase with respect to the polar cap contribution, but its amplitude is negligible.

The change in the $(2,1)$ harmonics gives rise to a shift in the principal axis of the greatest moment of inertia, or the figure axis. The angular departure of the figure axis from the $z$ axis is usually expressed in terms of its $x$- and $y$-components, $\Psi = \psi_x + i\psi_y$. This function $\Psi$ excites the polar motion and is thus known as the polar-motion excitation function (Munk and MacDonald, 1960). For any excitation mechanism that loads the planet (such as atmospheric variations), $\Psi$ can be found by

$$\Psi(t) = -\sqrt{5} \left[ \Delta C_{21}(t) + i\Delta S_{21}(t) \right] / (\sqrt{3} J_2).$$  \hspace{1cm} (21)

The excitation function in the present case has some interesting features, as can be shown from equation (21) and Table 2. First, it stays strictly along a fixed meridian ($125^\circ$E), simply because the topography is fixed. Second, its maximum amplitude is 26 milliarcsecond (mas), equivalent to a surface polar displacement as large as 45 cm. Compare this with that on Earth produced by the great 1960 Chilean earthquake, 23 mas (e.g., Chao & Gross, 1986) or the strong El Nino event of 1982-1983, ~40 mas (Gross and Chao, 1985). Unfortunately this rather strong excitation, at annual (669 sols) and semi-annual periods, is not likely to generate a large polar motion. This is because Mars, assuming an Earth-like rigidity, has a free wobble period of about 200 sols. Hence, unlike the Earth, no resonance should be expected from seasonal excitations.

The $(2,2)$ and $(3,3)$ harmonics are the only other components that have geoid height changes comparable to the above changes. They are exceptionally large on Mars simply because of the existence of the extensive Tharsis construct and its antipodal, but smaller, uplift.
Discussion

The variations in Mars' gravitational field due to the CO$_2$ mass exchange are generally very large compared with their terrestrial counterparts. Nevertheless, whether they can be observed by the upcoming MOM (Mars Observer Mission, due to be launched in 1990) is presently uncertain. A simple simulation made by F. Lerch and C. Wagner for an MOM proposal (1985) has indicated that, using Doppler tracking data of the MO orbit alone, the change in $J_2$ can be marginally detected. If in addition the surface altimetry data (as differences at crossovers) are available, then the accuracy in the gravitational field determination is expected to be improved. It may then be possible to detect changes in $J_2$ to a higher precision, thus placing constraints on the meteorological models of Mars. However, the simulation is done under the assumption that all favorable conditions prevail throughout the MOM lifetime (1 Martian year), and that no non-gravitational forces is acting on the spacecraft. The latter includes the atmospheric drag whose effect on the MO orbit, according to some estimates, may completely obscure the orbital effects caused by the small seasonal variations in the gravitational field. At the present time, simply too little about the Martian atmospheric density is known to warrant any further modeling.
References


On Seasonal Variations of Mars' Gravitational Field

A great quantity of CO₂ is exchanged between the Martian atmosphere and polar caps in the course of a Martian year. This exchange occurs in seasonal cycles: CO₂ condenses to form the polar caps in winter and sublimes into the atmosphere in summer. The mass involved is about 25% of the total mass of the Martian atmosphere. This paper studies the effects of the CO₂ mass redistribution on the Mars' gravitational field. Two mechanisms are examined: (i) the waxing and waning of solid CO₂ in the polar caps, and (ii) the geographical distribution of gaseous CO₂ in the atmosphere. The maximum changes produced by (i) in the low-degree zonal Jₖ harmonics in the Mars' gravitational field are found to be as much as 7.5 x 10⁻⁹ for J₄ (corresponding to 1.1 cm change in the geoid), and about half as much for J₃. The effect of (ii) on some most prominent low-degree harmonics (for k=1-3) in the "effective topography" (defined as the departure of the true topography from the geoid) is also evaluated. Their magnitudes are large by Earth standards. Whether they can be observed by the upcoming Mars Observer is presently uncertain.