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SOME CONSIDERATIONS ON MEASURING THE NEWTONIAN GRAVITATIONAL CONSTANT G IN AN ORBITING LABORATORY

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ABSTRACT

Some progress has been made on some of the objectives set for the NASA/ASEE Summer Fellowship, specifically: (1) Assemble a bibliography on proposals, reports, and suggestions for space-based laboratory measurements of G; (2) Become familiar with some of the activities in space science at Marshall Space Flight Center; (3) Identify and contact some workers with possible similar interests; (4) Investigate further the suggestion of orbiting two balls in a near-earth orbiting laboratory. With regard to the last of these, a manuscript entitled "Orbits inside a spacecraft: measuring the gravitational constant G," by Adam F. Falk and Stephen D. Baker has been drafted, the abstract of which reads as follows:

A common suggestion for measuring the Newtonian gravitational constant G in a near-earth orbiting laboratory is simply to put two balls in orbit around each other and observe the resulting motion, thereby determining G. However, the radial variation with distance of the gravitational field of the earth is so large that "tidal forces" on the balls in near-earth orbit can be several times greater than the gravitational attraction between the two masses, leading some writers to assume that two objects will not stably orbit about each other and that this method of measuring G in low-earth orbit is impossible, or at least impractical. We have, however, identified certain orbits which <u>are</u> stable (at least over many periods of the spacecraft about the earth). In this case, the objects experience their gravitational interaction for a long time, and it becomes reasonable to consider such orbits as candidates for measurements of G.

ACKNOWLEDGEMENTS

I would like to thank the scientists and staff in the High Energy Astrophysics Branch as well as in other branches of the Space Science Laboratory for their encouragement, hospitality, and support. My colleague, C. A. Meegan, good-naturedly went out of his way to be helpful with the details of finding one's way around the lab and to make me feel at home. Iwan Alexander, whose general expertise and whose help with the computations were invaluable, was a source of information and ideas and also fun to talk to. Finally, I appreciate the support--financial, intellectual, social, and administrative--of the NASA/ASEE Summer Faculty Fellowship Program.

INTRODUCTION

Since the time that it became reasonable to assume that orbiting laboratories would be available for physics experimentation, proposals have been put forward to measure the Newtonian gravitational constant G in such laboratories. Of the fundamental physical constants, G is by far the least well known (only about one part in a thousand). Since there have also recently been some suggestions that G is distance dependent or source composition dependent, new measurements of its value (with new sets of systematic errors to be understood) are needed. Of experiments proposed for orbiting laboratories, one suggestion is simply to put two balls in orbit around each other and measure the resulting orbital elements and period of the motion, thereby determining G. Another suggestion is to construct an oscillator whose restoring force is gravitational--a "gravitational clock."

With regard to the first suggestion, it turns out that a major complication is introduced in this method by the nonuniformity of the earth's gravitational field. The radial variation with distance of the gravitational field of the earth is so large (about a third of a part per million each meter at the surface of the earth) that "tidal forces" on the balls in near-earth orbit can be several times greater than the gravitational attraction between the two masses. The presence of these relatively strong tidal forces has led some writers to assume that two objects will not stably orbit about each other and that this method of measuring G in near-earth orbit is impossible, or at least impractical. We have, however, identified certain orbits which <u>are</u> stable (at least over many periods of the spacecraft about the earth). In this case, the objects experience their gravitational interaction for a long time, and it becomes reasonable to consider such orbits as candidates for measurements of G.

OBJECTIVES

The stated objective of the summer's work were to make some progress on some of the following tasks:

1. Preparation of a bibliography on proposals, reports, and suggestions for space-based laboratory measurements of G, to supplement the very complete bibliography prepared by Gillies on all determinations of G.

2. Determine the compatabillity of these suggestions and those I might have with current efforts in other ultra-low-g investigations.

3. Identify, contact, and perhaps propose a workshop for, workers with similar interests.

4. Propose and discuss theoretical and experimental feasibility studies needed before a space-based experiment can be carried out.

5. Other tasks as they become evident.

OBJECTIVES WHICH WERE REALIZED

It may go almost without saying that not all the objectives stated on the previous page were fully realized. On the other hand, some progress was made on some of them. As evidence, in the following pages I present (a) a draft manuscript entitled "Orbits inside a spacecraft: measuring the gravitational constant G" and (b) a short bibliography on measuring G in space. I have also this summer identified and spoken with a number of workers in the U.S. who might be interested in further consideration of measuring G in space, and I have realized one of the general objectives of the NASA/ASEE Summer Fellowship Program: to become familiar with the Marshall Space Flight Center.

DRAFT MANUSCRIPT

Orbits inside a spacecraft: measuring the gravitational constant G

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Abstract:

A common suggestion for measuring the Newtonian gravitational constant G in a near-earth space laboratory is simply to put two balls in orbit around each other and observe the resulting motion, thereby determining G. However, the radial variation with distance of the gravitational field of the earth is so large that "tidal forces" on the balls in near-earth orbit can be several times greater than the gravitational attraction between the two masses, leading some writers to assume that two objects will not stably orbit about each other and, therefore, that this method of measuring G in near-earth orbit is impossible, or at least impractical. We have, however, identified certain orbits which are stable (at least over many periods of the spacecraft about the earth). In this case, the objects experience their gravitational interaction for a long time, and it becomes reasonable to consider such orbits as candidates for measurements of G.

Introduction:

Since the time that it became reasonable to assume that orbiting laboratories would be available for physics experimentation, proposals have been put forward to measure

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the Newtonian gravitational constant G in such laboratories [see Bibliography]. A common suggestion is simply to put two balls in orbit around each other and measure the resulting orbital elements and period of the motion, thereby determining G. It turns out, however, that a major complication is introduced in this method by the nonuniformity of the earth's gravitational field. The radial variation with distance of the gravitational field of the earth is so large (about a third of a part per million each meter at the surface of the earth) that "tidal forces" on the balls in near-earth orbit can be several times greater than the gravitational attraction between the two masses. The presence of these relatively strong tidal forces has led some writers to assume that two objects will not stably orbit about each other and that this method of measuring G in near-earth orbit is impossible, or at least impractical. We have, however, identified certain orbits which are stable (at least over many periods of the spacecraft about the earth). In this case, the objects experience their gravitational interaction for a long time, and it becomes reasonable to consider such orbits as candidates for measurements of G. In any case, it is interesting to examine the motion of two gravitationally attacting masses in a space laboratory as a relevant application of elementary mechanics.

First we will characterize the gravitational environment of an ideal orbiting laboratory. Then we will describe the motion of single objects within that laboratory. Finally we will characterize the motion of two objects which are acting under their mutual attraction.

Orbiting Laboratory:

Let us consider that the laboratory is in a near-earth, circular orbit, and that the laboratory keeps one face toward the earth. That is, the laboratory rotates 2 each time it circles the earth. The forces on an object in the laboratory then depend on the gravitational force from the earth and the fictitious forces associated with the rotation of the laboratoy (centrifugal and Coriolis forces). We are not obliged to choose such a reference frame, of course, but this one is convenient, and we can always transform to another coordinate system which, for example, does not rotate.

Since the laboratory is in free-fall, one might expect that the earth's gravitational field may be neglected, but that is not the case. Before writing down any formulas, here is the situation.

Assume we have a spherically symmetric spacecraft with an empty space inside. Only at the center of mass of the spacecraft is the acceleration of the spacecraft equal to the acceleration due to the gravity of the earth. Above this point the field of the earth is weaker and below it is stronger, so that in the spacecraft there is a "tidal force" which tends to accelerate objects above the center of mass toward the top of the spacecraft and objects below the center of mass toward the floor of the spacecraft. In empty space, the divergence of the gravitational field is zero. Therefore, corresponding to the tidal tension which acts vertically, there is an tidal compression which tends to push objects that are displaced to the side of the center of mass sideways back toward the vertical line through the center of mass. When one adds the centrifugal force to these gravitational tidal forces, one finds that the net force in the radial direction increases, the net force in the horizontal direction parallel to the direction of motion of the spacecraft is zero, and the tidal compression force in the horizontal direction perpendicular to the direction of motion of the spacecraft is unaffected.

In the plane of the the orbit of the spacecraft, the resulting acceleration field for a particle instantaneously at rest in the spacecraft is schematically depicted in Fig. 1. The z-axis is chosen along the direction of motion of the spacecraft and the x-axis points radially away from the Earth. As one can see, there is a 'neutral line' (our z-axis) in the spacecraft on which an object released at rest will simply remain at rest. In this paper, we will confine our consideration to this plane, but it should be noted that at points off the zx plane, there is a y-component of force toward the zx plane. In drawing Fig.1 we have assumed that the spacecraft is much smaller than the earth, so that the acceleration vectors are practically parallel to the x-axis, and the neutral line is practically straight. An object anywhere in this field will move in response to this field as well as in response to a Coriolis force which acts in a direction $-\vec{v} \times \hat{y}$, where $\hat{y}$ is parallel to the axis of rotation of the spacecraft.

Motion of single particle:

As we show in the Appendix and in light of the above discussion, the equations of motion of a single particle moving in this field are

$$\ddot{z} = -2\dot{x}$$

$$\ddot{x} = 2\dot{z} + 3x \qquad (1)$$

$$\ddot{y} = -y$$

where dots indicate time derivatives, and time is measured in units of radians (one radian equals $T/2\pi$, where T is the orbital period of the spacecraft around the earth.) Note that the z-acceleration (parallel to the neutral line) comes only from the Coriolis force, while the x-acceleration (radial direction) has one term which is the Coriolis force and another term which is the sum of the tidal tension and the centrifugal force.

Solutions of these equations for motion in the zx plane are [Alexander and Lundquist]

$$\begin{aligned} z &= z(0) - \dot{z}(0)[3t - 4 \sin t] \\ &\quad -6x(0)[t - \sin t] - 2\dot{x}(0)[1 - \cos t] \qquad (2) \\ x &= x(0)[4 - 3 \cos t] + \dot{x}(0) \sin t + 2\dot{z}(0)[1 - \cos t] \end{aligned}$$

examples of which are shown in Fig.2(a) and Fig.2(b). As one can see, if a particle is launched so that it crosses the neutral line with a velocity perpendicular to the neutral line, it can remain in the spacecraft, executing a counterclockwise elliptical orbit. Such a trajectory is shown in Fig. 2(a), with the initial conditions chosen to give an orbit centered on the origin. (The orbit is marked at equal time intervals of one-sixth the spacecraft's period.) If the particle is launched in other ways, it eventually strikes the wall of the spacecraft. Such a trajectory is shown in Fig. 2(b), with initial conditions which give a non-zero z-component of the velocity as the particle crosses the neutral line. As one can see, the Coriolis force is very important, since without it the particle would simply move away from the neutral line under the influence of the tidal force.

We can get another perspective on this motion if we consider a non-rotating reference frame with its origin at the center of the Earth. The particle and the spacecraft are in independent Keplerian orbits. A particle placed on the neutral line is in the same orbit as the spacecraft; hence it remains at rest in the rotating, orbiting frame. A small initial velocity in the radial direction changes the eccentricity, but not the energy, of this orbit. Therefore the particle executes a closed orbit inside the spacecraft. An initial velocity along the neutral line, however, is an initial velocity parallel to the orbital velocity, so the total energy, and hence the period, of the particle's orbit is different from the spacecraft's. For example, an initial velocity in the <u>positive</u> z-direction will put the particle in an orbit of <u>higher</u> energy and longer period; thus in the spacecraft's frame it moves off in the <u>negative</u> z-direction.

Motion of two attracting particles:

Suppose now one launches two gravitationally attracting balls, each of mass m, in the spacecraft so that their center of mass coincides with the center of mass of the spacecraft; if one ball is at (z,x,y) then the other one is at (-z,-x,-y). Taking into account the gravitational attraction between the two balls, the equations of motion of the first ball are

$$\begin{aligned}\ddot{z} &= -2\dot{x} - z[x^2 + y^2 + z^2]^{-3/2} \\ \ddot{x} &= 3x + 2\dot{z} - x[x^2 + y^2 + x^2]^{-3/2} \\ \ddot{y} &= -y - y[x^2 + y^2 + z^2]^{-3/2}\end{aligned} \tag{3}$$

Here the unit of length has been chosen in such a way that the values of G and m do not explicitly appear in the equations of motion. This unit of length is the equal to $(m/M)^{1/3} a_o$, where M is the mass of the Earth and a_o is the radius of the orbit of the spacecraft.

Although we do not have solutions for the orbits in closed form, we may calculate the orbits numerically. We find several interesting cases, illustrated in Figs. 3. For concreteness, we have chosen to show the motion of 10 kg balls of radius 5 cm (density 19.1 gm/cm^3), and a spacecraft in a circular orbit of radius 6700 km (a typical Space Shuttle's near-earth orbit) for which the period T = 91.1 minutes. In Eq.(3), this corresponds to the unit of time equal to 870 seconds and the unit of length equa_ to 5.06 cm. We will restrict our attention to motion in the x-z plane. Again, to give some indication of the velocities, we have marked some of the orbits at intervals of 911 seconds, one sixth of the period of the spacecraft's motion about the earth.

As our first example of motion with mutual gravitational attraction between the two balls, we consider the case of two balls released from rest on the neutral line. Fig. 3(a) shows the trajectories. We have a somewhat paradoxical result. The gravitational force between them is attractive, yet they move apart! This can be understood, however, when we note that the balls initially accelerate toward each other, but as they pick up speed the Coriolis force moves them off the neutral line. The tidal force tends to move them further from the neutral line and they pick up speed until the tidal force is nearly balanced by the Coriolis force, and the balls continue to move apart and out of range of their gravitational attraction. (This is related to what is thought to be the behavior of some co-rotating moons of Saturn, which "exchange" orbits with each other.) [Spirig and Waldvogel 1985]

The motion can also be understood by returning to the non-rotating reference frame considered earlier. As the balls attract and are initially accelerated toward each other, they acquire velocities along the neutral line while remaining essentially on it. This effect changes the energies of their orbits. The ball on the right, for example, is decelerated in the non-rotating frame, drops into an orbit of lower energy and shorter period, and drifts away from the other ball which moved into an orbit of higher energy and lower period.

In Figs. 3(b,c,d) we show the motion of the center of only the first ball. The other ball moves symmetrically about the center of mass which is at the origin. We also show with a dashed line the excluded region around the origin, since at the dashed line the balls are in contact.

Fig. 3(b) indicates the motion corresponding to the same initial conditions as in Fig. 2(a). One can see that the attraction of the balls pulls them closer than they otherwise would have gone and that the balls then collide. The orbit is continued, however, to show the motion that would occur in the absence of a collision. Unless the balls are launched within within a small range of velocities, they will either collide or move apart, striking the walls of the spacecraft. Figs. 3(c) and 3(d) show cases in which the balls are launched within this range.

Figure 3(c) shows a trajectory with initial conditions chosen to make the balls move in closed orbits. Comparison with Fig. 2(a) shows that for the same z-intercept (15 cm), the balls must be launched with higher velocity and that the x-intercept is larger; i.e., the orbit is "fatter"--the limiting case of very massive balls would simply give circular orbits. In the example shown in Fig. 3(c) the period of the motion has been shortened by about 20%.

Finally, Figure 3(d) shows an orbit with the initial conditions of Fig. 2(b), showing that the gravitational attraction of the two balls actually stabilizes their motion and keeps them within the spacecraft for a long time. The source of this stability may be understood qualitatively as follows. When the orbit is almost closed but passes closer to the attractive center on one side of the orbit than the other, the ball receives an net impulse due to the gravitational force which causes a general drift in the opposite direction, an effect we have already observed in the example shown in Fig. 2(b).

This is a somewhat surprising result since the tidal forces in this example are typically several times as strong as the gravitational attaction of the two balls, and one does not have the sort of stability of orbit that one would have, say, of a satellite around the moon (which also experiences the tidal forces due to the Earth). Actually, such stability (satisfaction of the Hill criterion) [Szebehely 1967] would be attainable with laboratory sized masses if one were to use a spacecraft which orbits the earth at about two earth radii or beyond.

For the example that we have discussed so far, we may investigate the tolerances on the initial conditions of the orbit so that the balls neither collide with each other nor with the walls of the spacecraft. In Fig. 4 we show the "launch window" for a ball started out on the neutral line at z = 15 cm with the velocity components shown on the axes of the figure. Trajectories which result in the balls' striking each other or the walls of the spacecraft are represented by diamonds or crosses, respectively. Those which survive for 25T or longer are unmarked. One may note that the tolerance on v_z is much tighter than that on v_x, 6 micrometer/second vs. 90 micrometer/second, and while these tolerances are rather tight, they are not necessarily impractically so. Launching from other points besides those on the neutral line are, of course, possible, and we find that the allowable ranges of z- and x-velocities are very similar to those in Fig. 4.

Measurement of G:

We believe that the analysis of such motion could be one way to measure the value of the Newtonian graviatational constant G in an orbital laboratory. The balls spend considerable time in the neighborhood of each other, allowing the gravitational interaction to act for a long time. There are many practical problems which would have to be mastered before one could consider such an experiment to be a reasonable competitor in the measurement of G. However it should be noted that the value of G is not as well known as the other physical constants and that there may be some reason to believe that systematic errors in the measurement of G exist in the different methods of measurement. Since these differences are of the order of one part in 100 or 1000, an independant measure of G to only one part in 1000 would be of interest. And one might expect considerably better results than 1:1000 for an experiment based on the analysis of motion of the type illustrated in Fig. 3a.

Some of the practical problems requiring careful consideration are mentioned below. The gravitational field of the spacecraft itself would have to be carefully modeled, or known, or determined from the experiment itself. The gravitational field of the earth , which is well known, is nevertheless complicated and would have to be carefully taken into account. The electrical charge on the balls would have to be carefully monitored and controled so that electrical attraction or repulsion which mimics the gravitational force would not be a serious perturbation. The spacecraft would have to be flown around the center of mass of the two balls. The balls would have to be launched carefully, but it is interesting to note that they could be steered into place with the radiation pressure from modest sized lasers.

[To save space, the Appendix to the draft manuscript is deleted from this report.]

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References for this draft manuscript may be found in the bibliography for the entire report.

CONCLUSIONS AND RECOMMENDATIONS

Since it appears that there are several reasonable methods to measure G in space, further thought should be applied to the problem. In particular, a study of the practical problems to be encountered with an experimental realization of the orbiting balls method should be made. A further refinement of the "gravitational clocks" idea should also be initiated. It is probably time for a workshop to be organized to bring together workers who might be interested in participating in such a measurement.

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