Department of Geodetic Science and Surveying

BASIC RESEARCH FOR THE GEODYNAMICS PROGRAM

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Greenbelt, Maryland 20771

The Ohio State University
Research Foundation
Columbus, Ohio 43212

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PREFACE

This project is under the supervision of Professor Ivan I. Mueller, Department of Geodetic Science and Surveying, The Ohio State University. The Science Advisor is Dr. David E. Smith, Code 921, Geodynamics Branch, and the Technical Officer through June 30, 1986, was Mr. Jean Welker, Code 903, Technology Applications Center. From July 1, 1986, the Technical Officer is Dr. Gilbert D. Mead, Code 601, Crustal Dynamics Project, Space and Earth Sciences Directorate. All addresses are at Goddard Space Flight Center, Greenbelt, Maryland 20771.

Because of the change in the contract administration, from now on the semiannual reports will cover the periods January-June and July-December, instead of October-March and April-September.
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1. CURRENT TECHNICAL OBJECTIVES

1. Intercomparison of Different Space Geodetic Measurement and Computational Data

2. Regional Deformations and Relative Plate Motions from Robust Estimation
2. ACTIVITIES

2.1 Earth Rotation Parameter Determination from Different Space Geodetic Systems

This work has been nearly completed during this reporting period. After some further development of utility program software for analyzing final results, the main simulation experiments were performed, and results and conclusions compiled. Although awaiting final approval, the PhD dissertation [Archinal, 1987a] has been completed and describes the majority of the work and results of this study. Some additional work will probably be done, including performing simulation experiments using different orbital and observational weighting, and more normal observational data rates. It is planned that the just mentioned dissertation will be revised to reflect this additional work and published as [Archinal, 1987b].

During the latter part of the reporting period the results of the study were also presented at several meetings, as listed at the end of this report.

References

Archinal, Brent A. (1987a), Determination of Earth Rotation by the Combination of Data from Different Space Geodetic Systems, PhD dissertation, Ohio State Univ., Columbus, in preparation.

Archinal, Brent A. (1987b), "Determination of Earth Rotation by the Combination of Data from Different Space Geodetic Systems," Dept. of Geodetic Science and Surveying Rep., Ohio State Univ., Columbus, in preparation.
2.2 Utilization of Range-Difference Observations in Geodynamics

As anticipated in the 16th Semiannual Report, this study is close to being completed, and the final report is in preparation [Dedes, 1987]. For this reason, only a brief summary of the work accomplished in the last six months will be reported.

During this period the dynamic models employed by the GEOSPP software have been updated to be compatible with the MERIT Standards [Melbourne et al., 1983]. GEOSPP is a software used to implement the Simultaneous Range-Difference (SRD) semidynamic mode method [Pavlis, 1982]. This method was anticipated to yield baseline estimates whose accuracy would be insensitive to the overall orbital accuracy and sensitive to the accuracy of the observations, especially if certain geometric configurations are fulfilled. This in fact has proven to be the case. For instance, baselines close to being parallel with their observed arcs have been estimated with an accuracy of 2 cm. This accuracy not only represents one standard deviation but also reflects the agreement when these baselines are compared with the corresponding ones as estimated by other computational centers using the range dynamic mode method [Tapley et al., 1985]. This accuracy was achieved, despite the fact that the input initial state vectors were in error by 150 meters and that the standard deviations of the adjusted ones were as large as 50 meters.

These results demonstrate the sensitivity of the SRD method to the accuracy of the observations as well as the insensitivity of this method to the overall orbital accuracy. Furthermore, these baseline estimates were obtained by using only eight Lageos passes. Realistically, eight Lageos passes may be observed in about a week. Therefore, this method is ideal for rapid baseline determination especially when mobile laser systems are moved into an area with the aim of spending the least possible time collecting data for baseline determination.

There still needs to be established how the relative orientation between the baselines and their observed arcs is related to the number of arcs necessary for convergence, when the baselines are estimated through the SRD method. This will be revealed by estimating baselines having different lengths and different orientation with their observed arcs. Furthermore, in the geometric mode method (16th Semiannual Status Report) tidal effects should be accounted for. Both of these tasks are expected to be completed by the end of this calendar year.

Presently the work is focused on finishing with these tasks and on preparing the final report which is expected to be ready within the next three months.
References


2.3 An Algorithm for Crustal Deformation Analysis

Sequential Model Discrimination

In the previous five Semiannual Reports (12 through 16), a general algorithm for the analysis of crustal movement measurements was proposed. In this report, a method based on the Bayesian philosophy and entropy measure of information is given for the elucidation of time-dependent models of crustal motions as part of the proposed algorithm. The method, which is due to Box and Hill (1967), is first reproduced; then the strategy of model discrimination and design of measurements is illustrated in an example for the case of crustal deformation models.

In the analysis of statistical decision theory two problems are generally distinguished. One is the problem of making the best decision on the basis of a given set of data, and the other is the problem of designing the best experiments in order to get information upon which a decision will be made. In this respect, model discrimination procedures of the proposed algorithm can be examined within the scope of these two problems.

Assume that geodetic surveys are performed at different epochs. The data indicate that crustal motions occurred during these time intervals and a set of concurring descriptive models are postulated either as a result of the displayed dislocation patterns of the network points and/or previous information. The question is how to use the data to select the best model and to design new optimal observations to facilitate model discrimination.

Since in general every decision is a result of some type of decision rule, it is necessary at this point to define a discrimination criterion.

Consider a complete set of events, $E_i$, $i = 1, 2, ... m$ whose probabilities are $p_1, p_2, ... p_m$ such that $\sum_{i=1}^{m} p_i = 1$. The expected information of the message on the occurrence of one of these events is defined as

$$I = \sum_{i=1}^{m} p_i \ln \frac{1}{p_i}$$ (1)

which is also known as the entropy of the distribution whose probabilities are $p_i$, $i=1, 2, ... m$ (Shannon 1948). The least possible information occurs when $p_1 = p_2 = ... = p_m = 1/m$, which can be derived by maximizing the above equation subject to the constraint $\sum p_i = 1$. In this case the amount of information is small and the entropy as a measure of disorder is maximum. In other words, all events are equally likely. In situations where the probability of one event $p_i$ is larger than the probability of other events $p_j$, $j \neq i$, the amount of information is considered to be large and entropy is small.

This concept can be applied to the discrimination of different competing models. Let there be a set of $m$ competing models and the a priori probability of the $i^{th}$ model being true is $p_i$. If the observations are performed and the a posteriori probability for the $i^{th}$ model is computed, then the information gained by this experiment is specified, from (1) as
\[ \Delta I(H, x) = - \sum_{i=1}^{m} p_{i-1} \ln p_{i-1} + \sum_{i=1}^{m} p_{i} \ln p_{i} \]  

(2)

The maximum of \( \Delta I(H, x) \) is of interest in order to obtain the greatest amount of information out of the experiment. Now, using (2) and considering that there exists a finite number of models, the expected change in entropy (information) \( E(\Delta I) = \Delta J \) before and after the \( n \)th observation is

\[ \Delta J = \sum_{i=1}^{m} p_{i-1} \left\{ \sum_{k=1}^{m} p_{k} \ln p_{k} - \sum_{k=1}^{m} p_{k-1} \ln p_{k-1} \right\} \cdot p_{i}(y|x) \, dy_n \]  

(3)

where

\[ p_{ln} = \frac{p_{i-1} \cdot p_{i}(y|x)}{\sum_{i=1}^{m} p_{i-1} \cdot p_{i}(y|x)} \]  

(4)

Substituting (4) into (3)

\[ \Delta J = \sum_{i=1}^{m} p_{i-1} \int p_{i}(y|x) \ln \frac{p_{i}(y|x)}{\sum_{i=1}^{m} p_{i-1} \cdot p_{i}(y|x)} \, dy_n \]  

(5)

Now an observation which maximizes (5) is considered the optimum one. Evaluation of (5) however is quite complicated, but an upper bound for this expression leads to a tractable form. Consider

\[ \sum_{i=1}^{m} p_{j-1} \cdot p_{i}(y|x) \ln \frac{p_{i}(y|x)}{p_{j}(y|x)} \geq p_{i}(y|x) \ln \frac{p_{i}(y|x)}{\sum_{i=1}^{m} p_{i} \cdot p_{i}(y|x)} \]  

(6)

which is a result of Corollary 3.1 of Kullback (1959). Substitution of (6) into (5) gives an upper bound \( \Delta J_u \) for \( \Delta J \)

\[ \Delta J_u = \sum_{i=1}^{m} \sum_{j=1}^{m} p_{j-1} \left\{ \int p_{i}(y|x) \ln \frac{p_{i}(y|x)}{p_{j}(y|x)} \, dy_n + \int p_{j}(y|x) \ln \frac{p_{j}(y|x)}{p_{i}(y|x)} \, dy_n \right\} \]  

(7)

Let now a group of competing models is given by

\[ E(y^{(i)}) = A^{(i)}x^{(i)}, \quad i = 1, 2, \ldots, m \]  

(7a)

where \( y^{(i)} \) is the \( m \times 1 \) vector of observations for the \( i \)th model, \( A^{(i)} \) is the nonstochastic \( n \times u \) design matrix, and \( x^{(i)} \) is the \( u \times 1 \) unknown parameter vector for the \( i \)th model. \( u \) is not necessarily the same for all models. If the observations \( y_n \) are assumed to be distributed normally with mean \( E(y_n) \) and
known variance $\sigma^2$, the following relationships hold

\[ p_i(y_n \mid E(y_n), \sigma) = \frac{1}{\sqrt{2\pi}} \exp\left\{ -\frac{1}{2\sigma^2} \left[ y_n - E(y_n) \right]^2 \right\} \]  

(8)

\[ p_i(E(y_n) \mid \sigma) = \frac{1}{\sqrt{2\pi}} \exp\left\{ -\frac{1}{2\sigma} \left[ E(y_n) - \hat{y}_n^{(i)} \right]^2 \right\} \]  

(9)

where $\hat{y}_n^{(i)}$ is the predicted value of $y_n$ under model $i$ using $n-1$ observations and its variance is $\sigma_i^2$ which is given by

\[ \sigma_i^2 = \sigma^2 a_n^{(i)} \left(A^{(i)} A^{(i)}\right)^{-1} a_n^{(i)^T} \]  

(10)

where $a_n^{(i)}$ is the row vector of $A^{(i)}$. From the definition of the probability density function of $y_n$ under model $i$ given $\sigma$ and $n-1$ observations

\[ p_i(y_n \mid \sigma) = \int p_i(y_n \mid E(y_n), \sigma) \ p_i(E(y_n) \mid \sigma) \ d(E(y_n)) \]  

(11)

If (8) and (9) are substituted in (11) and integrated, then

\[ p_i(y_n \mid \sigma) = \frac{1}{2\pi(\sigma^2 + \sigma_i^2)} \exp\left\{ -\frac{1}{2(\sigma^2 + \sigma_i^2)} \left( y_n - \hat{y}_n^{(i)} \right)^2 \right\} \]  

(12)

Substitution of (12) into (7) results in the following operational form of discrimination function

\[ \Delta J_u = \frac{1}{2} \sum_{i=1}^{M} \sum_{j=1}^{M} P_{i,n-1} P_{j,n-1} \left[ \frac{(\sigma_i^2 - \sigma_j^2)^2}{(\sigma^2 + \sigma_i^2)(\sigma^2 + \sigma_j^2)} + \left( \hat{y}_n^{(i)} - \hat{y}_n^{(j)} \right) \right] \]  

(13)

where $\hat{y}_n^{(i)}$ and $\hat{y}_n^{(j)}$ are the predicted observations for model $i$ and $j$, $\sigma_i^2$ and $\sigma_j^2$ are the predicted variances obtained from (10) for these predicted observations and $\sigma^2$ is the known a priori variance of the observations.

It is now possible to design sequential geodetic surveys to discriminate the descriptive models of deformations using this entropy measure of information. The scenario which is depicted in Fig. 1 is as follows.

First an initial network design for the area under consideration is constructed, for instance, using the D-optimal design criteria which is discussed in the 12th Semiannual Report. Geodetic surveys are then performed at two different epochs covering the whole network. This is followed by the estimation of deformation parameters from the differences of observed quantities. At this point, information provided by the current estimates and prior qualitative information are examined, and prior
probabilities are assigned to each model. If no preference is inferred from
the existing information, each model is assigned equal probabilities \( p = 1/m \),
where \( m \) is the number of models. The next optimal observation (not
necessarily the resurvey of the whole network) that gives the maximum
expected discrimination among \( m \) rival models is sought in (13). Then the new
optimal measurement(s) is performed and posterior probabilities for each model
are computed using equation (4). Finally, the current standing of each model
is examined. This procedure is repeated each time using posterior
probabilities of previous observations as prior probabilities for the succeeding
observations, until one model emerges from the others. The following
numerical example illustrates the procedure.

**Numerical Example**

In the previous section, a method based on the entropy measure of
information is presented as a possible candidate for the discrimination of
several competing time-dependent models. In order to get a better feeling for
the applicability of the method to the crustal deformation analysis, this section
describes a numerical example.

Consider the case of a homogeneous deformation field. The following
descriptive models are postulated as a result of prior experiments to represent
possible network deformations,

\[
\begin{align*}
\text{model 1.} & \quad dx &= (e_x x + e_{xy} y) \Delta t \\
& \quad dy &= (e_{xy} x + e_y y) \Delta t \\
\text{model 2.} & \quad dx &= e_x x \Delta t \\
\text{model 3.} & \quad dx &= e_{xy} y \Delta t \\
& \quad dy &= e_{xy} x \Delta t \\
\end{align*}
\]

where \( dx \) and \( dy \) are the displacement components of network points for the
period \( \Delta t \); \( e_x \) and \( e_y \) are the extensional strains in \( X \) \( Y \) directions; \( e_{xy} \) is the
shearing strain. If the baselines are observed at different epochs, then the
baseline length \( t_{ij} \) at epoch \( t \) is given by the following expression,

\[
t_{ij}^2 = (x_j - x_i)^2 + (y_j - y_i)^2
\]

Linearizing this expression about the initial epoch \( t_0 \) and considering
equations (14), (15) and (16), results in the following mathematical models,

\[
\begin{align*}
\text{model 1:} & \quad t_{ijt} - t_{ijt_0} = \Delta t t_{ij} (\sin^2 \alpha_{ij} e_x + \cos^2 \alpha_{ij} e_y + \sin 2 \alpha_{ij} e_{xy}) \\
\text{model 2:} & \quad t_{ijt} - t_{ijt_0} = \Delta t t_{ij} \sin^2 \alpha_{ij} e_x \\
\text{model 3:} & \quad t_{ijt} - t_{ijt_0} = \Delta t t_{ij} \sin 2 \alpha_{ij} e_{xy}
\end{align*}
\]

where \( t_{ijt} \) and \( t_{ijt_0} \) are the observed baseline lengths at epochs \( t \) and \( t_0 \) re-
respectively, and \( \alpha_{ij} \) is the azimuth of the observed baseline \( i-j \). In this
example, model 3 is chosen to be the correct model and the pseudo-
observations $t_{ijt}$ and $t_{ijt_0}$ are derived using this model. They are also con-
taminated with a noise from a normal distribution with mean zero and variance
1 mm. Shearing strain, $e_{xy}$, in the correct model 3 is 0.50 ppm and the time
interval between observations is constant and equal to a month. The
sequential model discrimination procedure can now be performed using the
algorithm depicted in Fig. 1.

An initial design is set up for the measurement of deformation parameters
(Fig. 2a). This is a D-optimal design of model 1, which was derived in the
12th Semiannual Report, with some additional observations. It is initially
postulated that all three models are equally likely. In other words, prior
probabilities for each model are 1/3. Part of the network is resurveyed a
month later. This makes it possible to predict a new observation for the next
month that maximizes the discrimination function $\Delta J_u$ for each model and select
the possible observation which gives the maximum discrimination. In this
example three alternatives are possible: baselines which are in the north-east
direction, baselines which are in the south-west direction, and baselines which
are in the east-west direction. The predicted observation is then performed
and posterior probabilities for each model are computed using the new
observations and prior probabilities (4).

The rest of the experiment continues following this prediction and
observation procedure until computed posterior probabilities indicate that one
model is superior to the others (Figs. 2c, 2d and 2e). Fig. 2 shows that the
correct model 3 is identified effectively with 14 baseline observations after six
months.

As a comparison, null-hypothesis tests ($H_0$: Model 3 is the same as model
1, $H_0$: Model 3 is the same as model 2) are performed using the estimated
parameters obtained by the least squares method at 5% and 1% significance
levels. Neither hypothesis is rejected at the 1% significance level until $\Delta t = 5$
months (Fig. 2e). In the case of $\alpha = 0.05$, model 2 is rejected at $\Delta t = 5$.
Model 2 is rejected at $\Delta t = 6$ months at both levels. However, the results
were ambiguous in the sense that model 2 and model 3 were still likely
candidates until the last measurement was performed. This problem is clearly
eliminated by the proposed method due to the history of accumulated
measurements and calculated posterior probabilities of each model.

REFERENCES


START

SURVEY AT TWO EPOCHS

ASSIGN PRIOR PREFERENCES TO EACH MODEL

PERFORM PREDICTED OBSERVATION

ESTIMATE MODEL PARAMETERS

COMPUTE POSTERIOR PROBABILITIES. (EQUATION 29)

ASSIGN POSTERIOR PROB. AS PRIOR PROB.

MODEL IDENTIFIED?

DETERMINE NEW OBSERVATION THAT GIVES MAX INFORMATION. (EQUATION 39)

STOP

PRIOR INFORMATION (QUANTITATIVE)

DISTRIBUTION FUNCTION OF PARAMETERS

Fig. 1  Sequential model discrimination.
--- INITIAL DESIGN AND OBSERVATIONS

--- REPEATED OBSERVATIONS
--- PREDICTED OBSERVATION

b) \( \Delta t = 2 \)

c) \( \Delta t = 3 \)

d) \( \Delta t = 4 \)

e) \( \Delta t = 5 \)

f) \( \Delta t = 6 \)

Fig. 2 Results of sequential experiment for the discrimination of three competing models.
3. PERSONNEL

Ivan I. Mueller, Project Supervisor, part time
Brent Archinal, Graduate Research Associate, part time
George Dedes, Graduate Research Associate, part time
Stephen A. Hilla, Graduate Teaching Associate, without compensation
Huseyin Baki Iz, Graduate Teaching Associate, without compensation
Ziqing Wei, Visiting Researcher, without compensation

4. TRAVEL

Ivan I. Mueller, Ziqing Wei, Stephen Hilla
Austin, Texas April 26 – May 2, 1986
To attend 4th International Geodetic Symposium on Satellite Positioning
and to make presentations.

Ivan I. Mueller, Brent A. Archinal
Baltimore, Maryland May 20–23, 1986
To attend Annual Spring Meeting of the American Geophysical Union.
Mueller presented an invited paper. No project support.

Ivan I. Mueller
Toronto, Canada June 1-9, 1986
To attend 18th International Congress of Surveyors. No project support.

Ivan I. Mueller
Toulouse, France June 28 – July 11, 1986
To attend XXVI Plenary Meeting and associated activities of COSPAR and
present an invited paper. No project support.

Ivan I. Mueller
Tallinn, USSR September 7-12, 1986
Invited speaker at VI International Symposium on Recent Crustal
Movements. No project support.

Brent A. Archinal
Washington, D.C. September 11-12, 1986
Discussions at U.S. Naval Observatory. No project support.

Ivan I. Mueller, Brent A. Archinal, Huzeyin Baki Iz
Greenbelt, Maryland October 14-16, 1986
To make presentations at 11th Crustal Dynamics Principal Investigators
Meeting at Goddard Space Flight Center. Project support only for Mueller.

Brent A. Archinal, George C. Dedes
Coolfront, West Virginia October 20-24, 1986
To attend IAU/IAG Symposium on Earth Rotation and Reference Frames for
Geodesy. Archinal presented paper. No project support.

Ivan I. Mueller
Mendoza, Argentina October 24 – November 4, 1986
Invited speaker at 14th Meeting on Geophysics and Geodesy of the
Argentine Association of Geodesy and Geophysics. No project support.
Ivan I. Mueller
San Francisco December 6-10, 1986
To attend Annual Fall Meeting of the American Geophysical Union.
No project support.
5. REPORTS PUBLISHED TO DATE

OSU Department of Geodetic Science Reports published

262 The Observability of the Celestial Pole and Its Nutations
by Alfred Leick
June, 1978

263 Earth Orientation from Lunar Laser Range-Differencing
by Alfred Leick
June, 1978

284 Estimability and Simple Dynamical Analyses of Range (Range-Rate and
Range-Difference) Observations to Artificial Satellites
by Boudewijn H.W. van Gelder
December, 1978

289 Investigations on the Hierarchy of Reference Frames in Geodesy and
Geodynamics
by Erik W. Grafarend, Ivan I. Mueller, Haim B. Papo, Burghard Richter
August, 1979

290 Error Analysis for a Spaceborne Laser Ranging System
by Erricos C. Pavlis
September, 1979

298 A VLBI Variance-Covariance Analysis Interactive Computer Program
by Yehuda Bock
May, 1980

299 Geodetic Positioning Using a Global Positioning System of Satellites
by Patrick J. Fell
June, 1980

302 Reference Coordinate Systems for Earth Dynamics: A Preview
by Ivan I. Mueller
August, 1980

320 Prediction of Earth Rotation and Polar Motion
by Sheng-Yuan Zhu
September, 1981

329 Reference Frame Requirements and the MERIT Campaign
by Ivan I. Mueller, Sheng-Yuan Zhu and Yehuda Bock
June, 1982

337 The Use of Baseline Measurements and Geophysical Models for the
Estimation of Crustal Deformations and the Terrestrial Reference System
by Yehuda Bock
December, 1982
On the Geodetic Applications of Simultaneous Range-Differencing to Lageos
by Erricos C. Pavlis
December, 1982

A Comparison of Geodetic Doppler Satellite Receivers
by Brent A. Archinal
November, 1982
(partial support)

On the Time Delay Weight Matrix in VLBI Geodetic Parameter Estimation
by Yehuda Bock
July, 1983

Model Choice and Adjustment Techniques in the Presence of Prior
Information
by Burkhard Schaffrin
September, 1983

Positioning with NAVSTAR, the Global Positioning System
by Ziqing Wei
October, 1986

Determination of Earth Rotation by the Combination of Data from Different
Space Geodetic Systems
by Brent A. Archinal
Publications and Presentations Since Mid-1985


Wei, Ziqing, "GPS Positioning at The Ohio State University: Franklin County Results," Positioning with GPS—1985, Proc. of the 1st International Symp. on Precise Positioning with the Global Positioning System, Rockville, Maryland, April 15-19, 1985 (C.C. Goad, ed.) pp. 509-520, National Geodetic Information Center, NOAA, Rockville, MD 20852


Mueller, Ivan I., "From 100 m to 100 mm in (About) 25 Years," keynote address, Proc. 4th International Geodetic Symposium on Satellite Positioning, Austin, Texas, April 28 – May 2, 1986

Wei, Ziqing, "GPS Network Adjustment Using Nondifferenced Carrier Phase Measurements," Proc. 4th International Geodetic Symposium on Satellite Positioning, Austin, Texas, April 28 – May 2, 1986

Wei, Ziqing, "Mathematical Models and Results Comparison for Various Relative Positioning Modes," Proc. 4th International Geodetic Symposium on Satellite Positioning, Austin, Texas, April 28 – May 2, 1986


