FINAL REPORT

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February 18, 1987
Final Report

Under NAG8-490, we carried out a search in the HEAO A-1 Data Base (located at the Naval Research Laboratory in Washington, D.C.) for evidence of rapidly-rotating neutron stars that could be sources of coherent gravitational radiation. This search required developing a new data analysis algorithm that is described in the attached preprint (Appendix I). The algorithm was applied to data from observations of Cyg X-2, Cyg X-3 and 1820-30. Upper limits on pulsed fraction were derived and reported.

Final Report of Inventions and Subcontracts

There were no inventions or subcontracts under grant NAG8-490.
APPENDIX I
MILLISECOND X-RAY BINARY PULSARS

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ABSTRACT. Millisecond binary X-ray pulsars are expected theoretically, but remain undetected. We describe a novel search technique which applies a grid of quadratic time transformations to effect coherence recovery of smeared pulsar signals. We present upper limits on pulsed fluxes from the binary X-ray systems Cyg X-2, Cyg X-3, and 1820-30, and discuss how to improve future searches.

1. Coherence Recovery by Quadratic Time Transformations

We begin by assuming a pulsar with the following properties: 
P = pulse period = 1 - 100 milliseconds; f = pulsed fraction = 10^{-4} to 10^{-2}; \tau = P/(dP/dt) = 10^7 - 10^{10} seconds. In addition, the binary systems of interest are thought to have dimensions of lt-sec and orbital periods of a few hours. We first ask how to search effectively for such signals. The prime requirement for detection is preservation of phase information for large numbers of photons, several billion, in fact, if f < 10^{-4}. While the long timescale for intrinsic change in P is favorable, orbital motions of the source introduce a frequency modulation (FM) that poses a substantial impediment to detection.

To analyze the frequency modulation quantitatively, consider t' = t + c^{-1}D(t). Here, t is the time at which the observer would receive a particular pulse in the absence of source motions; t' is the time at which it actually is received. The goal is recovery of t given t', and the problem is that the corrections are not known to the precision required for adequate preservation of coherence. D(t) can be represented as A \cdot \sin(\Omega t + \phi), neglecting orbital eccentricity, where A is the projected orbital semimajor axis, in light seconds. The obvious procedure in the absence of information about A, \Omega, and \phi is a brute force, 3-dimensional search of all realistic binary orbits. Such a procedure would restore a perfectly coherent period.

Another approach, requiring less computational power, is to use integration times substantially less than a full orbit and to
approximate the true sinusoidal correction using a quadratic time transformation. This reduces the 3-D grid of sinusoid parameters to a 1-D grid which is of approximately uniform sensitivity over the entire orbital cycle, all parameters of which are presumed to be unknown except for limiting extreme values. Thus reconstruction is accomplished using \( t = t' + \alpha(t')^2 \). The parameter \( \alpha \) must be stepped by a value that is calculable from the integration time and the minimum period to be searched. The procedure works best if the sought pulsed signal is a sinusoid. For each distinct \( \alpha \) value, the search reduces to a standard fast Fourier transform, the sensitivity of which can be computed according to the formulae of Leahy et al. (1983). Expectation values also need to be adjusted for the number of independent searches conducted, i.e., essentially multiplication by the number of distinct \( \alpha \) values. This procedure has been described previously in Norris and Wood (1987). We refer to it as coherence recovery (CR). Figure 1 illustrates a simulation of the CR technique.

Optimization proceeds as follows: one first determines the plausible range of \( \alpha \), which may be constrained by some knowledge of likely orbits. It is then stepped over this range in increments \( \Delta \alpha = P/(2T^2) \) where \( T \) is integration time of the sample. The practical limit on \( T \) may come from computation cost or from a more basic consideration, the point at which the quadratic approximation becomes invalid. For a sample of \( N \) points, the computation cost consideration is that the number of flops (floating point operations) needed scales as \( N \cdot \log_2(N) \cdot (\alpha/\Delta \alpha) \), which is proportional to \( P^{-1}T^3\log_2(T) \). [Thus signal to noise achievable, which scales as (area \( \times \) time)\(^{1/2} \), is scaling approximately as (flops)\(^{1/6} \)]. As an example, the search for a 1 ms pulsar in an integration of \( T = 10^3 \) s will call for FFTs of length \( 10^6 \) data points, each taking on the order of \( 10^7 \) flops. The number of FFTs required will be on the order of a few hundred, so that the cost is about \( 10^{10} \) flops for one data segment from one source.

Integration times are limited by restrictions on the quadratic approximation technique, but often are a few thousand seconds and sometimes only a few hundred. There are various ways to achieve

![Fig. 1. A simulation of CR. A coherent signal has been modulated sinusoidally, simulating FM from an orbit. Quadratic CR is applied to recover the signal. The lower frame shows best recovered signal using the optimum \( \alpha \), which recovers ~ 90\% of the actual power, with the loss representing price of the quadratic approximation to the true orbit. Such simulations have been used to validate the CR method in detail.](image-url)
still longer integrations, all involving something more nearly like the 3-D parameter search described above, hence their computational cost grows drastically. Pulsed fractions less than 0.01 are often achievable with methods used here; \( f = 0.001 \) has not been reached.

Beyond a one-parameter quadratic approximation lie two recourses for improving CR sensitivity. The first is to use whatever knowledge of the orbit is available; this method is tailored to the source (see discussion of 1820-30, below), but it has considerable potential. The second is to increase collecting aperture by a large factor.

Note that in unfavorable situations, even after CR has been applied, considerable power may still be smeared over several channels. A limited improvement in signal-to-noise may then be obtained by using incoherent techniques as a post-process.

We now turn to specific examples, Cyg X-2, Cyg X-3, and 1820-30, in which CR has been applied, using data obtained from HEAO A-1. The HEAO A-1 proportional counters had thin windows which provided a photon yield per cm\(^2\) roughly double that obtained from these sources using detectors with beryllium windows (Wood et al. 1984). Routine HEAO A-1 telemetry modes limited resolution to 5 ms. Occasional 128kbps telemetry allowed resolution less than 1 ms.

**Cygnus X-2**

Cyg X-2 gives us nearly the standard form of the CR problem outlined above, except that the orbit is not totally unknown. Its radius is thought to be less than 60 lt-sec and the orbital period is \( \sim 10 \) days. There are 3 hr of HEAO data, with gaps, at a resolution of 5 ms. For pulsar searches, samples of \( 5 \times 10^5 \) pts have been used and the search cost is \( 3 \times 10^8 \) flops. The upper limit on pulsed fraction is \( 0.005 \) (\( P > 10 \) ms), at 98\% confidence. These are the data in which \( \sim 5 \) Hz QPO activity has been reported; a broad 50 Hz feature is also seen (Norris and Wood 1987). Integrations up to \( 2 \times 10^4 \) s are allowed here using CR. Computation costs and the lengths of the data gaps set practical limits on integration time.

**Cygnus X-3**

The orbital period for Cyg X-3 is well determined (4.8 hr) and the orbital radius is thought to be less than 8 lt-sec. There are 4 hr of HEAO data, with gaps, at 5 ms resolution, supplemented by short stretches of 128kbps data that have been re-binned to a resolution of 0.3 ms. Applying the standard procedure produces the upper limits \( f < 0.018 \) (\( P > 10 \) ms) and \( f < 0.065 \) (\( P > 0.6 \) ms), both at 98\% confidence. Supplementary searches were also conducted for periods near 12 ms, because of a reported period in ultra-high energy gamma rays (Chadwick et al. 1985). Integrations for the Cyg X-3 case were near the limit (\( \sim 10^3 \) s) allowed by the quadratic approximation.

**1820-30**

This source departs radically from the "standard form", because of the recent discovery of a 685 s period (Stella 1987), probably the orbital period of an unusual system. It is so short that the usual
quadratic CR procedure is unapt, being limited to very short integrations. The radius of the neutron star's orbit is small but its velocity is high. The best search is a trial grid of sinuosoids restricted by knowledge of orbital parameters (phase accuracy of ~ 5% and a precisely known orbital period). The assumption made is that the orbital radius is less than 0.32 lt-sec, equivalent to assuming a companion mass of 0.11 M or less. The search is computation-limited. For $2 \times 10^9$ flops (960 trials) the limit is $f < 0.009$, for $P > 10$ ms. A complementary search was also undertaken assuming that 685 s is not the orbital period. Then a limit of $f < 0.006$, for $P > 10$ ms and orbital periods greater than 8 hr, is obtained.

2. Discussion

Coherence recovery permits integration times of $10^3$ - $10^4$ s to be utilized in pulsar searching on systems with poorly-determined or unknown orbital parameters. The ideal utilization of CR is in conjunction with a very large collecting aperture. We have elsewhere proposed the construction of a 100 m$^2$ array, called XLA, on the NASA Space Station (Wood et al. 1985, 1986). Such a facility would permit pulsations to be detected at $f < 10^{-4}$.

It is extremely important for such a program to be pursued. Not only are millisecond spins becoming a central issue in the understanding of binary X-ray sources, there are significant links to gravitational physics. In particular, Wagoner (1984) has presented a scenario whereby a neutron star can be spun up until it becomes subject to gravitational radiation reaction instabilities. It then becomes simultaneously a pulsar in X-rays and gravitational waves. The sensitivities attainable for periodic signals with available gravity wave detector designs are greatest for narrow-band detectors tuned to a known frequency. The required program is to find the signal in X-rays and then build a special gravity wave antenna tuned to that frequency. The techniques presented here for finding frequency-modulated sinuosoids then have analogs in the processing of the frequency-modulated sinusoidal gravity wave signals as well.

This work was supported by NASA and the Office of Naval Research.

3. REFERENCES

Stella, L.: 1987, these proceedings.