Cloud-tracked wind measurements reported by Sromovsky et al. (1983, J. Geophys. Res. 88, 8650-8666), have been analyzed to determine meridional momentum transports in Saturn's northern middle latitudes. Results are expressed in terms of eastward and northward velocity components ($u$ and $v$), and eddy components $u'$ and $v'$. At most latitudes between 13 and 44 deg N (planetocentric), the transport by the mean flow ($\langle u \rangle \langle v \rangle$) is measureably southward, tending to support Saturn's large equatorial jet, and completely dominating the eddy transport. Meridional velocities are near zero at the peak of the relatively weak westward jet (centered at 34.5 deg N planetocentric latitude); along the flanks of that jet, measurements indicate divergent flow out of the jet. In this region the dominant eddy transport ($\langle u'v' \rangle$) is northward on the north side of the jet, but not resolvable on the south side. Eddy transports at most other latitudes are not significantly different from measurement error. The conversion of eddy kinetic energy to mean kinetic energy, indicated by the correlation between $\langle u'v' \rangle$ and $d\langle u \rangle/dy$ (where $y$ is meridional distance) is clearly smaller than various values reported for Jupiter, and not significantly different from zero. Both Jovian and Saturnian results may be biased by the tendency for cloud tracking to favor high-contrast (usually more active) features, and thus may not be entirely representative of the cloud level motions as a whole. Attempts to resolve a mean meridional velocity structure in three independent Jupiter data sets have discovered significant correlations between $\langle v \rangle$ and two different trial functions, depending on whether Voyager 1 data is included, or excluded. Neither correlation agrees with the form of Saturn's meridional profile, which has a positive correlation with $d\langle u \rangle/dy$. More measurements are needed to decide the issue.

Here is a preview of what I'm going to talk about. I'll begin with a brief summary of what we know about the Jovian transport results. In essence, there appears to be a large conversion of eddy kinetic energy into mean kinetic energy, but there's a question about sampling. The second topic is the eddy transport measurements for Saturn; these appear to show no significant eddy support of the jet structure. I'll talk about that just a little. The third topic is more of a progress report on my search for meridional transports by the mean flow. Potentially large mean transports have so far been obscured by really huge percentage errors in the meridional velocity measurements. I tried to increase the signal-to-noise ratio in these measurements by doing a lot of judicious averaging and looking for correlations between $\langle v \rangle$ and various functions of $\langle u \rangle$. Tentative results show statistically significant correlations, but Jupiter and Saturn show different kinds of correlations. I'll explain more about this as I go along here.
As a background for discussing transport results I'd first like to present a short, and somewhat sloppy, derivation of what momentum transport is and does. Consider a point at which the eastward velocity is $u$, the northward velocity is $v$, and the atmospheric density is $\rho$. At the same point we note that

$$\rho u = \text{Eastward momentum per unit volume, and}$$

$$v\rho u = \text{rate of northward transport of eastward momentum/unit area.}$$

Consider a volume bounded by two latitudes, labelled here by meridional distance coordinates $y_1$ and $y_2$, of thickness $H$ and of longitudinal width $W$. Assuming zonal symmetry and no vertical transports, the only net momentum transport is meridional. The momentum balance for this volume then becomes

$$\rho \cdot H \cdot W \cdot [(uv)_1 - (uv)_2] = \rho \cdot H \cdot W \cdot (y_2 - y_1) \frac{du}{dt},$$

which says that the difference between momentum flow rates into and out of the volume must equal the rate of change of the momentum within the volume. Writing this for a differential volume, and taking a zonal average, we obtain

$$\frac{d\langle u \rangle}{dt} = - \frac{d\langle uv \rangle}{dy} = -\frac{d\langle \langle u \rangle \langle v \rangle \rangle}{dy} - \frac{d\langle u'v' \rangle}{dy},$$

where $\langle u \rangle \langle v \rangle$ and $\langle u'v' \rangle$ are transports by mean and eddy flows respectively. The rate of change of kinetic energy, per unit mass, of the zonal mean flow is just the product of $\langle u \rangle$ and $d\langle u \rangle/dt$, i.e.

$$\frac{dK}{dt} = \langle u \rangle d\langle u \rangle/dt = -\langle u \rangle \frac{d\langle uv \rangle}{dy}$$

$$= - \langle u \rangle \frac{d\langle \langle u \rangle \langle v \rangle \rangle}{dy} - \langle u \rangle \frac{d\langle u'v' \rangle}{dy},$$

where the second term on the right states that net eddy transport of eastward momentum into an eastward jet tends to increase the kinetic energy of the jet. Thus it's interesting to see correlations between momentum transport derivatives and $\langle u \rangle$. But what we usually look at is the alternate product, which is momentum transport times the derivative of $\langle u \rangle$. These two products are related to the derivative of a product, namely

$$\langle u'v' \rangle d\langle u \rangle/dy = \frac{d}{dy} \langle \langle u \rangle \langle u'v' \rangle \rangle - \langle u \rangle \frac{d\langle u'v' \rangle}{dy}.$$ 

We usually talk only about the left hand side of this last equation in deciding whether eddy kinetic energy is being added to by the eddies. This is
justified only when integrated over a suitable atmospheric volume for which the derivative of \( \langle u'v' \rangle \) converts to a vanishing boundary term. At any rate, \( \langle u'v' \rangle \times d\langle u \rangle / dy \) has been popularly looked at (cf. Fig. 1.) Maybe we should be looking at \( \langle u \rangle \times d\langle u'v' \rangle / dy \) also, since it has more local significance.

\[
d\langle u \rangle / dy, \ 10^{-5} s^{-1} (- - -)
\]

Figure 1. Comparison of Jovian eddy momentum transports \( [\langle u'v' \rangle] \) with the latitudinal shear of the mean zonal component of motion \( [d\langle u \rangle / dy] \) for three data sets: Voyager 1 and 2 measurements by Ingersoll et al. (1981), denoted by V1 and V2, and our Voyager 2 measurements (Limaye et al., 1982; Sromovsky et al., 1982), denoted by V2M. A positive correlation between the solid and dashed curves indicates eddy kinetic energy conversion to mean flow kinetic energy.
Figure 1 displays measured eddy transports on Jupiter, where Voyager 1 and 2 results from Ingersoll et al. (1981) are shown in comparison with Voyager 2 results of Limaye et al. (1982). The solid curves represent $\langle u'v' \rangle$, the dashed curves $d\langle u \rangle/dy$. The correlation between them is pretty easy to see in our results; the local correlation is very strong except in the jet at 23 deg N latitude (this looks like 20 deg here because the latitude scale is planetocentric). Over all latitudes measured, the correlation coefficients are, from left to right, 0.54, 0.3, and 0.45. Those are all very significant considering the number of bins which we have here; the chance of random variables correlating this well is less than one percent.

Although the correlations look significant, the differences in these measurements have a disturbing pattern. Ingersoll et al.'s Voyager 1 measurements are noisier than their Voyager 2 measurements (the RMS deviations within a latitude bin are somewhat larger, and the differences between adjacent bins is considerably larger). Our measurements (Limaye et al., 1982) are also fairly noisy because we used low resolution images. Strangely, the noisier measurements are the ones that show the strongest correlation. Now, if you add random noise to correlated variables, you would expect a reduced correlation to appear, not a larger one. So this looks a little bit suspicious in itself; the most noise-free measurements (Ingersoll et al.'s Voyager 2 measurements) are the ones showing the least correlation. Also notable is that the product of these variables has a pretty large average; it's equivalent to an energy conversion rate 10-30 percent of the net flux emitted to space. That is definitely not Earth-like; on Earth the average conversion rate is only about 0.1% of the net flux emitted to space.

When we saw these large correlations we went looking in our data for the eddies that were responsible. What we hoped to see, maybe, were asymmetric eddies something like the upper eddy in Fig. 2, for which the $u'v'$ product is large in certain parts of the wave, and very small in others, similar to many baroclinic waves on Earth. We were looking for something like this, although we didn't have a specific target in mind. But we didn't really find anything in the way of obviously asymmetric eddies. Instead, we saw many fairly symmetric eddies and strong motions which we had not sampled uniformly. When we remeasured cloud motions with special efforts to achieve uniform sampling, our correlations became insignificant. We didn't remeasure the motions for the whole globe because it was too costly to do it. But where we did do it we couldn't maintain this fairly strong correlation we had found.

Figure 2 also shows a symmetric eddy imbedded in a mean flow with positive zonal shear; in certain parts of the eddy the $u'v'$ product has a positive correlation with $d\langle u \rangle/dy$, and at other places it has a negative correlation. Thus, in an average over latitude, these positive and negative contributions will result in no net contribution. To upset this balance all that's needed is a non-uniform sampling of the eddy motions. But, for a sampling problem to show up in large scale zonal means, the sampling must be not only non-uniform, it must also be consistently biased. Perhaps the eddies themselves do something to produce more targets in certain regions than others. Of course this is a very speculative suggestion.
Figure 2. Latitudinal eddy momentum transports are produced by asymmetric waves (a) because, at a fixed latitude, the $u'v'$ products on the gently sloping parts of the wave don't balance the $u'v'$ products on the steep part of the wave. But symmetric eddies (b) do balance, unless sampled asymmetrically. A symmetric eddy imbedded in a mean positive shear, indicated in the lower right, could provide eddy transports of the same sign as the shear itself if over-sampled in upper left and lower right regions of the circulation.

To summarize this situation, I think there is a problem with sampling in our data, and the data we used is more susceptible to it. We did obtain the largest correlation between $\langle u'v' \rangle$ and $\langle du/\rangle dy$, and when we tried to do equal area sampling the correlation became insignificant. Non-uniformity in sampling probably would be less of an influence on the Ingersoll et al. results because they are based on higher resolution images. So I still have doubts about what the true situation is. Now let me move to Saturn briefly.
Figure 3. Saturnian eddy momentum transports (shaded region) in comparison with the latitudinal shear in \(<u>\) (solid line). These results are from northern mid-latitude measurements by Sromovsky et al. (1983). The correlation coefficient over the plotted latitude range is only 0.02, with a 90 percent chance that uncorrelated variables would correlate this well, or better.
Measured eddy transports on Saturn are illustrated in Fig. 3. These results are from a new analysis of measurements described by Sromovsky et al. (1983), consisting of 800 wind vectors measured in northern middle latitudes. The calculated correlation coefficient for our measurements is 0.02 with a probability that random variables could have a correlation this large being about 90%. Independent measurements of about 1000 cloud targets distributed over both hemispheres of Saturn, were obtained by Ingersoll et al. (1984). (These sample numbers should be compared to 8,000 and 10,000 targets measured on Jupiter.) The correlation coefficient for the latter data set is also low, although there is a discrepancy that I haven't straightened out yet. In the Saturn book Ingersoll et al. gave the correlation coefficient as r=0.01, while my calculations for their data yield r=0.12; but, even for the larger value, the chance that this could be random error is 34%. Thus, there doesn't seem to be a significant correlation between \( u'v' \) and \( d<u>/dy \) on Saturn; the eddy transport of the jets is not as large as we seem to be measuring on Jupiter. Let me move on to the eddy comparison with the mean flow.

Here is the problem with mean transport measurements: because \( u \) is so large a little mean meridional flow can produce a huge transport, and the measured mean transports on Jupiter and Saturn often do overwhelm the eddy transports. This is especially dramatic in the comparison shown in Fig. 4. But the noise in \( \langle v \rangle \) is so large that these potentially large transports are highly uncertain, and not to be believed. So I started looking for ways to average results and try to put a better bound on the \( \langle v \rangle \) component, and on the associated meridional transport.

The search for mean meridional transports is based on the idea that there is a definite relation between meridional motions and zonal motions, and that the relation is the same for all (or nearly) all of the zonal jets on each planet. You've seen many examples of what kind of meridional flow might appear on Jupiter or Saturn: the typical picture, illustrated in Fig. 5, has rising motions in the zones and descending motions in the belts. A pressure gradient between these two is implied if the motions are geostrophic ("geo" is not quite the right word here); the horizontal pressure gradient provides a balance for the Coriolis force. Add some dissipation to upset the balance, and you might expect meridional motions in the direction of the pressure gradient (an example of "cross isobaric flow"). The steady meridional flow is presumably driven by convection.

If this picture is correct, we should expect the meridional velocities to be related to \( f \langle u \rangle \), where \( f \) is the Coriolis parameter. If, on the other hand, the meridional transport by mean flows is assumed to balance the measured eddy transports, then \( \langle u \rangle \langle v \rangle \) should have a negative correlation with \( u'v' \) or, since this appears to be correlated with \( d<u>/dy \), we might look for correlations between \( \langle u \rangle \langle v \rangle \) and \( -d<u>/dy \), or just between \( \langle v \rangle \) and \( -(1/\langle u \rangle)d<u>/dy \). Note that if \( \langle u \rangle \) is symmetric about the equator (it is, approximately) then both of these functions have a mirror symmetry about the equator: their signs reverse when the sign of the latitude is reversed. It is hard to imagine a combination of symmetric zonal motions and meridional flows which do not have this mirror symmetry. A third simple function with the proper symmetry is \( d<u>/d\phi \) (\( \phi \) is latitude), although there is no physical basis for picking a sign of the correlation.
Figure 4. Comparison of mean (shaded band) and eddy (solid curve) momentum transports in Saturn's northern middle latitudes. At low latitudes the mean transport is completely dominant, although the error band for the mean transport is only a formal estimate.
Figure 5. A popular conception of meridional flow on Jupiter and Saturn (after Ingersoll and Cuzzi, 1969). The Coriolis force on zonal jets is balanced by the latitudinal pressure gradient between zones and belts. The zones are pictured as warm high pressure regions with upwelling motions and meridional divergence at the level of observation, while the belts are associated with cooler temperatures, meridional convergence, and downwelling.

Thus we have three distinctly different patterns to look for, none really a theoretical prediction, but at least two having some physical connotation. The true patterns might easily be much more complex functions than those considered here.

The raw measurements of \( \langle v \rangle \) are a real jumble of noise, with meridional velocities reaching nearly 8 m s\(^{-1}\), and little correlation between different data sets. Just averaging that data and doing a three-point smoothing, returns a profile which ranges between -2 and +3 m s\(^{-1}\). This, it turns out, is fairly strongly correlated with \( \langle u \rangle \), a function which does not have the mirror symmetry described previously. That correlation could come about from a slight error in defining the angle of the Voyager camera relative to the planet's spin axis. If the images are slightly tilted, the velocity measurements can be resolved incorrectly along the u and v axes. Half a degree of tilt would explain the observed correlation. Since there is also no physical reason for this correlation to exist in true atmospheric motions, I removed it. The main effect of doing that is in the equatorial region; the other regions don't change too much. Now the \( \langle v \rangle \) profile excursions are down to ±2 m s\(^{-1}\) at the extremes.
Does this smoothed and corrected $v$-profile correlate with either of the physically suggested functions: $+ f\langle u \rangle$ from the belt-zone idea, or $-(1/\langle u \rangle) d\langle u \rangle/dy$, to balance the eddy transport into the jets? Neither of these provide a significant correlation on either Jupiter or Saturn, although on Jupiter we could obtain a significant correlation with the Coriolis force function if we eliminated the Voyager 1 data (the most noisy member of the three Jupiter data sets). If you calculate the correlation coefficient for the entire Jupiter data set, it's hopelessly insignificant.

The third function with the right kind of symmetry to allow correlations with $\langle v \rangle$ is $d\langle u \rangle/d\phi$, although I can't argue that $d\langle u \rangle/d\phi$ ought to correlate with $\langle v \rangle$. Using the complete set of Jupiter data, the correlation between these two is negative between latitudes of 40 deg and 15 deg or so, but positive in the equatorial region. [Between 15 deg and 42 deg latitude the correlation coefficient between $du/d\phi$ and the corrected $v$ is $-0.3$ which is significant at the 2% level.] To further increase the signal-to-noise ratio in this calculation I tried anti-symmetrizing both the $\langle v \rangle$ profile, and the $d\langle u \rangle/d\phi$ profile, under the assumption that the symmetric component must be erroneous (this is not quite proper because $\langle u \rangle$ is not quite symmetric). The anti-symmetrization does improve the correlation in mid-latitudes, but still does not show a significant correlation in the equatorial region.

Saturn is the next topic. As indicated in Fig. 6, meridional velocities in middle latitudes do not show a significant correlation with $f\langle u \rangle$. The correlation with $d\langle u \rangle/d\phi$, on the other hand, is significant: $r=0.59$, with a chance of random variables correlating this well being less than 1%. The apparent meridional flows on Saturn are divergent in the westward jets and convergent in the eastward jets. This is opposite to the sense that we saw for the complete Jupiter data set. But the picture on Jupiter would change if the noisy Voyager 1 data set were excluded.

To summarize, the conversion of eddy kinetic energy to mean kinetic energy on Jupiter appears to be large, but I think we'd have to be a little suspicious because of the possibility of asymmetric sampling. The conversion on Saturn does appear to be small, and so far not detectable.

There may be some evidence of a $\langle v \rangle$ component which is really measureable. These preliminary results seem to indicate that $\langle v \rangle$ is positively correlated with $d\langle u \rangle/dy$ on Saturn, but negatively correlated on Jupiter: that would imply a meridional convergence in the eastward jets on Saturn and convergence in the westward jets on Jupiter (cf. Fig. 7.) Saturn clearly did not show a significant correlation between $\langle v \rangle$ and $f\langle u \rangle$ which would contradict at least the sense, not the magnitude certainly, of the old belt zone interpretation. The same contradiction would be implied by the entire Jupiter data set, but if the noisier Voyager 1 data is excluded, then there is at least partial consistency with the old belt-zone interpretation. This latter statement points out something that needs to be emphasized: we have only tentative results, especially tentative results concerning mean meridional motions.

Unfortunately this tends to shed some darkness on the eddy question, and not too much light on the mean meridional flow. We hope we can do better with higher resolution imagery and more careful navigation.
Figure 6. Measured meridional motions on Saturn (shaded band) compared with $f' <u>$ and $d<u>/d\phi$ (solid curves). The correlation coefficient between $<v>$ and $d<u>/d\phi$ is significant at the 1 percent level.
Figure 7. Schematic illustration of tentatively inferred correlation between meridional motion and the latitudinal shear of the zonal wind on Jupiter and Saturn.

REFERENCES


DR. FLASAR: You made the point that if you're doing local latitude studies of jet structure, you really want to look at the meridional divergence $<u'v'>$ correlated with $<u>$ rather than $d<u>/dy$ correlated with $<u'v'>$. Can you get a correlation of the first thing I mentioned, the one that you didn't show, the one between $<u>$ and the meridional divergence of $<u'v'>$?

DR. SROMOVSKY: No.

DR. FLASAR: Its just hard to calculate that divergence, is that it?

DR. SROMOVSKY: Because $<u'v'>$ is so noisy its derivative looks really bad. We did do the integral over all latitudes of $<u>d<u'v'>/dy$ and compared it with the integral of the normal thing we look at ($<u'v'>d<u>/dy$) and the results were basically the same. But we didn't look at the detailed local distribution. It should be done but it hasn't been done. It's going to be noisy though, I can say that much.

DR. READ: Just to raise a few notes of healthy theoretical skepticism on interpreting some of these results. First of all, the kind of energy budget that you're talking about inferred from correlations between horizontal momentum fluxes and mean zonal shears, one has to emphasize, is fundamentally incomplete because you simply can't ignore the effect of heat fluxes in deriving the acceleration of the mean flow. You've also stressed the importance of the meridional circulation, which of course Mike Flasar referred to, the fact that that can bring about a cancellation of the effect of these momentum fluxes. So if you're going to do this excercise, you must have the complete information, otherwise you're simply not going to get anywhere in trying to interpret these results. There are some well established theoretical examples where you can write down simple analytical solutions of combinations of eddies and mean flows which have all sorts of exciting looking conversions but by definition do absolutely nothing. These are the so-called "free mode" solutions which satisfy the Charney-Drazin non-acceleration theorem. And these ideas are not just restricted to quasi-geostrophic arguments. Andrews and McIntyre for example, have generalized the whole theory well beyond just the simple quasi-geostrophic arguments. Just one last quick question. Having said that all these correlations may or may not have any real significance, I just want to ask you if you see any evidence of horizontal tilt in the very long lived eddies, the Red Spot and the like, because I gather Tony Maxworthy and Jim Mitchell have claimed to see some evidence of a systematic $<u'v'>$ conversion associated with these large eddies.

DR. SROMOVSKY: Has that been published?
DR. READ: I know of at least one place it's been published. In an I.U.T.A.M. conference proceedings which has only just appeared, I think.*

DR. STONE: Are you going to answer that question?

DR. SROMOVSKY: The answer is no. (We have not seen evidence of horizontal tilt in the long-lived eddies.)

DR. STONE: OK. Andy Ingersoll.

DR. INGERSOLL: First of all, all of us who have been trying to milk these measurements dry are fully aware of all the other terms. However, I disagree with one statement you (Dr. Read) made and that is that you can get nothing out of an incomplete picture. You get a diagnostic which, if it is statistically true—and I will admit there is controversy about that—but, if it is statistically true that there is a correlation between one thing and another, you may not have the whole picture of the whole momentum or energy conversion, but you have one number that is true. And if you have a theoretical model that disagrees with that number, you've got a problem with that theoretical model.

DR. LEOVY: I raise again my question about the isotropy of $u'$ versus $v'$ and let me mention the reason that I think it's important. It's because what one would like to do is to isolate the scale at which the energy is coming in if possible, and one would expect the eddies to be more or less isotropic at the scale at which the energy is coming in, and to get more and more longitudinal at the larger scales. Do you have any information on that?

DR. SROMOVSKY: Are you asking about the longitudinal distribution?

DR. LEOVY: I'm asking about first the relationship between RMS $u'$ and RMS $v'$, and second, is there any possibility of getting scale dependent information on that?

DR. SROMOVSKY: RMS $u'$ tends to be larger than RMS $v'$. I can't give you any information on scale dependence. I'm not sure how to answer anything more than that.

DR. POLLACK: A very naive question, and that is, does the fact that you get a correlation of one sign from Jupiter for your $v'$ correlation with $d<u>/dy$ and an opposite sign for Saturn make you potentially suspicious that all this is noise really and not real correlation?

DR. SROMOVSKY: Yes. Well, I'm less suspicious on Saturn actually; the results of the Saturn mean flow calculation are fairly consistent with the visual impression you get just looking at the images, and the noise on Saturn is pretty low in the one jet where we see a lot of eddy activity. On Jupiter the eddies are stronger, it's easier to make mistakes. The agreement between the three Voyager data sets is not quite as good as their estimated errors say they should be, so I'm more suspicious of the results on Jupiter and I think possibly larger error bars should be attached to those. And I'd like to see better measurements on Jupiter, which is possible. I am suspicious, but more suspicious of Jupiter actually.

DR. LIMAYE: Let me just add one thing to Conway Leovy's question. I guess I'll try to answer your question, at least one small part of it. I think the disagreement between the three previous Voyager estimates for Jupiter's mean $\langle u \rangle$ component is large enough so that it totally becomes a problem as to what is a $u$ eddy. But now I think the results I presented earlier suggest that the mean flow appears to be very stable. So we can redefine what the $\langle u \rangle$ component is. I was about to take the new mean $\langle u \rangle$, use the old measurements and calculate $\langle u'v' \rangle$. I resisted that temptation very greatly, because, frankly I don't know how to interpret what I would get. But we hope we can now use the mean flow and redefine some of the eddies.

DR. BEEBE: Some comments about those atmospheres. When you look at the atmosphere of Saturn, most of the turbulence you can see is in the westward jets. They're in the middle of the lowest albedo regions. You get the impression that you've got convection punching up through the cloud deck and you're actually seeing divergence of convection to the north and the south. The high speed jets on Saturn are much more spatially homogeneous with feathery structures that trail off along their edges. After you've worked in both north and south hemispheres looking at the morphology as you measure the features, you get the impression that the high speed jets are decoupled from the local convection. In the case of Jupiter's atmosphere all of the semi-permanent high albedo regions that we call zones are bounded on the poleward side by an eastward jet. There is chaotic structure along their equatorward side associated with the westward flows, and there seems to be divergence from those westward flows both north and south. But it looks as though the whole cloud deck that we're seeing on Jupiter is really still involved in the convective interface of the atmosphere and that doesn't appear to be the case in Saturn's atmosphere.