

ON THE OBLATENESS AND ROTATION RATE OF NEPTUNE'S ATMOSPHERE

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Recent observations of a stellar occultation by Neptune (Hubbard et al., 1985, *Astron. J.* 90, 655-667) give an oblateness of 0.022 ± 0.004 for Neptune's atmosphere at the 1-microbar pressure level. This result is consistent with hydrostatic equilibrium at a uniform atmospheric rotation period of 15 hours, although the error bars on quantities used in the calculation are such that an 18-hour period is not excluded. The oblateness of a planetary atmosphere is determined from stellar occultations by measuring the times at which a specified point on immersion or emersion occultation profiles is reached. We critically evaluate whether this standard procedure for deriving the shape of the atmosphere is consistent with what we know about vertical and horizontal temperature gradients in Neptune's atmosphere. We then consider the nature of the constraint placed on the interior mass distribution by an oblateness determined in this manner, considering the effects of possible differential rotation. A 15-hour Neptune internal mass distribution is approximately homologous to Uranus', but an 18-hour period is not. We discuss the remarkable implications for Neptune's interior structure if its body rotation period is actually 18 hours.

This morning, you heard Ellis Miner talking about two different values for the rotation period of Neptune. Some questions came up about which one was correct. The purpose of this talk is to try to elucidate that matter and to attempt to convince you that there is some possibility that maybe both are correct.

The first point I want to make is the reason that I, at least, would like to know the rotation period of Neptune is because of what it tells us about the interior. If we expand the external gravity potential V_e of the planet in the usual form where θ is the colatitude and J_2 is the second-degree zonal harmonic:

$$V_e = (GM/r) [1 - J_2 (a/r)^2 P_2(\cos \theta)] \quad , \quad (1)$$

where a is the equatorial radius; then for a rotating body in hydrostatic equilibrium, J_2 is going to be proportional to a small parameter given by

$$q = \omega^2 a^3 / GM \quad , \quad (2)$$

where ω is the angular rotation rate, G is the gravitational constant, and M is the mass. We express the proportionality in the form

$$J_2 = \Lambda_2 q \quad , \quad (3)$$

to lowest order in q . Λ_2 is the response coefficient. For the Jovian planets, we can look at these values which are known for Jupiter and Saturn, compare them with the homogeneous case, and we see that these planets are centrally condensed. The more centrally condensed they are, the smaller Λ_2 . For a sixteen-hour period of Uranus, which is sort of becoming a consensus value (although there may still be some holdouts), we obtain for Uranus, $\Lambda_2 = 0.09$.

Now the 18-hour period for Neptune, which was discussed this morning, and which comes from looking at cloud features going around on Neptune, gives us $\Lambda_2 = 0.20$, derived from the fact that the J_2 of Neptune is 4×10^{-3} . Now $\Lambda_2 = 0.20$ means that Neptune is less centrally condensed than any of the Jovian planets, and that leads to very weird interior models. For example, one may have to consider models that are composed entirely of water. Morris Podolak recently completed a study with Ray Reynolds and Rich Young, and he might want to say a word or two about this.

There is another way to measure the rotation period, and this is where the atmosphere comes in. One can look at the shape of the planet's surface, and the way one does this is to assume that the atmosphere defines an equipotential surface. Then we calculate the potential at the pole of the planet where b is the polar radius, and we equate that to the full corotating potential at the equator, where we add the centrifugal potential to expression (1). We then equate these potentials, use that to calculate the relative difference in the equatorial and polar radius, and that then can be expressed in terms of J_2 and this rotation parameter q , which can also be expressed in terms of these other variables as previously described:

$$\begin{aligned} e &= (a-b)/a = (3/2)J_2 + (1/2)q \\ &= q(3\Lambda_2 + 1)/2 \\ &= J_2(3 + \Lambda_2^{-1})/2 \quad . \end{aligned} \quad (4)$$

In 1983, we had a very good stellar occultation by Neptune, which was observed by our group, and we obtained data from eight stations over a rather large baseline (Hubbard et al., 1985). We were able to observe from extremely southern locations as well as extremely northern locations. The timings were used to define the equipotential surface. There was a very measurable difference between a spherical and oblate object. Thus, we were able to deduce the oblateness. One of the difficulties that comes in when one is trying to measure oblateness is that the planets with atmospheres have fuzzy edges.

We get a lot of scattering of light due to refractive focusing and defocusing. It's difficult to say where a particular pressure level occurs, but there is a standard procedure for doing this; namely, we look for the half intensity point as obtained by fitting an isothermal light curve (i.e., a theoretical light curve computed for a strictly isothermal atmosphere) to the observed data. That seems to work reasonably well, although I should point out that you see variations from one station to another which don't reproduce. So, in detail, the planet is not globally layered, at least beyond some limit. The immersion and emersion profiles are grossly similar: we have non-correlation in detail although the overall structure is similar.

If we considerably idealize the atmosphere, in terms of a perfectly isothermal quiet atmosphere, then the rays of light coming from the star to the observer will fall in a pattern such that the angle through which Neptune bends each ray will increase exponentially with depth of penetration of the ray into Neptune's atmosphere. At the ideal half intensity point, the linear deflection of the ray at the Earth is precisely equal to the scale height H , which in the case of Neptune is about 50 km. Now the argument in trying to say how accurately we can define the edge of the planet, goes as follows: We have density fluctuations (with respect to the ideal isothermal atmosphere) which are perturbing the rays so that they actually don't fall in the idealized path, but they, in fact, sometimes converge, producing a spike. At other times they may diverge by an extra amount. But the argument is that any extra bending that they will suffer due to imperfections in the isothermal atmosphere will be small compared with the value of the overall bending angle. The reason for that is that we assume that the density fluctuations are small compared to the background density, which is certainly quite reasonable. If that argument is true, then any residuals that we get in our fit to the overall profile of the atmosphere should be considerably less than the scale height. That is the type of accuracy for which we aimed in fitting the solution. With suitable discarding of anomalous data points, we succeeded.

Some of the data were affected by known systematic problems, and indeed it was those stations that turned out to have the largest residuals. On the ingress side, we got residuals less than a kilometer at our Taiwan station, and at our Hobart station it was again less than a kilometer (a very nice fit to the best fit solution). A portable station gave a 37 km residual and was thrown out in the final solution, though I could mention that we did include it in one solution, which gave a slightly shorter rotation period. On the emersion side, there was a measurable separation between the spherical profile and the oblate profile. The residual was -1 km at Chung Li, Taiwan. The Guam station was thrown out. The overall fit had all retained stations with residuals of substantially less than 50 km. The results of this solution I'll give to you in a moment. Let me also mention that as a by-product of this analysis, we obtained the temperature at the occultation level, which was computed by fitting an isothermal light curve to the data, and obtaining the scale height. The scale height was then corrected for the variable gravity, due to the oblateness of the planet. Finally, the temperature at the one microbar level was deduced as a function of the latitude. What we find is in general not really any indication of a large temperature gradient as a function of latitude. In fact, at this level in the atmosphere it looks like the temperature is reasonably constant, both with latitude and with time. The average value turns out to be

about 155 K. So if that is the case, then that is further support for the model that we are assuming, that we are dealing with a surface that is at a constant potential and a constant density.

We had to fit not only to the oblateness of course, but also solve simultaneously for the equatorial radius as well as the center of figure. Though we have a very nicely defined minimum in the rms residuals, because of the large number of the parameters that we had to vary, the probability that the solution actually lay within a narrow interval in the oblateness e is still rather small. But the deduced value is $e = 0.022 \pm 0.004$, which is an improved result over the one that I reported at the Uranus/Neptune meeting (which led to a 13.6 hour period, which was mentioned by Ellis Miner this morning). What we did in this analysis was to use Harris' (1984) improved pole position for Neptune. That leads to a reasonable value for Λ_2 : 0.12, and a period $P = 15^h (+3^h, -2^h)$. However, notice that because of the large error bars we still can't rule out an 18-hour period. Nevertheless, as I mentioned earlier, I don't like the 18-hour period.

Here is the model for the atmospheric dynamicist to figure out. Suppose that we had differential rotation on cylinders, and suppose that the bulk of the planet is actually rotating with a period consistent with the models and the J_2 value (say, $P = 15^h$). Suppose that we have an equatorial zone which is in differential rotation, but unlike Jupiter and Saturn, it's going backwards. In other words, it's rotating at a slower rate of, say, 18 hours. We have spots on it, so that's what we see going around. We want the q of the planet as a whole to be 0.033, of course, the value corresponding to a 15-hour rotation period. The oblateness is now going to be given by this formula:

$$e = (3/2)J_2 + (1/2)q_0 + (1/2) \Delta q \quad , \quad (5)$$

where $q_0 = q(15^h)$ is the q corresponding to the deep interior (about 0.033), and Δq is a correction due to the differentially rotating outer layer. But we want this oblateness to remain almost the same as the one we just got [eq. (4)]. Thus we want Δq to be much smaller than q_0 . It turns out that if you do the calculation, assuming the planet is rotating on cylinders so that you can derive the centrifugal force from a potential, then the correction $\Delta q/q_0$ is just given by the formula:

$$\Delta q/q_0 = -0.31 \cos^2 \theta_0 \quad , \quad (6)$$

where θ_0 is the colatitude where the bounding cylinder pierces the surface, the bounding cylinder being the one which divides the inner region which rotates with a period of 15^h from the outer region which rotates with a period of 18^h . Thus, by adjusting the parameters an appropriate amount (for example, taking θ_0 to be 60 deg which gives you a big band in latitude of ± 30 deg to have spots in), you can make $\Delta q = -0.076 q_0$, less than a tenth of q_0 . Thus one can construct a model which would do the job, i.e., reconcile a 15^h deep-interior period with an 18^h equatorial-atmospheric period. But as to whether that seems plausible or not, I would have to refer you to a dynamicist.

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DR. PODOLAK: I'd like to pick up on the speaker's first statements. In order to match models to Uranus with a 16-hour period you need very much different models from those required to match an 18-hour period for Neptune. The amount of ice necessary is not so unreasonable, but the distribution differences between Uranus and Neptune are great.

Briefly, there are three types of material you have to worry about: things that are always solid in the solar nebula (like things we call rock), things that are always gaseous like hydrogen and helium, things that could be solid or gaseous depending on the temperature that we call ice (it's just a generic name not meaning the stuff is solid and frozen) and that would be water, methane, ammonia or similar substances. Then if you try to construct models made up of those three materials that match Uranus with a 16-hour period and Neptune with an 18-hour period, what you find is that the amount of rock that you put in the core is similar for both planets. It's a little bit more for Neptune because Neptune has a slightly higher density. The amount of ice that you put in the planets is similar, and in fact the ice-to-rock ratio is about three, which is exactly what you'd expect for solar compositions. All that is very nice except that for Uranus, most of the ice sits in a shell around the core (it's very centrally condensed like Bill said). For Neptune, most of the ice has to spread out throughout the envelope because it's rotating so much more slowly that you still want to have a high moment of inertia. The problem is, why do these things have such different structure? Just to give you a feeling for how strange this is, if you were to say "O.K., suppose the stuff on Uranus just fell down and the stuff on Neptune is going to fall down in another couple of years." It turns out that the amount of gravitational energy you release is so large that even with an effective temperature of about 100 K, it would still take two billion years to radiate away. You're talking about a really substantial amount of energy and a really big difference in structure. It is hard to see how that came about. That is the reason that I would like a 15-hour period for Neptune too.

DR. ORTON: Does anyone else want to reconstruct the solar system?

DR. ALLISON: Bill, do you think that there is a realistic prospect of eventually refining these occultation measurements to the point where they could be used as a discriminant between models of rotation on cylinders versus thin-layer models?

DR. HUBBARD: I think that it could be done if we can find a way to pin down the location of the center of Neptune accurately. In the case of Uranus, we have the advantage of the rings. They don't have atmospheres, so you can determine exactly where they are. Now if we can get enough observations of a Neptunian ring, or portions of a Neptunian ring to do the same, then yes, in the long run it should be possible to do the same thing. In that case, I believe that enough precision would be possible to check this point out.

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