Tether Dynamics Simulation

Proceedings of a workshop held at
the Hyatt Regency Crystal City Hotel
Arlington, Virginia
September 16, 1986
Tether Dynamics Simulation

NASA Office of Space Flight
Washington, D.C.
and
George C. Marshall Space Flight Center
Marshall Space Flight Center, Alabama

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the Hyatt Regency Crystal City Hotel
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FOREWORD

The objective of the Tether Dynamics Simulation Workshop was to provide a forum for the discussion of the structure and status of existing computer programs which are used to simulate the dynamics of a variety of tether applications. A major topic of the workshop concerned itself with the purpose of having different simulation models and the process of validating them. Guidance on future work in these areas was obtained from a panel discussion consisting of resource and technical managers and dynamic analysts in the tether field. The conclusions of this panel are presented in Section III, Workshop Summary and Panel Discussion Results.

NASA Headquarters, Office of Space Flight, Advanced Programs, along with NASA Marshall Space Flight Center, sponsored the workshop. General Research Corporation and the American Institute of Aeronautics and Astronautics provided the workshop coordination. General Research Corporation also prepared the workshop proceedings. The workshop was held at the Hyatt Regency Crystal City Hotel in Arlington, Virginia in conjunction with the International Conference on Tethers in Space, which was held Sept. 17-19, 1986.
WORKSHOP ADMINISTRATIVE PERSONNEL

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- "Slack3," G. Gullahorn  
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I

WORKSHOP PRESENTATIONS
Tether Dynamics Simulation

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TETHER DYNAMICS SIMULATION

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PURPOSE OF SIMULATION

I. TO STUDY THE GENERAL DYNAMICS OF THE SHUTTLE-TETHER-SUBSATELLITE SYSTEM DURING (i) STATIONKEEPING,
   (ii) DEPLOYMENT OF THE SUBSATELLITE,
   (iii) RETRIEVAL OF THE SUBSATELLITE.

II. TO DETERMINE APPROPRIATE CONTROL LAWS AND SIMULATE THE CONTROLLED DYNAMICS, SPECIFICALLY DURING THE RETRIEVAL STAGE. MOST OF OUR SIMULATIONS HAVE DEALT WITH THIS ASPECT.
Figure 1. Geometry of motion.
SIMULATION PROCEDURE

EQUATIONS OF MOTION ARE OBTAINED USING HAMILTON'S PRINCIPLE. VIBRATIONAL EQUATIONS ARE DISCRETIZED USING RITZ-GALERKIN METHOD. THESE DISCRETIZED EQUATIONS ALONG WITH THE ROTATIONAL EQUATIONS ARE CONVERTED TO A SET OF FIRST ORDER EQUATIONS OF THE FORM $\mathbf{x} = \mathbf{f}(\mathbf{x}, \theta)$ WHERE $\theta$ IS A DIMENSIONLESS TIME (TRUE ANOMALY) AND $\mathbf{x}$ IS THE VECTOR OF STATE VARIABLES.

THE SYSTEM OF DIFFERENTIAL EQUATIONS IS INTEGRATED USING GEAR'S METHOD.
POSSIBLE DISCRETIZATION SCHEMES

PHYSICAL DISCRETIZATION: (i) LUMPED MASSES (OR POINT ELEMENTS) CONNECTED BY SPRINGS AND DASHPOTS - KALAGHAN ET AL. AT SMITHSONIAN

(ii) FINITE ELEMENTS - KOHLER ET AL. (FOR ESA)

MATHEMATICAL DISCRETIZATION: (i) FINITE DIFFERENCE - KOHLER ET AL. (FOR ESA)

- KULLA

(ii) RITZ-GALERKIN METHOD - BUCKENS

- BANERJEE & KANE
- BAINUM ET AL.
- GLAESER & PASTRICK
- MISRA & MODI

SEMI-ANALYTICAL PROCEDURES - BERGAMASCHI ET AL.
DEGREES OF FREEDOM

TETHER ROTATIONS:  

INPLANE (PITCH) \( \alpha \)  

OUT-OF-PLANE (ROLL) \( \gamma \)

TRANSVERSE VIBRATIONS:  

INPLANE  

\[ W = L \sum_{i=1}^{N} B_i(T) \phi_i(y) \]  

OUT-OF-PLANE  

\[ U = L \sum_{i=1}^{N} A_i(T) \phi_i(y) \]

LONGITUDINAL VIBRATIONS:  

\[ V = L \sum_{i=1}^{P} C_i(T) \psi_i(y) \]

Thus the generalized co-ordinates are

\( \alpha, \gamma, A_i, B_i, C_i \)

\( \phi_i(y) = 2 \sin \left( \frac{\pi y}{L} \right) \)

\( \psi_i(y) = \left( \frac{y}{L} \right)^i \)
ENVIRONMENTAL FORCES

GRAVITATIONAL PERTURBATIONS

DUE TO THE ASPHERICITY OF THE EARTH - \( J_2 \) PERTURBATIONS.
PERTURBATIONS DUE TO THE ATTRACTION OF THE SUN AND THE MOON.

THESE EFFECTS ARE SMALL.

SOLAR RADIATION PRESSURE

CAN BE IMPORTANT AT HIGH ALTITUDES.
EVEN AT LOWER ALTITUDES, HEATING EFFECT MAY BE SIGNIFICANT.

ELECTRODYNAMIC FORCE

FORCE ON A CONDUCTING TETHER MOVING IN THE EARTH'S
GEOMAGNETIC FIELD.

AERODYNAMIC DRAG

THIS IS THE MOST IMPORTANT ENVIRONMENTAL FORCE AT THE LOW
ALTITUDES (I.E., FOR TSS-1 AND TSS-2 MISSIONS).
\[ \Delta F = -\frac{1}{2} C_D \rho \Delta A \hat{v} |\hat{v}| \]

where \( \Delta F \) is the aerodynamic force on an element having projected area \( \Delta A \),

\( C_D \) is the drag coefficient,

\( \rho \) is the density of air at the altitude of the element under consideration,

\( \hat{v} \) is the velocity of the element relative to the atmosphere,

\( C_D \) is assumed constant in our simulation

\[ \rho = \rho_{\text{REF}} e^{-\left(\frac{H - H_{\text{REF}}}{H_0}\right)} = \rho_0 e^{-H/H_0} \]

where \( H = \) altitude above the surface of the Earth

oblateness of the Earth is taken into account (affects \( H \))

rotation of the atmosphere is taken into account (affects \( \hat{v} \))

the exponential relation above is an approximate one,

\( \rho_0 \) and \( H_0 \) are determined from the experimental data so as to fit the lower end of the tether well.
Program

written in FORTRAN IV to run on AMDAHL or IBM mainframe.

Core memory required - 512K

Software needed - IMSL

Inputs - orbital elements.
  semi-major and semi-minor axes of the earth
  air density parameters and drag coefficients
  tether parameters (density, diameter, Young's modulus,
  damping coefficient).
  subsatellite parameters (mass, area of cross-section)
  initial state vectors \( \mathbf{x}(0) \)
  deployment (retrieval) parameters
  control gains for rotations and vibrations

Outputs - state vector \( \mathbf{x}(\theta) \)
  length, length rate
SIMULATION OF DEPLOYMENT DYNAMICS

Fig. 5 Typical deployment dynamics (rotations and in-plane first modal co-ordinate).
CONTROLLED RETRIEVAL (a)
Length, Length Rate

\[ \eta' = \dot{\eta} \quad \text{(EXPONENTIAL)} \]

\[ L' = \text{const.} \quad \text{(UNIFORM)} \]

\[ \eta' = \eta'_0 (1 - 5 \alpha' - 18 \gamma' \omega' + 1.8 \omega_c (C_1 - C_{15}) - 5 \times 10^3 (B_1^L - B_1^L(0))) \]

FIG 1. A-F) DYNAMICAL RESPONSE DURING RETRIEVAL FROM 100 KM USING THRUSTERS FOLLOWED BY LENGTH CHANGE CONTROL LAW
CONTROLLED RETRIEVAL (b)

\[ \alpha(0) = 15^\circ \]

\[ \gamma(0) = 1^\circ \]

(B) VARIATION OF THE ROTATIONAL MOTION
\( C_1(0) = 0.47 \times 10^{-2} \)

\( C_2(0) = -0.19 \times 10^{-3} \)

(C) BEHAVIOUR OF LONGITUDINAL GENERALIZED CO-ORDINATES
CONTROLLED RETRIEVAL

(d)

Transverse Vibration
Out-of-plane

$A_1(0) = 0.5 \times 10^{-4}$

$A_2(0) = -0.5 \times 10^{-4}$

(D) Behaviour of Out-of-Plane Transverse Modal Co-ordinates

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CONTROLLED RETRIEVAL

\( B_1(0) = -1.6 \times 10^{-3} \)

\( B_2(0) = 0.48 \times 10^{-3} \)

Transverse Vibration
Inplane

\( \text{(E) Behaviour of Inplane Transverse Modal Co-ordinates} \)
CONTROLLED RETRIEVAL

\[ T_\alpha (N) \]

\[ T_\gamma (N) \]

KO=0.0 FOR L>5 KM

Thrusts

ORBITS
(F) VARIATION OF THRUSTS
Fig. 14. Retrieval dynamics using solely thruster control (Xu et al. [1984])

(a) Variation of length and rotations.
(b) Behaviour of longitudinal generalized co-ordinates.
(c) Behaviour of transverse modal co-ordinates.
(d) Variation of thrusts.
(e) Variation of tension.
STRENGTHS AND LIMITATIONS

STRENGTHS

° SIMULATION OF GENERAL DYNAMICS INCLUDING TRANSVERSE AND LONGITUDINAL VIBRATIONS OF THE TETHER TREATED AS A CONTINUUM.

° CAN HANDLE DYNAMICS DURING CHANGE OF LENGTH QUITE RIGOROUSLY. PROVISIONS FOR CONTROL OF RETRIEVAL DYNAMICS.

LIMITATIONS

° ROTATIONS OF THE SUBSATELLITE ARE NOT INCLUDED.

° SOME ENVIRONMENTAL FORCES ARE NOT INCLUDED.
FUTURE PLANS

- EXTENSION TO SPACE STATION APPLICATIONS

- SIMULATION ON MICROCOMPUTERS

- SIMULATION OF VIBRATION CONTROL LAWS

- MULTIPLE TETHER SIMULATION
Martin Marietta Simulation
HIFITS (Hi-Fidelity-Tethered-Satellite) Simulation

Howard Flanders
Martin Marietta Denver Aerospace
0  MARTIN MARIETTA HAS SEVERAL PROGRAMS AND SIMULATIONS

  0  POCC  (DEPLOY, RETRIEVAL, TRAJECTORY PROFILE DESIGN)
  0  TSSERO  (RELATIVE MOTION AND CONTROL LAW)
  0  SIM6  (SES VERIFICATION)
  0  MODEL1B  (PRELIMINARY SYSTEM DESIGN AND VERIFICATION)
  0  HIFITS  (HI-FIDELITY-TETHERED-SATELLITE) SIMULATION
  0  ETC

0  PURPOSE OF HIFITS

  0  SUPPORT THE TSS DEPLOYER PROGRAM BY PROVIDING A HIGH-FIDELITY SIMULATION TO

    -  VERIFY CONTROL LAWS
    -  PROVIDE DESIGN INFORMATION
    -  SUPPORT SYSTEM ANALYSIS STUDIES
COORDINATE SYSTEMS AND TRANSFORMATIONS

INERTIAL SYSTEM \((\mathbf{T}, \mathbf{J}, \mathbf{K})^T\) ASSUMED EARTH CENTERED

LOCAL VERTICAL LOCAL HORIZONTAL \((\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3)^T\) \((\alpha, \beta)\)

\[
\begin{bmatrix}
\mathbf{e}_1 \\
\mathbf{e}_2 \\
\mathbf{e}_3
\end{bmatrix} =
\begin{bmatrix}
1 & 0 & 0 \\
0 & C\beta & S\beta \\
0 & -S\beta & C\beta
\end{bmatrix}
\begin{bmatrix}
C\alpha & 0 & -S\alpha \\
0 & 1 & 0 \\
S\alpha & 0 & C\alpha
\end{bmatrix}
\begin{bmatrix}
\mathbf{I} \\
\mathbf{J} \\
\mathbf{K}
\end{bmatrix}
\]

LIBRATION SYSTEM \((\mathbf{T}, \mathbf{J}, \mathbf{K})\) \((\theta, \phi)\)

\[
\begin{bmatrix}
\mathbf{T} \\
\mathbf{J} \\
\mathbf{K}
\end{bmatrix} =
\begin{bmatrix}
1 & 0 & 0 \\
0 & C\phi & S\phi \\
0 & -S\phi & C\phi
\end{bmatrix}
\begin{bmatrix}
C\theta & 0 & -S\theta \\
0 & 1 & 0 \\
S\theta & 0 & C\theta
\end{bmatrix}
\begin{bmatrix}
\mathbf{e}_1 \\
\mathbf{e}_2 \\
\mathbf{e}_3
\end{bmatrix}
\]

BODY ONE & TWO SYSTEMS \((\mathbf{T}_1, \mathbf{J}_1, \mathbf{K}_1)\) \((\theta_i, \phi_i, \psi_i)\) EULER ANGLES

\[
\begin{bmatrix}
\mathbf{T}_1 \\
\mathbf{J}_1 \\
\mathbf{K}_1
\end{bmatrix} =
\begin{bmatrix}
C\psi_i & S\psi_i & 0 \\
-S\psi_i & C\psi_i & 0 \\
0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
1 & 0 & 0 \\
0 & C\phi_i & S\phi_i \\
0 & -S\phi_i & C\phi_i
\end{bmatrix}
\begin{bmatrix}
C\theta_i & 0 & -S\theta_i \\
0 & 1 & 0 \\
S\theta_i & 0 & C\theta_i
\end{bmatrix}
\begin{bmatrix}
\mathbf{T} \\
\mathbf{J} \\
\mathbf{K}
\end{bmatrix}
\]
COORDINATE SYSTEMS AND TRANSFORMATIONS (CONTINUED)

TETHER SYSTEM \( (i_T, j_T, k_T)^T (\theta_T, \phi_T) \)

\[
\begin{bmatrix}
  \tilde{i}_T \\
  \tilde{j}_T \\
  \tilde{k}_T
\end{bmatrix} =
\begin{bmatrix}
  1 & 0 & 0 \\
  0 & C\phi_T & S\phi_T \\
  0 & -S\phi_T & C\phi_T
\end{bmatrix}
\begin{bmatrix}
  C\theta_T & 0 & -S\theta_T \\
  0 & 1 & 0 \\
  S\theta_T & 0 & C\theta_T
\end{bmatrix}
\begin{bmatrix}
  \tilde{i} \\
  \tilde{j} \\
  \tilde{k}
\end{bmatrix}
\]
HIFITS SIMULATION - SUMMARY OF EQUILIBRIUM EQUATIONS

1) $\ddot{x}_1 \vec{e}_3 + 2\dot{x}_1 \ddot{\omega} \vec{x}_3 + x_1 \dddot{x}(\ddot{\omega} \vec{x}_3) + \frac{\mu}{x_1^2} \dddot{\vec{e}}_3 - \frac{\vec{r}_1}{m_1} = \vec{0}$ (ORBITER TRANSLATION)

2) $\dddot{x} + 2\dot{\omega} \ddot{x} + \dot{\omega} \dot{x} + \omega \vec{x}(\ddot{\omega} \vec{x}) - \vec{A} = \vec{0}$ (SAT. TRANSLATION REL TO ORBITER)

3) $\vec{J}_1 \cdot \dddot{\omega} + \vec{\omega} \times (\vec{J}_1 \cdot \ddot{\omega}) - \frac{\mu}{x_1^2} \vec{x}_1 \times (\ddot{\vec{J}}_1 \times \vec{x}_1) - \vec{Q}_1 = \vec{0}$ (ORBITER ROTATION)

4) $\vec{J}_2 \cdot \dddot{\omega} + \vec{\omega} \times (\vec{J}_2 \cdot \ddot{\omega}) - \frac{\mu}{x_2^2} \vec{x}_2 \times (\ddot{\vec{J}}_2 \times \vec{x}_2) - \vec{Q}_2 = \vec{0}$ (SATELLITE ROTATION)

5) $\dddot{\vec{n}} + \dddot{\omega} \times \vec{n} + 2\omega \ddot{\omega} \times \vec{n} + \omega^2 \vec{x}(\ddot{\omega} \times \vec{n})$

\[+ (1-\xi) \dddot{r}_a + \dddot{r}_b + \frac{\mu}{r_3^3} [(1-\xi) \dddot{r}_a + \dddot{r}_b + \dddot{\vec{n}}] - \frac{\vec{f}}{m} = \vec{0} \quad \text{(TETHER RE. MOTION)}

WITH

\[\vec{A} = \left(\frac{\dddot{\vec{J}}_1}{x_1^2} \frac{\dddot{\vec{J}}_2}{x_2^2} \right) - \frac{\dddot{r}_2}{m_2} - \frac{\dddot{r}_1}{m_1}

\[
\dddot{x}_2 = \dddot{x}_1 + \dddot{\omega} \vec{k}, \quad x_2^2 = x_1^2 + \dot{\omega}^2 + 2\omega \dot{\omega} \vec{k} \cdot \vec{k}
\]
WHERE

\( x_1 \) ... MAGNITUDE OF POSITION VECTOR FROM CENTER OF EARTH TO C.M. OF ORBITER
\( e_3 \) ... UNIT VECTOR IN RADIAL DIRECTION THRU C.M. OF ORBITER
\( \omega_{k} \) ... ANGULAR VELOCITY OF NADIR FRAME
\( r_{k} \) ... MAGNITUDE OF SEPARATION DISTANCE BETWEEN C.M. OF ORBITER AND C.M. OF SAT.
\( u_{k} \) ... UNIT VECTOR ALONG DIRECTION FROM C.M. OF ORBITER TO C.M. OF SATELLITE
\( \dot{\Omega} \) ... ANGULAR VELOCITY OF LIBRATION SYSTEM
\( j_{1} \) ... INERTIA DYADIC OF ORBITER
\( \dot{\omega}_{f} \) ... ANGULAR VELOCITY OF ORBITER FRAME
\( g \) ... EARTH GRAVITATION CONSTANT
\( Q_{1} \) ... MOMENT OF APPLIED FORCES ACTING ON ORBITER ABOUT C.M. OF ORBITER
\( Q_{2} \) ... MOMENT OF APPLIED FORCES ACTING ON SATELLITE ABOUT C.M. OF SATELLITE
\( x_{2} \) ... MAGNITUDE OF POSITION VECTOR FROM CENTER OF EARTH TO C.M. OF SATELLITE
\( \delta_{T} \) ... TETHER RELATIVE DISPLACEMENT
\( \beta_{T} \) ... ANGULAR VELOCITY OF TETHER SYSTEM
\( r_{a} \) ... POSITION VECTOR LOCATING THE BOOM TIP ON THE ORBITER
\( r_{b} \) ... POSITION VECTOR LOCATING THE TETHER ATTACH POINT TO THE SATELLITE
\( f \) ... LOCAL TETHER TENSION VECTOR
\( f_{1} \) ... APPLIED FORCES ON ORBITER
\( f_{2} \) ... APPLIED FORCES ON SATELLITE
ABSTRACT DIFFERENTIAL EQUATIONS OF MOTION:

\[
\begin{align*}
\mathbf{y} &= \begin{bmatrix} \alpha \\ \beta \\ \chi_1 \\ \theta \\ \phi \\ \xi \\ \theta_1 \\ \psi_1 \\ \theta_2 \\ \phi_2 \\ \psi_2 \\ \xi_\theta \\ \{q_x\} \\ \{q_y\} \\ \{q_z\} \end{bmatrix} \\
&= \begin{bmatrix}
\text{ORBITER POSITION} \\
\text{SPHERICAL - POLAR - COORDINATES} \\
\text{LOCATING ORBITER MASS CENTER IN INERTIAL FRAME} \\
\text{SATELLITE - POSITION} \\
\text{SPHERICAL - POLAR - COORDINATES} \\
\text{LOCATING SATELLITE MASS CENTER IN "NADIR" FRAME} \\
\text{ORBITER ATTITUDE} \\
\text{PITCH, ROLL, YAW EULER ANGLES} \\
\text{DESCRIBE ORBITER ATTITUDE REL. TO INERTIAL FRAME} \\
\text{SATELLITE ATTITUDE} \\
\text{PITCH, ROLL, YAW EULER ANGLES} \\
\text{DESCRIBE SATELLITE ATTITUDE REL. TO INERTIAL FRAME} \\
\text{SPOOLED-OFF TETHER LENGTH} \\
\text{GENERALIZED COORDINATES DEFINING} \\
\text{LOCAL X COMPONENT OF RELATIVE DISPLACEMENT} \\
\text{GENERALIZED COORDINATES DEFINING} \\
\text{LOCAL Y COMPONENT OF RELATIVE DISPLACEMENT} \\
\text{GENERALIZED COORDINATES DEFINING} \\
\text{LOCAL Z COMPONENT OF RELATIVE DISPLACEMENT} 
\end{align*}
\]
TETHER MODEL

0 20 EQUAL MASS BEADS
  (FIXED NUMBER OF VARIABLE MASS BEADS)

0 INERTIA, INTERNAL, AND ENVIRONMENTAL FORCES WRITTEN FOR EACH BEAD

0 UP TO 8 SINE SHAPE FUNCTIONS ASSUMED IN EACH DIRECTION

\[ \{n\}^T = [n_{1X} \ n_{1Y} \ n_{1Z} \ldots n_{20X} \ n_{20Y} \ n_{20Z}] \]

\[ n_X = \sum_{i=1}^{m} \sin \frac{\pi X}{x} \ q_i \ \{n\} = [T]\{q\} \]

(AERO
GEOMAGNETIC
GRAVITY)
ENVIRONMENTAL MODELS

1) GEOMAGNETIC FIELD: NASA TM 82478 (JAN 1983)

2) ATMOSPHERE: MSFC FA31 (83-159) STANDARD ATMOSPHERE MODEL (DEC 1983)

3) GRAVITY: (J2, J3, J4, J22) MSC/MPAD, 68-FM47-83
SYSTEM MODELS

0  TSSERVO MODEL
0  ORBITER DAP
0  COMMAND PROFILES FROM POLYNOMIALS
0  LIBRATION CONTROL
HIFITS SIMULATION (ROAD MAP)

IFLWIR (TETHER OR WIRE FLAG)

-1 ... STATIC STRETCH
0 ... ONE DYNAMIC ELASTIC STRETCH FREEDOM
1 ... UP TO 8 LONGITUDINAL AND 8 TRANSVERSE IN EACH DIRECTION MODES

IFLORA (ORBITER ATTITUDE FLAG)

$\delta$

0 ... ORBITER ATTITUDE LOCKED TO LIBRATION MOTION
1 ... ORBITER ATTITUDE FREE

IFLSAA (SATELLITE ATTITUDE FLAG)

0 ... SATELLITE ATTITUDE LOCKED TO LIBRATION MOTION
1 ... SATELLITE ATTITUDE FREE
**DEFINITION OF SYMBOLS**

<table>
<thead>
<tr>
<th>AL</th>
<th>... RANGE</th>
<th>PH2</th>
<th>... SATELLITE ROLL EULER ANGLE</th>
</tr>
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<tbody>
<tr>
<td>ALD</td>
<td>... RANGE RATE</td>
<td>PH2D</td>
<td>... TIME RATE OF CHANGE OF PH2</td>
</tr>
<tr>
<td>TH</td>
<td>... INPLANE LIBRATION ANGLE</td>
<td>PS1</td>
<td>... YAW ANGLE OF ORBITER</td>
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<tr>
<td>THD</td>
<td>... INPLANE LIBRATION ANGLE RATE</td>
<td>PS1D</td>
<td>... TIME RATE OF CHANGE OF PS1</td>
</tr>
<tr>
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<td>... OUT-OF-PLANE LIBRATION ANGLE</td>
<td>PS2</td>
<td>... SATELLITE YAW ANGLE</td>
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<tr>
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<td>... OUT-OF-PLANE LIBRATION ANGLE RATE</td>
<td>PS2D</td>
<td>... TIME RATE OF CHANGE OF PS2</td>
</tr>
<tr>
<td>TH1</td>
<td>... ORBITER PITCH EULER ANGLE</td>
<td>FSTAT</td>
<td>... STATIC TETHER TENSION AT ORBITER</td>
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<tr>
<td>TH1D</td>
<td>... TIME RATE OF CHANGE OF TH1</td>
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</tr>
<tr>
<td>TH2</td>
<td>... SATELLITE PITCH EULER ANGLE</td>
<td>FC</td>
<td>... TENSION COMMAND</td>
</tr>
<tr>
<td>TH2D</td>
<td>... TIME RATE OF CHANGE OF TH2</td>
<td>F</td>
<td>... TENSION</td>
</tr>
<tr>
<td>PH1</td>
<td>... ROLL EULER ANGLE OF ORBITER</td>
<td>DF</td>
<td>... TENSION ERROR</td>
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<tr>
<td>PH1D</td>
<td>... TIME RATE OF CHANGE OF PH1</td>
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LIMITATIONS

0 MACHINE TIME
0 NO MAN-IN-THE-LOOP

VALIDATION METHOD

0 HAND CHECKS WITH SIMPLE CASE
0 COMPARE WITH OTHER SIMULATIONS BOTH IN-HOUSE AND OUT-OF-HOUSE

ADDITIONAL WORK

0 UPDATE CONTROL SYSTEMS
0 UPDATE CLOSE-IN GEOMETRY
0 STUDY CLOSE-IN DEPLOYMENT/RETRIEVAL
The Tethered Satellite System on the Systems Engineering Simulator

Ron Humble
Lockheed Engineering and
Management Services Company, Inc.
The Tethered Satellite System
on the
Systems Engineering Simulator

by

Ron Humble
INTRODUCTION

The purpose of this presentation is to discuss the tethered satellite system (TSS) portion of the Systems Engineering Simulator (SES). The SES is a real time simulator, used at the Johnson Space Center (JSC), to evaluate space shuttle operations when there is a man in the control loop. The "on-orbit" portion of the SES consists mainly of a mock up of the shuttle aft cockpit, complete with functional controls. The operator can "fly" the orbiter using the cockpit controls and can respond to simulated external scenes and cockpit displays. Other portions of the SES include a front cockpit mock up for ascent/descent simulations and a manned maneuvering unit (MMU) for extravehicular activity (EVA) simulations.

DESCRIPTION

The on-orbit SES simulates the dynamical interactions between the shuttle orbiter, the MMU, and one other payload. (This capability will be increased to 4-6 individual payloads in the near future.) All of the TSS simulation is controlled from a mock up of the shuttle orbiter aft cockpit. This cockpit is complete with hand translational and rotational controllers, digital autopilot (DAP) console, radar console, remote manipulator system (RMS) console, and the generic CRT.
consoles/keyboards (MCDS). Status of the tethered satellite can be monitored in four ways:

1.) with simulated visual scenes out of the port rear and both overhead windows;
2.) with simulated camera scenes generated on television monitors;
3.) with simulated numerical data from the multifunction CRT display system (MCDS) display; and
4.) with simulated shuttle radar data.

Manipulation of the satellite control system is done via the MCDS keyboard.

Certain parameters and variables can be output, during a simulation run, to both a magnetic tape and an hardcopy printer. The tape may then be post processed to generate digital tables or plots.

During a simulation run, system faults (ie. an RCS jet failure) can be generated via an operator's console. This console will allow faults to be input to either the orbiter, the satellite, or the winch system.
TETHERED SATELLITE SYSTEM ON SES
SYSTEM MODELS

The TSS system is modeled as two free flying vehicles each with six degrees of freedom, constrained by an extensible tether with an additional degree of freedom. The vehicle translational equations include three second order differential equations containing the M50 spherical coordinates.

\[
\text{force} = \text{mass} \times \text{translational acceleration} \ (F = mA) - 3 \text{ dof}
\]

The vehicle rotational equations include three first order differential equations containing the body Euler angle rates and four first order differential equations containing inertial quaternions. The Euler angle accelerations are integrated to give Euler rates,

\[
\text{moment} = \text{inertia} \times \text{rotational acceleration} \ (M = I \alpha) - 3 \text{ dof}
\]

the Euler rates are then transformed into quaternion rates. The quaternion rates are integrated to give inertial quaternions which are then transformed back to Euler angles for output.

Shuttle equations of motion:

\[
A_{\text{shuttle}} = \frac{1}{m_{\text{shuttle}}} (F_{\text{gravity}} + F_{\text{aero}} + F_{\text{RCS}} + F_{\text{tether}} + F_{\text{payload}} + F_{\text{RCS plume}})
\]

\[
\alpha_{\text{shuttle}} = L^{-1} (M_{\text{gravity grad.}} + M_{\text{aero}} + M_{\text{RCS}} + M_{\text{tether}} + M_{\text{payload}} + M_{\text{RCS plume}})
\]
Satellite equations of motion:

\[ \mathbf{A}_{\text{sat}} = \left( 1/m_{\text{sat}} \right) \left( F_{\text{gravity}} + F_{\text{aero}} + F_{\text{APS}} + F_{\text{tether}} + F_{\text{shuttle}} + F_{\text{RCS plume}} \right) \]

\[ \alpha_{\text{sat}} = \mathbf{I}^{-1} \left( M_{\text{gravity grad.}} + M_{\text{aero}} + M_{\text{RCS}} + M_{\text{tether}} + M_{\text{shuttle}} + M_{\text{RCS plume}} \right) \]

The tether, as modeled in the SES, is massless but does include stretch. The necessity to keep track of both the stretched and unstretched length of the tether introduces one extra degree of freedom. The tether tension calculated during the previous time step is input to the natural length differential equation. This equation is then integrated twice to get the new natural length. The tether stretch (\( \Delta l \)) is calculated by subtracting the natural tether length from the difference between the positions of the orbiter and satellite. The tension is then calculated, by using the stretch value, for use on the next time step. In all, there are 13 dof.

Tether equations of motion:

- massless, 1/2 of the tether mass is placed at the orbiter CG and the other half at the satellite CG
- the tether mass lumped at the two body CGs is assumed constant
- the length (\( \Delta l \)) acceleration is integrated:

\[ \mathbf{A}_l = \left( R_R/J_T \right) \left( -T_D + R_R F_T \right) \]

\[ F_T = k \Delta l + \beta k m^* \mathbf{v}_{\Delta l} \]

where:

\[ R_R \] - radius of tether reel

\[ J_T \] - moment of inertia of tether reel

\[ T_D \] - deployer motor torque
k - spring constant (AE/l)
β - material viscous damping term
m* - generalized system mass
V_Δl - time rate of change of stretch
A - tether material cross sectional area
E - Young's modulus of the tether material

Orbiter systems:

The shuttle orbiter systems are an integral part of SES. These include systems such as the digital autopilot, star tracker, navigation, etc. All of these systems have been verified by other simulations and experimental data. Details of these systems are best found in the shuttle data manuals.

Rendezvous Radar:

The SES has a model of the orbiter's Hughes rendezvous radar. This model takes the satellite and orbiter state data and produces output similar to what is available on the orbiter:

1.) range / range rate
2.) inplane / outplane libration angles and rates
3.) line of sight angles and rates
4.) etc.

These data have bias, drift, and random errors introduced to them similar to those which occur on the on-board Hughes radar system.
At present, the TSS satellite is modeled as a point source with perfect reflectance etc. Some preliminary tests show that this may not be a good approximation and so some future modifications to the model may be required.

Tethered satellite systems:

The satellite, on the end of the tether, has three major dynamical components as far as the SES is concerned:

1.) mass properties - as previously discussed, the satellite is modeled as a free flying rigid body;
2.) attitude measurement - the attitude measurements, used for control purposes, are assumed to be perfect (ie. no errors are introduced); and
3.) auxilliary propulsion system - the APS is modeled as perfectly aligned cold (N₂) reaction jets with step force pulses and perfect duration. The libration control jets are controlled by the orbiter pilot via the MCDS keyboard while the satellite yaw is controled by an onboard control system. The APS fuel usage is monitored during usage.
ENVIRONMENTAL MODELS

Gravity model:

The Earth's gravitational field may be modeled, in the SES, in two different ways, depending on the setting of a digital switch:

1.) spherical Earth (Newtonian gravitation); or
2.) oblate spheroid including the first three zonal harmonics (J2, J3, J4) and the main Tesseral harmonic (C22, S22)

Gravity gradient model:

When an assymetrical body, having one moment of inertia less than the other two, is placed in a non-uniform (non-constant) gravitational field, it experiences a torque tending to align the axis of least inertia with the direction of the gravity gradient. SES is capable of determining gravity gradient torques on the orbiter and any free flying payloads. These torques may be "turned on" or "turned off" via a digital switch. The TSS satellite has essentially symmetrical mass properties and thus, gravity gradient torques are negligible.

Earth atmospheric density model:

The atmospheric density model used on the SES is the "Bab-Mueller" model. This is an approximation to the high fidelity Jacchia atmospheric density standard. (See ref. - Jacchia, L.G., "New Static Models of the
The Bab-Mueller model was chosen because the calculated densities are very close to the densities given by the Jacchia model but the calculation response is much quicker than for the Jacchia model. The quick response time is a fundamental requirement of real time simulations. The Bab-Mueller model is described in ref. - (Mueller, A.C., "Atmospheric Density Models", Analytical and Computational Mathematics Inc. Technical Report ACM-TR-106, June 1977). The classical Bab-Mueller coefficients, as given in the referenced report, are run through a Jacchia calibration algorithm in the initialization to improve fidelity. Basically the density ($\rho$) is calculated by:

$$\rho = \rho_o e^{(A + B + C)}$$

where:

- $\rho_o$ - reference density
- A - night time vertical profile exponent
- B - diurnal effect
- C - seasonal latitude coefficient

As in the Jacchia model, the sun's orbital position and the effect of the solar cycle are considered. In addition daytime/nighttime effects are considered.
Aerodynamic drag:

The orbiter drag model is a high fidelity model including the orbiter attitude effects on drag coefficient and reference area. The drag, pitching, and yawing coefficients are obtained by a table look up and linear interpolation routine. The roll moment coefficient for the orbiter is very small and is considered negligible.

The tethered satellite is modeled with a constant drag coefficient and reference area. This is because the tethered satellite has a very symmetric surface profile and the atmospheric drag is independent of attitude. (ie. a constant coefficient and reference area). Satellite attitude torques caused by drag are not modeled.

Tether drag is not modeled.

The atmospheric drag may be "turned on or off" with a digital switch.

RCS plume impingement:

During reaction control system operation, the RCS plume may impinge upon the surfaces of both the orbiter and the satellite. This can cause significant attitude and translational perturbations during proximity operations.
The plume impingement effects on the orbiter are modeled by curve fitting empirical data derived from test firings on an orbiter (i.e., curve fit constants were developed empirically). These data consider stationary orbiter surfaces as well as moving aerosurfaces.

The satellite plume impingement model can be made up of several elements. These elements may be flat, cylindrical, or spherical. If the target is more than 5000 ft. from the orbiter CG, no plume impingement effects are calculated. The plume flow is modeled as emanating radially from a point source, in a direction normal to the nozzle exit plane. The plume force/moment is calculated as a function of the jet type, distance from the source, magnitude of the thrust pulse, and the impingement surface geometry. Details of the jet selection geometry and layout are contained in the Shuttle Operational Data Book. Details of the plume impingement model are contained in the SES definition document.

The TSS satellite is modeled as a single spherical plate element.
LIMITATIONS

The TSS simulation has the following limitations:

- the tether is assumed massless
- the maximum tether length is 1200 meters
- the reel motor is not modeled
  ie. Reel torque = commanded reel torque
- external forces do not act on the tether directly
  ie. there is no tether drag
- a collision between the satellite and the docking ring freezes the sim
  ie. collision dynamics are not modeled

VALIDATION

The TSS simulation on the SES was validated by comparing run data with data generated by STOCS. STOCS is a non-real time simulation, using the TOSS software developed at JSC, with an orbiter digital auto pilot (DAP) modeled.

All of the shuttle systems were validated and are kept valid as an ongoing task on the SES. This is done by comparing run data with other "off line" simulation data and flight results.
FUTURE ENHANCEMENTS

The following enhancements are being planned:

1.) upgrading the present reel controller to present MMC standard
2.) modifying the MCDS CRT display to enhance useability
3.) implementing a tether cutting device model

The following are being considered as possible enhancements:

1.) modeling collision dynamics between the satellite and the docking ring
2.) modeling the reel dynamics with higher fidelity
GTOSS
Generalized Tethered Object Simulation System

David D. Lang
David D. Lang Associates
The GTOSS software system consists of approximately 410 subroutines representing 40,000 lines contained in 120 files. Code is constrained to a highly portable subset of Fortran 77, with over 700 pages of documentation describing user operation, equation derivation, and system software design. GTOSS runs on most computers (including: the Macintosh and PC's). GTOSS has been developed under the direction of the Avionics Systems Division, Johnson Space Center, NASA.

GTOSS represents a tether analysis-complex best described by addressing its family of modules, designed to be more or less tightly associated as a cooperative whole.

Tether dynamics: TOSS

TOSS is a portable software sub-system specifically designed to be introduced into the environment of any existing vehicle dynamics simulation to add the capability of simulating multiple interacting objects (via multiple tethers). These objects may interact with each other as well as with the vehicle into whose environment TOSS has been introduced. TOSS is incorporated by adherence to a straightforward set of interface rules set forth in the TOSS Interface Control Document. Without small motion assumption, and with complete generality, TOSS solves the tether dynamics problem relative to a reference point state defined for it by the host simulation.

Input is designed for easy data identification and entry, as well as to expedite parametric studies. Extensive initialization options (such as stabilized gravity gradient start-up, Euler angle type selection, etc.) allow user-friendly run setup.

General tethered system analysis: GTOSS

GTOSS is a stand-alone tethered system analysis program, representing an example of TOSS having been married to a host simulation. In order to verify the TOSS design concept and exercise the TOSS ICD ground rules, it was necessary to create a fully representative, yet easily managed simulation into whose environment TOSS could be incorporated. The resulting union was called GTOSS and has the properties of, and can be viewed as, a system tailored to the purpose of examining dynamic behavior of general tethered object configurations (space stations, constellations, etc). By contrast, TOSS has also been integrated into a Shuttle simulation (to study TSS), with the resulting association exhibiting (and rightfully so) the complexity and specificity of an Orbiter vehicle. The GTOSS host simulation represents a 6 DOF object with 8 tether attachment points. GTOSS has an executable code size of about 350k Bytes.
Input and initialization for GTOSS (as an entity separate from TOSS) is similar to that of TOSS. GTOSS provides output for itself as well as TOSS by invoking RTOSS (see below) to generate a Results Data Base to archive solution results for display post processing.

Solution-result archiving: RTOSS

RTOSS is the Results Data Base (RDB) Sub-system designed to archive TOSS simulation results for future display processing. While RTOSS was designed primarily to capture TOSS data, it offers a Wild-Card file which can be used by any users to capture data of their choice. For instance, GTOSS takes advantage of this feature to capture its host simulation data to an RDB for display post processing. The modular design of RTOSS requires minimal calls (from the host simulation) to invoke the creation and population of an RDB (for instance in GTOSS, this act is transparent to the user). At the end of a run, a set of files with unique names will have been created containing all pertinent time history data.

Also inherent in RTOSS are the routines which will extract data from the RDB and present it in a form most natural for display post-processing. These extraction routines insulate the user from structural knowledge of the RDB, thus rendering the user's display software invariant to future changes in RDB design. The RDB can be post-processed in a myriad of ways for engineering interpretation, as described below.

Simulation result display: DTOSS

DTOSS is the first of a growing family of display post processors designed to effectively utilize the RDB. DTOSS extracts data from the RDB for extensive multi-page printed time history displays. There are currently over 50 different display formats to choose from, each of which aggregates data selected to meet various display needs. Users are also invited to add new page formats to create output for specific needs.

Simulation result display: CTOSS

CTOSS is similar to DTOSS, but is designed to create ASCII plot files (as headers and data columns separated by delimiters). The same time history data formats provided by DTOSS (for printing) are available via CTOSS for plotting. In addition to time histories, repetitive tether shape snapshots can be taken in CTOSS. While the plot files are optimally configured for existing interactive graphics programs on the Macintosh computer, their plot format can be used on other PC's. Since CTOSS generates ASCII plot files (which are easily transported between different computers), it can be run on any mainframe to generate plot files for a PC or Macintosh.
Simulation result display: ITOSS

ITOSS differs from DTOSS and CTOS5 in two ways. First, it generates output display data designed for 3-D animated graphics display of simulation results. Second, its output is targeted specifically for the IMI graphics device (a large, high resolution, fast display device).

Simulation result display: GENERAL

The above RDB post-processors not only represent a significant existing display capability for GTOSS, but also function as convenient templates to spawn specialized display processors. For instance, hooks are clearly defined, and steps are documented to modify the existing DTOSS or CTOS5 to add new formats. These programs can also serve as a functional boilerplate structure for extracting RDB data to be post-processed in any fashion you wish.

B. COORDINATE SYSTEMS and DEGREES OF FREEDOM

Coordinate Systems

The following coordinate systems are used in TOSS:

A. TOSS Inertial Frame. The user is allowed to arbitrarily define this frame, however, its relationship to the planet-fixed frame must be explicitly stated in the standard TOSS routine invoked to transform inertial frame vector components to the planet-fixed frame. This routine (and its inverse-routine) can describe an arbitrarily complex relationship between inertial and planet-fixed frames. As delivered, the TOSS inertial frame is defined as one which is aligned with the planet-fixed frame at zero simulation time.

B. Planet-fixed Frame. This is typically an earth-fixed frame, and also the one in which all planetary environment calculations are defined. The user is allowed to arbitrarily define this frame, however, its relationship to the inertial frame must be explicitly stated in the TOSS routine which transforms planet frame vector components to the inertial frame, and environment calculations must be consistent with its definition. As delivered, GTOSS environment routines assume an earth-fixed frame, the +X axis of which is presumed to pass through the Greenwich meridian, +Z axis through the geocentric North pole.

C. Topocentric Frame. This is a frame aligned along local spherical longitude, latitude, and radius vector to the planet center. It is used for state initialization options and result interpretation.

D. Orbital Frame. This frame is defined by the kinematic state of a point, and is similar to the topocentric frame except the local longitude/latitude vectors are aligned to the current plane of Keplerian motion. Any TOSS entity can have an associated orbital frame, which is used for state initialization options and result interpretation.
E. **Body Axis Frame.** A body-fixed frame (and a body station reference point) is associated with each 6 DOF object. The frame and body station are arbitrary, but must be consistently used for defining all body attributes (CG location; tether attach-points; aerodynamic reference point; etc). This frame is the reference for body attitude interpretation.

F. **Tether Frame.** Each finite tether has its own tether frame. The X axis of this frame is aligned along the line of sight between the tether's attach points. The Z axis is orthogonal to the first, but lies *in the orbital plane* (of a preferred kinematic point). The Y axis is defined to complete the triad. This frame hosts the tether dynamics coordinate solution, and is used for both initialization options and result interpretation.

**Degrees of Freedom (DOF)**

Many of the DOF are modifiable via procedures included in the manuals. The nominally delivered GTOSS configuration provides:

1. Up to 9 bodies (each with up to 6 rigid-body DOF's).

2. A 3 DOF *particle dynamics option* to eliminate rotational dynamics overhead (for efficient study of overall topological behavior).

3. Concurrent simulation of up to 25 different tethers.

4. Tether inter-connection in *any conceivable fashion* at up to 8 attach-points per body. Each attach point is specified by its coordinates in an arbitrary body axis frame.

5. No practical constraints on tether/attach point connectivity.

6 Tether dynamics simulation as your mixed choice of:

- Massless models (ie. linear springs and dash pots)

- Modal Synthesis finite models (on a tether-by-tether, and axis-by-axis basis, up to 15 modal coordinates *per each* of 3 axes, with the evaluative resolution of each generalized force type specifiable at up to a maximum of 50 uniform spatial intervals). Tension is optionally evaluated either in terms of material strain, or Lagrangian multipliers.

- Point Synthesis (ie. Bead) type finite models (on a tether-by-tether basis, up to 48 collocation points *per* tether; with 3 DOF *per* point). Tether can break on tension limits, or be severed at multiple points.

7. Five *length-rate*, and five *tension-profile*, data-driven deployment scenarios:

8. Five *power-profile (amp-limited)* data-driven, power generation scenarios:

- All deployment and power scenarios can be arbitrarily assigned to any one or more tethers (a tether also has its own scenario *scaling factors*).
C. ENVIRONMENTAL MODELS

Planetary Environment

Subroutine calls to the planetary environment models are standardized within TOSS so that any level of environmental sophistication (you have available) can be easily incorporated into TOSS. Calling arguments to an environment model are: a sophistication-level flag; and, a position state in the planet-fixed (earth) frame. Vector results are returned in the planet-fixed frame. Currently delivered with GTOSS are the following:

- Gravitational Field Model: Earth; Inverse square central force field; also an Oblateness model with 2 anomaly terms.
- Globe Shape Geometry Model: Earth; Spherical globe.
- Atmospheric State Model: Earth; 1976 Standard earth density, speed of sound, and temperature model (3% accuracy to 1000 KM).
- Atmospheric Kinematics Model: Earth; Rotating atmosphere (with local wind perturbations currently zero).
- Magnetic Field Model: Earth; Tilted, shifted, vector dipole.
- Inertial Frame Model: Planet centered inertial point, with inertial frame aligned to planet-fixed frame at zero time.

Entity Attribute Environment

As delivered, all TOSS objects can experience simple aerodynamic drag. Tethers experience distributed aerodynamic lift and drag. Tethers carrying electric current experience electromagnetic forces.

TOSS is designed to facilitate incorporation of any degree of entity attribute simulation.

D. SYSTEM MODELS

The design philosophy of TOSS has been to provide a useful array of built-in system simulation features such as data driven scenarios for: control forces and moments, tether deployment, and electromagnetic power generation; as well as default aerodynamics options; etc. In addition, TOSS supports arbitrarily complex simulation of mass properties, control systems, aerodynamics, tether deployment, and power generation, etc. through its documented user-interface data structures, modularized code, and logical hooks which invite and assist in user modification.

Of course, when TOSS is incorporated into a host simulation, all the system models present in the host then function in the tether environment provided by TOSS.
E. LIMITATIONS

The following summarizes the currently known limitations to the use of GTOSS/TOSS (also see section below entitled: What should be done next).

a. Tether Frame Definition: The orientation of the Tether Frame is undefined if attach point line-of-sight becomes exactly perpendicular to an associated orbital plane. Near this state, frame orientation rates can become large, inducing large apparent rates of change of finite tether coordinates. This can be avoided in the Point synthesis model by integrating coordinates in the inertial frame (thus using the tether frame for interpretation only). To date, this has not been a practical restriction (due to the nature of engineering applications); if required, this restriction can be removed.

b. Modal Synthesis Finite Tether Model: This model is only valid during motions for which the distance between attach points is greater than the deployed tether length (a state of intrinsic tension). Furthermore, cases in which cyclic slack/taut states are a significant element of behavior may not be simulated well. In short, while the Modal synthesis model has some advantages, the Point synthesis (bead) model is significantly more robust and should be used to establish truth datum.

c. Stabilized Gravity Gradient Initialization: Currently this feature applies only to simple chains, limited to 3 objects and 2 tethers (as a mixture of massless and Point synthesis models). Multiple disconnected chains are allowed.

d. Certain Environmental Effects (for example, solar pressure effects) are not incorporated into TOSS.

F. VALIDATION METHODS

Validation of TOSS/GTOSS involves three distinct areas: Classical techniques (for those cases which are simple enough to be described by classical closed form solution); Comparative techniques (for those cases which defy classical substantiation); and Official techniques (to verify site installation and validate evolutionary changes to GTOSS).

Classical Verification

Official solution verification of TOSS has been accomplished from many directions. First, the simulation of the rigid body TOSS objects are verified via classically known solutions to Euler's rotational equations and Newton's law. Overall translational dynamics is further verified against classical Keplerian motion. Massless tether dynamics are verified against known gravity gradient as well as bolo type motions. Finite tether wave propagation and shape verification has also been accomplished against classical string theory.
Comparative Verification

Finite tether dynamics have been verified by comparing both the TOSS Modal Synthesis (MS) and Point Synthesis (PS, i.e., bead model) against each other and against a bead model independently developed by NASA JSC. These comparisons have addressed aerodynamic response, wave propagation, and slack/taut behavior. Electrodynamic finite tether response has been officially compared only internally to TOSS (between the MS and PS models).

There are at least 5 known, actively used, installations of GTOSS. When GTOSS is installed, users invariably compare its results to independent tether solutions previously verified by the installation. To date, there are no unexplained anomalies in result comparisons of this type.

Installation Verification

Delivered with GTOSS are 18 different input run decks (along with corresponding official results) which test all aspects of GTOSS. A user site compares the output of the newly installed GTOSS to these official results to verify site installation.

6. What Should Be Done Next

The following features are either planned, or recommended for future GTOSS/TOSS development:

1. Non-uniform point spacing for the Point Synthesis (PS) finite tether model: This will allow more efficient use of degrees of freedom for solving certain type finite tether problems.

2. Non-uniform material properties for the PS finite tether model: This will allow simulation of tethers which are purposely constructed of different materials as a function of length.

3. Auto-transition (on user defined criteria) from finite to massless to finite tether models: This will allow full mission simulation continuity without the large computation overhead associated with very short tethers.

4. Improved aero force model for finite tethers: The current model does not represent the consensus standard for distributed air loads on tethers

5. Flexible boom attachment simulation: This will allow a certain amount of attach structure flexibility without full involvement in simulating body flexibility of a TOSS object.

6. While the latest version of TOSS reflects an emission-end-sensitive plunging aspect of tether deployment (under small strain), the author does not admit to knowledge of an ultimate expression for deployment kinematics or dynamics. TOSS will be continually improved in this area as understanding is gained.
H. CASE STUDY EXAMPLES

Examples of Host/TOSS Integration

There are two different instances of TOSS having been successfully introduced into a host simulation environment: the first of these is GTOSS (described above); the second is called STOCS (Shuttle Tethered Object Control Simulation). The host simulation for STOCS is a full engineering fidelity flight control simulation of the Orbiter. STOCS is being used to perform mission verification tasks for the TSS experiment (under the responsibility of NASA JSC). STOCS is an excellent example of complex control systems (TSS on-board control) becoming associated with a TOSS object (which represents the TSS), as well as the introduction of specialized tether deployment systems (TSS deployer) into TOSS.

Examples of TOSS/GTOSS Application

TOSS has been used to study enumerable engineering applications of tethers. Some of these are:

1. TSS mission study (STOCS/TOSS, 2 rigid-bodies, both massless and bead model tethers).
2. A spinning, orbital station (6 rigid-bodies, 15 massless tethers).
3. Gravity gradient/Gyroscopic orienting spinning dumbell (2 particles, 1 massless tether).
4. Electrodynamic day/night power generation/orbital boost with tethered counter balance (3 particles, 1 bead model and 1 massless tether).
5. Real-time Shuttle Engineering Simulator (SES) verification (comparison runs made with both GTOSS and STOCS).
6. Planetary exploration maneuvers using a slingshot mechanism.
7. Electrodynamic pulse maneuvers (2 particles, 1 bead model tether).
8. Space Station docking devices (3 particles, 2 massless tethers).
9. Orbiting, spinning carousel (3 rigid-bodies, 1 massless, 1 modal synthesis, and 1 bead model tether).
10. Gravity gradient stabilized orbiting platform (3 rigid-bodies, 1 modal synthesis and 4 massless tethers).
11. Simulated bead model by chain-connecting 9 TOSS rigid-bodies with 8 massless tethers (for bead model verification study).
12. Comparative solution verification studies between GTOSS and an independent bead model simulation (wave propagation, aerodynamic response, symmetrical slack/taut gravity gradient behavior of a system exhibiting TSS physical properties).
GT OSS TERMINOLOGY

TOSS  Tethered Object Sub-System

GT OSS  Generalized Tethered Object Simulation System

RTOSS  Result Data Base sub-system

DTOSS  Display print RDB post-processor

CTOSS  Chart/Graphics RDB post-processor

IT OSS  Imi graphics RDB post-processor
No Orbital State Assumptions

No Small Motion Assumptions
GTOSS OPERATION

GTOSS Input file INDATA

Optional CRT interactive input
Optional CRT run Monitoring

Quicklook output file OUTDATA

Results Data Base file set run 1
Results Data Base file set run 2
Results Data Base file set run N

ARCHIVED RDB's

DTOSS Input file INDIS

Optional CRT interactive input
Optional CRT run Monitoring

Multi-Page Print Facility output file OUTDIS

(Similar RDB processing for CTOSS and ITOSS)

means ASCII file
DEGREES OF FREEDOM

9 Bodies (3 or 6 DOF)

25 Different Tethers

8 Tether attachment points per body

Totally General Connectivity

Tether models, your mixed choice of:

- Massless
- Modal Synthesis (3-D)
  Up to 15 Modal Coordinates (Legendre) per axis
  Up to 30 Generalized Force evaluation intervals
  Tension via strain or Lagrangian multipliers
- Point Synthesis (3-D)
  Up to 50 collocation points per tether
  3 DOF per collocation point
  Tether sever (on time) or break (on tension)

Data-driven, arbitrarily assignable scenarios

5 length-rate deployment scenarios
5 tension control deployment scenarios
5 power profile (amp limited) generation scenarios
INTEGRISTIC ENVIRONMENT

Input to all TOSS environment routines:

1. Fidelity level flag
2. Simulation Time
3. Position in Planet-fixed frame

- Inertial Frame Model EFTEI, EITEF
  To evaluate this environment, use standard routine

- Gravity Model GRAV

- Globe Shape Model GEOD

- Magnetic Field Model GAUSS

- Atmospheric State Model ATMOS

- Atmospheric Kinematics Model WINDS
CURRENT LIMITATIONS

Tether-Frame is Un-defined when line of attach points is 90 degrees out of the orbital plane.

Modal Synthesis model valid only for a state of intrinsic tension.

Modal Synthesis model dubious for cyclic slack-taut states.

Stabilized Gravity Gradient start-up valid only for simple chains of up to 3 bodies and 2 tethers (mixed bead and/or massless).
CLASSICAL VERIFICATION
- Keplarian/Newtonian behaviour
- Classical String Theory

COMPARATIVE VERIFICATION
Participants:
- JSC developed bead model
- GTOSS bead model
- GTOSS modal synthesis model

Study parameters:
- TSS Type Parameters
- Aero Response
- Transverse Wave Response
- Symmetrical, Un-forced Response

OFFICIAL VERIFICATION

UN-OFFICIAL VERIFICATION
TOSS/GTOSS APPLICATION

- TSS/Orbiter Compatibility (2 RB, 1 TH)
- Spinning, Orbital station (6 RB, 15 TH)
- Gravity gradient/Gyro dumbell (2 P, 1 TH)
- Day/Night electro pwr gen (3 P, 2 TH)
- SES verification (GTOSS/STOCS)
- Planetary exploration studies
- Electro pulse maneuvers (2 P, 1 th)
- Space Station docking device (3 P, 2 TH)
- Orbiting spinning carousel (3 RB, 3 TH)
- Gravity gradient platform (3 RB, 5 TH)
- 9 Body equivalent bead model (8 TH)
- Verification studies (20 - 50 beads)
WHIRLING DERVISH
FUTURE GTOSS PLANS

- Non-uniform Point Spacing
- Non-uniform Tether Properties
- Auto-phase transition
- Improved aero-force model
- Flexible boom simulation
- Improved deployment fidelity
- Expert, Friendly User interface
BEAD MODEL vs MODAL SYN AERO RESPONSE

BEAD MODEL WAVE PROPAGATION (48 Beads)

T = 0.0 sec; Shape = (1-cos) wave

T = 60.0 sec

T = 170.0 sec

Bead Positions
Tethered Satellites Simulation

John R. Glaese
Control Dynamics Company
CONTROL DYNAMICS COMPANY
HUNTSVILLE ALABAMA

TETHERED SATELLITES SIMULATION

Dr. John R. Glaese
- Introduction
- Simulation Overview / Assumptions
- Equations of Motion
- Force Models
- Sample Data
- Amiga Graphics Workstation
- Summary
Introduction

- Tethered Satellites Simulation (TSSIM)
  - Developed by ESA/ACM
  - Available by license from Software Applications, Inc.
  - Modifications and additions by Control Dynamics
  - Versions available on VAX, HP9000, IBM-XT, IBM-AT

- Amiga Graphics Workstation
  - Run time monitor/terminal mode
  - Post-run processor mode
  - "Walking" Plots
  - Animation
  - X vs. Y plots
Simulation Coordinate Systems

- Inertial Reference
  
  \[ X \rightarrow Y \rightarrow Z \quad \text{(Geographic North Pole)} \]

- Local Vertical
  
  \[ X \rightarrow \text{Y (Orbit Normal)} \rightarrow \text{Z (Local Vertical)} \]

- Special Local Vertical
  
  \[ X \rightarrow Z \rightarrow \text{Y (Walking Plots, Tether Shape)} \]
Partial Differential Equation (PDE)
Tethered Satellites Dynamics
Model

L – Deployed tether length

$L_T$ – Total tether length

\( \lambda \) – Mass point on tether measured from satellite
Tether Segment

Tension: \[ N(t) = \beta \left[ \left| \epsilon'(t) \right| - 1 \right] \frac{\epsilon'}{|\epsilon'|} ; \epsilon' = \frac{3r}{3t} ; |\epsilon'| = \sqrt{\epsilon'^2 - \epsilon^2} \]

Shear: \[ Q(t) = -\alpha \left[ \frac{1}{|\epsilon'|} \left( \frac{1}{|\epsilon'|} \right) \right]' - \alpha \left[ \frac{1}{|\epsilon'|} \left( \frac{\epsilon'}{|\epsilon'|} \right) \right] \frac{\epsilon'}{|\epsilon'|} \]

External Forces \( f(t) \)

1. Spherical Earth Gravity
2. Non-spherical Earth perturbations
3. Sun & Moon Gravity
4. Aerodynamic Drag
5. Solar Radiation pressure
Dynamics Equations of Motion

Boundary Conditions

\[ \mu \frac{dl}{dt} \ddot{t}(t) = N(t + dt) - N(t) + Q(t + dt) - Q(t) + f \, dt \]
\[ = \frac{dl}{dt} \frac{\partial}{\partial t} \left[ N(t) + Q(t) \right] + f \, dt \]
\[ \Rightarrow \quad \mu \ddot{t}(t) = \frac{\partial}{\partial t} \left[ N(t) + Q(t) \right] + f : \text{Interior tether point} \]

Boundary Conditions:

\[
\begin{align*}
    m_1 \ddot{\dot{r}}_1 &= F_1 + \left[ N(t) + Q(t) \right]_{t=0} \\
    m_2 \ddot{\dot{r}}_2 &= F_2 + \left[ \mu L^2 \ddot{t}(t) - N(t) - Q(t) \right]_{t=L} \\
    \text{for } \quad & = 0 \\
    \left( \dddot{r} - \frac{r^\prime}{|r^\prime|} \frac{\partial}{\partial t} |r^\prime| \right) \quad |r = 0 &= 0 \\
    \left( \dddot{r} - \frac{r^\prime}{|r^\prime|} \frac{\partial}{\partial t} |r^\prime| \right) \quad |r = L &= 0
\end{align*}
\]

End body dynamics

Bending moment must vanish at tether ends
DISCRETIZATION

Define normalized length $\xi = \frac{\ell}{L}$

Change to tether parameter from $\ell$.

\[ r (\ell, t) = \hat{r} (\xi, t) \]

\[ r' = \frac{\partial r}{\partial \ell} ; \hat{r}' = \frac{\partial \hat{r}}{\partial \xi} \]

\[ \ddot{r} = \left( \frac{\partial \hat{r}}{\partial t} \right) \left[ \text{const} = \left( \frac{\partial \hat{r}}{\partial t} \right) \xi \text{ const} + \xi \frac{\partial \hat{r}}{\partial \xi} \right] \text{ const} \]

\[ \dot{\xi} = -\xi \frac{\dot{L}}{L} \]

\[ \ddot{r} = \ddot{\hat{r}} + 2 \dot{\hat{r}} \dot{\xi} + \dddot{\hat{r}} \dot{\xi}^2 + \dddot{\hat{r}} \dddot{\xi} \]

\[ \dddot{\xi} = -\dddot{\xi} \left( \frac{\xi}{L} \right)^2 + 2 \dddot{\xi} \left( \frac{\xi}{L} \right)^2 \]
DISCRETIZATION

(Continued)

Let \( I = \hat{r}' \)
\[ V = \hat{r} \]

To discretize the spatial equations assume \( n \) uniformly distributed nodes including boundary nodes.

\[
\begin{align*}
\lambda_i &= \frac{i - 1}{n - 1} L \quad \Rightarrow \quad \Delta \lambda = \frac{L}{n - 1} \\
\xi_i &= \frac{i - 1}{n - 1} \quad \Rightarrow \quad \Delta \xi = \frac{1}{n - 1} \\
I_{i+1/2} &= \frac{\hat{r}_{i+1} - \hat{r}_i}{\Delta \xi} = (n - 1)(\hat{r}_{i+1} - \hat{r}_i) \\
I_i &= \frac{1}{2}(I_{i+1/2} + I_{i-1/2}) \\
W_{i+1/2} &= V_{i+1} - V_i
\end{align*}
\]
VISCOSOUS TETHER DAMPING

\[ \cdot N = \beta (|\mathbf{r}'| - 1) + Y \frac{\mathbf{r}' \cdot \dot{\mathbf{r}}'}{|\mathbf{r}'|} \]

\[ = \beta \left( \frac{|\mathbf{l}'|}{L} - 1 \right) + Y \frac{\mathbf{r}}{L^2 |\mathbf{r}'|} \left( L \dot{\mathbf{l}}' - L \dot{\mathbf{r}}' - L \xi \dot{\mathbf{r}}'' \right) \]

\[ N = N \frac{\mathbf{r}}{|\mathbf{r}'|} \]

\[ L = \frac{\beta \left| \mathbf{r}'_{n-1/2} \right| (L \left| \mathbf{r}'_{n-1/2} \right| - L^2) + L \mathbf{r}'_{n-1/2} \cdot \mathbf{r}'_{n-1/2} - T_c L^2 \left| \mathbf{r}_{n-1/2} \right|}{Y \left( \mathbf{r}'_{n-1/2} \cdot \mathbf{r}'_{n-1/2} + \xi_{n-1/2} \mathbf{r}'_{n-1/2} \cdot \mathbf{r}''_{n-1/2} \right)} \]
TENSION CONTROLLED DEPLOYMENT

- Original Sim. Assumed $L$, $L'$, $L''$ Constrained

\[
\dot{y} = f_*(y) + L' f_i(y) \\
L = L(t) \\
L' = L'(t) \\
L'' = L''(t)
\]

- Modified Sim.

\[
\begin{align*}
\dot{y} &= f_*(y) + L' f_i(y) \\
L' &= L''(y) + \ddot{L}(y) \dot{y} \\
\end{align*}
\]

\[
N_{n-1/2} = N_{n-1/2,0} + L' N_{n-1/2,1} \Rightarrow L' = \frac{F - N_{n-1/2,0}}{N_{n-1/2,1}}
\]

Fully Extensible, Homogeneous

Inextensible
Amiga Graphics Worksatation Development

Work Sponsored by NASA/MSFC

Objective: Develop tether simulation graphical monitoring and analysis tool.

Current Capabilities:

- Engineering plots
- Tether shape projection plots
- Animated Tether
- "Walking plots"
- Run time environment (smart terminal)
- Post processing

Video Tape Example Problem

- SEDS Low Tension Deployment
  90,000 kg orbiter
  150 kg satellite
  upward deployment
  $\sim 24$ km deployment
Summary

TSSIM is a versatile/accurate tether dynamics simulation
Large variety of cases can be analyzed
Model complexity selectable by user
Graphics workstation software developed
Skyhook Program

David A. Arnold
Smithsonian Astrophysical Observatory
SKYHOOK PROGRAM

A. PURPOSE OF SIMULATION

• GENERAL PURPOSE PROGRAM FOR SIMULATING THE DYNAMICS OF TETHERED SATELLITE SYSTEMS
• FEASIBILITY AND DESIGN STUDIES
• STUDY SCIENTIFIC AND OTHER APPLICATIONS OF TETHERED SYSTEMS
B. COORDINATE SYSTEMS AND DEGREES OF FREEDOM

- RECTANGULAR GEOCENTRIC COORDINATE SYSTEM
- POSITION AND VELOCITY OF EACH MASS
- TEMPERATURE OF THE WIRE
- ORIENTATION OF THE SUBSATELLITE (DIRECTION COSINES)
- ELECTRIC CHARGE ON EACH MASS
C. ENVIRONMENTAL MODELS

- SPHERICAL HARMONIC REPRESENTATION OF EARTH'S GRAVITY
- LUNAR & SOLAR GRAVITY PERTURBATIONS
- ATMOSPHERIC DRAG FORCE
- SOLAR, EARTH INFRARED, AND ATMOSPHERIC DRAG HEATING
- EARTH'S MAGNETIC & ELECTRIC FIELDS
- IONOSPHERIC DENSITY
D. SYSTEM MODELS

- BEAD MODEL OF THE TETHER WITH VISCO-ELASTIC FORCES
- AREA TO MASS RATIO OF SUBSATELLITE, SHUTTLE AND WIRE
- TORQUES ON SUBSATELLITE
- THERMAL ABSORPTIVITY, EMISSIVITY, AND HEAT CAPACITY OF THE WIRE
- THERMAL EXPANSION COEFFICIENT
- CAPACITANCE AND RESISTANCE OF THE WIRE
- CHARGE COLLECTION MODELS OF THE WIRE, SHUTTLE, AND SUBSATELLITE
- ELECTRODYNAMIC FORCE ON THE WIRE
- THRUSTER MODELS
- VARIOUS REEL CONTROL ALGORITHMS
E. LIMITATIONS

- Computer time and memory requirements increase rapidly with the number of mass points.
- Resolution of wire dynamics is limited by number of mass points.
- Resolution varies during deployment & retrieval as beads are added or subtraction.
- Inductive effects not included in electrodynamic model.
F. VALIDATION METHODS

- COMPARISON OF SIMULATIONS WITH ANALYTIC SOLUTION FOR THE BEHAVIOR
- COMPARISON OF RESULTS WITH DIFFERENT NUMBERS OF MASSES
- COMPARISON OF RESULTS FROM DIFFERENT SIMULATION PROGRAMS
- EXAMINATION OF INTERMEDIATE CALCULATIONS DURING DEBUGGING AND COMPARISON WITH HAND CALCULATIONS
G. WHAT SHOULD BE DONE NEXT

- MODIFY SKYHOOK TO MAINTAIN RESOLUTION AS WIRE LENGTH CHANGES
- COMPARE RESULTS OF BEAD MODEL OF WIRE WITH MODAL REPRESENTATION
OF POOR QUALITY
SLACK3

Gordon E. Gullahorn
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SLACK3

High Resolution Slack Tether Simulator

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PREPARED FOR: NASA MARSHALL SPACE FLIGHT CENTER
CONTRACTS NAS8-35036, NAS8-36160

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PURPOSE: SITUATIONS IN WHICH TETHER IS DE-TENSIONED:
- TETHER BREAK
- REEL 'JAM

MODEL TYPE: LUMPED MASS.

HOST SYSTEM: VAX/VMS. FORTRAN.

TETHER DYNAMICS SIMULATION WORKSHOP, ARLINGTON, VA, 16 SEPTEMBER 1986.
NEED FOR SLACK TETHER SIMULATOR

SITUATIONS IN WHICH GRAVITY GRADIENT AND OTHER FORCES NO LONGER KEEP THE TETHER IN TENSION:
- BROKEN TETHER
- REEL JAM DURING RETRIEVAL
- MASS RELEASE

SUCH FAILURE MODES MAY HAVE UNDESIRABLE CONSEQUENCES:
- TANGLING OF DEPLOYER MECHANISM AS RECOILING TETHER IMPACTS.
- COMPLETE WRAP-AROUND OF SHUTTLE.
- LOSS OF SATELLITE ATTITUDE STABILITY.

SIMULATOR IS NEEDED TO
- CONFIRM AND QUANTIFY CONSEQUENCES.
- DESIGN AVOIDANCE PROCEDURES.

PREVIOUS SIMULATORS (E.G. SKYHOOK) CAN MODEL LOSS-OF-TENSION SITUATIONS ONLY WITH LOW PRECISION AND AT HIGH COMPUTATIONAL COST.
THE MODEL

THE SHUTTLE

- ASSUMED CIRCULAR ORBIT.
- DEPLOYMENT BOOM VIBRATES IN RESPONSE TO TENSION LOSS.
- SHUTTLE UNDERGOES PRESCRIBED MANEUVERS:
  - ROTATION:
    - ANY AXIS
    - CONSTANT OR ACCELERATING RATE
  - LINEAR ACCELERATION:
    - UP TO FIVE BURNS
    - ARBITRARY VECTORS, DURATIONS
- THIS DEFINES A FIXED MOTION OF THE TETHER ATTACHMENT.
  THERE IS NO BACK REACTION OF THE TETHER ON THE SHUTTLE.

THE TETHER

- PROPERTIES: DENSITY, ELASTICITY, DAMPING.
- LUMPED MASS DISCRETIZATION:
  - POINT MASSES
  - MASSLESS TETHER SEGMENTS
  - VARIABLE SEGMENT LENGTHS, CLUSTERED AT SHUTTLE END
  - IN REEL JAM CASE, THE SATELLITE IS THE TIP MASS
- MASS MOTION GIVEN BY
  - LINEARIZED GRAVITY GRADIENT AND CORIOLIS FORCE
  - ATMOSPHERIC DRAG
- SEGMENTS EXERT FORCE ONLY WHEN THE CONNECTED MASSES BECOME SEPARATED BY THE SEGMENT NATURAL LENGTH. THE RELATIVE VELOCITY COMPONENT ALONG THE TETHER IS THEN INSTANTANEOUSLY REFLECTED. ENERGY LOSS IF DAMPED.
- THIS LATTER ASSUMES THAT THE TETHER REMAINS DE-TENSIONED, ALTHOUGH SOME ACQUISITION OF TENSION IS HANDLED REASONABLY.
- TETHER MAY BE CUT LOOSE FROM SHUTTLE ATTACHMENT

INITIAL CONDITIONS
- TETHER INITIALLY WITH SPECIFIED LENGTH, DIRECTION.
- REEL JAM CASE: INITIAL RETRIEVAL VELOCITY GIVEN BY CONTROL LAW.
- TETHER BREAK CASE: AT ARBITRARY POINT.
- UNIFORM RECOIL VELOCITY IS GIVEN BY THEORETICAL CONSIDERATIONS.
- TETHER IS INITIALLY SLIGHTLY SLACK, TO AVOID IMMEDIATE RE-TENSIONING BY GRAVITY GRADIENT.
- SEGMENT DIRECTIONS AND MASS SEPARATIONS ARE SLIGHTLY RANDOMIZED.
SLACK3 INPUT PROMPTS

These prompts define the parameters available for specification by the program user.

THE FOLLOWING PARAMETERS AND FLAGS HAVE DEFAULT VALUES AND MAY BE CHANGED BY ENTERING THE DESCRIPTOR IN THE FIRST COLUMN.

ORIENTATION
TETHER_PROP
TETHER_ANGLE
TETHER_TENS
DISCRETIZE
ORBIT
DEBUG
SEG_LENGTH_RANDOM
SEG_SLACK
SEG_DIR
DRAG_FUDGE
BOOM_PROP
BOOM_ANGLE
BOOM_PHASE
ROT_AXIS
ROT_RATE
OFFSET
REL_ABS
BURNS
CUT_FREE
RUN_TYPE

UP/DOWN TOGGLE IS: UP
1.40E+12 CV 0.30D+09 DIAM 0.30
PITCH 0.00 ROLL 0.00
ORIG LENGTH(KM) 20.0 SS MASS(TONS) 0.50
TENSION FUDGE 1.00
LENGTH RATIO, ATTACHMENT/TIP: 0.50
HEIGHT (KM): 235.0 ATM. DENS.(G/CC): 0.27D-13
DEBUG OUTPUT TOGGLE: F
% SEGMENT LENGTH RANDOMIZATION: 5.0
SEGMENT DIRECTION RANDOM: 5.0
BALLOON DRAG FUDGE FACTOR: 0.75
EI,LENGTH,WEIGHT (NASA UNITS): 0.13D+09 849.0 106.2
PHASE AT T=0: 0.0
DEFINING ANGLES, PITCH & ROLL 90.0 0.0
RATE & ACCELERATION (DEG,SEC): 0.00 0.00
X,Y,Z (METERS): 0.00 0.00 0.00
RELATIVE/ABSOLUTE OUTPUT TOGGLE: REL
CUT TETHER FREE AT T= 9999.9
BREAK

THE FOLLOWING INPUTS ARE ALWAYS REQUIRED:

ENTER RANDOM SEED, ODD INTEGER < 2147483647
NUMBER OF TETHER SEGMENTS? 3 TO 50
LENGTH AT TIME OF REEL JAM (KM)?
A CRUDE TIME SCALE FOR RECOIL, NEGLECTING GRAVITY GRADIENT, IS: 1329.333973151609
HOW LONG (TETHER-SECONDS) TO COMPUTE?
HOW OFTEN (SECONDS) TO OUTPUT?
SET A LIMIT ON # OF BOUNCES (INTEGER):
LIMITATIONS

THE MODEL ASSUMES COMPLETELY DE-TENSIONED TETHER. ALTHOUGH PARTIAL OR BRIEF ACQUISITION OF TENSION IS OFTEN HANDLED WITHOUT PROGRAM FAILURE, GIVING A RAPID SEQUENCE OF BOUNCES AND SEEMINGLY REASONABLE POST-TENSION RESULTS, SUBSTANTIAL ACQUISITION OF TENSION LEADS TO EXCESSIVE NUMBERS OF BOUNCES AND PROGRAM FAILURE.

COMPUTATION TIME CONSIDERATIONS LIMIT SIMULATIONS TO ABOUT FIFTY MASSES (THE CURRENT PROGRAM LIMIT), AND OFTEN TO ABOUT THIRTY.

A MINIMUM NUMBER OF MASSES, GENERALLY ABOUT TWENTY, IS REQUIRED. BELOW THIS NUMBER, THE PHYSICALLY DEMONSTRATED "TRAVELLING WAVE" PHENOMENON IS NOT SIMULATED, AND THE OVERALL BEHAVIOR IS LESS SMOOTH THAN FOR HIGHER RESOLUTION MODELS. PAQUETTE SEES THIS WITH FEWER MASSES

TETHER BENDING STIFFNESS, HYSTERESIS AND NON-LINEAR AXIAL PROPERTIES ARE NOT INCLUDED. THESE WILL BE IMPORTANT, AT LEAST FOR THE SHORT-SCALE, "COLUMN BUCKLING" BEHAVIOR.
VALIDATION METHODS

SLACK2 (A VERSION RESTRICTED TO TWO DIMENSIONS) HAS BEEN VALIDATED AGAINST SKYHOOK WITH A SMALL NUMBER OF MASSES.

SIMULATIONS REVEALED AN UNEXPECTED PHENOMENON WHICH WE CALL A "TRAVELLING WAVE." AFTER THE RECOILING TETHER FIRST INTERACTS WITH THE ATTACHMENT AT THE SHUTTLE, A PULSE OF ENHANCED VELOCITY PROPAGATES RAPIDLY ALONG THE TETHER LEAVING THE BROKEN TIP RECOILING AT A HIGHER VELOCITY THAN THE REMAINDER. A SIMPLE PHYSICAL ANALOG WAS CONSTRUCTED, AND VERIFIED THIS BEHAVIOUR.

RANDOMIZATION OF THE INITIAL CONDITIONS ALLOWS STUDY OF MODEL STABILITY. THE OVERALL RESULTS APPEAR STABLE TO I.C. PERTURBATIONS, BUT THE DETAILED CONFIGURATIONS ARE LESS SO. IT IS UNCLEAR WHETHER THIS IS A FEATURE OF THE LUMPED MASS MODEL OR OF THE PHYSICAL SYSTEM.
OTHER SLACK TETHER WORK

THEORETICAL EFFORTS (CONTINUUM MODELS):

- UNDER CONTRACT TO MARTIN MARIETTA; R.G. HOHLFELD, CO-I.
- STRESS WAVES.
- SHOCK WAVE ANALOGY, JUMP CONDITIONS.
- PROVIDES JUSTIFICATION FOR SLACK3 INITIAL CONDITIONS, ALTHOUGH GRAVITY GRADIENT EFFECTS NOT YET INCLUDED.
- WHAT IS "LOSS OF TENSION?" COLUMN BUCKLING ANALOGY.
- PRELIMINARY, ONE-DIMENSIONAL SIMULATORS OF LOSS OF TENSION.

MMC/MIT LUMPED MASS MODEL (K. PAQUETTE MASTER'S THESIS).

- EXTENSIBLE TETHER SEGMENTS.
FUTURE DIRECTIONS

SLACK3:
- FUNDAMENTALLY COMPLETE. ADD DETAILS AS REQUIRED.
- EXERCISE FOR ENGINEERING STUDIES.
- STABILITY STUDIES.

CONTINUUM THEORY:
- IMPROVE SLACK3 INITIAL CONDITIONS; INCLUDE GRAVITY GRADIENT.
- STABILITY OF PHYSICAL SYSTEM.
- INPUT FOR POSSIBLE NEW GENERATION OF SLACK TETHER SIMULATORS.
- VALIDATE WITH ACTUAL EXPERIMENT.
CASE STUDY: THRUSTER AVOIDANCE MANEUVER.

TETHER: DIAMETER 0.3 CM, LINEAR DENSITY 0.08 GM/CM,
AXIAL STIFFNESS (AE) $0.6 \times 10^{10}$ DYNE.
INITIAL LENGTH 20 KM, CUT AT 1 KM.

DEPLOYED STRAIGHT UP, WITH A 500 KG SATELLITE.

AVOIDANCE MANEUVERS:
- ACCELERATION 12 CM/SEC$^2$. (THREE THRUSTERS.)
- INITIATED 5 SECONDS AFTER BREAK.
- TERMINATED AFTER 20 SECONDS.
- TWO CASES FOR VECTOR:
  - ALONG ORBIT
  - ACROSS ORBIT
Figure 2.3.1. 20 km tether cut at 1 km. Output at 25 second intervals, total run of 1350 sec.
Figure 2.4.4. As for Fig. 2.2.1, except: Thrusters directed along-orbit, cutoff after 20 second burn, total run of 1200 seconds.
Figure 2.4.3. As for Fig. 2.3.1, except: Thrusters directed perpendicular to orbital plane, cutoff after 20 second burn, total run of 1075 seconds.
Artificial Gravity Laboratory

Enrico C. Lorenzini
Smithsonian Astrophysical Observatory
PURPOSE OF SIMULATION

- LUMPED MASS MODEL FOR SIMULATING THE DYNAMICS OF N MASSES CONNECTED SEQUENTIALLY BY TETHERS

- INVESTIGATIONS TO DATE

  - DEPLOYMENT OF A 3-MASS TETHERED SYSTEM (2-D MODEL)
    - DAMPING OF VIBRATIONAL MODES
    - DEPLOYMENT STRATEGY

  - STATION-KEEPING OF A 3-MASS TETHERED SYSTEM (2-D MODEL)
    - DAMPING OF VIBRATIONAL MODES
    - CONTROLLED SINUSOIDAL G-PROFILES
    - G-TUNING

  - STATION-KEEPING OF A 4-MASS TETHERED SYSTEM (3-D MODEL)
    - FREE DYNAMICS
    - DAMPING OF VIBRATIONAL MODES
    - FORCED DYNAMICS
    - ASSESSMENT OF ACCELERATION LEVELS
SYSTEM MODELS. 3-MASS TETHERED SYSTEM

- SYSTEM CHARACTERISTICS
  - SS MASS: $m_1 = 90.6 \times 10^3$ kg
  - G-LAB: $m_2 = 4530$ kg
  - END MASS: $m_3 = 9060$ kg
  - ORBIT ALTITUDE: 500 km
  - 2 MM DIA KEVLAR TETHER
  - OVERALL TETHER LENGTH: 10 km
  - TWO LONGITUDINAL PASSIVE DAMPERS
    (SPRING-DASHPOT) IN SERIES WITH THE
    TWO TETHER SEGMENTS

- APPLICATIONS
  - MICRO-G/VARIABLE-G EXPERIMENTS AT
    THE G-LABORATORY ($M_2$)
SYSTEM MODELS. 4-MASS TETHERED SYSTEM

- **SYSTEM CHARACTERISTICS**
  - BALANCING MASS: \( m_1 = 10^4 \text{ kg} \)
  - SS AT SYSTEM C.M.: \( m_2 = 306.752 \times 10^3 \text{ kg} \)
  - VARIABLE-G LAB: \( m_3 = 5 \times 10^3 \text{ kg} \)
  - END MASS: \( m_4 = 10^4 \text{ kg} \)
  - 2 MM DIA KEVLAR TETHER
  - ORBIT ALTITUDE: 450 km
  - UPPER TETHER: 10 km
  - LOWER TETHER: 10.15 km
  - THREE LONGITUDINAL PASSIVE DAMPERS IN SERIES WITH THE THREE TETHER SEGMENTS

- **APPLICATIONS**
  - MICRO-G EXPERIMENTS AT THE SS
  - VARIABLE-G EXPERIMENTS AT THE G-LAB
MATHEMATICAL MODEL

- LUMPED MASS MODEL
  - POINT MASSES
  - ELASTIC TETHERS
  - LUMPED MASS MODEL OF TETHERS (OPTIONAL)
  - GENERIC ORBIT
  - 3-D (SIMPLIFIED VERSION: 2-D)

- LONGITUDINAL DAMPERS

- DAMPING ALGORITHMS OF LIBRATIONAL AND LATERAL OSCILLATORY MODES

- NUMERICAL INTEGRATORS
  - 4TH ORDER VARIABLE STEP RUNGE-KUTTA
  - VARIABLE STEP PREDICTOR - CORRECTOR
REFERENCE FRAMES AND COORDINATES

• INTEGRATION VARIABLES
  - CARTESIAN COORDINATES OF THE N MASSES WRT. LH-LV

• STATE VARIABLES
  - TETHER LENGTHS: $L_i$
  - LATER DEFLECTIONS
    • IN-PLANE COMP: $\epsilon_{ii}$
    • OUT-OF-PLANE COMP: $\epsilon_{0i}$
  - IN-PLANE ANGLE: $\theta$
  - OUT-OF-PLANE ANGLE: $\phi$
ENVIRONMENTAL MODELS

• GRAVITY
  - \( J_0 \) AND OPTIONALLY \( J_2 \)

• DRAG
  - ANALYTICAL FIT OF JACCHIA'S 1977 MODEL
    INDEPENDENT PARAMETERS: HEIGHT, EXOSPHERIC TEMPERATURE

• THERMAL MODEL OF THE WIRE
  - SOLAR ILLUMINATION (ECLIPSES)
  - EARTH ALBEDO
  - IR EARTH RADIATION
  - EMITTED RADIATION
  - NO DRAG HEATING
ATMOSPHERIC DENSITY

THERMAL BALANCE

KEVLAR 29
DIA = 2 mm
ε = 0.9
α = 0.4
α_{IR} = 0.045
MATHEMATICAL MODEL LIMITATIONS

- INTRINSIC LIMITATIONS
  - HIGH RESOLUTION OF TRANSVERSE WIRE VIBRATIONS (STRING-LIKE)
    IMPLIES VERY LONG CPU TIME

- REMOVABLE LIMITATIONS
  - ROTATIONAL DYNAMICS OF PLATFORMS
  - SPACE STATION ORBITAL MANEUVERS
  - IDEAL CONTROL SYSTEMS
VALIDATION METHODS

- IN-HOUSE VALIDATION
  - RESULTS FROM 3-D MODEL CONSISTENT WITH RESULTS FROM TWO SEPARATE 2-D SIMPLIFIED MODELS
  - FREQUENCIES OF VARIOUS VIBRATIONAL MODES CONSISTENT WITH SIMPLE ANALYTICAL COMPUTATIONS

- COMPARISON WITH EXTERNAL MODELS NOT PERFORMED BECAUSE OF UNAVAILABILITY OF COMPARABLE SIMULATION RUNS
CASE STUDY EXAMPLE

- 4-MASS TETHERED SYSTEM FREE DYNAMICS
- SIMULATION A
  - LONGITUDINAL PASSIVE DAMPERS ONLY
- SIMULATION B
  - LONGITUDINAL PASSIVE DAMPERS
  - IN-PLANE LIBRATIONAL/LATERAL DAMPING ALGORITHMS (REEL-CONTROL)
    \[
    l_{1c} = l_{10} \left( 1 - k_s \theta + k_{s_{11}} \frac{e_{11}}{e_{10}} \right) \\
    l_{2c} = l_{20} \left( 1 - k_s \theta + k_e_{12}/e_{03} - k_e_{11} e_{11}/e_{02} \right) \\
    l_{3c} = l_{30} \left( 1 - k_s \theta + k_e_{12} e_{12}/e_{02}/e_{03} \right)
    \]
- INITIAL CONDITIONS FOR SIMULATION A AND B
  - SYSTEM INITIALLY ALIGNED WITH LV
  - NON-ZERO INITIAL VELOCITIES OF THE PLATFORMS WRT. LH-LV
- NO J₂
SIMULATION B. ACCELERATIONS

![Graphs showing ACV, ACL, and ACH accelerations over time.](image-url)
SIMULATION A. SIDE-VIEW

- Expanded Scale, Side-View Plotted Every 40 Sec for 7200 Sec.
Original page is of poor quality

Simulation B. Side-View

- Expanded Scale, Side-View Plotted Every 40 Sec for 7200 Sec.
  Longitudinal + In-Plane Librational/Lateral Dampers On.
SIMULATION A AND B. FRONT-VIEW

- Expanded Scale, Front-View Plotted Every 40 Sec for 7200 Sec.
PLAN OF FUTURE ACTIVITY

- FURTHER IMPROVEMENT OF THE MATHEMATICAL MODEL
  - ROTATIONAL DYNAMICS OF THE PLATFORMS
  - MODEL OF ROTATIONAL DAMPERS

- DEVELOPMENT OF CONTROL/NOISE-ATTENUATION STRATEGIES
  - OUT-OF-PLANE LIBRATIONAL/LATERAL DAMPING ALGORITHMS
  - DAMPING OF TRANSVERSE WIRE VIBRATIONS
  - AUGMENTATION OF ISOLATION OF G-LABORATORY FROM NOISE GENERATED ON BOARD THE SS BY MEANS OF REEL CONTROL
SUMMARY

- 3-D LUMPED MASS MODEL FOR SIMULATING THE DYNAMICS OF N MASSES CONNECTED SEQUENTIALLY BY TETHERS.

- OPTIONALLY SOME LUMPED MASSES CAN MODEL THE PLATFORMS WHILE THE OTHER LUMPS MODEL THE TETHER SEGMENTS

- ENVIRONMENTAL MODEL: \( J_0 \) + \( J_2 \), DRAG, THERMAL

- SPECIALIZED CONTROL ALGORITHMS FOR DAMPING OSCILLATORY MODES

- SO FAR THE FREE-DYNAMICS OF 3-MASS AND 4-MASS TETHERED VERTICAL CONSTELLATIONS HAS BEEN SIMULATED

- FUTURE PLAN FORSEES IMPLEMENTATION OF ROTATIONAL DYNAMICS OF PLATFORMS, FURTHER DEVELOPMENT OF CONTROL/NOISE-ATTENUATION STRATEGIES, INVESTIGATION OF FORCED DYNAMICS OF 4-MASS TETHERED SYSTEM.
Elasticity Effects in Tether Dynamics

Silvio Bergamaschi
University of Padova, Italy
OBJECTIVES OF THE ITALIAN PROPOSAL

The overall purpose is to understand the relevance of tether elasticity in TSS dynamics, because:

- long tethers have never been flown; experience is lacking
- high frequency dynamical noise might preclude future experiments (*gravity gradiometry, atmospherics*)
- non-nominal situations can occur where elastic effects are of primary importance (*tether slackness, impact with a micrometeoroid*)

Thus, the work plan is:

- To investigate *natural frequencies* of elastic vibrations of TSS
- To study *amplitude of response* to excitations
- To analyze *data from gyros and accelerometers* mounted on the satellite
- To check theoretical and experimental results
- If needed, to improve the simulation models

THE MATHEMATICAL MODEL

**Assumptions:**

- Space Shuttle in circular, unperturbed orbit. Earth oblate-ness, atmospheric and electrodynamic drag neglected
- Perfectly elastic tether. No material damping included
- Point mass satellite
- Constant tether length
- Constant tether diameter
- Ambient temperature changes ignored

**Method**

Lagrangian formulation adopted
THE LAGRANGIAN DENSITY

\[ \mathbf{\ddot{r}} = (a + x) \mathbf{i} + y \mathbf{j} + z \mathbf{k} \]

**Kinetic Energy:**

\[ T = \frac{1}{2} \mu \int_0^l (\mathbf{\ddot{r}} \cdot \mathbf{\ddot{r}}) \, ds = \frac{1}{2} \mu \int_0^l \left( (\dot{x} - my)^2 + \left[ y + m(a + x) \right]^2 + z^2 \right) \, ds \]

where \( \mu \) is the linear mass per unit length.

**Elastic Energy:**

\[ V_{el} = \frac{1}{2} EA \int_0^l \left[ (x'^2 + y'^2 + z'^2) - 1 \right] \, ds \]

**Gravitational Energy:**

\[ V_g = -\mu_0 \mu \int_0^l \mathbf{r} \cdot d\mathbf{l} \approx \mu m^2 a^2 \int_0^l \left( 1 - \frac{x}{a} + \frac{2x^2 - y^2 - z^2}{2a^2} \right) \, ds \]

with: \( \mu_0 = 3.986008 \times 10^{14} \, \text{m}^3 \, \text{kg}^{-2} \)
Perturbation solution assumed having the form:

\[
\begin{aligned}
  x(t, 1) &= x_1(t) + \varepsilon x_2(t) \\
y(t, 1) &= \varepsilon y_2(t) \\
z(t, 1) &= \varepsilon z_2(t)
\end{aligned}
\]

where \( \varepsilon \) is a small parameter.

Lagrangian density:

\[
L = \frac{\mu}{2} \left\{ 3 m^2 (\ddot{x}_1 x_1 - \frac{E^A}{\mu} (x_1' - 1)^2) + 2 \varepsilon \left[ 3 m^2 x_1 x_2 + m \dot{y}_2 (a - x_1) - \frac{E^A}{\mu} x_1' (x_1' - 1) \right] + \varepsilon^2 \left[ \dot{x}_2^2 + \dot{y}_2^2 + \dot{z}_2^2 + m^2 (3 x_1^2 - \frac{1}{2} z_2^2) + 2 m (x_1 y_2 - x_2 y_1) - \frac{E^A}{\mu} \left[ (x_2^2 + y_2^2 + z_2^2) (1 - \frac{1}{x_1'}) + \frac{x_2^2}{x_1'} \right] \right]\right\}
\]
STATIC EQUILIBRIUM

From 0-th order Lagrangian density:

\[ x''_i + x^2 x_i = 0 \]

\[ k = m \left( \frac{3p}{E} \right)^{\frac{1}{2}} \]

with boundary conditions:

\[
\begin{align*}
  x_i(0) &= 0 \\
  x_i'(l) &= 1 + 2e \\
  2 &= \frac{3mm^2e}{EA}
\end{align*}
\]

Solution:

\[ x_i'^2 = \frac{1 + d}{l(e + 2e)} \text{ mm/sec} \]

Numerical values:

\[ m = 500 \text{ kg} \quad E = 7 \times 10^9 \text{ N/m}^2 \quad p = 1.5 \times 10^3 \text{ kg/m}^3 \]

\[ d = 2 \text{ mm} \quad m = 1.15795 \times 10^{-3} \text{ rad/sec} \]
DYNAMICAL EQUATIONS

From the 2nd order Lagrangian density:

\[
\begin{align*}
\ddot{x}_2 - \frac{EA}{\mu} x_2'' - 2m \dot{y}_2 - 3m^2 z_2 &= 0 \\
\ddot{y}_2 + 2m \dot{x}_2 - \frac{EA}{\mu} \left[ (1 - \frac{1}{x_i}) y_2 \right]' &= 0 \\
\ddot{z}_2 - \frac{EA}{\mu} \left[ (1 - \frac{1}{x_i}) z_2 \right]' + m^2 z_2 &= 0
\end{align*}
\]

with boundary conditions:

\[
\begin{align*}
{x}_2 (0, t) &= 0 \\
{y}_2 (0, t) &= 0 \\
{z}_2 (0, t) &= 0 \\
{\dot{x}}_2 (e, t) - 2m \dot{y}_2 (e, t) - 3m^2 \dot{x}_2 (e, t) + \frac{EA}{m} x_i^' (e, t) &= 0 \\
{\dot{y}}_2 (e, t) + 2m x_2 (e, t) + \frac{EA}{m} \left[ 1 - \frac{1}{x_i^' (e)} \right] y_i^' (e, t) &= 0 \\
{\dot{z}}_2 (e, t) + m^2 \dot{z}_2 (e, t) + \frac{EA}{m} \left[ 1 - \frac{1}{x_i^' (e)} \right] z_i^' (e, t) &= 0
\end{align*}
\]
METHOD OF SOLUTION
(details for the $Z_2$ equation)

Solution by variable separation:

$$Z_2(\lambda_1) = C(\lambda) \cdot e^{q \lambda}$$

The general solution for the amplitude $c(\lambda)$ can be written as:

$$c(\lambda) = A c_1(\lambda) + B c_2(\lambda)$$

with $c_1(\lambda)$ and $c_2(\lambda)$ linearly independent integrals, i.e.:

\[
\begin{align*}
  &\begin{cases} 
    c_1(0) = 0 \\
    c_1'(0) = 1 
  \end{cases} & \begin{cases} 
    c_2(0) = 1 \\
    c_2'(0) = 0 
  \end{cases}
\end{align*}
\]

but $c_2(\lambda)$ is not admissible
TRIAL AND ERROR PROCEDURE TO COMPUTE THE EIGENVALUES

- To assume a tentative value for \( q \)
- To integrate numerically the resulting equation (in nondimensional variables) in the interval 0-1 with \( c(o)=0 \) and \( c'(o)=1 \) as initial conditions
- To check if the boundary condition at the satellite end of the tether is satisfied. If not, to reflect the sequence with an increased \(|q|\) value.
FIRST THREE MODES OF OUT OF PLANE MOTION
TETHER STRUCTURAL DAMPING

Purpose: To investigate transients in longitudinal vibrations

Assumptions:
- coupling with orbital motion and lateral vibrations neglected
- experimental data from Martin Marietta used

Dynamical equation:

\[ M \frac{\partial^2 \gamma}{\partial t^2} = EA \frac{\partial^2 \gamma}{\partial x^2} + C \frac{\partial^3 \gamma}{\partial x^3 \partial t} \quad (0 \leq x \leq l) \]

Boundary conditions:

\[ \gamma(0, t) = 0 \quad EA \gamma'(l, t) = -m \ddot{y}(l, t) \]

Initial conditions:

\[ \gamma(x, 0) = \kappa x \quad \dot{y}(x, 0) = 0 \]
Purpose of the investigation:

- To analyze system stability during one elevator transfer from the Space Station to the SATP.
- To evaluate orbital perturbations on the Space Station.
- To devise velocity control laws, if needed.

Assumptions:

- Tether elasticity neglected
- Characteristic dimensions of Space Station, elevator and SATP negligible with respect to tether length
- In plane motion only (Coriolis force)
- No external perturbations taken into account (Earth oblateness, atmospheric drag, etc.)

Method:

- Lagrangian formulation of the equations of motion

THE SYSTEM

Retrievable platforms tethered to the Space Station in order to:

- conduct experiments in a dynamic noise force environment (not attainable on the Space Station)

Elevator envisaged as:

- platform for microgravity experiments at system center of gravity
- servicing module for the SATP

Tether intended to:

- sustain SATP gravity gradient force
- act as structural damper
- transfer power and information (umbilical concept)
**Coordinates:**

\[
\begin{align*}
\{ x_0 &= r \cos \theta \\
\{ y_0 &= r \sin \theta \\
\{ x_1 &= x_0 + d \cos (d + \theta) \\
\{ y_1 &= y_0 + d \sin (d + \theta) \\
\{ x_2 &= x_1 + (l - d) \cos (d + \theta + \phi) \\
\{ y_2 &= y_1 + (l - d) \sin (d + \theta + \phi)
\end{align*}
\]

**Kinetic energy:**

\[
T = \frac{1}{2} m_0 (\dot{x}_0^2 + \dot{y}_0^2) + \frac{1}{2} m_1 (\dot{x}_1^2 + \dot{y}_1^2) + \frac{1}{2} m_2 (\dot{x}_2^2 + \dot{y}_2^2) + \\
+ \frac{1}{2} \mu \int_0^d (\dot{z}_3^2 + \dot{z}_5^2) \, ds + \frac{1}{2} \mu \int_0^\ell (\dot{z}_3^2 + \dot{z}_5^2) \, d\eta
\]

**Potential energy:**

\[
V = -\frac{\mu c m_0}{r} - \frac{\mu c m_1}{r_1} - \frac{\mu c m_2}{r_2} - \mu c \mu \int_0^d r_3^{-1} \, ds - \mu c \mu \int_0^\ell r_5^{-1} \, d\eta
\]

**Series expansion of distances:**

\[
r_0^{-1} = \frac{1}{r} \left(1 + \frac{d^2}{r_0^2} + \frac{3d^4}{2r_0^4} \sin^2 \alpha \right)^{-\frac{1}{2}} \approx \frac{1}{r} \left(1 - \frac{d^2}{r_0^2} - \frac{d}{r_0^2} \omega_1^2 + \frac{3d^4}{2r_0^4} \omega_2^2 \right)
\]

\[r_2^{-1} = \ldots \ldots \ldots \]
FREE MOTION

To calculate \( v_0 \), let:

\[ \ddot{x} - 3m^2 x = 0 \]

with the first integral:

\[ x^2 - 3m^2 x^2 = v_0^2 - 3m^2 x^2(0) \]

Initial velocity subject to:

\[ \dot{x}(0) = m\sqrt{3} x(0) \]

It is expected that:

\[ x_f = (3m^2 x_f^2 + v_n^2)^{\frac{1}{2}} \]

where \( v_n \) is the residual elevator velocity at c.o.g.

Slight dependence of final velocity on initial conditions.
CONTROLLED TRANSFER

Control laws affected by:

- Coriolis force, with impact on average translation velocity
- Transients at beginning and end of each maneuver, because of singularities of the dynamical equations at both ends of the tether

Linear Control:

- In some cases numerical instabilities at the very beginning of maneuvers

Exponential Control:

- At the beginning: \[ \dot{d}_0 = v_0 \frac{1}{\sqrt{g}} \sin \theta (d) \]
- At the end: \[ \dot{d}_1 = v_0 \frac{1}{\sqrt{g}} \sin \theta (l-d) \quad (d > d^*) \]
- Smoother motion
- Reduced time for a transfer
- Slightly increased perturbations on Space Station orbit
Purpose: To evaluate tether motion around the local vertical, forced by Earth oblateness

Assumptions:
- tether elasticity neglected
- first order theory used for the changes of the oscillating elements
- three-dimensional motion simulated
TSS Subsatellite Attitude Dynamics
and Control Laws Verification Programs

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TSS SUBSATELLITE ATTITUDE DYNAMICS
AND CONTROL LAWS VERIFICATION PROGRAMS

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Abstract- This paper deals with the presentation of a dynamic model of the Tethered Satellite System and of the relevant simulation program, developed in order to provide the dynamic analysis support for the design verification of the Subsatellite attitude control. A special care has been spent in the Satellite attitude dynamic analysis and the model has been specifically conceived to this aim.

The way in which the simulation results can be utilized for the verification and testing of the attitude control is also presented.

1 - INTRODUCTION

The design of the Attitude Measurement and Control Subsystem (AMCS) of the TSS-Subsatellite for the electrodynamics...
namic mission has been completed and now it is going to be verified and tested. A representative dynamics of the system is one of the main items that has to be implemented for the design verification phase and in such an optic a dedicated program has been developed.

The aims of this program are:
- to investigate the dynamics of the system, with a particular interest for the Satellite attitude behaviour, and to identify potentially critical areas for the AMCS performance verification;
- to provide realistic input data for the verification and testing phases.

To this purposes a model representing the overall Orbiter-tether-Satellite system has been considered and developed, while the adoption of a simplified, provided representative, AMCS model is enough for the goal.

On the other side, to the AMCS performance verification aim, a complete and accurate model is needed for the AMCS itself, requiring a small integration step (128 msec), tied to the high on-board data handling sampling rate. The need for the implementation of a simplified dynamic model (3 satellite rotational d.o.f + the possible elastic ones) arises then, in order to avoid unaffordable CPU times.

The "dynamic-simplified" model can be called an 'open
loop' approach because it assumes that the overall system
dynamics impacts on the Satellite attitude, while the fact
that the attitude is controlled has a negligible effect on
the overall behaviour, this resulting in the tether tension
at the swivel point which does not depend on the Satellite
attitude. This seems to be a reasonable hypothesis, to be
anyhow validated by means of the "AMCS-simplified" model,
which, in turn, can be called the 'closed loop' approach, as
it accounts for the coupling between the satellite attitude
and the overall dynamics, while being not burdened by low
integration step requirements.

The simulation results of the 'closed loop' model will
then be used as input data for the "dynamic-simplified"
model runs; in fact, in order to provide an adequate capa-
bility to represent the system, the "dynamic-simplified"
model requires, for each run, the force time history at the
tether attachment point and the firing sequence of the
Satellite thrusters not driven by the AMCS (in plane and out
of plane thrusters) in input.

The results of the 'closed loop' simulations will also
be used for the integration and testing phase: the AMCS
hardware and software will work, by means of gyros and
sensors stimulators, in closed loop with a simulated real-
time-computed system dynamics. The same 'open loop' model
as for the verification analyses will be used to provide the necessary input to the stimulators.

At the moment, the 'closed loop' model has been implemented and the program is undergoing integration tests, while the first simulation runs will be performed in a short time.

Hereafter the 'closed loop' model and program are described.

2 - MODEL DESCRIPTION
2.1 - Configuration

The electromagnetic mission will be supported by the Shuttle in an approx 300 km orbit; the Satellite will be deployed upward up to 20 km from the Orbiter; the Satellite mass will be approx 500 kg.

Only the Satellite yaw axis is controlled, stabilization on the other two axes being assured by the gravity gradient. During on-station phase, the Satellite spins about the yaw axis and the spin rate is controlled (1 rpm). During retrieval, the yaw angle is controlled in such a way to allow for the in-plane and out of plane thrusters to fire in the correct direction for libration control. During the on-station phase, two deployable booms, part of the scientific
experiment, will be extended to approx 4m.

2.2 - Tether Model

Some preliminary analyses showed different kinds of behaviour of the system depending on the tether length. This leads to different representations of the system, depending on the deployed length. In figure 1 the main system frequencies are given as a function of tether length ($l$).

The frequencies of the Satellite oscillations about its roll and pitch axes decrease as $l$ decreases; the gravity gradient stabilizing effect for these axes also decreases, support in tensioning the tether being provided at short distances by in-line thrusters.

Fig. 1
The longitudinal frequencies of the overall system due to tether elasticity decrease as $\lambda$ increases. It can be seen that the axial frequencies may couple with the pitch/roll oscillations. The number of modes to be taken into account depends on the tether length: at short distance only the first axial mode is significant, while the others could be neglected, leading to a reduction of the integration step without affecting the system behaviour representation; at the longest tether extension also the second mode has to be considered.

The string vibration too may have a significant coupling with the Satellite oscillation at short tether length.

These considerations lead to the need of considering in any case the tether mass and elasticity: a lumped mass approach for describing the tether was chosen, where the number of masses is selectable as a function of tether length.

The tether can be modelled as a massless spring connecting the Orbiter and the Satellite; in such a way only the first axial mode is simulated. To include the other modes, the tether is described by mass points connected by springs whose stiffness depends on the undeformed tether length between adjacent bodies.
During deployment and retrieval the tether mass changes: this is represented by changing the mass of the mass point closer to the Orbiter; the other mass points maintain their mass. The nominal (unstressed) tether length between the Orbiter and the closer mass point also changes (and so its stiffness does), according to tether reeling, while the unstressed tether length between the other masses remains unchanged.

The number of mass point cannot be changed during the simulation in the current version of the program. The fixing of this number at the beginning of each run allows for introducing only the frequencies of interest during each part of the mission.

Fig. 2
When, during deployment, the need for considering another frequency appears, the simulation requires a restart with a new description of the tether which takes into account new mass points. The I.C. for such a new configuration can be derived from the results of the previous simulation, under the assumption that the new modes are not excited; thus, the new masses must be added until the associated mode effects are negligible. Reverse considerations can be applied for the retrieval phase.

An alternative/complementary model is being developed, to be used for cross validation purposes: the mass points are maintained uniformly spaced along the tether length to represent the actual mass distribution, during both deployment and retrieval. The system frequencies are now closer to the actual ones, but the mass points "move" along the tether.

2.3 System Model

The Orbiter and the Satellite are considered rigid bodies connected by a tether represented by series of springs and point masses. The system is considered orbiting around the Earth.

The references frames, showed in figure 3, are:

- Inertial reference frame (IRF), located in the centre of the Earth.
Fig. 3
- Accelerating reference frame (ARF), in the centre of mass of the overall system; it is always parallel to the IRF. The motion of ARF origin is described with respect to IRF, integrating the orbit equations of the system. The motion of the parts of the system is described with respect to ARF. This is to avoid possible numerical problems when the motion of a system is referred to a very far reference frame (small numbers added to large numbers).

- Orbiter body reference frame, located in the Orbiter c.m.

- Satellite body reference frame, in the Satellite c.m.; z-axis is directed toward the tether attachment point.

- Satellite attitude reference frame, in the Satellite c.m., with the z-axis always directed toward the Earth centre. The Satellite attitude is referred to this frame (local vertical).

2.4 Dynamic Equations

The motion of each body is computed with respect to ARF. The Orbiter has 3 translational rigid d.o.f.'s; the 3 rotational d.o.f.'s can be taken into account or neglected, according to the figure of an input flag. The Satellite has all the 6 rigid d.o.f.'s. The mass points describing the tether have only 3 translational d.o.f.'s.
Three differential equations describe the orbital motion of the origin of ARF about the Earth.

The translational differential equations with respect to ARF are obtained for each body considering all the forces acting on it (actual and apparent ones):

\[ \ddot{\mathbf{X}} = -m\ddot{A} + m\ddot{\mathbf{g}} + T + F \]

where:
- \( X \) : body c.m. position in ARF
- \( A \) : acceleration of ARF origin in IRF
- \( \mathbf{g} \) : local gravity acceleration
- \( T \) : force(s) coming from the tether
- \( F \) : other external forces (thrusters firing, imposed perturbations).

No Coriolis force is present because the ARF does not rotate. The rotational differential equations for the Orbiter or the Satellite are represented by the Euler equations. The rotational motion is described by means of 3 Euler angles, deriving from the integration of the Euler equations (motion about the centre of mass).

2.5 Other Features

No environmental model has been implemented: aerodynamic drag and electromagnetic forces on the tether were considered negligible, even if the possibility to input forces on
the mass points is foreseen.

In general arbitrary forces can be introduced to analyse the system behaviour under specified perturbations.

All the Satellite thrusters are modelled and their misalignements, as source of attitude perturbations, considered.

The control laws of the tether length and in-plane & out-of-plane libration developed for the TSS program have been implemented.

The Orbiter attitude may have an indirect but important impact on the Satellite behaviour during some mission phases, so it has been included in the model together with a very simplified model of the digital autopilot and reaction control system.

Due to lack of data on tether characteristics, no damping has been considered for the system oscillation, this resulting in a conservative hypothesis. The tether torsional stiffness is negligible and so it is not taken into account in this program.

A new feature which will be introduced in the program is represented by the addition of flexible modes on the Satellite, simulating two deployable booms.

The need to model in an easy way the flexibilities of the deployer boom on the Orbiter, especially at very short
tether length, is being investigated.

3 - SOFTWARE DESCRIPTION

The general architecture of the program is presented in figure 4. The simulation program is coded in FORTRAN 77 and runs on a VAX 785 machine.
The YLT routine computes, time by time, the state vector derivative to be integrated. It determines all the force and torque vectors acting on each part of the system, via the management of the call of several subroutines, each describing a particular item (see Table 1), allowing to build the differential equations.

<table>
<thead>
<tr>
<th>Subroutine</th>
<th>Function</th>
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<td>AMCS</td>
<td>AMCS model</td>
</tr>
<tr>
<td>ACS</td>
<td>Simplified AMCS model</td>
</tr>
<tr>
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<td>In-line thrusters command logic</td>
</tr>
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<td>PTCHCMD</td>
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</tr>
<tr>
<td>LIBRCTL</td>
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</tr>
<tr>
<td>VARMAS</td>
<td>Tether variable characteristics computation</td>
</tr>
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<td>ORBEQS</td>
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<td>TENSION</td>
<td>Tether tension force acting on each body</td>
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</tr>
<tr>
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<td>Translational equations for each body</td>
</tr>
<tr>
<td>ROTEQ</td>
<td>Rotational equations (Satellite, Orbiter)</td>
</tr>
</tbody>
</table>

Table 1
The program has been conceived in a modular way so that the change of the generic module can be friendly performed, as well as new routines introduced and other bypassed.

The integrator is a 4th order Runge-Kutta algorithm.

The output of the program are:
- diagrams of selected variables;
- print-outs;
- restart file, which allows for the restart of the simulation from the end of a previous one;
- output file, stored on tape, containing the time history of all the variables needed as input for the verification and testing programs.

The dynamic part of the program is being checked via comparison runs to be performed with a qualified simulation program.

4 - REFERENCES

1 - C.S. Bodley, A.C. Park - TSS Orbital Dynamics Model 1B, Martin Marietta Corporation, June 1984, Contract no. NAS8-36000

2 - C.S. Bodley, A.C. Park, P.J. Grosserode - TSS Orbital Dynamics Model 1B (Revision A), Martin Marietta Corp.,
February 1985, Contract no. NAS8-36000

3 - DCAP User Manual - Vol 1 Theory, Aeritalia

ESA Contract no. 4045/79/NL/AK (SC)
TSS SUBSATELLITE
ATTITUDE DYNAMICS
AND
CONTROL LAWS VERIFICATION
PROGRAMS

TDS WORKSHOP '86
FRAMEWORK: VERIFICATION AND TESTING OF THE TSS SUBSATELLITE ATTITUDE MEASUREMENT AND CONTROL SUBSYSTEM (AMCS) DESIGN TO BE PERFORMED BY AERITALIA

SELECTED MISSION: 1ST ELECTRODYNAMIC MISSION
A dynamic analysis of the TSS system is required to support:
- The verification by analysis of the AMCS design;
- The AMCS integration/testing phase.
DYNAMIC ANALYSIS PROGRAM

* IT HAS BEEN DEVELOPED IN ORDER TO:

- INVESTIGATE THE SYSTEM DYNAMICS (WITH A PARTICULAR CARE FOR THE SATELLITE ATTITUDE), AND IDENTIFY POTENTIALLY CRITICAL AREAS FOR AMCS PERFORMANCE VERIFICATION;

- PROVIDE REALISTIC INPUT DATA FOR VERIFICATION AND TESTING PHASES.
SYSTEM FREQUENCIES

1st ax freq
Sat. roll freq
Sat. pitch freq
2nd ax freq
3rd ax freq
SYSTEM RESPONSE TO IN-LINE THRUSTERS FIRING

SYSTEM RESPONSE TO AN EXCITATION OF THE SAME FREQUENCY OF SUBSATELLITE OSCILLATIONS

TDS WORKSHOP '86
over 15000 m:
\[ l_2 = 4500 \text{ m} \]
\[ l_3 = 4500 \text{ m} \]
\[ l_4 = 2500 \text{ m} \]

over 17000 m:
\[ l_2 = 6600 \text{ m} \]
\[ l_3 = 6600 \text{ m} \]
\[ l_4 = 3300 \text{ m} \]

ACTUAL SYSTEM AND
MODEL FREQUENCIES

TDS WORKSHOP '86
* THE MOTION OF THE SYSTEM IS DESCRIBED WRT THE ACCELERATING REFERENCE
  FRAME (ARF).

* THE DIFFERENTIAL EQUATIONS ARE DERIVED IMPOSING FOR EACH BODY THE CONDITION
  OF DYNAMIC EQUILIBRIUM.

* THE TRANSLATION EQUATION FOR THE GENERIC BODY IS:

\[ M \ddot{\mathbf{x}} = - M \ddot{\mathbf{a}} + M \ddot{\mathbf{g}} + \mathbf{T} + \mathbf{f} \]

\( \mathbf{x} \) = BODY'S C.M. POSITION IN ARF
\( \ddot{\mathbf{a}} \) = ACCELERATION OF ARF ORIGIN IN THE INERTIAL FRAME
\( \ddot{\mathbf{g}} \) = LOCAL GRAVITY ACCELERATION
\( \mathbf{T} \) = FORCE(S) COMING FROM THE TETHER
\( \mathbf{f} \) = OTHER EXTERNAL FORCES (THRUSTER FIRING, IMPOSED PERTURBATIONS)

* THE ROTATIONAL MOTION IS DESCRIBED ABOUT THE CENTER OF MASS OF EACH BODY BY
  MEANS OF THE EULER EQUATIONS.
* OTHER FEATURES

- NO ENVIRONMENTAL MODEL
- ARBITRARY FORCES CAN BE IMPOSED ON EACH BODY
- SATELLITE THRUSTERS MISALIGNMENTS CONSIDERED
- ORBITER ATTITUDE AND SIMPLIFIED MODEL OF DAP/RCS
- NO DAMPING (CONSERVATIVE HYPOTHESIS)
- TETHER CONTROL LAW AND LIBRATION CONTROL LAW IMPLEMENTED
- IN PROGRESS:
  - DEPLOYABLE BOOMS ON THE SATELLITE
  - FLEXIBILITY OF THE DEPLOYER BOOM ON THE ORBITER
Program General Flow Chart

TDS Workshop '86
Validation of TSS Engineering Simulation

Zachary J. Galaboff
NASA Marshall Space Flight Center
BACKGROUND

MID '84 SES modification for Tethered Satellite System (TSS)

EARLY '85 SES requested test cases for comparison and validation
  about 25 test conditions were proposed
  Subsequently Martin's validation effort was deemed not
  in the scope of their contract

MAY '85 SES made some runs for comparison with the Shuttle Tethered
  Object Control Simulation (STOCS) Reasonable agreement
  SES began operations with crew participation
  Loss of tension/slack tether encountered

JULY '85 TSS Project Office requested assistance to corroborate
  validity of SES results
  Six cases were selected for Phase I-A comparison

AUGUST '85 Martin was authorized to produce results for the six cases

SEPTEMBER '85 Validation meeting for Phase I-A
PROBLEMS

Complexity of problem

- Highly non-linear
- "Stiff" equations (for short tether lengths)
- Computer time to real time greater than one

Dissimilarities of simulations

- Underlying assumptions
- Reference frames
- Models - atmosphere, tether, etc.
- Integration techniques
- Constraints
- Units

Dissimilarities of computer equipment/capabilities

- Word length/precision
- Plot capabilities
### TABLE I - TETHERED SATELLITE SYSTEM TEST ICs

<table>
<thead>
<tr>
<th>Set ID</th>
<th>ORBITER STATE</th>
<th>SATELLITE STATE</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>M50 position (ft)</td>
<td>M50 velocity (ft/sec)</td>
</tr>
<tr>
<td>1C1</td>
<td>21.8980048E6</td>
<td>0.0</td>
</tr>
<tr>
<td></td>
<td>0.0</td>
<td>22281.50</td>
</tr>
<tr>
<td></td>
<td>0.0</td>
<td>12097.87</td>
</tr>
<tr>
<td>1C1*</td>
<td>21.8980048E6</td>
<td>0.0</td>
</tr>
<tr>
<td></td>
<td>0.0</td>
<td>22281.50</td>
</tr>
<tr>
<td></td>
<td>0.0</td>
<td>12097.87</td>
</tr>
<tr>
<td>1C3</td>
<td>21.8980048E6</td>
<td>0.0</td>
</tr>
<tr>
<td></td>
<td>0.0</td>
<td>22281.50</td>
</tr>
<tr>
<td></td>
<td>0.0</td>
<td>12097.87</td>
</tr>
<tr>
<td>1C4</td>
<td>21.8980048E6</td>
<td>0.0</td>
</tr>
<tr>
<td></td>
<td>0.0</td>
<td>22281.50</td>
</tr>
<tr>
<td></td>
<td>0.0</td>
<td>12097.87</td>
</tr>
<tr>
<td>RUN ID</td>
<td>TEST CONDITIONS</td>
<td>TEST AND OBJECTIVES</td>
</tr>
<tr>
<td>--------</td>
<td>---------------------------------------------------------------------------------</td>
<td>----------------------------------------------------------</td>
</tr>
<tr>
<td>EOM1-A</td>
<td>IC1, no aerodynamics, central force field, reel active, $L_c$ = constant, tether stretch = 0.148 ft, tether mass = 0, attach point at center of mass (CM) of both the orbiter and satellite</td>
<td>Gravity Gradient Mode Check equations of motion, State propagation</td>
</tr>
<tr>
<td>EOM2-A</td>
<td>Same as EOM1-A, except tether stretch = 0.051137 ft and reel locked</td>
<td>Bob Weight Mode Two-body radial frequency response</td>
</tr>
<tr>
<td>EOM3</td>
<td>Same as EOM1-A, except use IC3 and tether stretch = 0.1377 ft</td>
<td>Pendulum Mode In-plane frequency and displacement</td>
</tr>
<tr>
<td>EOM5</td>
<td>Same as EOM1-A, except tether slack = 0.25 ft</td>
<td>Slack-Taut Mode Transition from zero to non-zero tension</td>
</tr>
</tbody>
</table>
## TABLE II - TETHERED SATELLITE SYSTEM DYNAMICS TEST RUN MATRIX - CONTINUED

<table>
<thead>
<tr>
<th>RUN ID</th>
<th>TEST CONDITIONS</th>
<th>TEST AND OBJECTIVES</th>
<th>TEST PROCEDURES</th>
</tr>
</thead>
<tbody>
<tr>
<td>SYST3-A</td>
<td>ICI*, no aerodynamics, central force field, reel active, tether stretch = 0.148 ft, tether mass = 0, attach point at center of mass (CM) of both orbiter and satellite, no thrusters</td>
<td>Servo Step Response Satellite and reel dynamics</td>
<td>Doublet in commanded length rate with hold on $L_C$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$0 &lt; t &lt; 150 \quad \dot{L}_C = \dot{L}_C(t)$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$L_C = L_C(t)$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$150 &lt; t &lt; 300 \quad \dot{L}_C = \dot{L}_C(t) + .5$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$L_C = L_C(t = 150)$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$300 &lt; t &lt; 450 \quad \dot{L}_C = \dot{L}_C(t)$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$L_C = L_C(t)$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$450 &lt; t \quad \dot{L}_C = \dot{L}_C(t) - .5$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$L_C = L_C(t = 450)$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Run 1200 seconds</td>
</tr>
<tr>
<td>TRAJ1</td>
<td>ICI, full environment, reel active, orbiter in vernier LVLH track, tether stretch = 0.148 ft, orbiter attach point at (1077.3, -.6, 847.24) in O-frame, satellite attach point (0, 0, -2.625) in body frame, no plume impingement</td>
<td>Nominal trajectory Satellite trajectory during retrieval with no initial dispersions</td>
<td>Run 3800 seconds</td>
</tr>
</tbody>
</table>

*IC1*
SUMMARY

Comparison of computer results is not the only way to validate a simulation. Ideally physical tests would be better. Impractical in this case. These four simulations are not all-inclusive and were written for specific purposes.

Phase I-A validation did not include the dynamics of the orbiter and the satellite. Phase I-B test cases/which include end body effects have been determined.

SES and STOCS have completed them
MSFC working on them
Martin awaiting funding

Since the Phase I-A validation meeting additional dynamic concerns have surfaced

- coning/pendulous motion of the satellite about tether
- coupling with slack in tether
- coupling with transverse wave dynamics
- tether damping tests
TSS-1 Dynamics Flight Experiments

Gordon E. Gullahorn
Smithsonian Astrophysical Observatory

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TSS-1 DYNAMICS FLIGHT EXPERIMENTS

G. GULLAHORN
SMITHSONIAN ASTROPHYSICAL OBSERVATORY

PURPOSE:
- DEVELOP MODELS OF TSS DYNAMIC NOISE.
- VERIFY AND REFINE THESE MODELS USING ACCELEROMETER AND
  GYRO DATA FROM THE FIRST TSS FLIGHT.

INVESTIGATION TEAM:
  PI: G.E. GULLAHORN, SAO.
  CO-I'S: M.D. GROSSI AND E.C. LORENZINI, SAO.
  W. KAHN, NASA-GSFC.

IN COOPERATION WITH THE PARALLEL INVESTIGATION OF PROF. S. BERGAMASCHI OF
THE UNIVERSITY OF PADOVA, ITALY.

TETHER DYNAMICS SIMULATION WORKSHOP, ARLINGTON, VA, 16 SEPTEMBER 1986.
TSS-1 EXPERIMENT ON DYNAMIC NOISE IN TETHERED SATELLITE SYSTEMS

- OBJECTIVES:
  (1) TO LEARN ABOUT THE DYNAMICS OF THIS NOVEL SYSTEM;
  (2) TO ASSESS SUITABILITY OF TETHERED SATELLITES TO
      ACCOMMODATE IN LATER FLIGHTS, SENSORS THAT REQUIRE
      LOW-DYNAMIC-NOISE ENVIRONMENT.

- INSTRUMENTATION SYSTEM USED IN EXPERIMENT WILL BE ACCELEROMETERS
  AND CYROS ALREADY ON BOARD TSS-1 SUBSATELLITE

- PRE-FLIGHT ACTIVITY
  - ANALYTICAL INVESTIGATION OF TSS
  - DETAILED MODELS AND COMPUTER SIMULATIONS
  - PREPARATION FOR POST-FLIGHT DATA ANALYSIS

- POST-FLIGHT ACTIVITIES
  - DATA REDUCTION, PROCESSING AND ANALYSIS
  - MODEL VERIFICATION AND IMPROVEMENT
  - OBTAIN ESTIMATES OF SYSTEM DAMPING
TSS-1 FLIGHT SYSTEM

SATELLITE:
- 500 KG
- 0.8 M RADIUS
- I = 80 KG M² ALONG ORBIT, 96 KG M² ACROSS ORBIT

TETHER:
- 20 KM AT FULL DEPLOYMENT
- MULTILAYER, CONDUCTING, KEVLAR STRENGTH MEMBER
- $\mu = 2$ G/CM, AE = $0.6 \times 10^{10}$ DYNE
- DAMPING AE' UNCERTAIN; ~ 10° DYNE-SEC (?)

DYNAMICS INSTRUMENTS:
- LINEAR ACCELEROMETERS
  - THREE AXES
  - PRECISION $10^{-5}$ G
  - TIME RESOLUTION < 1 SEC
- THREE AXIS RATE INTEGRATING GYROS

MISSION PROFILE:
- 36 HOURS ACTIVE, DEPLOYED
- 20 HOURS STATION KEEPING AT FULL DEPLOYMENT
- INTERMEDIATE STOPS DURING DEPLOYMENT AND/OR RETRIEVAL
TETHERED SATELLITE SYSTEM FREQUENCY SPECTRUM

- **DISCRETE: VIBRATION MODES**
  - **MASS/SPRING MODE** 0.03 Hz
  - **TETHER VIBRATION MODES**
    - **LATITUDINAL (STRING)** 0.002 n Hz  n = 1, 2, 3 ---
    - **LONGITUDINAL (BAR)** 0.2 n Hz
  - **SATELLITE ATTITUDE** 0.1 Hz
  - **DEPLOYMENT BOOM VIBRATION** 0.5 Hz

- **MAGNITUDES DEPEND ON EXCITATION:** E.G., CREW MEMBER "KICKING OFF"
  SHUTTLE WALL, ~ 10^-4 g

- **CONTINUUM: EXTERNAL PERTURBATION**
  - **ATMOSPHERIC DRAG FLUCTUATIONS**
    - **SPECTRUM:** UNKNOWN
    - **MAGNITUDE:** ~ 10^-3 g (ATMOSPHERIC MISSION)
VIBRATION MODES OF TSS
(EXCITED BY ATMOSPHERIC IRREGULARITY)

Orthogonal Component

Tangent Component

Log spectral magnitude

Tangent component
Orthogonal component

Frequency (Hz)
MODAL RESONANCES

THE (DECOUPLED) MODAL FREQUENCIES DEPEND ON TETHER LENGTH AND TENSION.

AS THE TETHER LENGTH IS VARIED DURING DEPLOYMENT OR RETRIEVAL, VARIOUS MODAL FREQUENCIES WILL BECOME MOMENTARILY EQUAL, I.E. THEY WILL TUNE THROUGH RESONANCE. ENERGY MAY BE EXCHANGED BETWEEN MODES.

THE (DECOUPLED) MODAL FREQUENCIES HAVE BEEN COMPUTED AND PLOTTED FOR:

• SATELLITE ATTITUDE OSCILLATIONS,
  \[ \omega = \sqrt{\frac{b}{I}} \frac{T^{1/2}}{L^{1/2}} = \sqrt{\frac{3bgm}{R_{orbit}I}} L^{1/2} \]

• SPRING-MASS OSCILLATIONS \[ \omega = \sqrt{\frac{EA/m}{L^{1/2}}} \]

• LONGITUDINAL TETHER OSCILLATIONS \[ \omega = n \pi \sqrt{\frac{AE/\mu}{L}} \]

• LATITUDINAL TETHER OSCILLATIONS
  \[ \omega = \frac{n\pi}{L} \sqrt{\frac{T}{\mu}} = n \pi \sqrt{\frac{3mg}{R_{orbit}\mu}} L^{-1/2} \]

(THE ACTUAL VALUES ARE PRELIMINARY.)

BELOW 1 KM DEPLOYED LENGTH, A 2 NEWTON IN-LINE THRUSTER HAS BEEN ASSUMED. SUCH A THRUSTER WILL BE USED ON THE SATELLITE TO ENSURE TENSION.
MODEL DEVELOPMENT

ANALYTIC AND SEMI-NUMERICAL:
- HIGHLIGHT INDIVIDUAL DECOUPLED MODES OR INTERACTIONS
- RESPONSE TO PERTURBATIONS
- EMPHASIZE PHYSICAL INSIGHT
- A COHERENT SET OF MODELS, SIMPLE TO MODERATELY COMPLEX

DETAILED SIMULATOR; ONE OR TWO APPROACHES FROM AMONG:
- EXTENSION OF ANALYTIC MODELS
- STREAMLINED LUMPED MASS (SKYHOOK) OR FINITE ELEMENT
- STOCHASTIC MODELING

SIMULATOR MUST INCLUDE SUCH PHENOMENA AS:
- ORBITER AND SATELLITE ATTITUDE DYNAMICS
- SATELLITE AERODYNAMICS, INCLUDING INSTRUMENT BOOMS
- DEPLOYER DYNAMICS, INCLUDING BOOM VIBRATIONS
- SATELLITE AND ORBITER THRUSTERS
- TETHER CONTROL

TETHER MATERIAL PROPERTIES MUST BE MODELED, INCLUDING DISSIPATION AND POSSIBLE NON-LINEARITIES DUE TO YARN-LIKE STRUCTURE.
EXPERIMENT LIST

MANY OF THESE EXPERIMENTS OR OBSERVATIONS, THOUGH CONCEPTUALLY DISTINCT, WOULD INVOLVE THE SAME OR OVERLAPPING DATA SPANS. THIS DETAILED BREAKDOWN HAS BEEN DONE PARTLY IN CASE POWER CONSIDERATIONS PROHIBIT OPERATION OF THE ACCELEROMETERS FOR THE ENTIRE MISSION.

- BASELINE ORBITER/DEPLOYER MOTIONS
- IMPULSIVELY PERTURBED ORBITER/DEPLOYER MOTIONS
- COMPLETE MISSION DYNAMIC PROFILE
- VARIABLE LENGTH SYSTEM DYNAMICS
- INITIAL DEPLOYMENT DYNAMICS
- TSS BEHAVIOR WITH IN-LINE THRUSTER
- OSCILLATIONS INDUCED BY THRUSTER CUTOFF
- VARIOUS MODAL RESONANCES
- SLOW TUNING THROUGH SPRING-MASS/SUBSATELLITE-ATTITUDE RESONANCE
- DYNAMICAL EFFECTS OF TETHER CURRENT JxB FORCES
- DYNAMICAL MODES EXCITED BY DEPLOYMENT/RETRIEVAL:
  - CESSION (FIXED LENGTH SYSTEM)
  - INITIATION (VARIABLE LENGTH SYSTEM)
- OBSERVATION OF DYNAMIC NOISE IN FIXED LENGTH SYSTEM
  - BACKGROUND, WITH OTHER EXPERIMENTS, OPERATIONS
  - DYNAMICALLY QUIET PERIOD
- OBSERVATION OF LONG-PERIOD MODES (BACKGROUND AND QUIET)
- IMPULSIVE PERTURBATION, IN-LINE
- IMPULSIVE PERTURBATION, TRANSVERSE
- PERIODIC PERTURBATION, IN-LINE
• PERIODIC PERTURBATION, TRANSVERSE
• DYNAMIC EFFECTS OF TERMINATOR CROSSING
• TETHER LIBRATION DURING RETRIEVAL
• RESPONSE TO SATELLITE SIDE-THRUSTERS DURING RETRIEVAL
Tether Simulation Design for Mission Planning and Analysis

Richard M. Deppisch
NASA Johnson Space Center

Yashvant Jani
LINCOM Corporation
AGENDA

- PURPOSE OF SIMULATION
- SIMULATION CHARACTERISTICS
- SIMULATION COMPONENTS
- CASE STUDY: VALIDATION
  - PROCEDURE DESCRIPTION
  - TESTS - SUBSYSTEM
    - SYSTEM
    - INTEGRATED
  - RESULTS
- FUTURE PLANS
  - SIMULATION UPDATES
  - FUTURE STUDIES
PURPOSE OF SIMULATION

- OBJECTIVE IS TO PROVIDE A TESTBED WITH THE FOLLOWING CAPABILITIES
  - VARIED FIDELITY DYNAMICS (SEVERAL EQUATIONS OF MOTION (EOM) AVAILABLE)
  - SEVERAL CONTROL LAWS AND SUBSYSTEM MODELS
  - DETAILED STUDY OF VARIOUS SEGMENTS OF THE MISSION TIMELINE
  - INTERACTIVE MAN-IN-THE-LOOP GRAPHICS FOR COMPLETE MISSION TIMELINE

- MISSION PLANNING AND ANALYSIS
  - TRADE STUDIES AND PERFORMANCE ENVELOPE
  - MISSION TIMELINE ANALYSIS
  - DETAILED DYNAMICS ANALYSIS

- CREW PROCEDURE DEVELOPMENT AND SUPPORT
  - MANUAL CONTROL PROCEDURES USING MAN-IN-THE-LOOP GRAPHICS

- PARTICIPATION IN PAYLOAD INTEGRATION PLAN (PIP) ANNEX TWO SUPPORT
SIMULATION CHARACTERISTICS

- SIMULATION RECONFIGURED QUICKLY AND EASILY
- FLEXIBLE, MENU-DRIVEN INPUT AND OUTPUT
- INTERFACE CAPABILITY
  - GRAPHICS
  - ON-BOARD DISPLAYS (*)
  - HAND-CONTROLLED (*)
- TRANSPORTABLE TO OTHER MACHINES (LANGUAGE EXTENSIONS NOT USED)
- USER CAN SELECT MODELS OF VARYING FIDELITY
  - MEDIUM FIDELITY MODELS (MODEL 1B, TETHER_1, TETHER_2)
  - HIGH FIDELITY MODELS (FINITE ELEMENT/LAGRANGIAN FORMULATION) (*)
  - INTEGRATION SCHEMES (*) AND VARIABLE INTEGRATION STEP-SIZE
SIMULATION COMPONENTS

- ENVIRONMENT AND SYSTEM MODELS
  - GRAVITY, AERODYNAMICS, GEOMAGNETIC POTENTIAL (*)
  - END-MASS THRUSTERS, REEL-MOTOR DRIVE
  - LENGTH AND LENGTH-RATE SENSORS, Tensiometer (*)
  - PHASE-PLANE LOGIC FOR THRUSTERS
  - IDEAL AND OPERATIONAL CONTROL LAWS

- COORDINATE SYSTEMS
  - EARTH CENTERED SPHERICAL
  - EARTH CENTERED CARTESIAN
  - KEPLERIAN
  - EARTH-FIXED (LATITUDE, LONGITUDE, ALTITUDE, INCLINATION FOR A CIRCULAR ORBIT)
  - TETHER FRAME (ORIGIN AT ONE ATTACH POINT, Z-AXIS ALONG TETHER LENGTH)
  - NADIR FRAME (ORIGIN AT CENTER-OF-MASS, Z-AXIS ALONG RADIUS VECTOR)
CASE STUDY EXAMPLE: VALIDATION

- OVERALL VALIDATION PROCEDURE
  
  - SUBSYSTEM TESTS SHOULD CHECK OUT COMPONENTS AND SUBROUTINES (UNIT TEST PROCEDURE)
  
  - SYSTEM TESTS SHOULD PRODUCE EXPECTED RESULTS UNDER APPROPRIATE SCENARIOS (e.g., TWO-BODY TRAJECTORY, PENDULUM MOTION, ETC.
  
  - INTEGRATED TESTS WITH MISSION TIMELINE SHOULD MATCH REFERENCE DOCUMENT (#TSS-01-M1B)
    
    - MUST ALSO CONFORM TO ENGINEERING ANALYSIS
CASE STUDY: VALIDATION

- TEST DESCRIPTION

  - SUBSYSTEM LEVEL TESTS
    - DRAG FORCES
    - DENSITY
    - LATITUDE, LONGITUDE, ALTITUDE
    - REEL MOTOR DRIVE
    - BOOM DYNAMICS
    - LENGTH CONTROL
    - INITIAL CONDITIONS

  - SYSTEM LEVEL TESTS
    - TRAJECTORY WITHOUT DRAG (TEST 1)
    - TRAJECTORY WITH DRAG (TEST 2)
    - LIBRATIONAL MOTION (TEST 3)
    - LIBRATIONAL CONTROL VIA REEL-MOTOR DRIVE AND THRUSTERS (TEST 4)
    - CONTROLLED DEPLOYMENT AND STOPPING AS A FUNCTION OF MISSION TIME (TEST 5)

  - INTEGRATED TESTS
    - ELECTRODYNAMIC MISSION:
      SATELLITE DEPLOYED UPWARD TO 20km (30 HOUR MISSION TIMELINE)
    - ATMOSPHERIC MISSION:
      SATELLITE DEPLOYED DOWNWARD TO 100km (30 HOUR MISSION TIMELINE)
<table>
<thead>
<tr>
<th>TEST ID</th>
<th>TEST OBJECTIVE</th>
<th>TEST CONDITIONS</th>
<th>DATA REQUIREMENTS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) Trajectory without</td>
<td>Verify that the Model 1B properly simulates the two-body trajectory</td>
<td>All external perturbing forces set to zero; circular orbit, spherical earth</td>
<td>Input - IREEL = ISIDT = ILENCOM = 0</td>
</tr>
<tr>
<td>perturbations</td>
<td></td>
<td></td>
<td>IAERO = 0</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>CLV = THR = 0</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Sim time = 10,000 sec</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Output - ground track, altitude</td>
</tr>
<tr>
<td>2) Trajectory with drag</td>
<td>Verify the TSS trajectory with aero drag</td>
<td>Same as 1 except aero drag on</td>
<td>Input - IREEL = ISIDT = ILENCOM = 0</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>IAERO = 1</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>CLV = THR = 0</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Sim time = 10,000 sec</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Output - ground track, altitude</td>
</tr>
<tr>
<td>3) Natural Libration</td>
<td>Verify that the Model 1B properly simulates natural librations (pendulum mode)</td>
<td>TSS orbital flight; All perturbing forces inactive; initial in-plane libration set at 30° offset; tether length = 10 km</td>
<td>Input - ω = 30°, ω = 15°</td>
</tr>
<tr>
<td>Test</td>
<td></td>
<td></td>
<td>IREEL = ISIDT = ILENCOM = 0</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>IAERO = CLV = THR = 0</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Output - ground track, altitude, tether parameters, rates and accelerations as applicable, tension, commanded length, length rate [Set 1]</td>
</tr>
<tr>
<td>4) Librational Control</td>
<td>Verify the performance of control laws, reel/motor drive, and thruster system</td>
<td>TSS orbital flight; All perturbing forces active; initial in-plane libration angle set at 30° offset; tether length = 10 km</td>
<td>Input - ω = 30°, ω = 15°</td>
</tr>
<tr>
<td></td>
<td>for libration control</td>
<td></td>
<td>IREEL = ISIDT = ILENCOM = 1</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>IAERO = 1</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Output - [Set 1]</td>
</tr>
<tr>
<td>5) Reel Motor drive</td>
<td>Verify tether deployment to 2 km, verify stopping of deployment, and verify in-plane and out-of-plane libration damping during deployment (via the reel-motor drive and side thrusters)</td>
<td>All perturbing forces are active, circular orbit, side thruster active, ideal sensors</td>
<td>Input - IAERO = ISIDT = ILENCOM = 1</td>
</tr>
<tr>
<td>and Inline thruster test</td>
<td></td>
<td></td>
<td>Sim time = 40,000 sec</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>XLF = 2000 m</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Output - [Set 1]</td>
</tr>
</tbody>
</table>
SYSTEM LEVEL TEST RESULTS

- **TRAJECTORY WITHOUT DRAG**
  - PROPER GROUNDTRACK WAS GENERATED (GROUNDTRACK SHIFTED BY $T_{orb} \times W_{Earth}$)
  - CIRCULAR ORBIT WAS MAINTAINED [RADIUS RATE = 0]
  - PROPER TRUE ANOMALY BEHAVIOR WAS OBSERVED
  - TEST RESULTS VERIFIED IMPLEMENTATION OF EOMs AND USER INTERACTIVE Routines

- **TRAJECTORY WITH DRAG**
  - PROPER RADIUS RATE AND DECREASE IN ALTITUDE OBSERVED
  - AERO TORQUES DEPEND ON INCLINATION
  - IMPLEMENTATION OF AERO AND DENSITY VERIFIED

- **LIBRATIONAL MOTION**
  - PROPER IN-PLANE AND OUT-OF-PLANE LIBRATIONAL MOTION OBSERVED
  - COUPLING BETWEEN IN-PLANE AND OUT-OF-PLANE LIBRATION SHOWN PROPERLY
  - VERIFIED IMPLEMENTATION OF EOM/MARTIN FOR $\ell$, $\theta$, AND $\phi$, ALONG WITH USER INTERACTIVE SUBROUTINES FOR INITIAL CONDITIONS AND PARAMETER SET-UP
MODEL 1B: TEST ONE

LATITUDE VS LONGITUDE

SHIFT = \( T_{\text{obs}} \times \omega_{\text{Earth}} \)

DATE: 7/29/86
MODEL 1B: TEST TWO  XINC=0.0

ORBITAL RADIUS (M x1.E+06) VS TIME

DATE: 8/05/86

MODEL 1B: TEST TWO  XINC=0.0

ORBITAL RADIUS RATE VS TIME

DATE: 8/05/86
MODEL 1B: TEST THREE
IN-PLANE LIBRATION vs TIME

MODEL 1B: TEST THREE
OUT OF PLANE LIBRATION vs TIME
SYSTEM LEVEL TEST RESULTS

- CONTROL OF LIBRATIONAL MOTION
  - REEL-MOTOR DRIVE AND THRUSTERS PROPERLY DAMP IN-PLANE LIBRATIONAL MOTION AND THRUSTERS SLOWLY DAMP OUT-OF-PLANE LIBRATIONAL MOTION
  - TENSION CHANGED PROPERLY TO CONTROL IN-PLANE LIBRATION
  - VERIFIED IMPLEMENTATION OF REEL-MOTOR DRIVE, SATELLITE THRUSTERS, AND LENGTH CONTROL ROUTINES.
  - PHASE-PLANE GAINS AND THRUST VALUE ARE TOO WEAK TO DAMP OUT LARGE EXCURSIONS IN LIBRATIONAL MOTION
  - REEL-MOTOR DRIVE IS NEEDED TO QUICKLY DAMP OUT IN-PLANE LIBRATIONAL MOTION

- CONTROLLED DEPLOYMENT
  - TEST RESULTS SHOW DEPLOYMENT TO 2km AND STABILIZATION AT COMMANDED LENGTH
  - VERIFIED IMPLEMENTATION OF LENGTH CONTROL AND IN-LINE THRUSTERS ALONG WITH REEL-MOTOR DRIVE TO STOP LENGTH RATE
  - INTERRELATIONSHIP BETWEEN THRUSTERS AND REEL-MOTOR DRIVE WAS ESTABLISHED VERY CLEARLY FOR PROPER DEPLOYMENT
MODEL 1B: TEST FOUR

IN-PLANE LIBRATION VS TIME

MODEL 1B: TEST FOUR

TENSION VS TIME
MODEL 1B : TEST FOUR
THD VS TH (PHASE PLANE)

DEADBAND

BURN 1

BURN 2

BURN 3

IN-PLANE LIBRATION RATE (DEG/SEC)

IN-PLANE LIBRATION, DEG

DATE: 8/4/86

MODEL 1B: TEST FOUR
X THRUST VS TIME

DATE: 8/04/86
MODEL 1B: TEST FOUR

PHID VS PHI (PHASE PLANE)

OUT-OF-PLANE LIBRATION RATE (DEG/SEC)

DEADBAND

OUT-OF-PLANE LIBRATION, DEG  DATE: 8/4/86

MODEL 1B: TEST FOUR

Y THRUST VS TIME

Y THRUST (N)

TIME (SEC)  (Thousands)  DATE: 8/04/86
INTEGRATED TEST RESULTS

- RESULTS WERE COMPARED WITH CARL BODLEY'S JUNE 1984 RESULTS (REPORT: TSS-M1B-01)

- PLOTS COMPARE VERY WELL
  - NO NOTICEABLE DIFFERENCES
  - ACCURACY OF COMPARISON LIMITED TO PLOTS AVAILABLE

- PROPORTIONAL PHASE-PLANE AND PROPORTIONAL JET THRUSTERS WERE REQUIRED IN ORDER TO MATCH THE RETRIEVAL PART OF THE ATMOSPHERIC MISSION (100KM DOWNWARD DEPLOYMENT)
MODEL 1B: UPWARD MISSION  XINC = 57

TETHER LENGTH VS TIME

TIME (HRS)  DATE: 8/11/86

Deployment  Retrieval

On Station

271
MODEL 1B: UPWARD MISSION  XINC = 57

LENGTH RATE VS. TIME

TIME (HRS)  DATE: B/11/85

Deployment

On Station

 Retrieval

LENGTH RATE (km/hr)

272
MODEL 1B: DOWNWARD MISSION

TETHER LENGTH VS TIME

DATE: 8/14/86

TIME (HRS)
MODEL 1B: DOWNWARD MISSION

LENGTH RATE VS. TIME

TIME (HRS)  DATE: 8/14/86

LENGTH RATE (KM/HR)

Deployment  On Station  Retrieval

140 130 120 110 100 90 80 70 60 50 40 30 20 10 0 -10 -20 -30 -40 -50 -60

0 4 8 12 16 20 24 28

1.4 1.0 0.6 0.2 0.0 -0.2

0 1.0

HOURS
FUTURE PLANS

- SIMULATION UPDATES
  - MATH MODELS
    - GOEMAGNETIC POTENTIAL
    - HIGH-FIDELITY SENSORS, TENSOMETER
  - FINITE ELEMENT TETHER
  - ON-BOARD DISPLAYS AND HAND CONTROLLERS

- PERFORMANCE STUDIES
  - SENSITIVITY OF SENSOR MEASUREMENT TO MISSION TIMELINE
  - SENSITIVITY OF CONTROL LAW GAINS
  - NAVIGATION MEASUREMENTS AND TRACKING
  - DETAILED DYNAMICS SIMULATION DURING INITIAL DEPLOYMENT AND FINAL RETRIEVAL
  - MANUAL CONTROL PROCEDURES
II

ADDITIONAL PRESENTATIONS
Preliminary Analysis of Damping Effect on Tethered Satellite

Giovanni Bianchini
University of Padova, Italy
PRELIMINARY ANALYSIS OF DAMPING EFFECT ON TETHERED SATELLITE

BY

G. BIANCHINI    UNIVERSITY OF PADOVA

A PRELIMINARY STUDY OF THE DAMPING EFFECT ON TSS HAS BEEN
PERFORMED BY NUMERICAL SIMULATIONS USING THE 'SKYHOOK'
PROGRAM.

SIMULATION PARAMETERS

- TETHER LENGTH       20 KM
- DISCRETIZATION      20 MASSES
- ROPE DIAMETER       2.5 MM
- YOUNG'S MODULUS     $7 \times 10^{11}$ DYNES/CM$^2$
- WIRE DENSITY        1.63 GR/CM$^3$
PRELIMINARY ANALYSIS OF DAMPING EFFECTS ON TETHERED SATELLITES

1. Plots of tension magnitude and the radial component of the sub-satellite motion have been produced.

2. From this data, frequency analysis has been performed for different initial conditions and values of the damping ratio.
CONCLUSION

IF THE DAMPING RATIO DECREASES TO ONE PERCENT, ALL THE LONGITUDINAL VIBRATION MODES PROVIDE A SIGNIFICANT CONTRIBUTION TO THE SUB-SATELLITE PERTURBATIONS.
### Test Cases of Tethered Satellite Damping Effect

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<th>Run No.</th>
<th>Date</th>
<th>Duration</th>
<th>VAX 750 CPU (SEC)</th>
<th>C-Value (Dyne/cm/sec)</th>
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284
3/8/86 20-masses step=0.1 sec c=0.01
Radial Component cm

TIME (sec)

9/3/86 20 mass run C=1.04, excit. = 20 cm mass 2 & 15
HANNING WINDOW

9/9/86 20mass run. C=1.e4. 200s
Subsat radial comp.
*** LOG. POWER SPECTRUM ***

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RUN # 3

SUBSAT TENSION

DATI SELEZIONATI

TIME SEC
Run #3  3/9/86 dati del 7/8 -- 18 sec c=1.75 e3 tension. Subsat
8/9/86  

run #4

mass 2  +20 cm  mass 15  -?0 cm
9/9/86 -- 20 mass run. C=1.e3 . 200s
Subsat radial component.
**ORIGINAL PAGE IS OF POOR QUALITY**

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**LOG. POWER SPECTRUM**

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***** AMPLITUDE SPECTRUM ******

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Rigorous Approaches to Tether Dynamics in Deployment and Retrieval

Ettore Antona
University of Torino, Italy
ETTORE ANTONA

DIPARTIMENTO DI INGEGNERIA AERONAUTICA
E SPAZIALE DEL POLITECNICO DI TORINO

RIGOROUS APPROACHES TO TETHER
DYNAMICS IN DEPLOYMENT
AND RETRIEVAL

AIAA - NASA - PSN - FIRST INTERNATIONAL
CONFERENCE ON TETHER IN SPACE

Arlington, VIRGINIA - September 1986

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Dynamics of Tethers in a linearized analysis can be considered as the superposition of propagating waves. This approach permits to have a new way for the analysis of a Tether behaviour during deployment and retrieval, were a Tether can be considered composed by a part at rest and a part subjected to propagation phenomena, being the separating section depending on time.

The dependence on time of the separating section requires the analysis of the reflection of the waves travelling toward the part at rest. Such a reflection generates a reflected wave, whose characteristics are determined.

The propagation phenomena of major interest in a Tether are transverse waves and longitudinal waves, all mathematically modelled by the "vibrating chord" equations, if the tension is considered constant along the Tether itself. An interesting problem also considered is concerned with the dependence of the Tether tension from the longitudinal position, due to microgravity, and the influence of this dependence on propagation waves.
INTRODUCTION

Dynamics of Tethers('), as well as of any structure, in a linearized analysis can be considered as the superposition of propagating waves. This requires the study of dynamic propagation along the Tether with the appropriate boundary conditions. During deployment and retrieval, with reference to a lagrangian reference system("), Tether can be considered as composed of two parts - one at the rest and one subjected to the propagation phenomena. These two parts are separated by a section that changes with time, i.e. the Tether section that bounds the part constrained to rest is changing with time (the other part being free to move and vibrate and having the opposite end section subjected to the boundary conditions imposed by the satellite).

The propagation phenomena of major interest from a practical point of view are the following.

1) Transverse waves, mainly a "vibrating chord" behaviour, where inertial forces and the tension in the Tether - in combination with its local curvature - are the most important elements of the dynamic equilibrium.

2) Longitudinal waves, mathematically modelled by the "vibrating chord" equation, where inertial forces and longitudinal internal forces, due to elastic deformations, are the most important elements of the dynamic equilibrium.

The dependence on time of the section which bounds the part at rest requires the analysis of the reflection of the wave travelling toward the part at rest. Such a reflection generates a wave travelling outward, whose characteristics are to be determined.

The A. had previously considered from a theoretical point of view such problem in particular in order to analyze the behaviour of deployable booms subjected to longitudinal and flexural dynamic phenomena (see (1), (2), (3) and (4)). Also in the case of the problems concerning a structure like the Tether, the theoretical analysis gives rigorous solutions and permit an insight into experimentally observed effects,

(') It can be suggested Ref. (8) for a general presentation of Tether concept and its (dynamic) problems.

(" ) A reference system which introduces a bi-univocal correspondence between a longitudinal coordinate and each Tether section.
that by some authors were erroneously thought to be "continuous" changes of frequencies and amplitudes of the proper modes.

This paper belongs to a series of works having the scope of opening a new way in the approach by means of mathematical models of several mechanical problems of deployable systems of telescopic and Tether type.

Recently several attempts have been made to solve the problem of the telescopic structures behaviour. Such attempts are mainly based in changes of the coordinates in order to take into account the changes during the time of the space where the problem is defined. As a matter of fact these attempts don't seem obtain good results. They don't take into account energetic balances.

On the contrary this work introduces and develops to some extent the basic idea of considering each dynamic motion in a structure as the results of wave propagation, taking also into account energetic exchanges at the ends.

In the case of vibrating chord the problem of the time dependence of the definition space can't be resolved by means of the Cauchy, Goursat and Darboux results, (5). These results deal with the problems of time depending location of the sections where are imposed the boundary conditions. In the part that is external to such sections dynamic phenomena take place that are coherent with phenomena acting in the internal part and contribute to supply or spillover energy in it.

This work deals with the request of having external parts at the rest (not only the boundary sections). Therefore boundary sections have the behaviour of surface where internal dynamic phenomena "reflect". Obviously reference is made to a constant section unaxial structure, as Tether can be considered. Longitudinal tension loads due to microgravity permits to consider additive small tension or compression loads without critical phenomena.
FUNDAMENTAL EQUATIONS

Let us have first a brief recall of concepts, with reference only to the case of longitudinal waves making use of Ref. (1). The reader can easily do the extension to transverse waves. The problem can be analyzed by means of an equilibrium and a continuity equation. If \( \sigma(x,t) \) is the stress at point \( x \) and time \( t \) and \( \mu = \mu(x,t) \) is the velocity of the motion, the equilibrium linearized equation is

\[ \frac{\partial \sigma}{\partial t} = -\frac{E}{c^2} \frac{\partial \mu}{\partial t} . \]

The continuity linearized equation on the other hand is

\[ \frac{\partial \mu}{\partial x} = -\frac{1}{E} \frac{\partial \sigma}{\partial t} . \]

If we put \( c^2 = E/\rho \), operating we obtain

\[ \frac{\partial^2 \sigma}{\partial x^2} = \frac{1}{c^2} \frac{\partial \sigma}{\partial t} . \]

\[ \frac{\partial^2 \mu}{\partial x^2} = \frac{1}{c^2} \frac{\partial \mu}{\partial t} . \]

The general solutions of eq.s 1') and 2') are

\[ \sigma = \psi_1(t - \frac{x}{c}) + \psi_2(t + \frac{x}{c}) , \]

\[ \mu = \mu_1(t - \frac{x}{c}) + \mu_2(t + \frac{x}{c}) . \]

Eqs. 3) and 4) indicate that the motion of the bar is composed of two waves: one travelling in the increasing \( x \) direction and another travelling in the decreasing \( x \) direction.

On the base of these considerations it is possible to obtain a relation between \( p \) and \( u \) of each travelling wave.
If \( t_1 = t - \frac{x}{c} \) and \( t_2 = t + \frac{x}{c} \), from 3) and 4) one obtains

5) \[
\frac{\partial \rho}{\partial x} = \frac{1}{c} \left( \frac{\partial \rho_1}{\partial t_2} - \frac{\partial \rho_2}{\partial t_1} \right),
\]

6) \[
\frac{\partial \mu}{\partial t} = \frac{\partial \mu_1}{\partial t_1} + \frac{\partial \mu_2}{\partial t_2}.
\]

Taking into account eq. 1) from 5) and 6) we have the relation

7) \[
\frac{1}{c} \left( \frac{\partial \rho_1}{\partial t_1} - \frac{\partial \rho_2}{\partial t_2} \right) = \frac{\mu_1}{\mu_2} \left( \frac{\partial \mu_1}{\partial x_1} + \frac{\partial \mu_2}{\partial x_2} \right).
\]

If \( \rho_1 = 0 \), \( \mu_2 = 0 \), eq. 7), gives the relation

\[
\frac{\partial \rho_1}{\partial t_1} = \frac{\mu_1}{\mu_2} \frac{\partial \mu_1}{\partial t_1} = \frac{E}{c} \frac{\partial \mu_1}{\partial t_1}.
\]

On the other hand if \( \rho_1 = 0 \), \( \mu_2 = 0 \), we have the relation

\[
\frac{\partial \rho_2}{\partial t_2} = -\frac{\mu_2}{\mu_1} \frac{\partial \mu_2}{\partial t_2} = -\frac{E}{c} \frac{\partial \mu_2}{\partial t_2}.
\]

With the initial conditions \( \mu_1(t_3) = 0 \), \( \rho_2(t_3) = 0 \) or (separately) \( \rho_1(t_3) = 0 \), \( \mu_2(t_3) = 0 \) the following relations between \( u \) and \( p \) hold for each travelling wave

8) \[
\mu_1(t - \frac{x}{c}) = \frac{c}{E} \rho_1(t - \frac{x}{c}),
\]

9) \[
\mu_2(t + \frac{x}{c}) = -\frac{c}{E} \rho_2(t + \frac{x}{c}).
\]

Such relations enable us to determine one of the two values \( \mu_1/\rho_1 \) when the other is known (eq. 8) and also to determine one of the values \( \mu_2/\rho_2 \) when the other is known (eq. 9). They are connected respectively with the separate behaviour of the two travelling waves.
REFLECTION ON THE TIME DEPENDENT SECTION

Also in this paragraph longitudinal waves are considered as a sample problem, still making use of Ref (1). The extension to transverse wave is easy. In practical applications the boundary conditions that are usually considered for uniaxial extensional bars are free edge \((\varphi=0)\) and fixed edge \((\varphi=\varphi_z)\). At the edge where deployment is done, it is \((\varphi(t))=0\), where \(\varphi_c=\varphi_c(t)\) is the time dependent section that can be considered as fixed and \(\frac{d\varphi_c}{dt}=\varphi\) is the velocity of displacement of the constraint. Here \(\varphi_c\) is an abscissa on the undeformed bar.

As a first analysis of the behaviour at a time dependent fixed section we can consider the problem of an extensional bar having a free edge at \(\varphi=0\) and constrained with \(\varphi(\varphi_c)=0\) at \(\varphi_c=\ell_0+\varphi,\) where \(\ell,\varphi\) are constant. The bar is subjected in \(\varphi=0\) to an external extensional specific force \(\varphi_1\) independent of the time \(t\).

Such force produces a wave travelling in the direction of the increasing \(\varphi\) when such wave reaches the moving constraint section \(\varphi_c\), a reflected wave of specific force \(\varphi_2\) is generated that runs in the direction of the decreasing \(\varphi\).

We will now determine the characteristic of the reflected wave, before it reaches the section \(\varphi=0\).

In order to determine the reflected wave by means of an energy balance is necessary to dispose of an evaluation of the energy exchanged at the constrained end. A discussion on this subject is performed in ref. (1). The conclusion is that the constraint has an energy exchange different from zero \(e\) that the reflected wave can be determined by means of a behavioural analysis like the following.

During the time we have the already introduced displacement of the section where the constraint is imposed. Such displacement corresponds to the internal deformation of the rod.

The interval \(dx=vt\) during \(dt\), withstands a length change \(dl=-(u_1-u_2)dt\) with a strain \(\varepsilon=\frac{dx}{\sqrt{v dt}}\).

On the other hand the final stress in \(\frac{v dt}{\ell}\) must be \(\sigma=\sigma_1+\sigma_L=\frac{1}{c}(u_1-u_2)\), that means a strain \(\varepsilon=\frac{u_1-u_2}{c}\).
Equating the two expressions of the strain gives

\[ \mu_2 = -\mu_1 \frac{c - \omega}{c + \omega} . \]

This result coincides with that of the application of the Goursat Darboux and Cauchy problem solution, (5), with the condition \( \mu = 0 \) at \( z = C + \sqrt{a} \).

In spite of the observed coincidence with the well known results of the vibrating chord analysis, the proposed model presents the advantage of the applicability to more complex problems as dispersive systems, (see for instance (6) and (3)).
EXTENSION TO TIME DEPENDENT AMPLITUDE (OF THE TRAVELLING WAVE) AND SPEED (OF THE BOUNDARY CONDITION)

Let us consider in a non dispersive system the wave $u_1$, travelling inward the time variable restraint and the $u_2$, travelling outward expanded as follows, (see also (2)),

$$
\begin{align*}
  u_1 &= \alpha_0 + \alpha_1(x-ct) + \alpha_2(x-ct)^2 + \cdots, \\
  u_2 &= \beta_0 + \beta_1(x+ct) + \beta_2(x+ct)^2 + \cdots.
\end{align*}
$$

If at a time $t$ the restraint condition is

$$
  u(x,t) = 0 \quad (x \geq x_1)
$$

and at the time $t + \Delta t$ where $\Delta t$ will tends to zero,

$$
  u(x,t + \Delta t) = 0 \quad (x \geq x_1 + c\Delta t),
$$

without loss in generality we can put $x_1 = 0$ and introduce a dummy variable $\tau$ such that $\tau < t < t + \Delta t$.

During $\Delta t$ at the section $x_1$, the displacement

$$
\Delta \beta = \int \left[ u_1(0, \tau) + u_2(0, \tau) \right] d\tau
$$

take place. During the same $\Delta t$ in the region $x > 0$ the internal deformation generates a change in length

$$
\Delta \beta = \int \left[ u_1(\tau, \Delta t) - u_2(\tau, \Delta t) \right] \frac{\tau}{c} d\tau.
$$

The equation between such two $\Delta \beta$, substituting (14) and taking into account only the terms of the lower order for respect to the principal $\Delta t$, operating gives,

$$
(1 + \frac{\tau}{c}) \beta_0 = \alpha_0 \left( 1 - \frac{\tau}{c} \right)
$$
or

$$
\beta_0 \frac{\tau}{c} = -\alpha_0 \left( 1 - \frac{\tau}{c} \right).
$$

Relation (14) means that, beside the higher order terms,

$$
  u_2 \approx -u_1 \left( 1 - \frac{\tau}{c} \right),
$$

like in the case of $u_1$ and $u_2$ of constant value.
The present paragraph concerns the extension of the analysis to the case where eq. 1') and 2') become as follows
\[
\frac{\partial^2 \sigma}{\partial x^2} = \frac{k}{c^2(x)} \frac{\partial^2 \sigma}{\partial t^2}, \quad 1'')
\]
\[
\frac{\partial^2 \mu}{\partial x^2} = \frac{k}{c^2(x)} \frac{\partial^2 \mu}{\partial t^2}. \quad 2'')
\]
This case includes the dynamics of transverse waves in Tethers where longitudinal load is depending on the position due to microgravity. Because \(c(x)\) is not constant but a function of \(x\), \(c = c(x)\), expressions 3) and 4) are not still valid, and it is necessary to find an appropriate way of solution.

The A. in a previous paper, (7) here largely recalled, proposed that the general solution of eqs 1'') were composed by means of two waves in opposite directions travelling and having speed depending on \(x\). Such waves reduce to eqs 3) and 4) when \(c(x)\) reduce to a constant \(C\). If \(\phi\) indicates \(\phi_1\) or \(\phi_2\) as necessary, the following expression was adopted (')
\[
\phi = \phi_1\left(t - \int \frac{dx}{k\epsilon(x)} \right) + \phi_2\left(t + \int \frac{dx}{k\epsilon(x)} \right), \quad 1'''
\]
where \(k\) is an arbitrary constraint and \(\pm k \in \epsilon(x)\) could be the propagation speed at \(x\).

Obviously, not all the functions \(\phi_1\) and \(\phi_2\) are useful to satisfy 1''). The problem is now reduced to the determination of the functions \(\phi_1\) and \(\phi_2\) if any, that can satisfy 1'').

The functions \(\phi_1\) and \(\phi_2\) can be examined separately.

If we put \(\phi(x) = t - \int \frac{dx}{k\epsilon(x)}\), we have
\[
\frac{\partial \phi_1}{\partial x} = \frac{d\phi}{dt} \frac{\partial t}{\partial x} = \frac{d\phi_1}{dt} \left(-\frac{1}{k\epsilon(x)} \right),
\]
\[
\frac{\partial^2 \phi_1}{\partial x^2} = \frac{d^2\phi_1}{dt^2} \left(1 \right) + \frac{d\phi}{dt} \left( \frac{1}{k} \frac{c'(x)}{c^2(x)} \right), \quad (c'(x) = \frac{d c(x)}{d x}),
\]
\[
\frac{\partial \phi_2}{\partial t} = \frac{d\phi}{dt} \frac{\partial t}{\partial t} = \frac{d\phi_2}{dt} \left(1 \right), \quad \frac{\partial^2 \phi_2}{\partial t^2} = \frac{d^2\phi_2}{dt^2}.
\]

(') When \(c(x)\) const. and \(k=1\), the proposed expression reduces to 3) and 4).
Recalling now 1) we obtain
\[ \ddot{\phi} + \frac{K}{m^2} \dot{\phi} + \frac{C}{m} \phi = 0. \quad (16) \]
The function \( \phi \); if it there exists, must satisfy eq. (16). To the same conclusion we lead if we consider \( \phi_2 \).

As an application we can now restrict to the case \( C = C_0 x \).
We have \( C = C_0 \) and
\[ (\lambda - \frac{1}{k^2}) \frac{d^2 \phi_1}{d^2} - \frac{C_0}{k} \frac{d \phi_1}{d^2} = 0 \quad (48') \]
This is a linear ordinary second order equation.
Its solution is of the form \( \phi_i e^{\lambda x} \) and precisely
\[ \phi_1 = \exp \left( -\frac{1}{1-k^2} \ln (x) \right) \quad (12) \]
If we consider \( \phi_2 \) we obtain also
\[ \phi_2 = \exp \left( \frac{1}{1-k^2} (kC + \ln x) \right) \quad (12') \]
In the previous analysis \( k \) is an arbitrary constant. If we let \( k \) assume all the values \( 0 \sim k < \infty \) we obtain \( \phi_1 \) as \( \phi_2 \) and complete sets of functions, which allow us to expand by integral whatever function. Each dynamic phenomenon in a structure where eq.s 1) are valid and \( C = C_0 x \) can be analysed as, (see 17) and 17'),

\[ \phi = \int_{0}^{\infty} \phi_1 (k) \exp \left( \frac{1}{1-k^2} (kC + \ln x) \right) dk + \]
\[ + \int_{0}^{\infty} \phi_2 (k) \exp \left( \frac{1}{1-k^2} (kC + \ln x) \right) dk. \quad (18) \]

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Obviously 18) is not the only way by which to expand by integral such a dynamic phenomenon, but this way allows us to consider component functions when the propagation speed at any $x$ is well known. Such speed, as we know, is a fundamental datum in order to evaluate the speed of a reflected wave, in particular in the case of time depending restraint conditions.
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(3) E. ANTONA: Dynamic problems on time variable restraints in dispersive systems. (To be Published).


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III

WORKSHOP SUMMARY
AND
PANEL DISCUSSION RESULTS
SUMMARY OF THE SEPTEMBER 16
TETHER DYNAMICS SIMULATION WORKSHOP

Charles C. Rupp
George C. Marshall Space Flight Center

The objective of the Tether Dynamics Simulation Workshop is to provide a forum to discuss the structure and status of existing computer programs which are used to simulate the dynamics of a variety of tether applications. The reasons for having different simulation models and how the simulations are verified is discussed. Recommendations made during a panel discussion regarding the direction of future work are presented.

INTRODUCTION

This summary was prepared by a Tether Dynamics Simulation Review Committee consisting of the following members:

Peter M. Bainum  American Astronautical Society/Howard University
William A. Baracat  General Research Corporation
Silvio Bergamaschi  University of Padova, Italy
Paul A. Penzo  Jet Propulsion Laboratory
Charles C. Rupp  George C. Marshall Space Flight Center

The workshop featured presentations on eleven simulation models and special presentations on the validation of the Tethered Satellite System Engineering Simulation at the Johnson Space Center and the dynamics flight experiments to be conducted on the first flight of the Tethered Satellite System. A panel composed of preselected workshop participants discussed five issues regarding future simulation validation activities. Approximately 120 people participated in the workshop.

RECOMMENDED FUTURE SIMULATION DEVELOPMENT

Comments that were received during the presentations of the simulation models are summarized as follows:

1. Incorporate sensor hardware and observer measurement dynamics.
2. Improve simulation of tether transient thermodynamic effects such as rapid heating and cooling during terminator crossing.

3. Resolve different approaches to atmospheric drag models.

4. More experimental information is required on tether material damping and thermoelastic effects.

5. Simplify simulations where possible.

PANEL DISCUSSION TOPICS

Five topics were selected for the panel discussion with the intent of provoking opposing viewpoints. However, little disagreement was found among the panel members and the members of the audience who participated. These topics, the names of the panel members leading the discussion, and the conclusions follow.

TOPIC 1:

Howard Flanders
Martin Marietta Denver Aerospace
Henry Wolf
Analytical Mechanics Associates

Should a universal simulation program be developed?

a. General consensus is no.
b. Desirable in theory, but not practical at this time.
c. A universal program will be difficult to verify, costly to maintain, not likely to be widely and readily accepted.
d. There may not exist such a program—various applications may require their own "best" program.
e. A library of special purpose application programs or subroutines might be useful.
f. These programs should be mini or micro-computer usable, well documented, and user-friendly.
g. An alternative to a universal program development would be better documentation and use of existing programs.

TOPIC 2:

David A. Arnold
Smithsonian Astrophysical Observatory
Silvio Bergamaschi
University of Padova, Italy

What accuracy is required in tether dynamics simulations?

a. The simulation model should be selected based on the mission goals and the mission phase.
b. The influence of various factors should be studied individually to determine the magnitude of their effect.

c. Effects should be included in the simulation according to their influence on the particular problem.

d. Accuracy verification of the selected simulation should be made with a sample case of the complete simulation, and comparison of results with an independent program.

e. Results depend not only on the accuracy of the computer algorithms, but on the accuracy of the physical parameters.

TOPIC 3:

Joe Carroll       Energy Science Laboratory
Charles C. Rupp   George C. Marshall Space Flight Center

What methods should be used to verify simulations?

a. Mathematical confirmation can be used to compute simple cases whose results are known, compare with other simulation programs, and provide a self-test on the level of accuracy.

b. Physical confirmation tests can be performed in a laboratory to determine tether physical characteristics, perform dynamics and control system tests, and test tether motion in a large vacuum chamber. The KC-135 can provide zero-G tests of tethers and system components.

TOPIC 4:

Enrico Lorenzini       Smithsonian Astrophysical Observatory
Paul A. Penzo          Jet Propulsion Laboratory

Who should be responsible for simulation verification?

a. The sponsoring agency should ultimately be responsible for verification.

b. Possible options for the verification effort include in-house, unbiased third party, and/or a formal professional committee (e.g. AIAA-AAS connected).

c. The methodology should include providing sufficient updated documentation for each model and establishing, case by case, a set of test runs.
TOPIC 5:

Peter M. Bainum  
American Astronautical Society/Howard University

Bill Djinis  
NASA Headquarters

Should a peer process be established? How?

a. Numerous simulation programs have been developed under various contracts.

b. The complexity and accuracy required of simulations will depend on use (preliminary design, particular phase of mission support, experimental flight dynamics verification, future applications).

c. Small new projects may not be able to afford development of their own simulations and could benefit from a group of expert peers.

d. The panel recommends formation of a working group to study peer selection, availability, and financial support.

RECOMMENDATIONS

The final recommendations of the workshop are as follows:

1. Establish a joint tether applications working group with the following objectives:

   a. Serve as a focal point for dynamics studies for tether applications.
   b. Advocate simulation validation.
   c. Maintain a simulation capabilities catalog.
   d. Perform periodic reviews of dynamics issues.
   e. Provide recommendations to management on resource requirements.

2. Pursue ground and flight dynamics experimentation.
TETHER DYNAMICS SIMULATION WORKSHOP
Tuesday, 16 September 1986
PROGRAM

7:45 Registration/Continental Breakfast

8:15 Introduction
Chris Rupp
NASA Marshall Space Flight Center
William A. Baracat
General Research Corporation

Simulation Descriptions

8:30 Tether Dynamics Simulation
Vinod J. Modi
University of British Columbia, Canada
Arun K. Misra
McGill University, Canada

9:00 HIFITS Simulation
Howard Flanders
Martin Marietta Denver Aerospace

9:30 Tethered Satellite System on the Systems Engineering Simulator
Ron Humble
Lockheed Missiles and Space Company
David Harshman
Rockwell Space Operations Company

10:15 GTOSS
David D. Lang
David D. Lang Associates

10:45 Tethered Satellites Simulation
John R. Glaese
Control Dynamics Company

11:20 Skyhook Program
David A. Arnold
Smithsonian Astrophysical Observatory

11:40 Slack3
Gordon E. Gullahorn
Smithsonian Astrophysical Observatory

12:00 Artificial Gravity Laboratory
Enrico Lorenzini
Smithsonian Astrophysical Observatory

12:20 Lunch/Computer Simulation Demonstrations

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**Simulation Descriptions - Continued**

**PM**

1:30 Elasticity Effects in Tether Dynamics  
Silvio Bergamaschi  
University of Padova, Italy

2:00 TSS Subsatellite Attitude Dynamics and Control Laws Verification Programs  
Floriano Venditti  
Aeritalia Space Systems Group, Italy

2:45 Validation of TSS Engineering Simulation  
Zachary Galaboff  
NASA Marshall Space Flight Center

3:15 TSS-l Dynamics Flight Experiments  
Gordon E. Gullahorn  
Smithsonian Astrophysical Observatory

3:30 Tether Simulation Design for Mission Planning and Analysis  
Richard Deppisch  
NASA Johnson Space Center  
Yashvant Jani  
LinCom Corporation

**4:00 Panel Discussion on Future Validation Activities**

**Topic 1:** Howard Flanders, Henry Wolf: Should a single universal simulation program on tether dynamics be built?

**Topic 2:** Dave Arnold, Silvio Bergamaschi: How accurately must simulations be able to compute tether system dynamics?

**Topic 3:** Joe Carroll, Chris Rupp: What verification methods should be employed?

**Topic 4:** Enrico Lorinzini, Paul Penzo: Who should be responsible for simulation verification?

**Topic 5:** Peter Bainum, Bill Djinis: Should a peer process be established?

**5:30 Adjournment**

**6:00 Executive Committee Meeting - Report to be presented Friday 10:00.**
APPENDIX B

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<td>The proceedings of the first Tether Dynamics Simulation Workshop are presented. The workshop was held at the Hyatt Regency Crystal City Hotel in Arlington, Virginia, September 16, 1986, in conjunction with the International Conference on Tethers in Space. The objective of this workshop was to provide a forum for the discussion of the structure and status of existing computer programs which are used to simulate the dynamics of a variety of tether applications in space. A major topic of the workshop was the purpose of having different simulation models and the process of validating them. Guidance on future work in these areas was obtained from a panel discussion; the panel was composed of resource and technical managers and dynamic analysts in the tether field. The conclusions of this panel are also presented in this document.</td>
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