FEASIBILITY ANALYSIS OF RECIPROCATING MAGNETIC HEAT PUMPS

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Objective:

The goal of the project is to study the commercial feasibility of magnetic heat pumps and refrigerators operating near room temperature. An analytical Phase Zero study is being conducted to quantify the expected performance and economics of commercial magnetic thermodynamic devices and to recommend whether a Phase One program should be started to include experimental R and D.

Background:

Until recently, interest in magnetic heat pumps and refrigerators was limited to temperatures under 20K. In 1976, Brown [1,2] at NASA/Lewis suggested the possibility of practical magnetic devices operating near room temperature. Noting that all heating/cooling effects in magnetic materials are greatest near the Curie point, he considered available materials, e.g. gadolinium, with Curie points near room temperature. Several thermodynamic cycles were considered. From those, the magnetic Stirling cycle with regeneration was selected for further study. A reciprocating, porous gadolinium core in a column of regenerative fluid inside the bore of a superconducting magnet was envisioned. A proof-of-concept laboratory device was successfully demonstrated.

Status:

The conceptual design selected for detailed system analysis and optimization is the reciprocating gadolinium core in a regenerative fluid column within the bore of a superconducting magnet. The thermodynamic properties of gadolinium are given by Griffel, et al. [3], Brown [4], and Benford and Brown [5]. The general thermodynamic relations for magnetic materials are given by Hatsopoulos and Keenan [6] and Booker [7]. The relation between the applied fields (no magnetic material present) and the internal fields in the gadolinium in place is taken to be that of an ellipsoid of gadolinium [8,9]. Initially, it is assumed that the gadolinium does not perturb the sources of the applied field. The forces are calculated on the basis of the approximation of a small dipole in a field gradient [8].

A computerized literature search for relevant papers has been conducted and is being analyzed. Contact has been made with suppliers of superconducting magnets and accessories, magnetic materials, and various types of hardware. A description of the model for the thermal analysis of the core and regenerator fluids is included in the following section.

Thermal Analysis of Regenerator Column:

The thermal analysis of the regenerator fluid and gadolinium core is presently based upon a one-dimensional, transient model. This model will be used to identify important design and performance parameters and to identify areas which require further investigation.
The assumptions listed for the model in the previous status report [10] have been altered:

1. Viscous forces are neglected.

2. Thermal equilibrium has been assumed between the gadolinium and the fluid at any cross section in the regenerator column. A one-temperature model follows in which the heat transfer coefficient is not required.

To facilitate the computation, the coordinate systems have been changed. Now two coordinate systems are used. The first (x,y) is fixed to one end of the regenerator column. The second (x', y') is fixed at the magnet center.

The energy equation based on the assumption of local thermal equilibrium is:

\[
\rho_s (1-\varepsilon) T \frac{\partial s}{\partial T} \left|_{\rho, vH} + \rho_f c_f \right] \frac{\partial T}{\partial t} + \left[ \rho_s T \frac{\partial s}{\partial T} \left|_{\rho, vH} - \rho_f c_f \right] (1-\varepsilon) V \frac{\partial T}{\partial t} \right]
\]

\[
+ \left[ \rho_s (1-\varepsilon) T \frac{\partial s}{\partial (vH)} \right] \left( \frac{\partial (vH)}{\partial t} + V \frac{\partial (vH)}{\partial x} \right)
\]

\[
= [k_f \varepsilon + k_s (1-\varepsilon)] \frac{\partial^2 T}{\partial x^2}
\]

where

- $C_{pf}$ - constant pressure specific heat of the fluid
- $\varepsilon$ - porosity
- $H$ - intensity of the magnetic field
- $\rho_s(f)$ - density of the Gd (fluid)
- $s$ - entropy of Gd
- $T$ - temperature
- $t$ - time
- $V$ - velocity of the plug relative to the regenerator column
- $v$ - specific volume of Gd
- $x$ - column fixed coordinate along the regenerator axis.
\[
\frac{\partial (v\vec{H})}{\partial t} + \vec{v} \frac{\partial (v\vec{H})}{\partial x}
\]
is the total change in the field intensity observed at the magnetic material. The first term appears because the field appears to be time varying relative to the column due to the column motion. This term can be referenced to the magnet fixed coordinates which removes the time varying component. Then

\[
\left[ \frac{\partial (v\vec{H})}{\partial t} + \vec{v} \frac{\partial (v\vec{H})}{\partial x} \right]_{x,y} = \left[ \frac{\partial (v\vec{H})}{\partial x} \right]_{x,y} \vec{V}_{abs}
\]

where \( \vec{V}_{abs} = \vec{V}_{col} + \vec{V}_{plug/col} \)

If \( \varepsilon = 1 \) we are outside the Gd and the equation is

\[
\rho_f C_p \frac{\partial T}{\partial t} = k_f \frac{\partial^2 T}{\partial x^2}
\]

As expected, the energy equation reduces to conduction in the fluid.

**Boundary Conditions:**

1. Initially adiabatic at the regenerator column ends:
   \[
   \frac{\partial T}{\partial x} = 0
   \]
   At the plug ends:
   \[
   T_{plug} = T_{fluid}
   \]
2. \[ T_{plug} = T_{fluid} \]
3. \[
\left[ k_s (1-\varepsilon) + k_f \varepsilon \right] \frac{\partial T}{\partial x} \bigg|_{plug} = k_f \frac{\partial T}{\partial x} \bigg|_{fluid}
\]

These conditions represent the continuity of temperature and heat flux.

The energy equation was implemented on the computer using a finite difference scheme. Non-dimensionalizing the equation was attempted. This proves to be very difficult due to non-constant coefficients and the lack of global geometric scales that can be used for references. A dimensional approach was adopted for the present.

**Problems in implementation:**

(1) **Node Types.**
   The nodes are picked to be fixed relative to the regenerator column. As the plug moves along the column the character of the nodes changes. Also, the plug-fluid boundary is in general not at a node location. The numerical scheme must recognize different node types and use various schemes to calculate new temperatures. This leads to bookkeeping problems with the nodes.
(2) Stability and Convergence.
This is the usual problem with the ratio of step sizes. The space increment $\Delta t$ is computed to keep the coefficients in the numerical scheme positive. The increment $\Delta t$ is also checked so that the plug advances by fractions of a space step.

(3) Convective Terms.
The convective terms (those with $\frac{\partial T}{\partial x}$) had to be treated as one-sided upwind difference to improve stability. As $V$, the relative velocity between column and plug changes sign, the $\frac{\partial T}{\partial x}$ terms change relative to the upwind directions. Higher order difference was also tried but did not improve stability.

Results:

Some initial predictions from the model are given in Figures 1-3.

The regenerator column has a length of 1m, the gadolinium plug, 0.2m. The fluid is half water, half methyl alcohol by volume. The porosity of the gadolinium (open area/section area) is 80%. The cylinder and ends of the regenerator are adiabatic. The vacuum field of the superconducting magnet is assumed constant in time and given by American Magnetics for their 8 Tesla unit operating at 6T maximum.

At the start of the first cycle, the fluid and gadolinium are uniform in temperature at 295K. The end of the regenerator nearest to the magnet is 1m from magnet center and the gadolinium is at the near end. After a half-cycle, the gadolinium and magnet are concentric, and the gadolinium is at the other end of the regenerator.

Figure 1 shows the temperature profile within the regenerator after 14.5 and 15 cycles. The cycle period is 37 seconds with no pauses and with velocities given by appropriate step functions. During this computer run, the minimum separation between the gadolinium and regenerator end was 5cm at each end. The column position, 0.00, marks the near end of the column at the start of a cycle. The drop in temperature at the other end between the two profiles shows the cooling effect of removing the gadolinium from the magnet (followed by repositioning the column to complete the cycle). At present the model includes thermal conduction in the column, but excludes mixing. The gradients at the column ends reveal that axial conduction has a minor effect.

Figure 2 shows the maximum and maximum temperatures in the column after each half-cycle. The nearly isothermal sections again reveal the minor effect of axial conduction. The curvatures suggest that asymptotes will be approached, but the $\Delta T$ is already close to the value found in the experiment by Brown and Pappel.

Figure 3 gives the results of a run in which there is no dead space between the gadolinium and the ends of the regenerator. The temperatures at the column ends are now changing with each cycle.

Current Plans:
Next we plan to incorporate mixing into the model and also the transfer of energy between load and sink.
REFERENCES


FIGURE 1: COLUMN TEMPERATURE PROFILE AT 14.5 AND 15 CYCLES.
FIGURE 2: MAXIMUM AND MINIMUM TEMPERATURE VERSUS TIME
FIGURE 3: COLUMN TEMPERATURE PROFILE AFTER 14.5 CYCLES

[Temperature vs. Column Position Graph]

0.00  0.12  0.25  0.50  0.62  0.87  1.00
0  320.00  310.00  300.00  290.00  280.00  270.00

TEMPERATURE (K)

COLUMN POSITION (M)