Expansion of Epicyclic Gear Dynamic Analysis Program

Final Report

by

Linda Smith Boyd
and
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Hamilton Standard
Division of United Technologies Corporation
Windsor Locks, CT. 06096

Prepared for
National Aeronautics and Space Administration
NASA Lewis Research Center
Contract NAS3-24614
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The modifications refine the options for the flexible carrier and flexible ring gear rim and adds three new options: a floating sun gear option, a natural frequencies option, and a finite element compliance formulation for helical gear teeth. The option for a floating sun incorporates two additional degrees of freedom at the sun center. The natural frequency option evaluates planetary, star or differential systems' frequencies as well as the effect of additional springs at the sun center and those due to a flexible carrier and/or ring gear rim. The helical tooth pair calculated finite element compliance is obtained from an automated element breakup of the helical teeth and then is used with the basic gear dynamic solution and stress postprocessing routines. The flexible carrier or ring gear rim option for planetary and star spur gear systems allows the output torque per carrier and ring gear rim segment to vary based on the dynamic response of the entire system, while the total output torque remains constant.
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The modifications refine the options for the flexible carrier and flexible ring gear rim and adds three new options: a floating sun gear option, a natural frequencies option, and a finite element compliance formulation for helical gear teeth. The option for a floating sun incorporates two additional degrees of freedom at the sun center. The natural frequency option evaluates planetary, star or differential systems' frequencies as well as the effect of additional springs at the sun center and those due to a flexible carrier and/or ring gear rim. The helical tooth pair calculated finite element compliance is obtained from an automated element breakup of the helical teeth and then is used with the basic gear dynamic solution and stress postprocessing routines. The flexible carrier or ring gear rim option for planetary and star spur gear systems allows the output torque per carrier and ring gear rim segment to vary based on the dynamic response of the entire system, while the total output torque remains constant.
ABSTRACT

The multiple mesh/single stage dynamics program is a gear tooth analysis program which determines detailed geometry, dynamic loads, stresses and surface damage factors. The program can analyze a variety of both epicyclic and single mesh systems with spur or helical gear teeth including internal, external, and buttress tooth forms.

The modifications refine the options for the flexible carrier and flexible ring gear rim and adds three new options: a floating sun gear option, a natural frequency option, and a finite element compliance formulation for helical gear teeth. The option for a floating sun incorporates two additional degrees of freedom at the sun center. The natural frequency option evaluates the frequencies of planetary, star or differential systems as well as the effect of additional springs at the sun center and those due to a flexible carrier and/or ring gear rim. The helical tooth pair finite element calculated compliance is obtained from an automated element breakup of the helical teeth and then is used with the basic gear dynamic solution and stress postprocessing routines. The flexible carrier or ring gear rim option for planetary and star spur gear systems allows the output torque per carrier and ring gear rim segment to vary based on the dynamic response of the entire system, while the total output torque remains constant.
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I. SUMMARY

The epicyclic gear program is a multiple mesh/single stage, gear dynamics program. It is a versatile gear tooth dynamic analysis computer program which determines detailed geometry, dynamic loads, stresses and surface damage factors. The program can analyze a variety of both epicyclic and single mesh systems with spur and helical gear teeth including internal, external, and buttress tooth forms. This NASA Lewis sponsored contract called for four improvements: refinement of the option for the flexible carrier or flexible ring gear rim, a floating sun gear option, a natural frequencies option, and a finite element compliance formulation for helical gear teeth.

Task I was to add an option for a floating sun to account for flexible sun gear mounting, which incorporates two additional degrees of freedom at the sun center. Generally, soft mounted sun gears are used to minimize the effects of gear runout, etc. The test case used for the program checkout was a lightly loaded, three planet, planetary system. The floating sun case results showed similar loads for the sun-planet meshes and slightly higher loads for the ring-planet meshes when compared to the rigidly mounted sun case. Other cases, with higher loads and various spring rates, should be examined to analyze the effect more thoroughly.

Task II was to add an option to determine the natural frequencies of the system. This option is desirable to predict system critical speeds without having to run the complete dynamic analysis for a range of input speeds. The information can also aid the user in running the dynamic solution routines. At the system critical speeds the dynamic loads will be much more sensitive to the input variables and the user will know where these speeds occur beforehand. Planetary, star or differential systems can be investigated as well as the effect of the additional springs at the sun center (floating sun) and those due to a flexible carrier and/or ring gear rim. In addition, the effect of variation of the tooth pair stiffness on the natural frequencies due to load position can be investigated. The frequency results are consistent with the program's dynamic response solution.

Task III was to generate a helical tooth compliance routine based on finite element modeling. The previous version divided the tooth into ten equivalent spur gear teeth and used the spur tooth routines for the dynamic solution. However, this technique did not allow for coupling between the equivalent spur teeth segments. The program also had provisions to input a general compliance matrix for the helical gear tooth to be analyzed, but the user had to know the matrix before running the gear dynamics program. This added finite element routine eliminates these prior shortcomings. The new option internally generates a finite element breakup of the helical teeth and the necessary data for the internal finite
element routines. The routines use a four noded, quadrilateral, higher order plate element with five degrees of freedom per node. The results are used to obtain a general tooth pair compliance curve which is then used by the basic dynamic solution routines.

Task IV was to refine the flexible carrier and ring gear rim options for planetary and star spur gear systems. The frequency equations were expanded to account for nonrigid carriers and ring gear rims. In addition, some minor modifications were made with respect to the numerical solution tolerances. These modifications result in more stable solutions, thus allowing the user to investigate the effects of various spring rates for both the carrier and the carrier/planet pin, e.g., a bearing, or for the ring gear rim and the corresponding pin.
II. INTRODUCTION

A. PROGRAM HISTORY

The multiple mesh gear dynamic analysis computer code has been under development at Hamilton Standard for about five years. The program can determine detailed geometry, dynamic loads, stresses, and surface damage factors for epicyclic gear systems and single mesh systems with internal, external, buttress, or helical tooth forms. The significant parameters can be plotted through the entire mesh in addition to the maximum values which are tabulated as output from the program.

The initial program, a single spur gear mesh, was written for high contact as well as low contact ratio gearing. The basic concept was an extension of that developed by Richardson in 1958. Since the basic program was developed, many enhancements and refinements have been made. Buttress, internal and external involute tooth forms can be analyzed for spur and helical gear teeth. Tooth spacing errors, runout errors, involute profile modifications, etc. can be accounted for in the dynamic solution.

The program has been developed to operate over a wide range of contact ratios, and to allow the gear teeth to have a different pressure angle on the drive side and coast side (buttress form). The teeth of the meshes being analyzed may have modified profiles and spacing errors may be specified. Influence coefficients may be input for rim and web geometry to determine the effect on peak dynamic stress caused by non-uniform stiffness along the width of the gear, dependent on the design configuration of the foundation under the gear tooth. More recent additions to the program include variable contact friction throughout each mesh, user friendly options, dynamic side bands, a speed survey option and the option of solving non-planetary or single mesh systems. See References 1 through 6 for more details.

B. PROGRAM ENHANCEMENTS

This NASA contract refines the option for the flexible carrier or flexible ring gear rim and adds three new options: a floating sun gear option, a natural frequency option, and a finite element compliance formulation for helical gear teeth.

Task I was to add an option to allow the sun to float at the center. This allows for investigation of the effects of different spring rates and damping at the sun center on the dynamic tooth load behavior. This was accomplished by adding two global translational degrees of freedom at the sun center. These translational degrees of freedom were transformed into degrees of freedom along the respective meshing lines of action, to remain consistent with the existing code. The theoretical development was aided by the work of Hidaka et al., Reference 7.
Task II was to develop a routine for calculating the natural frequencies of the gear system. This was accomplished by solving the classical eigenvalue problem. This option will solve for the frequencies of the system using tooth pair compliances at the pitch diameter. The designer can also utilize the load position varying gear tooth compliance formulation and evaluate the frequencies at other mesh positions. This option allows the user to investigate the effects of different spring rates and masses throughout the gear system on the critical speeds.

Task III was to refine the helical gear tooth compliance routines. The refinement was to incorporate convective (coupled) compliance effects which were not previously included. This involved adding routines to build finite element models of the helical gear teeth. Reference 8. The models are used to obtain a spur gear compliance formulation which utilizes the basic gear dynamic routines and stress postprocessors. This refined approach uses much less CPU time, as well as using a smaller time step for the numerical solution, which will lead to a more stable dynamic solution than the previous uncoupled spur tooth segment approach. However, for helical gears with large helix angles the stress postprocessing will give unconservative results.

Task IV was to refine the flexible carrier/ring gear rim option. This option allows the user to investigate the effects of various stiffnesses for the carrier or ring gear rim as well as the pin stiffness between the planet gear and the carrier or the ring gear rim and the output shaft. This involved investigation of previous work, Reference 1, in order to determine the cause of an instability. The numerical solution technique was evaluated, as well as review of the mathematical model. The most significant change was in the mathematical model, where the forcing functions of the carrier or ring gear segment equations were modified to vary with respect to time while the total output torque remained constant. In addition, some minor changes were made to the numerical solution parameters to increase stability.

These four improvements further enhance the flexibility of the multiple mesh gear dynamics program and the variety of applications that can be modeled. The modifications allow the user to investigate a wider variety of system complexities. For example, a simple planetary can now be evaluated utilizing various additional degrees of freedom such as the floating sun gear or the flexible carrier, which are likely to be influential in a real system. The finite element compliance formulation lays the groundwork for a more exact modeling of the helical gear teeth.
III. FLOATING SUN GEAR

A. PROGRAM MODIFICATIONS

The floating sun gear option adds two additional degrees of freedom at the sun center. Springrates are required for the two orthogonal directions at the sun center as well as the translational mass of the sun gear and the additional boundary conditions. The equations of motion for the sun center are included in the system of equations being solved.

The equations of motion for the sun gear center in Cartesian coordinates, x and y directions, using the model of Figure 1, can be written:

\[
\begin{align*}
\ddot{x}_s + d_x \dot{x} + k_x x - \sum_{i=1}^{N} d_{sp_i} \dot{y}_{sp_i} \sin a_i &= 0 \\
\ddot{y}_s + d_y \dot{y} + k_y y + \sum_{i=1}^{N} d_{sp_i} \dot{y}_{sp_i} \cos a_i + \sum_{i=1}^{N} \frac{V_{sp_i}}{L_{sp_i}} \cos a_i &= 0
\end{align*}
\]

(1)  (2)

where

\[
\begin{align*}
a_i &= a_0 + \tau_1 - \delta \\
\alpha_i &= \tau_1 - \delta - \theta_2
\end{align*}
\]

(planetary)  (star)

The angle, \(a_i\), is for resolution of the tooth pair forces from along the line of action to the Cartesian coordinates, x and y. The pressure angle, \(\alpha\), was assumed to remain constant through the mesh; however, the more precise formulation would account for the varying angle at different mesh times. It is believed this assumption has secondary effects on the results. However, the information is available in the code such that the varying angle could be included during future enhancements.

In addition to the sun center equations, the carrier or ring rotational displacements, corresponding to a planetary or star system respectively, are also required to obtain the angle, \(a\). Thus the following equations must also be solved for a rigid carrier or rigid ring gear rim in conjunction with the tooth pair mesh equations, Reference 2, and equations (1) and (2).
The numerical solution requires these equations be reduced to first order differential equations; thus equations (1), (2), and (3) or (4) are added to the system of equations via six first order equations, see Appendix B.

The sun center displacements must be resolved in the direction of the line of action to determine the effect on the tooth pair meshes. This is accomplished via:

\[ x_{LOA} = y \cos \alpha - x \sin \alpha \]  

The tooth pair meshing loads of Reference 2, then become:

\[ L_{sp1} = \sum_{j=1}^{m} \left( (\gamma_{sp1} - x_{spj1} - x_{spj1} + x_{LOA}) \eta_{spj1} \phi_{spj1} \right) \]  

\[ L_{rp1} = \sum_{j=1}^{m} \left( (\gamma_{rp1} - x_{rpj1} - x_{rpj1} + x_{LOA}) \eta_{rpj1} \phi_{rpj1} \right) \]

where \( m \) is the number of teeth in contact at mesh \( i \), \( \eta_{sp} \) is the tooth pair stiffness and \( \phi_{sp1} \) is a tooth pair contact identity function.

To obtain a steady state solution, the solution is iterated until the boundary conditions converge for the tooth pair meshes. Convergence is determined by comparing the displacements along the lines of action due to the sun center movement to the largest sun-planet tooth pair displacement. Convergence is faster when the spring rates at the sun center are of similar order of magnitude to the tooth pair stiffnesses.
B. DISCUSSION OF FLOATING SUN RESULTS

The addition of the floating sun gear option allows the user to analyze various sun center spring rates and damping and the resulting dynamic tooth loads. Most epicyclic gear systems have a sun gear that can move in the in-plane translational directions, thus this option incorporates degrees of freedom that are of practical interest.

Two test cases were run using a lightly loaded planetary gear system. The description of the spur gear test case, Task I, Example 1.1, is described in Table 1 and was run using both the three planets of the system and reducing the number of planets to two. Several spring rates at the sun center were examined. The results for the three planet cases are summarized in Table 2.

The three planet cases included phasing constants to account for the different location of each planet mesh on its respective line of action. Because of the interactive dynamics, the total distances moved along the lines of action, due to the additional sun center movement, can be different for each mesh. In the three planet case with sun center springs 4.4 times the sun-planet tooth pair stiffness (10,000,000 lb./in. and damping of 5%) the maximum loads increased from 2 to 7 percent for the sun-planet loads and from 14 to 15% for the ring-planet meshes. The ring-planet meshes also indicated a decrease in the dynamic contact ratio, which is calculated by determining the total tooth contact time for one tooth pass during the dynamic solution. A low dynamic contact ratio represents tooth pair separation, which indicates tooth bouncing may be occurring, possibly causing the load increase. The additional degrees of freedom at the sun center may have introduced natural frequencies near the operating speed which could also cause the load increase.

The sun center spring rates were increased to 30,000,000 lb/in, or 13 times the sun-planet tooth pair stiffness. The no tooth error case showed the sun-planet loads less than 3% different and the ring-planet loads were generally a little higher, 4-5%, for 2% damping. Thus, as the springs become stiffer, the loads approach the non-flexible sun center mount solution as expected.

The two planet case showed no variation in the maximum tooth pair loads. As expected, the diametrically opposed planets (equal phasing constants) moved in equal and opposite directions along their lines of action due to equal tooth pair stiffnesses. The two cases executed to confirm these results were for sun center spring rates of 300,000 lb/in and 30,000 lb/in. Both cases yielded results identical to the non-floating sun gear.
IV. NATURAL FREQUENCY OPTION

A. PROGRAM MODIFICATIONS

The natural frequency option allows the user to investigate natural frequencies of the system through a classical eigenvalue solution. The effects of various spring rates at the sun center for a floating sun, or the effect of various stiffnesses of a planet carrier and/or ring gear rim on the frequency response can readily be determined. In addition, by using the planet phasing constants, the range of frequencies due to the nonlinear tooth meshing action can be investigated.

The general form of the dynamic equations for the eigenvalue solution is:

\[ [M] \{\ddot{\mathbf{x}}\} + [K] \{\mathbf{x}\} = 0 \]  \hspace{1cm} (7)

The mass and stiffness matrices \([M]\) and \([K]\), are derived from the system of equations of References 1 and 2, as well as equations (1) to (4). The eigenvector/eigenvalue solution, which assumes harmonic motion, then solves for the roots of the determinant:

\[ \begin{vmatrix} [K] - \lambda [M] \end{vmatrix} = 0 \]  \hspace{1cm} (8)

A standard eigenvalue/eigenvector numerical solution routine for real, symmetric matrices was used to solve the determinant, Reference 9.

The program also calculates the gear mesh frequencies using the following equation.

\[ f_i = \text{rpm/60} \times \frac{\lambda_i}{1 + \lambda_i^2 + \lambda_i^4 + \lambda_i^6 + \lambda_i^8 + \lambda_i^{10}} \]  \hspace{1cm} (9)

These gear mesh frequencies are printed with the natural frequencies to aid the user in generating critical speed diagrams.

The natural frequency option can be used for eight system types: planetary, star, differential, single meshes, planetary with flexible carrier, star with flexible ring gear rim, or a differential system with both a flexible ring gear rim and a flexible planet carrier. In addition, the floating sun degrees of freedom can be included in the natural frequency solution.

This option is initiated by a trigger with two choices of output format, either natural frequencies and gear mesh frequencies or
full output which also includes the eigenvectors. After the frequencies have been calculated, the program ends and does not continue with the dynamic load solution.

The frequency solution uses tooth pair stiffnesses from the nonlinear compliance formulation of the gear program, Reference 4 and 5. This compliance formulation also includes the Hertzian effect; thus, the torque that is input will influence the natural frequencies. A zero torque case will eliminate the Hertzian effect if it is not desired.

The natural frequency solution is for a specific instant in time, with the corresponding tooth pair stiffnesses. If no phasing constants are included, the tooth pair stiffnesses will be those at the pitch diameter. The phasing constants can be used to simulate different times and therefore different stiffnesses in the mesh. These phasing constants tell the program that the different planet meshes are at different positions with respect to each other along their lines of action. This indicates that all the planet meshes may not be at the pitch diameter initially. See User's Manual for details on calculation of the phasing constants.

B. DISCUSSION OF NATURAL FREQUENCY RESULTS

The natural frequency option is a useful tool for predicting critical speeds. This option allows for investigation of the effect of mesh position dependent tooth pair stiffnesses on the natural frequencies, as well as the additional frequencies due to carrier and ring flexibilities and floating sun flexibilities. The eigenvalue/eigenvector solution executes rapidly, as the program does not continue with the dynamic load solution. Thus, this is an economical approach to investigate the effects of various spring rates or masses on the natural frequencies.

The natural frequency option can be used to calculate the speed ranges where high dynamic loads could occur. This can also assist the user in reducing the number of boundary condition iterations that are necessary for convergence for a regular dynamic load solution by avoiding these critical speed areas. The effects of different spring rates throughout the gear system on the critical speeds can also be investigated. It also allows the designer to increase the number of degrees of freedom and observe the additional frequencies. For example, a simple planetary system will have \( N + 2 \) degrees of freedom, where \( N \) is the number of planets. The designer can add two spring rates at the sun center for \( N + 4 \) degrees of freedom, then could increase the total degrees of freedom again to \( 2N + 4 \) via the carrier flexibility.

Several test cases were run, and two examples are presented in Figures 3 and 4. Both figures show the critical speeds predicted by the natural frequency option, and the critical speeds predicted by running speed surveys with the dynamic response solution, Reference 1. Both cases agree in predicting the natural frequencies when the pitch diameter tooth pair stiffnesses were
used for the eigenvalue solution.

Figure 3 shows a critical speed diagram for Example 2.1, Table 1, and Figure 4 shows Example 2.2. The horizontal bands illustrate the range of frequencies that result from the variation of tooth pair stiffnesses through the mesh. The variation in frequency due to tooth pair stiffnesses versus percent of total mesh positions is shown in Figure 5 and corresponds to Figure 3. The higher frequencies of the band are when the mesh is at the pitch diameter and the tooth pair stiffness is maximum. The lower frequencies of the bands are at other positions in the mesh. These were simulated by adjusting the planet phasing constants until the minimum stiffness for the sun-planet meshes and the minimum stiffness for the ring-planet meshes were obtained.

The case in Figure 3 showed reasonable correlation between the natural frequency option and the results of an analytical speed survey using the dynamic solution. This was a two planet case with unequal phasing. The speed survey response results indicated peak dynamic loads in the range of sun gear input speeds of 23,000 to 25,000 rpm, a distinct peak near 7,000 rpm and a smaller load increase near 12,000 rpm. This case was previously shown to have reasonable correlation to test data, Reference 1.

By varying the stiffnesses the frequencies can be easily associated with particular degrees of freedom; e.g., in Example 2.2, when the sun-planet stiffness decreased significantly, the highest frequency decreased and when the ring-planet stiffness decreased, the two lower frequencies decreased. Thus, the highest of the gear mesh frequencies is due to the sun-planet meshes, while the lower frequencies are due to the ring-planet meshes. This may not be immediately obvious from the eigenvectors, because they indicate the motion along the lines of action for each gear and not the relative motion of the sun-planet or ring-planet meshes.

For planetary, star, and differential systems, the carrier and/or ring gear are treated as rigid bodies, unless the flexible carrier or flexible ring gear rim options are selected. The rigidity is also evident in the resulting natural frequencies. The eigenvalue solution yields one rigid body mode for planetary and star systems corresponding to the rigid carrier or ring gear, and the differential system results in two rigid body modes corresponding to both the rigid carrier and ring gear. These rigid body modes are not shown in the table of natural frequencies; however, if the user requests full output, all of the frequencies are printed as well as the mass and stiffness matrices and all eigenvectors.

Systems with a flexible carrier and/or ring gear rim may yield rigid body modes if the stiffnesses of the carrier and/or ring gear rim or pin stiffnesses are relatively high. For these
situations, either the carrier and/or the ring are acting as rigid bodies. For these systems all the frequencies are output, but if the first two modes are orders of magnitude less than the other frequencies, the pin stiffnesses and/or the program results should be carefully examined.
V. REFINED HELICAL COMPLIANCE ROUTINES

A. PROGRAM MODIFICATIONS

An option for helical gear analysis was added which incorporates a finite element analysis to obtain a tooth pair compliance curve. Several new subroutines have been added to the code to generate finite element models of the helical gear teeth. These models are then used to calculate tooth pair compliance curves. The model generation is internal to the code, so the user need only input an additional trigger to initiate the option.

The finite element option uses two finite element plate routines, Reference 8, to generate the stiffnesses for both in-plane and out-of-plane loads, as well as a routine to process the information to and from the multiple mesh program. Figure 6 summarizes the procedure via a flow chart.

For both routines, four noded isoparametric plate elements, with a total of five degrees of freedom per node, are used. The routine for out-of-plane loads includes transverse shear effects. The model for any tooth will have 9 elements along the face width and 9 elements along the tooth centerline (10 by 10 nodes), see Figure 7. The plate thicknesses are average thicknesses determined from the existing tooth geometry subroutines.

The tooth model is fixed at the root and displacements are applied along seven equally spaced load lines via boundary conditions for the finite element solution. The reaction forces along the load lines are used in conjunction with the applied displacements to obtain average stiffnesses for seven load line positions.

The existing spur tooth pair compliance used in the program includes axial bending, Hertzian, and fillet and foundation effects, Reference 5 and 6. These are determined for seven load positions for each tooth pair and combined to obtain total compliances. A curve fitting routine is then used to obtain the following fourth order polynomial for compliance as a function of position along the line of action.

\[
C - C_o = 1 + A(S/S_o) + B(S/S_o)^2 + C(S/S_o)^3 + D(S/S_o)^4
\]  

The finite element model accounts for the axial bending and fillet effects and an average Hertzian effect. The load line position at the center of the helical tooth face width is used to calculate an average Hertzian compliance.
This approach accounts for the helix angle, but uses the spur gear dynamic solution technique. The previous helical solution divided the tooth into ten equivalent spur gear tooth segments. Each of the segments was evaluated dynamically and the CPU running time was quite long to solve for each segment at each of 100 time steps. Because of this, the number of time steps had been reduced to ten for the previous helical solution. However, because the finite element approach is more direct than the segment approach, the time step was increased back to 100 for this option to improve the accuracy and response definition.

B. DISCUSSION OF REFINED HELICAL GEAR COMPLIANCE RESULTS

This option offers the user an alternative helical gear tooth analysis. A compliance curve was generated using the existing formulations for Hertzian effects and detailed geometry to obtain plate thicknesses to be used with two finite element routines, one for in-plane loads and one for out-of-plane loads, see Figure 8 for example output of Task III, Example 3.1, Table 1.

Future improvements should include further refinement of the stress postprocessing, where the finite element routines could be utilized more fully. The current finite element configuration utilizes the spur gear stress postprocessing routines by using the calculated dynamic load and applying it to the center of the tooth face width. This could lead to unconservative stress results when the helix angle is large, because the load line is parallel to the axis of rotation for spur gear postprocessing. A more complete solution would utilize the flexibility matrix generated by the finite element routines in conjunction with the dynamic solution. This more precise method would involve calling the finite element routines directly during the dynamic solution, which would require significant amounts of additional computer time. Similarly, finite element stress sensitivity routines that use the plate element directly for postprocessing could be included.

The number of elements that are chosen for the mesh breakup should be changed to be a user input in future work. The current constant breakup could lead to width to height ratios that exceed acceptable limits for finite element aspect ratios for wide teeth. The tooth fillet elements could also be refined. It was assumed that the first row of elements would extend from the root to the first point of contact, while the plate thicknesses were modified to estimate the additional fillet material, see Figure 9. However, this does not accurately define this area and could lead to significant element size variation and therefore error for cases with large fillet radii relative to the tooth height.
VI. FLEXIBLE CARRIER EVALUATION

A. PROGRAM MODIFICATIONS

During the previous contract work (CR # 174747) an attempt had been made to add the flexible carrier option to the dynamics model. However, numerical instabilities had occurred in the check case solutions which were not resolved during that contract. Therefore, the flexible carrier/ring gear rim dynamic solutions were reviewed with respect to the numerical solution, the equations, and the FORTRAN code. The major improvement was to the output torque term of the flexible carrier or flexible ring gear equations, i.e. the forcing function for the differential equation. This term was modified to calculate the output torque from the carrier or ring segment displacements and an interface stiffness, such as a pin or bearing stiffness, while the total output torque was constrained to remain constant.

The theoretical model is shown in Figure 10, and can be used to write the carrier or ring segment equations of motion as follows.

\[ m_i \ddot{y}_i - d_{p_i} (\dot{y}_i - \dot{y}_{p_i} - \dot{y}_{c_i}) - d_{r_{p_i}} (\dot{r}_{p_i} - \dot{r}_{c_i}) - L_{sp_i} - L_{r_{p_i}} + K_{o_i} (y_{o_i} - y_{o_{i+1}}) + K_{i-1} (-y_{o_{i-1}} + y_{o_i}) = - \bar{K} y_{o_i} \]

(11)

\[ m_r \ddot{r}_i - d_{p_i} (\dot{y}_i - \dot{y}_{c_i} - \dot{r}_{r_i}) - L_{r_{p_i}} + K_{r_{i-1}} (-y_{r_{i-1}} + y_{r_i}) + K_{r_i} (y_{r_i} - y_{r_{i+1}}) = - \bar{K} r_{r_i} \]

(12)

\[ \sum_{i=1}^{N} \bar{K}_i y_{o_i} = \tau_{out} / R_b \]  

(13)

Equation (13) adds one more equation than there are unknowns; thus, a constraint must be simulated in the numerical solution routines by introducing another parameter. The numerical routine used to solve the system of first order equations does not have the direct capability of a constraint on a primary variable, i.e. \( y_{o_i} \). Therefore, an artificial mass which simulates a large inertia, assumed to be 1000 times the largest mass of the system and attached to the carrier or ring, was added to the system of equations. This results in \( \ddot{y} = 0 \), analogous to the system shown in Figure 12. The additional equation of motion is:

\[ m \ddot{x} + \sum_{i=1}^{N} \bar{K}_i y_{o_i} = \tau_{out} / R_b \]

(carrier)

(14)
This increases the number of second order equations by \(1\) to \((3N + 2)\). The first order equations used for the numerical solution are in Appendix B.

The numerical solution routine was also investigated with respect to stability. A simple example was used in conjunction with the numerical solution routines to determine the effect of the size of the integration time steps on the solution. The example used a system of differential equations similar to the equations for a planetary system. They were used directly in the numerical solution routines, independent of the gear program. It was verified that the time step used for the numerical integration could lead to erroneous results if the step size was too large. This step size was previously a constant value in the multiple mesh code, but the total mesh time varies for different systems. A better choice for the integration time step is a function of the meshing time. Therefore, the integration time step was then changed to be 0.001 percent of the time for each mesh cycle.

This step size change can actually reduce the number of iterations internal to the numerical routines, due to the way the solution routine handles the step size if it is too large. If the routine is given a step size that is too large, it reduces the integration step size and tries again until it is small enough to yield a good solution. If it is given a step size that is adequate initially, it will obtain a good solution more rapidly. As this occurs for each of the 100 time steps in the dynamic solution, this could lead to significant reductions in computation time for some cases.

The size of the mesh time step can also have an effect on the solution. This is currently set to be equal to the total mesh time divided by 100. For very low speeds this time step can become relatively large and could lead to an unstable solution.

**B. DISCUSSION OF FLEXIBLE CARRIER RESULTS**

The flexible carrier modifications allow the user to investigate the effect of a carrier with additional flexibility, along the line of action, on the dynamic tooth loads of a spur gear system. Similarly, additional flexibility in the ring gear rim along the line of action. Two test cases with different carrier stiffnesses for a planetary spur system (flexible carrier) were investigated. They indicated higher maximum loads for the more flexible systems. This is due to the increased tooth pair bouncing that occurred, as was indicated by lower dynamic contact ratios.
Figures 12a and 12b illustrate the non-flexible test case numerical results and a case, Example 4.1, Task IV, Table 1, with a pin stiffness of 2,500,000 lbs/in. and a carrier stiffness of 5,000,000 lbs/in. The maximum tooth loads increased with the increased flexibility of the planet carrier. For this case, the ring-planet loads increased more than the sun-planet loads, 12.5% and 5.9%. Results from another case, with equal carrier and pin spring rates showed that the sun-planet mesh loads increased 9.3%, while the ring-planet loads increased by 5.4%.

Figures 13a and 13b show the plots generated by the multiple mesh program for the first planet mesh for the nonflexible case and for the case with pin stiffness of 2,500,000 lbs/in., respectively. There were no planet phasing constants for this test case; therefore, all three planet mesh results are identical and only the results for one planet are shown. Comparison of the flexible and nonflexible load plots indicated that while the maximum loads increased, the minimum loads remained nearly the same. There was a similar effect observed in the other plotted parameters; that is, the curves maintained the same general shape, the minimum points did not noticeably change, but the maximums increased with the addition of the flexible carrier.

Another test case, Example 4.2, Table 1, was executed for a lightly loaded planetary system with unequal phasing and with a carrier stiffness of 3,000,000 lbs/in and a pin stiffness of 5,000,000 lbs/in. The results for this case, Figure 12c, show very little variation from the rigid carrier results, less than 2%. This variation is most likely due to the very small loads.

It should be noted that this solution consumes considerably more CPU time than a simple planetary or star system due to the larger system of dynamic equations and particularly the increased number of boundary conditions that must converge. It is recommended that the nonflexible solution be run first and the converged boundary conditions be used in conjunction with the output torque term to estimate the new boundary conditions, see Appendix A for details.

Due to funding limitations, a flexible ring rim case was not executed to completion. It has been coded and a test of the first few boundary condition iterations indicated that there should be no significant problems with this code.
A number of new options have been added to the multiple mesh gear program, but only a few test cases were run for each of these due to the minimum amount of epicyclic data available for comparison. It is recommended that the program be more thoroughly tested and evaluated via parametric studies as well as comparison to test data. Follow on work should also include refinement of the finite element stress sensitivity postprocessing.

A floating sun gear option was added which will allow the user to investigate the effects of various spring rates and damping at the sun center. Several test cases were executed and the program gives reasonable results, with the solution approaching the rigid mount solution as the spring rates become 10 to 15 times the tooth pair stiffnesses.

The critical speeds predicted by the new natural frequency option, and the critical speeds indicted by running speed surveys with the dynamic response solution agreed well with the frequencies for tooth pair stiffnesses at the pitch diameter.

The refined helical gear option lays the foundation for an alternative method of analyzing helical gear dynamics. Due to funding limitations, the potential for the finite element solution was not fully accomplished. However, the finite element results were used to generate a general tooth pair compliance similar to the original spur gear compliance formulation. The stress postprocessing uses the spur gear stress postprocesser which inherently assumes the load line is parallel to the axis of rotation. Thus, the stress postprocessing could have significant errors for large helix angles.

The flexible carrier and flexible ring gear rim options modified the output torque for each segment to vary with the dynamic results while the total output torque remains constant. The test case results were reasonable, with the maximum loads increasing and the minimum loads remaining nearly the same when compared to the rigid carrier results.
VIII. REFERENCES


IX. BIBLIOGRAPHY


-19-
X. TABLES
### TABLE 1: DESCRIPTION OF TEST CASES

<table>
<thead>
<tr>
<th>Example Case</th>
<th>Number Of Planets</th>
<th>Diametral Pitch</th>
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<th>Number of Teeth</th>
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<th>Helix Angle</th>
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XI. FIGURES
Figure 1. Floating Sun Model
Subroutine FREQ is called from READ2

```
subroutine FREQ

    calculate # of degrees of freedom

    calculate the mesh stiffnesses

    call subroutine GFREQ
    set up stiffness and mass matrices
    call eigenvalue/eigenvector solution
    normalize eigenvectors by largest value

    calculate gear mesh frequencies & natural frequencies

    print results

RETURN
    RETURN
```
Figure 3. Natural Frequency Interference Plots, Example 1

F178 Peak Response from Dynamic Load Speed Survey

1X Gear Mesh Frequency

2X Gear Mesh Frequency

F178 - Multiple Mesh Gear Dynamic Analysis Program

Frequency range

Frequency range

Sun Gear Speed, rpm x 10^-3
Figure 4. Natural Frequency Interference Plots, Example 2

- Original page is of poor quality.
- Frequency ranges.
- F178 Peak Response from Dynamic Load Speed Survey.
- F178 = Multiple Mesh Gear Dynamic Analysis Program.

Sun Gear Speed, rpm \( \times 10^{-3} \)
Figure 5. Natural Frequency vs. Percent Mesh Time

Natural Frequency (KHz)

% Mesh Cycle
(for instantaneous stiffness)
Figure 6. Flowchart for Helical Tooth Compliance Modifications

- Call FEASTF for each of 7 load positions
  - Calculate nodal locations for load line
  - Calculate average plate thicknesses
  - Determine FE degrees of freedom for initial conditions
  - Choose plane stress or plane strain for IP
    - Call in-plane FE routine (FEAPI)
  - Determine additional boundary condition for OOP
    - Call out-of-plane FE routine
  - Resolve cartesian stiffnesses to along the line of action
    - Transform stiffnesses to consistent compliance form
    - Return

Abbreviations
FE = Finite Element
IP = In-Plane
OOP = Out-of-Plane

Original page is of poor quality
Figure 7. Helical Gear Tooth Finite Element Model
Figure 8. Program Output from Finite Element
Helical Gear Option

1. CALCULATES DYNAMIC LOAD RESPONSE FOR
   A. SINGLE MESH SPUR GEARING
   B. MULTIPLE MESH SPUR GEARING,
      STAR OR PLANETARY
   C. SINGLE MESH HELICAL GEARING
   D. MULTIPLE MESH HELICAL GEARING
   E. DOUBLE HELICAL GEARING
   F. THE CARRIER OR GEAR RIM FLEXIBILITIES
      ALONG THE RESPECTIVE LINES-OF-ACTION CAN BE
      ACCOUNTED FOR IN THE DYNAMIC LOAD SOLUTION
   G. A FLOATING SUN GEAR CAN BE INCLUDED IN THE DYNAMIC
      SOLUTION FOR MULTIPLE MESH SPUR GEARS

2. GEOMETRIC PREPROCESSOR CAN BE USED FOR
   A. INVOLUTE SPUR AND HELICAL TOOTH FORMS
   B. INVOLUTE BUTTRESSED TOOTH FORMS
   C. EXTERNAL AND INTERNAL TOOTH FORMS
   D. MEASUREMENT OVER WIRE DATA
   E. TOLERANCE AND INTERFERENCE CHECKING
   F. INVOLUTE MODIFICATION TABLES

3. STRESS POSTPROCESSOR CAN BE USED FOR
   A. DYNAMIC LOAD SUMMARY
   B. MODIFIED HEWWOOD STRESS SENSITIVITY
   C. HERTZ STRESSING
   D. FLASH TEMPERATURE SUMMARY
   E. PRESSURE-SLIDING VELOCITY(PV) SUMMARY

4. DYNAMIC LOAD SOLUTION ASSUMPTIONS
   A. CIRCUMFERENTIALLY STIFF RING GEAR
   B. CIRCUMFERENTIALLY STIFF PLANET CARRIER
   C. EQUILIBRIUM EQUATIONS DO NOT INCLUDE FRICTION
   D. INPUT TORQUE IS CONSTANT

5. GEOMETRIC DATA IN ROTATIONAL PLANE UNLESS NOTED
   A. ANGLES ARE IN DEGREES
   B. FORCES ARE IN POUNDS(#)
   C. LENGTHS ARE IN INCHES(IN)
   D. MASSES ARE IN 1B-SEC**2/IN
   E. STRESSES ARE IN PSI(#/IN**2)
   F. TEMPERATURES ARE IN DEGREES FAHRENHEIT

THIS PROGRAM WAS DEVELOPED AT HAMILTON STANDARD DIVISION OF UNITED TECHNOLOGIES, WINDSOR LOCKS, CONN. BY
J. PIKE, R. CORNELL, N. WESTERVELT, AND L. BOYD. FUNDING HAS GIVEN UNDER CONTRACT BY NASA-LEWIS,
CLEVELAND AND MONITORED BY D. TOWNSEND.
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## HELICAL GEAR DATA FOR SUN-PLANET MESH

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<td>Lead of Planet Gear (Inches)</td>
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<tr>
<td>Crowned Edge Relief of Diseng</td>
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### Number of Planets
- Number of Planets: 2
- Number of Boundary Condition Iterations: 10
- Tolerance for Boundary Condition Convergence: 0.10000E+00

### Equivalent Mass of Gear
- Equivalent Mass of Sun Gear: 0.16000E-01
- Equivalent Mass of Planet Carrier: 0.97400E-01
- Equivalent Mass of Ring Gear: 0.00000E+00
- Equivalent Mass of Planet #1: 0.21600E-01

### Compliance Constants

#### SUN-PLANET
0.1563E-06 * (1 + -0.8254E-01 * (S/SO) + 0.6360E+00 * (S/SO)^2 + 0.4108E-01 * (S/SO)^3 + -0.2646E+00 * (S/SO)^4)

#### RING-PLANET
0.1730E-06 * (1 + 0.5929E-01 * (S/SO) + 0.6488E+00 * (S/SO)^2 - 0.1420E+00 * (S/SO)^3 + -0.2578E+00 * (S/SO)^4)

************** PLANETARY GEAR SYSTEM **************
**Boundary Condition Iteration Results**

**Sun-Planet: Meshes 1 thru N, Left to Right**

<table>
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<tr>
<th>Iteration, End Displacement</th>
<th>2</th>
<th>0.15479E-02</th>
<th>0.18250E-02</th>
</tr>
</thead>
<tbody>
<tr>
<td>Starting Displacement</td>
<td>2</td>
<td>-0.19363E-02</td>
<td>0.16605E-02</td>
</tr>
<tr>
<td>Iteration, Ending Velocity</td>
<td>2</td>
<td>-0.21040E+02</td>
<td>-0.26328E+02</td>
</tr>
<tr>
<td>Starting Velocity</td>
<td>2</td>
<td>-0.21040E+02</td>
<td>-0.26328E+02</td>
</tr>
</tbody>
</table>

**Ring-Planet: Meshes 1 thru N, Left to Right**

<table>
<thead>
<tr>
<th>Iteration, End Displacement</th>
<th>3</th>
<th>0.15479E-02</th>
<th>0.18250E-02</th>
</tr>
</thead>
<tbody>
<tr>
<td>Starting Displacement</td>
<td>3</td>
<td>-0.19363E-02</td>
<td>0.16605E-02</td>
</tr>
<tr>
<td>Iteration, Ending Velocity</td>
<td>3</td>
<td>-0.21040E+02</td>
<td>-0.26328E+02</td>
</tr>
<tr>
<td>Starting Velocity</td>
<td>3</td>
<td>-0.21040E+02</td>
<td>-0.26328E+02</td>
</tr>
</tbody>
</table>

**Planet Number 1**

RPM = 10000.00

Maximum Load for Sun-Planet = 15200.17

Maximum Load for Ring-Planet = 12795.10

**Planet Number 2**

RPM = 10000.00

Maximum Load for Sun-Planet = 15207.16

Maximum Load for Ring-Planet = 12502.19
### NO TOOTH ERRORS SOLUTION

**Figure 8 (cont)**

#### MAXIMUM VALUES FOR SUN-PLANET MESH 1

<table>
<thead>
<tr>
<th></th>
<th>SUN</th>
<th>PLANET</th>
</tr>
</thead>
<tbody>
<tr>
<td>FILLET STRESS CONCENTRATION (KSUBT)</td>
<td>1.48134</td>
<td>1.48045</td>
</tr>
<tr>
<td>MAXIMUM HERTZ STRESS</td>
<td>255256.7</td>
<td>255256.7</td>
</tr>
<tr>
<td>MAXIMUM HERTZ STRESS AT PD</td>
<td>206045.6</td>
<td>206045.6</td>
</tr>
<tr>
<td>MAXIMUM BENDING STRESS</td>
<td>107946.4</td>
<td>94638.1</td>
</tr>
<tr>
<td>MAXIMUM BENDING STRESS AT PD</td>
<td>55945.5</td>
<td>55044.5</td>
</tr>
<tr>
<td>DEPTH TO MAXIMUM SHEAR</td>
<td>0.01406</td>
<td>0.01406</td>
</tr>
<tr>
<td>MAXIMUM DYNAMIC P/V(MILLIONS OF PSI*FT/MIN)</td>
<td>560.14429</td>
<td>560.14429</td>
</tr>
<tr>
<td>MAXIMUM FLASH TEMPERATURE</td>
<td>276.3</td>
<td>1258.0</td>
</tr>
<tr>
<td>MAXIMUM NORMAL LOAD</td>
<td>15280.2</td>
<td></td>
</tr>
<tr>
<td>AVERAGE COEFFICIENT OF FRICTION</td>
<td>0.06640</td>
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<tr>
<td>RPM FOR STRESSES</td>
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</table>

**THE EFFECTIVE CONTACT RATIO = 1.5800**

#### MAXIMUM VALUES FOR SUN-PLANET MESH 2

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</tr>
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<td>FILLET STRESS CONCENTRATION (KSUBT)</td>
<td>1.52691</td>
<td>1.45766</td>
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<td>MAXIMUM HERTZ STRESS</td>
<td>255319.1</td>
<td>255319.1</td>
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<tr>
<td>MAXIMUM HERTZ STRESS AT PD</td>
<td>193971.0</td>
<td>193971.0</td>
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<tr>
<td>MAXIMUM BENDING STRESS</td>
<td>108625.4</td>
<td>96478.6</td>
</tr>
<tr>
<td>MAXIMUM BENDING STRESS AT PD</td>
<td>74564.4</td>
<td>36543.2</td>
</tr>
<tr>
<td>DEPTH TO MAXIMUM SHEAR</td>
<td>0.01406</td>
<td>0.01406</td>
</tr>
<tr>
<td>MAXIMUM DYNAMIC P/V(MILLIONS OF PSI*FT/MIN)</td>
<td>502.11499</td>
<td>502.11499</td>
</tr>
<tr>
<td>MAXIMUM FLASH TEMPERATURE</td>
<td>277.7</td>
<td></td>
</tr>
<tr>
<td>MAXIMUM NORMAL LOAD</td>
<td>15287.2</td>
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<tr>
<td>AVERAGE COEFFICIENT OF FRICTION</td>
<td>0.02433</td>
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<td>RPM FOR STRESSES</td>
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<td></td>
</tr>
</tbody>
</table>

**THE EFFECTIVE CONTACT RATIO = 1.5800**

#### MAXIMUM VALUES FOR RING-PLANET MESH 1

<table>
<thead>
<tr>
<th></th>
<th>PLANET</th>
<th>RING</th>
</tr>
</thead>
<tbody>
<tr>
<td>FILLET STRESS CONCENTRATION (KSUBT)</td>
<td>1.52831</td>
<td>1.56517</td>
</tr>
<tr>
<td>MAXIMUM HERTZ STRESS</td>
<td>178244.5</td>
<td>178244.5</td>
</tr>
<tr>
<td>MAXIMUM HERTZ STRESS AT PD</td>
<td>139438.2</td>
<td>139438.2</td>
</tr>
<tr>
<td>MAXIMUM BENDING STRESS</td>
<td>92648.5</td>
<td>75782.4</td>
</tr>
<tr>
<td>MAXIMUM BENDING STRESS AT PD</td>
<td>71376.2</td>
<td>21364.0</td>
</tr>
<tr>
<td>DEPTH TO MAXIMUM SHEAR</td>
<td>0.01690</td>
<td>0.01690</td>
</tr>
<tr>
<td>MAXIMUM DYNAMIC P/V(MILLIONS OF PSI*FT/MIN)</td>
<td>157.38115</td>
<td>157.38115</td>
</tr>
<tr>
<td>MAXIMUM FLASH TEMPERATURE</td>
<td>208.6</td>
<td></td>
</tr>
<tr>
<td>MAXIMUM NORMAL LOAD</td>
<td>12795.1</td>
<td></td>
</tr>
<tr>
<td>AVERAGE COEFFICIENT OF FRICTION</td>
<td>0.02975</td>
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</table>
### Maximum Values for Ring-Planet Mesh

<table>
<thead>
<tr>
<th>Fillet Stress Concentration ($K_{SUBT}$)</th>
<th>Planet</th>
<th>Ring</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum Hertz Stress</td>
<td>177345.0</td>
<td>177345.0</td>
</tr>
<tr>
<td>Maximum Hertz Stress at PD</td>
<td>159500.1</td>
<td>159500.1</td>
</tr>
<tr>
<td>Maximum Bending Stress</td>
<td>87345.4</td>
<td>68110.6</td>
</tr>
<tr>
<td>Maximum Bending Stress at PD</td>
<td>54211.2</td>
<td>44304.4</td>
</tr>
<tr>
<td>Depth to Maximum Shear</td>
<td>0.01660</td>
<td>0.01660</td>
</tr>
<tr>
<td>Maximum Dynamic PV (Millions of PSI*ft/min)</td>
<td>152.18002</td>
<td>152.18002</td>
</tr>
</tbody>
</table>

| Maximum Flash Temperature | 206.3 |
| Maximum Normal Load        | 12502.2 |
| Average Coefficient of Friction | 0.09525 |
| RPM for Stresses            | 10000.00 |

---

**THE EFFECTIVE CONTACT RATIO = 1.6100**

---

**THE EFFECTIVE CONTACT RATIO = 1.6200**

---

Figure 8 (cont)
Figure 9. Fillet Element Thickness

45° for fillet element thickness calculations

![Diagram showing fillet element thickness calculations]

- $R_r$ = root radius
- $R_B$ = base radius
- $R_p$ = pitch radius
- $R_t$ = tip radius
- $r$ = fillet radius
- $L$ = load
- $\phi$ = pressure angle
- $\phi'$ = load line angularity
- $t_f$ = fillet element thickness
Figure 10. Flexible Carrier/Ring Gear Rim Model
Figure 11. Torque Constraint Model Analogy

\[ F = \text{output torque} \]
\[ b = \text{base radius} \]
\[ M = \text{artificial mass} \]

\[ m_1, m_2, \ldots, m_n = \text{planet n effective mass} \]
\[ K_{ci} = \text{carrier pin stiffness} \]
\[ K_{ci} = \text{carrier rim segment stiffness} \]
### Flexible Carrier Results

#### Maximum Values for Sun-Planet Mesh 1

<table>
<thead>
<tr>
<th>Description</th>
<th>Mesh 1</th>
<th>Mesh 2</th>
<th>Mesh 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fillet Stress Concentration (KPSI)</td>
<td>1.09690</td>
<td>1.09690</td>
<td>1.09690</td>
</tr>
<tr>
<td>Maximum Hertz Stress</td>
<td>107556.9</td>
<td>107556.9</td>
<td>107556.9</td>
</tr>
<tr>
<td>Maximum Bending Stress at PD</td>
<td>601.60</td>
<td>601.60</td>
<td>601.60</td>
</tr>
<tr>
<td>Maximum Bending Stress at PD</td>
<td>864.57</td>
<td>1424.06</td>
<td>1424.06</td>
</tr>
<tr>
<td>Depth to Maximum Shear</td>
<td>0.00179</td>
<td>0.00179</td>
<td>0.00179</td>
</tr>
<tr>
<td>Maximum Flash Temperature</td>
<td>199.7</td>
<td>199.7</td>
<td>199.7</td>
</tr>
<tr>
<td>Normal Load</td>
<td>46.0</td>
<td>46.0</td>
<td>46.0</td>
</tr>
<tr>
<td>Average Coefficient of Friction</td>
<td>0.07036</td>
<td>0.07036</td>
<td>0.07036</td>
</tr>
<tr>
<td>RPM for Stresses</td>
<td>36000.00</td>
<td>36000.00</td>
<td>36000.00</td>
</tr>
</tbody>
</table>

#### Maximum Carrier Displacement

<table>
<thead>
<tr>
<th>Description</th>
<th>Valve 1</th>
<th>Valve 2</th>
<th>Valve 3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.62540-03</td>
<td>0.62540-03</td>
<td>0.62540-03</td>
</tr>
</tbody>
</table>

The Effective Contact Ratio = 1.0000

### Non-flexible Carrier Results

#### Maximum Values for Sun-Planet Mesh 1

<table>
<thead>
<tr>
<th>Description</th>
<th>Mesh 1</th>
<th>Mesh 2</th>
<th>Mesh 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fillet Stress Concentration (KPSI)</td>
<td>1.09690</td>
<td>1.09690</td>
<td>1.09690</td>
</tr>
<tr>
<td>Maximum Hertz Stress</td>
<td>107556.9</td>
<td>107556.9</td>
<td>107556.9</td>
</tr>
<tr>
<td>Maximum Bending Stress at PD</td>
<td>601.60</td>
<td>601.60</td>
<td>601.60</td>
</tr>
<tr>
<td>Maximum Bending Stress at PD</td>
<td>864.57</td>
<td>1424.06</td>
<td>1424.06</td>
</tr>
<tr>
<td>Depth to Maximum Shear</td>
<td>0.00179</td>
<td>0.00179</td>
<td>0.00179</td>
</tr>
<tr>
<td>Maximum Flash Temperature</td>
<td>199.7</td>
<td>199.7</td>
<td>199.7</td>
</tr>
<tr>
<td>Normal Load</td>
<td>46.0</td>
<td>46.0</td>
<td>46.0</td>
</tr>
<tr>
<td>Average Coefficient of Friction</td>
<td>0.07036</td>
<td>0.07036</td>
<td>0.07036</td>
</tr>
<tr>
<td>RPM for Stresses</td>
<td>36000.00</td>
<td>36000.00</td>
<td>36000.00</td>
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</table>

#### Maximum Carrier Displacement

<table>
<thead>
<tr>
<th>Description</th>
<th>Valve 1</th>
<th>Valve 2</th>
<th>Valve 3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.62540-03</td>
<td>0.62540-03</td>
<td>0.62540-03</td>
</tr>
</tbody>
</table>

The Effective Contact Ratio = 1.0000
**Figure 12b. Flexible Carrier Results, Example 4.2**

### Flexible Carrier Results

<table>
<thead>
<tr>
<th>Maximum Values for Sun-Planet Mesh 1</th>
<th>Sun</th>
<th>Planet</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fillet Stress Concentration (ksi)</td>
<td>1.43x17</td>
<td>1.39x15</td>
</tr>
<tr>
<td>Maximum Hertz Stress</td>
<td>1976.6</td>
<td>1976.0</td>
</tr>
<tr>
<td>Maximum Hertz Stress at PD</td>
<td>1964.7</td>
<td>1961.7</td>
</tr>
<tr>
<td>Maximum Bending Stress at PD</td>
<td>277.0</td>
<td>251.0</td>
</tr>
<tr>
<td>Depth to Maximum Hertz</td>
<td>0.0373</td>
<td>0.0373</td>
</tr>
<tr>
<td>Maximum Dynamic Pivellen (psi*ft/tn)</td>
<td>18.72x15</td>
<td>18.72x15</td>
</tr>
<tr>
<td>Flash Temperature</td>
<td>19.0</td>
<td></td>
</tr>
<tr>
<td>Shank Pivellen</td>
<td>0.074x02</td>
<td>0.00x00</td>
</tr>
<tr>
<td>Average Coefficient of Friction</td>
<td>4.00x00</td>
<td>4.00x00</td>
</tr>
<tr>
<td>RPM for Stresses</td>
<td></td>
<td></td>
</tr>
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</table>

**The Effective Contact Ratio = 1.4500**

### Flexible Carrier Results

<table>
<thead>
<tr>
<th>Maximum Values for Sun-Planet Mesh 2</th>
<th>Sun</th>
<th>Planet</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fillet Stress Concentration (ksi)</td>
<td>1.49x02</td>
<td>1.38x94</td>
</tr>
<tr>
<td>Maximum Hertz Stress</td>
<td>1964.6</td>
<td>1964.8</td>
</tr>
<tr>
<td>Maximum Hertz Stress at PD</td>
<td>1944.7</td>
<td>1944.7</td>
</tr>
<tr>
<td>Maximum Bending Stress at PD</td>
<td>386.0</td>
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</tr>
<tr>
<td>Depth to Maximum Hertz</td>
<td>0.0399</td>
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<tr>
<td>Maximum Dynamic Pivellen (psi*ft/tn)</td>
<td>18.50x39</td>
<td>18.50x39</td>
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<tr>
<td>Flash Temperature</td>
<td>19.0</td>
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</tr>
<tr>
<td>Shank Pivellen</td>
<td>0.014x32</td>
<td>0.00x00</td>
</tr>
<tr>
<td>Average Coefficient of Friction</td>
<td>4.00x00</td>
<td>4.00x00</td>
</tr>
<tr>
<td>RPM for Stresses</td>
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<td></td>
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</table>

**The Effective Contact Ratio = 1.4500**

### Flexible Carrier Results

<table>
<thead>
<tr>
<th>Maximum Values for Sun-Planet Mesh 3</th>
<th>Sun</th>
<th>Planet</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fillet Stress Concentration (ksi)</td>
<td>1.41x30</td>
<td>1.41x32</td>
</tr>
<tr>
<td>Maximum Hertz Stress</td>
<td>1944.3</td>
<td>1944.3</td>
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<td>Maximum Hertz Stress at PD</td>
<td>1354.7</td>
<td>1354.7</td>
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<tr>
<td>Maximum Bending Stress at PD</td>
<td>379.8</td>
<td>379.8</td>
</tr>
<tr>
<td>Maximum Bending Stress at PD</td>
<td>379.8</td>
<td>379.8</td>
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<tr>
<td>Depth to Maximum Hertz</td>
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<td>0.00041</td>
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<tr>
<td>Maximum Dynamic Pivellen (psi*ft/tn)</td>
<td>18.49x96</td>
<td>18.49x96</td>
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<tr>
<td>Flash Temperature</td>
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</tr>
<tr>
<td>Shank Pivellen</td>
<td>0.014x99</td>
<td>0.00x00</td>
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<tr>
<td>Average Coefficient of Friction</td>
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<td>4.00x00</td>
</tr>
<tr>
<td>RPM for Stresses</td>
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</table>

**The Effective Contact Ratio = 1.4500**

### Non-Flexible Carrier Results

<table>
<thead>
<tr>
<th>Maximum Values for Sun-Planet Mesh 1</th>
<th>Sun</th>
<th>Planet</th>
</tr>
</thead>
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<tr>
<td>Fillet Stress Concentration (ksi)</td>
<td>1.47x01</td>
<td>1.39x03</td>
</tr>
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<td>Hertz Stress at PD</td>
<td>1976.6</td>
<td>1976.0</td>
</tr>
<tr>
<td>Maximum Dynamic Pivellen (psi*ft/tn)</td>
<td>18.72x15</td>
<td>18.72x15</td>
</tr>
<tr>
<td>Flash Temperature</td>
<td>19.0</td>
<td></td>
</tr>
<tr>
<td>Average Coefficient of Friction</td>
<td>4.00x00</td>
<td>4.00x00</td>
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<tr>
<td>RPM for Stresses</td>
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**The Effective Contact Ratio = 1.4500**

### Non-Flexible Carrier Results

<table>
<thead>
<tr>
<th>Maximum Values for Sun-Planet Mesh 2</th>
<th>Sun</th>
<th>Planet</th>
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<tbody>
<tr>
<td>Fillet Stress Concentration (ksi)</td>
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<td>1.38x94</td>
</tr>
<tr>
<td>Hertz Stress at PD</td>
<td>1964.6</td>
<td>1964.8</td>
</tr>
<tr>
<td>Maximum Dynamic Pivellen (psi*ft/tn)</td>
<td>18.50x39</td>
<td>18.50x39</td>
</tr>
<tr>
<td>Flash Temperature</td>
<td>19.0</td>
<td></td>
</tr>
<tr>
<td>Average Coefficient of Friction</td>
<td>4.00x00</td>
<td>4.00x00</td>
</tr>
<tr>
<td>RPM for Stresses</td>
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<td></td>
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</tbody>
</table>

**The Effective Contact Ratio = 1.4500**

### Non-Flexible Carrier Results

<table>
<thead>
<tr>
<th>Maximum Values for Sun-Planet Mesh 3</th>
<th>Sun</th>
<th>Planet</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fillet Stress Concentration (ksi)</td>
<td>1.41x30</td>
<td>1.41x32</td>
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<tr>
<td>Hertz Stress at PD</td>
<td>1944.3</td>
<td>1944.3</td>
</tr>
<tr>
<td>Maximum Dynamic Pivellen (psi*ft/tn)</td>
<td>18.49x96</td>
<td>18.49x96</td>
</tr>
<tr>
<td>Flash Temperature</td>
<td>19.0</td>
<td></td>
</tr>
<tr>
<td>Average Coefficient of Friction</td>
<td>4.00x00</td>
<td>4.00x00</td>
</tr>
<tr>
<td>RPM for Stresses</td>
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<td></td>
</tr>
</tbody>
</table>

**The Effective Contact Ratio = 1.4500**
### Flexible Carrier Results

<table>
<thead>
<tr>
<th>Planet</th>
<th>Ring</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fillet Stress Concentration (ksi)</td>
<td>1.44136</td>
</tr>
<tr>
<td>Mean Hertz Stress</td>
<td>12017.4</td>
</tr>
<tr>
<td>Mean Hertz Stress at PD</td>
<td>3061.9</td>
</tr>
</tbody>
</table>

### Non-flexible Carrier Results

<table>
<thead>
<tr>
<th>Planet</th>
<th>Ring</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fillet Stress Concentration (ksi)</td>
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<tr>
<td>Mean Hertz Stress</td>
<td>12017.4</td>
</tr>
<tr>
<td>Mean Hertz Stress at PD</td>
<td>3061.9</td>
</tr>
</tbody>
</table>

### Maximum Values for Ring-Planet Mesh 1

<table>
<thead>
<tr>
<th>Planet</th>
<th>Ring</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fillet Stress Concentration (ksi)</td>
<td>1.44136</td>
</tr>
<tr>
<td>Mean Hertz Stress</td>
<td>12017.4</td>
</tr>
<tr>
<td>Mean Hertz Stress at PD</td>
<td>3061.9</td>
</tr>
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</table>

### Maximum Values for Ring-Planet Mesh 2

<table>
<thead>
<tr>
<th>Planet</th>
<th>Ring</th>
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<tbody>
<tr>
<td>Fillet Stress Concentration (ksi)</td>
<td>1.44136</td>
</tr>
<tr>
<td>Mean Hertz Stress</td>
<td>12017.4</td>
</tr>
<tr>
<td>Mean Hertz Stress at PD</td>
<td>3061.9</td>
</tr>
</tbody>
</table>

### Maximum Values for Ring-Planet Mesh 3

<table>
<thead>
<tr>
<th>Planet</th>
<th>Ring</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fillet Stress Concentration (ksi)</td>
<td>1.44136</td>
</tr>
<tr>
<td>Mean Hertz Stress</td>
<td>12017.4</td>
</tr>
<tr>
<td>Mean Hertz Stress at PD</td>
<td>3061.9</td>
</tr>
</tbody>
</table>

---

**Figure 12b.** (cont)
**Figure 13a. Non-flexible Carrier Gear Mesh Plots**

PV = pressure sliding velocity

pitch point = 0.0 distance along line of action

sun-planet mesh
Figure 13a (cont)

sun-planet mesh

ring-planet mesh
Figure 13b. Flexible Carrier Gear Mesh Plots

sun-planet mesh

sun-planet mesh
Figure 13b (cont)

sun-planet mesh

ring-planet mesh
Figure 13b (cont)
XII. APPENDICES
APPENDIX A: USER'S MANUAL

Part 1: Summary of New Option Information

The new options can be easily added to any existing data set. The new inputs corresponding to the new options are summarized below. An updated version of the User's Manual of Reference 1 follows in Part 2 with all the input descriptions. In addition, some comments are included that may be useful for input or interpretation of output. It should be noted that the locations to request plots have been changed.

Floating Sun:

Springrates (locations 115 and 116) and damping (locations 117 and 118) are input for two Cartesian directions, horizontal and vertical or x and y, at the sun center. The translational mass of the sun gear (location 119) and the additional boundary conditions (locations 561 to 644) are also inputs. The additional boundary conditions that must be input are for the sun center and the carrier or ring displacements and velocities, see the updated location listing, Part 2, for details on input.

Note that some damping should be input or numerical instabilities may develop.

The convergence criteria should not initially be too stringent, especially if the spring rates at the sun center are soft. This can lead to diverging boundary conditions, especially for systems with unequal phasing constants.

Natural Frequencies:

The user can utilize existing input data sets by simply adding a trigger (location 805). In general, minimum input (Level I) is all that is necessary; however, any existing data set can be used with the addition of the trigger. The trigger is also used to indicate the type of output -- frequencies only or eigenvalues/eigenvectors, etc. After the frequencies have been calculated, the program ends and does not continue with the dynamic solution.

A list of the various system types that can be investigated and the corresponding number of degrees of freedom follows.
$k =$ type of epicyclic spur gear system  
$n =$ number of planets  
$ndf =$ number of degrees of freedom

$k = 1$ planetary system, i.e. ring gear fixed and rigid planet carrier  
$ndf = n + 2$

$k = 2$ star system, i.e. planet carrier fixed and rigid ring gear rim  
$ndf = n + 2$

$k = 3$ differential system, i.e. rigid ring gear rim and rigid planet carrier  
$ndf = n + 3$

$k = 4$ single mesh, sun-planet(s) or external/external mesh  
$ndf = n + 1$

$k = 5$ single mesh, ring-planet(s) or external/internal  
$ndf = n + 1$

$k = 6$ planetary system with flexible carrier  
$ndf = 2n + 1$

$k = 7$ star system with flexible ring gear  
$ndf = 2n + 1$

$k = 8$ *differential system with both flexibar and flexible planet carrier*  
$ndf = 3n + 1$

*NOTE:* For $k=8$, a differential system with both a flexible carrier and a flexible ring, there is only a natural frequency solution, i.e. there is no dynamic solution.

To include the floating sun option or for systems with $k$ of 6, 7, or 8, note that the corresponding additional parameters must be input, e.g. the floating sun mass and sun center springs. Two additional degrees of freedom are added to any system if the floating sun parameters are input.

**Phasing constants for mesh time location simulation:**

For a phasing constant of 0 the stiffnesses will be at the pitch diameter or time zero, while for a phasing constant of 1, the stiffnesses will again be at the pitch diameter but at the end of the meshing time. Thus, for phasing constants between 0 and 1, the teeth will be located at a corresponding percentage of the total mesh time.
Other program inputs affecting the output:

The normalized eigenvectors or mode shapes can be printed if desired. These will indicate displacements along the lines of action for the rotational modes, which are all modes except when there is a floating sun. The additional equations for the floating sun are for translational movement of the sun center in the x and y directions and will lead to translational modes.

If the full output is requested, the user may find rigid body modes that do not always correspond to a natural frequency of exactly zero, but are several orders of magnitude less than the actual frequencies and are the first frequencies output. This is caused by the numerical eigenvalue/eigenvector solution.

Systems 6, 7 and 8 may yield rigid body modes if the stiffnesses for the carrier and/or ring gear rim or pin stiffnesses are insufficient. For these situations either the carrier and/or the ring are acting as rigid bodies. For these systems all the frequencies are output, but if the first two frequencies are orders of magnitude less than the other frequencies, the pin stiffnesses and the results should be carefully examined.

Input geometry can affect the tooth pair stiffnesses, as well as the input torque.

Finite Element Helical Gear Tooth Analysis:

This option is initiated by a trigger in location 111. Note that the default for helical gear tooth analysis is 0., or the original option of 10 independent axial segments, and that no provision is made for double helical tooth forms in the finite element solution.

Another point of interest is the numerical solution stability. The size of the mesh time step has an effect on the numerical solution. This is currently set to be equal to the total mesh time divided by 100 for spur gears and the finite element helical option. For the original helical option the total mesh time is divided by 10. For very low rpms this time step can become relatively large and could lead to an unstable solution.

Flexible Carrier or Flexible Ring Gear Rim Option:

The flexible carrier/ring gear rim option is triggered by the input system type, for either a planetary or star system. The additional inputs required are stiffnesses (locations 800-803) In addition, the boundary conditions must now be input for each individual component (locations 481 to 621). The following procedure is recommended to reduce the iterations for convergence:
1. Run the case without a flexible carrier or ring gear rim \((k = 1 \text{ or } 2)\).

2. Estimate the carrier segment or ring gear rim displacement (assuming equal segment displacements) from:

\[
XCO(I) = \frac{\text{total output torque}}{(\text{carrier base radius} \times \text{pin stiffness})}
\]

or:

\[
XRO(I) = \frac{\text{total output torque}}{(\text{ring gear base radius} \times \text{pin stiffness})}
\]

3. Using the converged displacement boundary conditions from Step 1, solve for the remaining displacements recalling the following tooth pair displacement (along the line of action) relationships:

\[
XOSP(I) = XSO - XPO(I) - XCO(I)
\]

\[
XORP(I) = XPO(I) - XCO(I) - XRO(I)
\]

Recall that for planetary systems \(XRO(I) = 0\). and for star systems, \(XCO(I) = 0\).

4. For the initial velocity conditions, one value must be assumed (e.g. carrier velocity = 0.) as there are two equations and three unknowns. The same relations hold as in Step 3 replacing the displacements with velocities.
Part 2: List of Input Variables and Corresponding Location Numbers
** USER'S MANUAL FOR MULTIPLE MESH **
** SPUR AND HELICAL GEAR SYSTEMS **

** USER'S MANUAL FOR MULTIPLE MESH **
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DESCRIPTIONS), AND DATA ITEMS ARE IN 5 FIELDS OF 12 SPACES.

CASE TERMINATION CARD: TO TERMINATE A CASE THE LAST LINE MUST CONTAIN 0-1. IN COLUMNS 1-4.

SUBSEQUENT CASES: ANOTHER DATA SET OF THE SAME FORMAT—TITLE CARD, DATA CARDS, AND CASE TERMINATION CARD—MAY FOLLOW THE CASE TERMINATION CARD.

JOB TERMINATION CARD: AFTER THE LAST SUBSEQUENT CASE TERMINATION CARD, A BLANK CARD (LINE) MUST BE INCLUDED, OTHERWISE AN ERROR MESSAGE IS GENERATED.

** NOTE ** ALL NUMBERS MUST BE REAL EXCEPT THE NUMBER OF ITEMS, THAT IS, A DECIMAL POINT MUST BE INCLUDED. IT IS NOT NECESSARY TO INPUT ZERO VALUES AS THEY WILL DEFAULT TO ZERO UNLESS OTHERWISE SPECIFIED.

A BRIEF DESCRIPTION OF OUTPUT FOLLOWS THE GEAR SYSTEM PARAMETER SECTION.

*** THE FOLLOWING EXAMPLE IS FROM THE 1984 NASA CR ***

IN APPENDIX C THE TITLE CARD READS 'EXAMPLE DATA SET'. THE NEXT LINE IN THE DATA SET CONTAINS, FROM LEFT TO RIGHT, 5, CORRESPONDING TO THE FIVE DATA ITEMS ON TO BE ON THAT LINE, THEN THE LOCATION NUMBER (1.) CORRESPONDING TO THE FIRST INPUT DATA TO BE PUT IN COLUMNS 13-24, FOLLOWED BY THE DATA ITEMS: LEVEL (2.), THE DIAMETRAL PITCH (8.46671), THE PRESSURE ANGLE (22.51), COAST SIDE PRESSURE ANGLE AND HELIX ANGLE (BOTH COAST SIDE PRESSURE ANGLE AND HELIX ANGLE - 0 FOR STANDARD SPUR GEARS AND WOULD DEFAULT TO 0 IF LEFT BLANK)

MOST OF THE INPUT VARIABLES HAVE SUFFICIENT EXPLANATION IN THE GEAR SYSTEM PARAMETERS SECTION, HOWEVER THE FOLLOWING PARAMETERS SHOULD BE CALCULATED AS FOLLOWS.

*** EQUIVALENT MASSES:

SUN EQUIVALENT MASS = J / (BASE RADIUS OF SUN)**2

WHERE J IS THE MASS MOMENT OF INERTIA FOR THE SUN. THE OTHER COMPONENTS ARE CALCULATED SIMILARLY, USING THE CORRESPONDING MOMENTS AND BASE RADII.

*** PHASING CONSTANTS:

FOR EQUALLY SPACED PLANETS THE SUN PHASING CONSTANTS ARE DETERMINED BY ASSUMING THE FIRST PLANET MESH HAS A PHASING
CONSTANT OF ZERO. THE REMAINING SUN-PLANET PHASING CONSTANTS ARE DETERMINED BY:

\((\text{PLANET}\ # - 1) \times \text{THE FRACTIONAL REMAINDER FROM DIVIDING THE NUMBER OF SUN TEETH BY THE TOTAL NUMBER OF PLANETS WHERE THE 'FRACTIONAL REMAINDER' INDICATES THE SPACING DIFFERENCE BETWEEN PLANETS.}\)

THE PHASING CONSTANTS FOR THE RING-PLANET MESHES ARE DETERMINED THE SAME WAY IF THE PLANET HAS AN ODD NUMBER OF TEETH, USING THE NUMBER OF RING TEETH. IF THE PLANETS HAVE AN EVEN NUMBER OF TEETH THE CONSTANTS ARE CALCULATED FOR THE SUN MESHES THEN 0.5 IS ADDED TO EACH TO OBTAIN THE RING-PLANET MESH PHASING CONSTANTS (KRP), DUE TO THE RING GEAR BEING \(0.5\) OFFSET FROM THE SUN GEAR.

EXAMPLE CORRESPONDING TO APPENDIX C:

SUN PLANET MESH— \(14/3 = 12\ 2/3\)

PLANET 1, \(\text{KSP}(1) = 0.\)

PLANET 2, \((2-1) \times (2/3) = 0.6666667\)
\(\text{KSP}(2) = 0.6666667\)

PLANET 3, \((3-1) \times (2/3) = 1.3333333\)
\(\text{KSP}(3) = 1.3333333\)

RING PLANET MESH— (EVEN NUMBER OF PLANET TEETH)

PLANET 1, \(\text{KRP}(1) = \text{KSP}(1) + 0.5 = 0.5\)

PLANET 2, \(\text{KSP}(2) + 0.5 = 1.6666667\)
\(\text{KRP}(2) = 1.6666667\)

PLANET 3, \(\text{KSP}(3) + 0.5 = 0.8333333\)
\(\text{KRP}(3) = 0.8333333\)

*** BOUNDARY CONDITIONS:

AN INITIAL ESTIMATE FOR DISPLACEMENT BOUNDARY CONDITIONS MAY BE OBTAINED BY DIVIDING THE STATIC TOOTH LOAD AT THE PITCH RADIUS BY THE TOOTH SPRING RATE AT THE PITCH RADIUS.
1986 UPDATE

SEE THE 1986 CR REPORT FOR A SUMMARY OF THE LOCATIONS FOR THE ADDITIONAL INPUTS REQUIRED FOR THE NEW OPTIONS. THE SAME LEVELS APPLY, WHERE THE NEW OPTIONS CAN BE USED WITH ANY LEVEL OF INPUT. NOTE THAT THE LOCATIONS TO TRIGGER THE PLOTS HAVE BEEN CHANGED. ALSO NOTE THAT THE NEW OPTIONS THAT ADD DEGREES OF FREEDOM ARE SENSITIVE TO THE INITIAL BOUNDARY CONDITIONS, SEE CR FOR INITIAL FLEXIBLE CARRIER BOUNDARY CONDITION INPUT RECOMMENDATIONS.

LEVELS OF INPUT

LEVEL I INPUT
THE FIRST INPUT ITEM WILL BE THE LEVEL DESIRED, FOLLOWED BY ITEMS 2 THROUGH 51 AND WHERE NOTED. THE OTHER VALUES WILL DEFAULT. THE MATERIAL PROPERTIES, LOCATIONS 22 TO 27, DEFAULT FOR STEEL. THE TOLERANCES ARE SET TO ZERO OR DEFAULT VALUES BELOW, ERRORS ARE SET TO 0. THERE ARE NO PROFILE MODIFICATIONS. OTHER NONZERO DEFAULT VALUES ARE:

- SURFACE ROUGHNESS = 25
- OIL TEMPERATURE = 180 F
- MATERIAL CONSTANT = 0.0528
- DAMPING = .02
- OIL TYPE MIL-L-23699

ANY DESIRED PLOTS CAN BE OBTAINED, SEE LOC 651-658, A CHECK ON INPUT DATA CAN BE MADE, LOC # 699.

IF CONTACT RATIO IS HIGH (GREATER THAN 2) IT MUST BE INPUT IN LOC 54 & 55.

LEVEL II INPUT
THE MINIMUM INPUT FOR THIS LEVEL WOULD BE LEVEL 1 DATA PLUS DAMPING, FLASH TEMPERATURE DATA, SOLUTION ITERATION DATA, AND PHASING CONSTANTS. DEFAULT VALUES ARE ZERO UNLESS INDICATED OTHERWISE BELOW. THE MAXIMUM INPUT WOULD INCLUDE THE ITEMS REQUIRED FOR LEVEL 1 PLUS ALL OTHER ITEMS EXCEPT LOCATIONS 64, 65, 91-100, AND 175-192.

LEVEL III INPUT
LEVEL 3 REQUIRES ALL ITEMS TO BE INPUT, UNLESS DEFAULT VALUES ARE INDICATED BELOW. THE MINIMUM INPUT FOR THIS LEVEL WOULD BE LEVEL 2 DATA.

**GEAR SYSTEM PARAMETERS**********

<table>
<thead>
<tr>
<th>LOC</th>
<th>NAME</th>
<th>DESCRIPTION</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>LEVEL</td>
<td>TRIGGER FOR LEVEL OF INPUT DATA</td>
</tr>
<tr>
<td>2</td>
<td>DP</td>
<td>DIAMETRAL PITCH (NORMAL PLANE)</td>
</tr>
<tr>
<td>3</td>
<td>PSANG</td>
<td>SUN-PLANET DRIVE SIDE PRESSURE ANGLE @ PD</td>
</tr>
<tr>
<td></td>
<td></td>
<td>RING-PLANET COAST SIDE PRESSURE ANGLE @ PD</td>
</tr>
<tr>
<td>4</td>
<td>PSANGB</td>
<td>SUN-PLANET COAST SIDE PRESSURE ANGLE @ PD</td>
</tr>
<tr>
<td></td>
<td></td>
<td>RING-PLANET DRIVE SIDE PRESSURE ANGLE @ PD</td>
</tr>
<tr>
<td></td>
<td>** 0.0 IF PRESSURE ANGLES FOR DRIVE &amp; COAST SIDES EQUAL**</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>PSI0</td>
<td>HELIX ANGLE @ PD</td>
</tr>
<tr>
<td></td>
<td>** 0.0 FOR SPUR GEARS**</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>N1</td>
<td>NUMBER OF TEETH ON SUN GEAR</td>
</tr>
<tr>
<td>7</td>
<td>N2</td>
<td>PLANET GEARS</td>
</tr>
<tr>
<td>8</td>
<td>N3</td>
<td>RING GEAR</td>
</tr>
<tr>
<td>9</td>
<td>FW1</td>
<td>AXIAL FACE WIDTH OF SUN GEAR</td>
</tr>
<tr>
<td>10</td>
<td>FW2</td>
<td>PLANET GEARS</td>
</tr>
<tr>
<td>11</td>
<td>FW3</td>
<td>RING GEAR</td>
</tr>
<tr>
<td>12</td>
<td>N</td>
<td>NUMBER OF PLANET GEARS</td>
</tr>
<tr>
<td>13</td>
<td>K</td>
<td>IF K=1 PLANETARY SYSTEM, I.E., RING GEAR FIXED</td>
</tr>
<tr>
<td></td>
<td>IF K=2 STAR SYSTEM, I.E., PLANET CARRIER FIXED</td>
<td></td>
</tr>
<tr>
<td></td>
<td>*** IF K=3 DIFFERENTIAL SYSTEM, I.E. NEITHER RING OR CARRIER FIXED</td>
<td></td>
</tr>
<tr>
<td></td>
<td><strong>Z</strong> IF K=4 NON PLANETARY, I.E., NO RING AND NO CARRIER</td>
<td></td>
</tr>
<tr>
<td></td>
<td>SUN-INPUT, PLANET-OUTPUT</td>
<td></td>
</tr>
<tr>
<td></td>
<td><strong>Z</strong> IF K=5 NON PLANETARY, I.E., NO SUN AND NO CARRIER,</td>
<td></td>
</tr>
<tr>
<td></td>
<td>PLANET INPUT, RING OUTPUT</td>
<td></td>
</tr>
<tr>
<td></td>
<td>* IF K=6 PLANETARY SYSTEM WITH FLEXIBLE PLANET CARRIER</td>
<td></td>
</tr>
<tr>
<td></td>
<td>* IF K=7 STAR SYSTEM WITH FLEXIBLE RING GEAR RIM</td>
<td></td>
</tr>
<tr>
<td></td>
<td>****************** K - 6 AND K - 7 CURRENTLY UNSTABLE RESULTS **********</td>
<td></td>
</tr>
</tbody>
</table>

* IF K=6 OR K=7 LOC 88 & 89 ARE REQUIRED FOR PLANET CARRIER OR RING GEAR RIM STIFFNESS VALUE ALONG THE LINE-OF-ACTION AND AN INTERFACE STIFFNESS.
** FOR K=4 AND RUNOUT ERROR, INPUT LOC 200.
*** FOR K=3, INPUT LOC 18 - 21.
**Z** FOR K=4 OR 5, THE SINGLE MESH PROGRAM WILL GIVE THE RESULTS IN LESS CPU TIME DUE TO AN EXACT SOLUTION. ALSO, FOR K = 5, THERE IS A BUG IN THE DEPTH TO MAX SHEAR CALCULATION IN THE MULTI-MESH PROGRAM.

<table>
<thead>
<tr>
<th>LOC</th>
<th>NAME</th>
<th>DESCRIPTION</th>
</tr>
</thead>
<tbody>
<tr>
<td>14</td>
<td>TORQ</td>
<td>AXIAL INPUT TORQUE ON SUN GEAR (OR PLANET IF K=5)</td>
</tr>
<tr>
<td>15</td>
<td>RPM</td>
<td>INITIAL AXIAL ROTATIONAL SPEED OF SUN GEAR (OR PLANET IF K=5)</td>
</tr>
<tr>
<td>16</td>
<td>RPMF</td>
<td>FINAL AXIAL ROTATIONAL SPEED OF SUN GEAR FOR SPD RANGE</td>
</tr>
<tr>
<td>17</td>
<td>INTVL</td>
<td>NUMBER OF MAIN INTERVALS THE SPEED RANGE DIVIDED INTO</td>
</tr>
<tr>
<td></td>
<td>** FOR ONE RPM ONLY, RPM=RPMF AND INTVL=1.**</td>
<td></td>
</tr>
</tbody>
</table>
STEP SIZE - (RPMF - RPM)/INTVL, THIS STEP SIZE WILL BE AUTOMATICALLY BY 5 IN AREAS OF PEAK LOADS, THUS REFINING THE INCREMENT.

******** DIFFERENTIAL SYSTEM INPUTS *************

18  TOUTC  OUTPUT TORQUE FROM CARRIER
19  TOUTR  OUTPUT TORQUE FROM RING
20  RPMC  CARRIER RPM
21  RPMR  RING RPM

******** GEAR MATERIAL PROPERTIES *************

22  E1  YOUNGS MODULUS * E-06 OF SUN GEAR  (DEFAULT = 30.)
23  E2  PLANET GEARS  "  "  "
24  E3  RING GEAR  "  "  "
25  MU1  POISSONS RATIO OF SUN GEAR  (DEFAULT = .30)
26  MU2  PLANET GEARS  "  "  "
27  MU3  RING GEAR  "  "  "

******** GEAR EQUIVALENT MASSES ************

28  MS  EQUIVALENT MASS OF SUN GEAR ABOUT ROTATIONAL AXIS
29  MC  PLANET CARRIER
30  MR  RING GEAR

31-50 MP(I)  PLANET GEARS
I = PLANET NUMBER
*** NOTE: FOR K-6 OR K-7, CARRIER OR RING MASS FOR TOTAL UNIT, NOT SEGMENTS.

******** TRIGGER FOR DOUBLE HELICAL GEARING ************

51  DBHEL  IF = 0.0 SINGLE HELICAL GEARING IF PSIO .NE. 0.0
     IF = 1.0 DOUBLE HELICAL GEARING IF PSIO .NE. 0.0

** ** ** ** ** ** ** ** ** ** ** ** **
END OF LEVEL 1 INPUT (UNLESS HIGH CONTACT RATIO)
** ** ** ** ** ** ** ** ** ** ** ** **

******** GEAR MESHING DAMPING RATIOS ************

52  ZSP  DAMPING RATIO OF SUN-PLANET MESHES
53  ZRP  "  "  " RING-PLANET MESHES
** ** NOTE: IF CONTACT RATIO > 2, CRSP AND CRRP MUST BE INPUT, BUT THE PROGRAM WILL CALCULATE FOR CR < 2 ONLY.
54  CRSP  INVOLUTE CONTACT RATIO OF SUN-PLANET MESHES
55  CRRP  "  "  " RING-PLANET MESHES
**NOTE:** LOC 56 - 59 FOR HELICAL GEARS ONLY, NOT NECESSARY FOR FINITE ELEMENT OPTION

<table>
<thead>
<tr>
<th>Location</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>56</td>
<td>L1SP INACTIVE SUN-PLANET FACE WIDTH ON LEFT</td>
</tr>
<tr>
<td>57</td>
<td>L2SP</td>
</tr>
<tr>
<td>58</td>
<td>L1RP INACTIVE RING-PLANET FACE WIDTH ON LEFT</td>
</tr>
<tr>
<td>59</td>
<td>L2RP</td>
</tr>
</tbody>
</table>

**NOTE:** LOCATIONS 66-68 AND 75-78 ARE ONLY USED FOR PROFILE MODIFICATION TABLES, THEY ARE NOT USED IN THE DYNAMICS.

<table>
<thead>
<tr>
<th>Location</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>60</td>
<td>PCTSOD SD AS A PERCENT OF SOD (%)</td>
</tr>
<tr>
<td>61</td>
<td>PCTSOE SE AS A PERCENT OF SOE (%)</td>
</tr>
<tr>
<td>62</td>
<td>DELD DISENGAGEMENT TIP RELIEF (IN.), MINIMUM</td>
</tr>
<tr>
<td>63</td>
<td>DELE ENGAGEMENT TIP RELIEF (IN.), MINIMUM</td>
</tr>
</tbody>
</table>

**NOTE:** SIGN CONVENTION—POSITIVE INPUT, MATERIAL REMOVED

**NOTE:** XNVDX AND XNVEX ARE NOT REQUIRED FOR LEVEL 2

<table>
<thead>
<tr>
<th>Location</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>64</td>
<td>XNVDX DISENGAGEMENT PROFILE MODIFICATION SHAPE FACTOR</td>
</tr>
<tr>
<td>65</td>
<td>XNVEX ENGAGEMENT</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Location</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>66</td>
<td>PMODSP PERCENT OF TIP RELIEF ON SUN GEAR, ENGAGEMENT</td>
</tr>
<tr>
<td>67</td>
<td>PMDSPD PERCENT OF TIP RELIEF ON SUN GEAR, DISENGAGEMENT</td>
</tr>
</tbody>
</table>

**NOTE:** LOCATION 73 AND 74 NOT REQUIRED FOR LEVEL 2

<table>
<thead>
<tr>
<th>Location</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>73</td>
<td>XNVDX3 DISENGAGEMENT PROFILE MODIFICATION SHAPE FACTOR</td>
</tr>
<tr>
<td>74</td>
<td>XNVEX3 ENGAGEMENT PROFILE MODIFICATION SHAPE FACTOR</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Location</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>75</td>
<td>PMODRP PERCENT OF TIP RELIEF ON PLANET GEAR, ENGAGEMENT</td>
</tr>
</tbody>
</table>
PMDRPD PERCENT OF TIP RELIEF ON PLANET GEAR, DISENGAGEMENT
- TIP RELIEF ON RING GEAR

DLRP TOLERANCE AT START OF PROFILE MODIFICATION

DLTOL TOTAL TIP RELIEF TOLERANCE, FOR BOTH SUN+PLANET
AND/OR RING + PLANET

***** MESH MODIFICATION DUE TO FACE WIDTH CROWNING ************

LECSP LENGTH OF FACE WIDTH CROWN OF ENGAGEMENT FOR SP MESH
LDCSP " " " " " " DISENGAGEMENT FOR SP MESH
DLECSP ENGAGEMENT EDGE RELIEF FOR SP MESH
DLDCCP DISENGAGEMENT EDGE RELIEF FOR SP MESH

LECRRP LENGTH OF FACE WIDTH CROWN OF ENGAGEMENT FOR RP MESH
LDCRP " " " " " " DISENGAGEMENT FOR RP MESH
DLECRP ENGAGEMENT EDGE RELIEF FOR RP MESH
DLDCRP DISENGAGEMENT EDGE RELIEF FOR RP MESH

***** TOOTH PAIR COMPLIANCE DATA ******************************

WIOK IF = 0.0 PREPROCESSOR CALCULATES TOOTH PAIR COMPLIANCE
IF = 1.0 INPUT COMPLIANCE DATA IN LOCATIONS 91-101

(FOR FINITE ELEMENT HELICAL COMPLIANCE, LOC 111)

** 91 - 100 NOT REQUIRED FOR LEVEL 2 **

SPKSP SINGLE TOOTH PAIR SPRINGRATE OF SUN-PLANET MESHES
COMASP COMPLIANCE CONSTANT (S/S0)**1 OF SUN-PLANET MESHES
COMSP 2
COMCSP 3
COMDSP 4

SPKRSP SINGLE TOOTH PAIR SPRINGRATE OF RING-PLANET MESHES
COMARP COMPLIANCE CONSTANT (S/S0)**1 OF RING-PLANET MESHES
COMBRP 2
COMCRP 3
COMDRP 4

WHERE S = IS THE LENGTH ALONG THE LINE OF ACTION TO THE POINT
OF ENGAGEMENT
AND SO = THE LENGTH OF THE LINE OF ACTION

** ** ** ** ** ** ** ** ** ** ** ** ** ** ** ** ** ** ** **

HRTZSP HERZT STRESS FOR COMPLIANCE CALCULATION FOR SUN-PLANET
HRTZRP RING-PLANET

** THE PROGRAM WILL USE 101 AND 102 AS CONSTANTS FOR THE COM-
PLIANCE CALCULATIONS IF INPUT. DEFAULT WILL CALCULATE
THE STATIC LOAD/PITCH RADIUS FOR THE CONSTANTS

103  XPSPS  IF  0.0  PLANE STRESS IS ASSUMED FOR SUN-PLANET
     IF  1.0  PLANE STRAIN IS ASSUMED FOR SUN-PLANET

104  XPSPRP  IF  0.0  STRESS  RING-PLANET
     IF  1.0  STRAIN  RING-PLANET

111  CONVEK  IF  0.0  HELICAL TOOTH IS DIVIDED INTO 10 INDEPENDENT
     AYTAI SEGMENTS
     IF  1.0  FINITE ELEMENT ROUTINES USED TO GENERATE AN
     EQUIVALENT SPUR GEAR COMPLIANCE CURVE

******* FLOATING SUN PARAMETERS ********************

115  KFSX  LINEAR SPRING IN THE X DIRECTION AT SUN CENTER
116  KFSY  LINEAR SPRING IN THE Y DIRECTION AT SUN CENTER
117  DFSX  LINEAR DAMPER IN THE X DIRECTION AT SUN CENTER
118  DFSY  LINEAR DAMPER IN THE Y DIRECTION AT SUN CENTER
119  FSMS  ACTUAL SUN GEAR MASS (NOT EQUIVALENT MASS)

******* TOOTH PAIR GEOMETRIC DATA ********************

120  BYPASS  IF  0.0  GEOMETRIC PREPROCESSOR IS USED
     IF  1.0  GEOMETRIC DATA MUST BE INPUT IN LOCATIONS
     121-132

121  RADIOS  MAX. RADIUS TO BASE OF FILLET OF SUN GEAR (ROOT
     PLANET GEARS RADIUS)
122  RADTIP  PLANET GEARS
123  RADTIR  RING GEAR

124  RADTIS  MAX. RADIUS TO TIP OF TOOTH OF SUN GEAR
125  RADTIP  PLANET GEARS
126  RADTIR  RING GEAR

127  TOOTHS  NOMINAL TOOTH THICKNESS AT PD OF SUN GEAR
128  TOOTHP  PLANET GEARS
129  TOOHR  RING GEAR

130  RADFIS  MAX. FILLET RADIUS OF SUN GEAR
131  RADFIP  PLANET GEARS
132  RADFIR  RING GEAR

******* FLASH TEMPERATURE DATA ********************

140  SROU  SURFACE ROUGHNESS (RMS, MICROINCHES)
141  TOIL  OIL INLET TEMPERATURE
142  CFMAT  MATERIAL CONSTANT
143  OILTYP  TYPE OF OIL IN GEARBOX
     1 FOR MIL-L-23699
     2 FOR MIL-L-7808
     3 TO BE DETERMINED

-63-
4 TO BE DETERMINED
(O DEFAULTS TO TYPE 1)

****** SOLUTION ITERATION DATA ****************************

150  TOLER  ITERATION TOLERANCE FOR BOUNDARY CONDITION CONVERGENCE
       (% * 100, DEFAULT FOR LEVEL 1 = .01)
151  NLOOP  NUMBER OF ITERATIONS FOR BOUNDARY CONDITION CONVERGENCE
       NLOOP MAXIMUM = 20, DEFAULT = 20.

****** TOOTH PAIR GEOMETRIC TOLERANCE DATA ********************

160  DLROS  TOLERANCE ON TIP RADIUS OF SUN GEAR (DEFAULT = .002)
161  DLRP  PLANET GEARS
162  DLRR  RING GEAR
163  BSE  EDGE BREAK ON TOPLAND OF ALL GEARS (DEFAULT = .010)
164  DLRR  ROOT RADIUS TOLERANCE OF ALL GEARS (DEFAULT = .005)
165  DLCDN  CENTER DISTANCE TOLERANCE (TOWARD TIGHT MESH) ALL GEARS
166  DLCDP  CENTER DISTANCE TOLERANCE (AWAY FROM TIGHT MESH)
167  DLMBT  MACHINE BACKLASH TOLERANCE ALL GEARS (DEFAULT = .002)

NOTE: THE TOOTH ROOT FILLET RADIUS TOLERANCE = .005 IN THE PREPROCESSOR

****** 3-DIMENSIONAL FACTORS FOR SPUR GEARs ONLY ***************

* * * * * 175 - 192 NOT REQUIRED FOR LEVEL 2 * * * * * * * * * *

175  SPRNBS  IF = 0. RIM BENDING EFFECTS NOT INCLUDED IN SUN GEAR
         (DEFAULT = 10.E+10)
         IF > 0. BENDING SPRINGRATE(IN-N/RAD) OF SUN RIM
176  SPRNPB  IF = 0. RIM BENDING EFFECTS NOT INCLUDED IN PLANET GEAR
         (DEFAULT = 10.E+10)
         IF > 0. BENDING SPRINGRATE(IN-N/RAD) OF PLANET RIM
177  SPRNBR  IF = 0. RIM BENDING EFFECTS NOT INCLUDED IN RING GEAR
         (DEFAULT = 10.E+10)
         IF > 0. BENDING SPRINGRATE(IN-N/RAD) OF RING RIM
178  SPRNLS  RADIUS FROM RIM TO PD (REQUIRED WITH SPRNBS) SUN
179  SPRNLP  RADIUS FROM RIM TO PD (REQUIRED WITH SPRNPB) PLANET
180  SPRNLR  RADIUS FROM RIM TO PD (REQUIRED WITH SPRNBR) PLANET
181  EFWD1S  EFFECTIVE SUN FACE WIDTH FACTOR AT TOOTH TIP
         IF = 0. (DEFAULT = 1.0)
182  EFWD2S  EFFECTIVE SUN FACE WIDTH FACTOR AT FILLET
         IF = 0. (DEFAULT = 1.0)
183  EFWD1P  EFFECTIVE PLANET FACE WIDTH FACTOR AT TOOTH TIP
         IF = 0. (DEFAULT = 1.0)
184  EFWD2P  EFFECTIVE PLANET FACE WIDTH FACTOR AT FILLET
         IF = 0. (DEFAULT = 1.0)
185  EFWD1R  EFFECTIVE RING FACE WIDTH FACTOR AT TOOTH TIP
         IF = 0. (DEFAULT = 1.0)
**EFWD2R**  EFFECTIVE RING FACE WIDTH FACTOR AT FILLET
187  STRSES  STRESS DISTRIBUT. FACTOR FOR END EFFTS AT SUN WIDTH EDGE
188  STRSCS  STRESS DISTRIBUT. FACTOR FOR END EFFTS AT SUN WIDTH CENT
189  STRSEP  STRESS DISTRIBUT. FACTOR FOR END EFFTS AT PLAN WDTH EDGE
190  STRSCP  STRESS DISTRIBUT. FACTOR FOR END EFFTS AT PLAN WDTH CENT
191  STRSER  STRESS DISTRIBUT. FACTOR FOR END EFFTS AT RING WDTH EDGE
192  STRSCR  STRESS DISTRIBUT. FACTOR FOR END EFFTS AT RING WDTH CENT

**HELIX ANGLE ERRORS**

195  DELPSP  HELIX ANGLE ERROR FOR SUN-PLANET MESH
196  DELPRP  "    "    "    "    RING-PLANET MESH
197  DELPS2  IF DBHEL - 0.0 DELPS2 = 0.0
198  DELPR2  IF DBHEL - 0.0 DELPR2 = 0.0

**TOOTH PAIR SPACING ERRORS**

200  DR  SUN RUNOUT ERROR FOR EXTERNAL-EXTERNAL SINGLE MESH ONLY

**NOTE:** A WRITE(7,5029) STATEMENT NEEDS A COMMENT STATEMENT REMOVED FOR TIME AND LOADS TO BE OUTPUT, OTHERWISE CARDS ARE PUNCHED WHEN A TAPE IS WRITTEN. IT IS IN SUBROUTINE STRESS.

**NOTE:** FOR DOUBLE HELICAL GEARS (DBHEL = 1.0) THE FIRST 10 VALUES OF I IN THE E(I,J) ARRAYS ARE FOR THE LEFT HALF OF THE DOUBLE HELICAL GEARS AND THE LAST 10 LOCATIONS ARE FOR THE RIGHT HALF OF THE DOUBLE HELICAL GEARS, WHERE I = THE PLANET NUMBER.

THE ERRORS SHOULD BE PUT ON THE SECOND OR THIRD TOOTH TO ILLUSTRATE THE DYNAMICS.

221 TO 240  ESP(I,1) SUN-PLANET ERROR ARRAY FOR TOOTH 1
241 TO 260  ESP(I,2)
261 TO 280    ESP(I,3)
281 TO 300 ESP(I,4)
301 TO 320 ESP(I,5)
321 TO 340 ERP(I,1) RING-PLANET ERROR ARRAY FOR TOOTH 1
341 TO 360 ERP(I,2)
361 TO 380 ERP(I,3)
381 TO 400 ERP(I,4)
401 TO 420 ERP(I,5)

****** PLANET GEARS PHASING CONSTANTS **********

421 TO 440 KSP(I) SUN-PLANET PHASING CONSTANTS ARRAY
441 TO 460 KRP(I) RING-PLANET PHASING CONSTANTS ARRAY

****** INITIAL BOUNDARY CONDITIONS **********

481 TO 500 XOSP(I) SUN-PLANET INITIAL DISPLACEMENT BOUNDARY COND
OR SUN DISPLACEMENT FOR K-6 OR K-7.
501 TO 520 XORP(I) RING-PLANET INITIAL DISPLACEMENT BOUNDARY COND
OR PLANET DISPLACEMENT FOR K-6 OR K-7.
521 TO 540 XISP(I) SUN-PLANET INITIAL VELOCITY BOUNDARY COND
OR SUN VELOCITY FOR K-6 OR K-7.
541 TO 560 XIRP(I) RING-PLANET INITIAL VELOCITY BOUNDARY COND
OR PLANET VELOCITY FOR K-6 OR K-7.

FLOATING SUN INITIAL CONDITIONS

561 CARRIER DISPLACEMENT
581 CARRIER VELOCITY
601 RING DISPLACEMENT
621 RING VELOCITY
641 SUN CENTER DISPLACEMENT IN X DIRECTION
642 SUN CENTER DISPLACEMENT IN Y DIRECTION
643 SUN CENTER VELOCITY IN X DIRECTION
644 SUN CENTER VELOCITY IN Y DIRECTION

****** PROGRAM PLOT SELECTIONS FOR SPUR GEARS ONLY **********

651 PLILD IF = 0.0
      IF = 1.0 LOAD PLOTS, NORMALIZED LOAD—DYNAMIC/STATIC
652 PLTPV IF = 0.0
      IF = 1.0 PV PLOTS, PRESSURE SLIDING VELOCITY
653 PLTHS IF = 0.0
      IF = 1.0 HERTZ STRESS PLOTS
654 PLIFT IF = 0.0
      IF = 1.0 FLASH TEMPERATURE PLOTS
655 PLTSS IF = 0.0
      IF = 1.0 SUN GEAR HEYWOOD STRESS PLOTS
656 PLTPSS IF = 0.0
IF = 1.0 SUN-PLANET PLANET GEAR HEYWOOD STRESS PLOTS
657 PLTPRS IF = 0.0
IF = 1.0 RING-PLANET PLANET GEAR HEYWOOD STRESS PLOTS
658 PLTRPS IF = 0.0
IF = 1.0 RING GEAR HEYWOOD STRESS PLOTS

******* PROGRAM CHECK RUN ****************************

699 CHECK IF = 0.0 REGULAR RUN TO COMPLETION
IF = 1.0 EXITS PROGRAM BEFORE DYNAMIC SOLUTION
TO ALLOW CHECKING OF INPUT DATA AND THE
GEOMETRIC PREPROCESSOR RESULTS

******** FLEXIBLE PLANET CARRIER OR RING GEAR RIM DATA ********

800 CARRK1 IF K-6, PLANET CARRIER STIFFNESS ALONG-THE-LINE-OF-
ACTION (#/IN)
801 CARRK2 IF K-7 RING GEAR RIM STIFFNESS ALONG-THE-LINE-OF-
ACTION (#/IN)
802 KFLX1 PLANET/CARRIER 'PIN' OR INTERFACE STIFFNESS
803 KFLX2 PLANET/RING 'PIN' OR INTERFACE STIFFNESS

****************** NATURAL FREQUENCIES OPTION ******************

805 NFREQ TRIGGER FOR NATURAL FREQUENCY OPTION
- 1, OUTPUTS FREQUENCIES IN TERMS OF INPUT RPM
- 2, OUTPUTS FREQUENCIES IN TERMS OF INPUT RPM
AS WELL AS THE FREQUENCIES WITH EIGENVECTORS

** SEE USER'S MANUAL IN 1986 CR FOR ADDITIONAL SYSTEM
TYPES THAT ARE AVAILABLE
**** OUTPUT DESCRIPTION

THE FOLLOWING LIST INDICATES THE INFORMATION AND RESULTS THAT MAY APPEAR IN THE OUTPUT, IN THE ORDER THEY WILL APPEAR.

CASE TITLE—THE TITLE AND/OR OTHER INFORMATION INPUT ON THE FIRST DATA CARD.

TIP MODIFICATION—IF THERE IS INSUFFICIENT TIP CLEARANCE, AN ADJUSTMENT IS MADE INTERNALLY AND A MESSAGE OUTPUT UNTIL SUFFICIENT CLEARANCE IS OBTAINED.

INVOLUTE MODIFICATION TABLE—SHOWS THE PROCESSED RESULTS OF ANY INPUT PROFILE MODIFICATIONS. TABLES ARE PRINTED FOR ENGAGEMENT AND DISENGAGEMENT WHICH INCLUDE MINIMUM AND MAXIMUM INVOLUTE MODIFICATIONS, DIAMETER AND CORRESPONDING ROLL ANGLE FOR THE MODIFIED PORTION OF THE PROFILE. THIS WILL BE OUTPUT FOR THE SUN-PLANET MESH AND/OR RING-PLANET MESH IN CONJUNCTION WITH THE CORRESPONDING INPUT DATA.

INPUT DATA—THE INPUT DATA AND PREPROCESSED GEOMETRIC DATA IS PRINTED FOR SUN-PLANET MESH AND/OR RING-PLANET MESH.

FLEXIBILITY—IF THE FLEXIBLE PLANET CARRIER OPTION IS IN EFFECT A MESSAGE APPEARS THAT INDICATES THIS.

ADDITIONAL INPUT DATA—NUMBER OF PLANETS, EQUIVALENT MASSES, ETC.

COMPLIANCE CONSTANTS—CALCULATED CONSTANTS FOR FOURTH ORDER COMPLIANCE EQUATION.

ITERATED BOUNDARY CONDITIONS—BOUNDARY CONDITIONS PRINTED, FOLLOWED BY THE CURRENT SPEED BEING EXAMINED AND CORRESPONDING MAXIMUM LOADS (FOR SUN-PLANET AND/OR RING-PLANET MESHES). IF A SPEED SURVEY WAS RUN, THE SPEED CORRESPONDING TO THE OVERALL MAXIMUM LOAD FROM THE RANGE CALCULATED IS OUTPUT WITH THE MAXIMUM LOAD. THIS IS THE SPEED USED FOR THE REMAINING CALCULATIONS, I.E. STRESS.

MAXIMUM VALUES—TABLE(S) OF MAXIMUM VALUES CALCULATED ARE PRINTED FOR EACH SUN-PLANET MESH AND/OR RING-PLANET MESH. THESE WILL APPEAR FOR THE NO ERROR SOLUTION AND EACH ERROR SOLUTION. TWO ABBREVIATIONS APPEAR—PV-PRESSURE SLIDING VELOCITY AND PD-PITCH DIAMETER.

ERROR MATRIX—IF TOOTH SPACING ERRORS ARE INPUT, OR GENERATED FOR RUNOUT SOLUTION, A TABLE OF ERRORS IS PRINTED BEFORE THE TABLES OF MAXIMUM VALUES.

PLOTS — WILL BE FOR EACH PLANET MESH.
IF THERE ARE TOOTH PAIR SPACING ERRORS INCLUDED, PLOTS ARE GENERATED FOR THE NO ERROR CASE ONLY, I.E. THERE ARE NO PLOTS GENERATED FOR THE ERROR CASES.
APPENDIX B: FIRST ORDER DIFFERENTIAL EQUATIONS

A. Floating Sun Equations:

The equations can be transformed to first order equations as follows.

Let:

\[
\begin{align*}
\dot{x}_1 &= \ddot{x}
\quad \dot{x}_3 = \ddot{y} \\
\dot{x}_2 &= x_1 - \dot{x} 
\quad \dot{x}_4 = x_3 - \dot{y} \\
\dot{x}_5 &= \ddot{r}_c 
\quad \dot{x}_7 = \ddot{r} \\
\dot{x}_6 &= \ddot{r}_c - x_8 
\quad \dot{x}_8 = \ddot{r} - x_7 \\
\end{align*}
\]

\[\begin{align*}
\dot{x}_1 &= - \frac{1}{m} \left( \sum_{i=1}^{N} d_{sp_i} \ddot{y}_{sp_i} \sin a_i + \sum_{i=1}^{N} L_{sp_i} \sin a_i \\
&\quad - d_x x_1 - k_x x_2 \right) \\
\dot{x}_2 &= x_1 \\
\dot{x}_3 &= - \frac{1}{m} \left( \sum_{i=1}^{N} d_{sp_i} \ddot{y}_{sp_i} \cos a_i + \sum_{i=1}^{N} L_{sp_i} \cos a_i \\
&\quad + d_y x_3 + k_y x_4 \right) \\
\dot{x}_4 &= x_3 \\
\dot{x}_5 &= \frac{1}{m_c} \left( \sum_{i=1}^{N} d_{sp_i} \ddot{y}_{sp_i} + \sum_{i=1}^{N} d_{rp_i} \ddot{y}_{rp_i} \\
&\quad + \sum_{i=1}^{N} L_{sp_i} + \sum_{i=1}^{N} L_{rp_i} + F_c \right) \\
\dot{x}_6 &= x_5 \\
\dot{x}_7 &= - \frac{1}{m_r} \left( \sum_{i=1}^{N} d_{rp_i} \ddot{y}_{rp_i} + \sum_{i=1}^{N} L_{rp_i} + F_r \right) \\
\dot{x}_8 &= x_7 \\
\end{align*}\]
B. Flexible Carrier Equations

The carrier and ring gear equations are reduced to first order equations as follows.

Let

\[ x_{c01} = y_{c1} \]  \hspace{1cm} (6a)
\[ x_{c11} = y_{c1} \]  \hspace{1cm} (6b)
\[ \dot{x}_{c11} = y_{c1} \]  \hspace{1cm} (6c)

\[ \dot{x}_{c11} = \frac{1}{\zeta_{c10}} \frac{d}{dt} \left( X_{s1} - X_{p11} - X_{c11} \right) - \frac{1}{\zeta_{c1}} \frac{d}{dt} \left( X_{p11} - X_{c11} - X_{r11} \right) \]
\[ - \frac{1}{\zeta_{c10}} \frac{L_{sp1}}{m_{c10}} \left( X_{c0} - X_{c10} \right) + \frac{K_{c1-1}}{m_{c10}} \left( -X_{c1-1} + X_{c10} \right) \]
\[ - \tau_{out} / m_{c1} N B_{po} \quad \text{(carrier)} \]

\[ \dot{x}_{c10} = x_{c11} \]

Let

\[ x_{r10} = y_{r1} \]  \hspace{1cm} (8a)
\[ x_{r11} = y_{r1} \]  \hspace{1cm} (8b)
\[ \dot{x}_{r11} = y_{r1} \]  \hspace{1cm} (8c)

\[ \dot{x}_{r11} = \frac{1}{\zeta_{r1}} \frac{d}{dt} \left( X_{r11} - X_{r11} - X_{r11} \right) - \frac{1}{\zeta_{r1}} L_{rp1} \]
\[ + \frac{1}{\zeta_{r1-1}} \frac{k_{r1-1}}{m_{r1}} \left( -X_{r1-1} - X_{r10} \right) + \frac{1}{\zeta_{r1}} \frac{k_{r1}}{m_{r1}} \left( X_{r10} - X_{r11} - X_{r11} \right) \]
\[ - \tau_{out} / m_{r1} N B_{r} \quad \text{(ring)} \]

\[ \dot{x}_{r10} = x_{r11} \]
The equation for the torque constraint reduces as follows.

Let

\[\begin{align*}
  \ddot{z} - & \ X \\
  \dot{z} - & \dot{X} - R \\
  \ddot{R} - & \dot{R} \\
  \dot{R} - & \dot{X}
\end{align*}\]  

\[\dot{R} = \left( \frac{\tau_{out}}{R_{dc}} - \sum_{i=1}^{N} \frac{K_i Y_{i} \omega}{M} \right)\]  

(10a) \quad (10b) \quad (10c) \quad (10d)
APPENDIX C: FORTRAN LISTING

The program listing is available from NASA Project Manager or Contractor upon request.
APPENDIX D: NOMENCLATURE

A,B,C,D = gear tooth coefficients
C = single tooth pair compliance (in/lb)
C₀ = single tooth pair compliance at pitch radius
     (Reciprocal of spring rate) (in/lb)
dₚₚᵢ = ring-planet tooth pair damping (lb s/in)
dₛₛᵢ = sun-planet tooth pair damping (lb s/in)
dₓ = damping at sun center in x-direction (lb s/in)
dᵧ = damping at sun center in y-direction (lb s/in)
eₚᵢj = tooth spacing error, ring-planet mesh
eₛₛᵢj = tooth spacing error, sun-planet mesh
fᵢ = gear mesh frequencies
I = integer multiplier of gear mesh frequencies
[K] = stiffness matrix (lb/in)
Kᵢ = planet/carrier pin stiffness (lb/in)
Xᵢ = carrier segment stiffness (lb/in)
Xᵢ = planet/ring stiffness (lb/in)
Kᵢ = ring gear rim segment stiffness (lb/in)
kₚᵢ = ring-planet tooth pair stiffness (lb/in)
kₛₛᵢ = sun-planet tooth pair stiffness (lb/in)
kₓ = spring rate at sun center in x-direction (lb/in)
kᵧ = spring rate at sun center in y-direction (lb/in)
\( L_{rp_i} \) = ring–planet tooth pair load for planet mesh \( i \)

\( L_{sp_i} \) = sun–planet tooth pair load for planet mesh \( i \)

\( [M] \) = mass matrix, a diagonal matrix (lb.)

\( M \) = artificial mass for total torque constraint

\( m \) = translational mass of the sun gear (lbs \( \text{s}^2/\text{in} \))

\( m_c \) = rotational (equivalent) mass of the planet carrier (lbs \( \text{s}^2/\text{in} \))

\( m_r \) = rotational (equivalent) mass of the ring gear (lbs \( \text{s}^2/\text{in} \))

\( N \) = number of planets

\( R_{bc} \) = Carrier Base Radius (in)

\( R_{br} \) = Ring Gear Base Radius (in)

\( \text{rpm} \) = speed of driving gear

\( S \) = motion along line of action from pitch line

\( s_0 \) = Reference distance along line of contact for tooth pair compliance coefficients

\( x \) = displacement of the sun center in \( x \)-direction (in)

\( X_{LOA_i} \) = Sun–planet displacement along the line of action due to the floating sun (in)

\( XN \) = Number of teeth on driver

\( Y \) = displacement of the sun center in \( y \)-direction (in)

\( Y_{rp_i} \) = ring–planet tooth pair displacement for planet mesh \( i \)

\( Y_{sp_i} \) = sun–planet tooth pair displacement for planet mesh \( i \)

\( \alpha_i \) = \( \theta_c + \psi_i - \Phi \) (planetary)

\( \alpha_i \) = \( \psi_i - \Phi - \theta_r \) (star)

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$\beta_{r pj i}$ = Cam modification, ring-planet mesh

$\beta_{s pj i}$ = Cam modification, sun-planet mesh

$\eta_{r pi}$ = Ring-Planet tooth pair spring rates

$\eta_{s pi}$ = Sun-Planet tooth pair spring rates

$\theta_c$ = carrier angle of rotation

$\theta_r$ = ring angle of rotation

$\lambda$ = $\omega^2$

$\tau_{out}$ = Output Torque

$\phi$ = pressure angle

$\phi_{r pj i}$ = Identify function for ring-planet tooth pair contact

$\phi_{s pj i}$ = Identify function for sun-planet tooth pair contact

$\psi_i$ = relative angular position of planets

$\omega$ = natural frequency (rad/s)

$\chi_{s pj i}$ = 0 or 1 depending on whether the tooth contact is on the profile modification cam or not

Subscripts

c = carrier

i = planet mesh

j = tooth pair mesh

p = planet gear

r = ring gear

s = sun gear